Effect of Strain on Stripe Phases in the Quantum Hall Regime

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Preferential orientation of the stripe phases in the quantum Hall (QH) regime has remained a puzzle since its discovery. We show experimentally and theoretically that the direction of high and low resistance of the two-dimensional (2D) hole gas in the QH regime can be controlled by an external strain. Depending on the sign of the in-plane shear strain, the Hartree-Fock energy of holes or electrons is minimized when the charge density wave (CDW) is oriented along the [110] or [1$\bar{1}$0] directions. We suggest that shear strains due to internal electric fields in the growth direction are responsible for the observed orientation of CDW in pristine electron and hole samples.

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The interplay between kinetic energy and electron-electron interactions in two-dimensional electron gases in magnetic fields leads to a rich variety of ground states, ranging from incompressible Laughlin liquids, the Wigner crystal, charge density waves (CDW) to exotic non-Abelian states. The possibility of the formation of a CDW state was suggested [1] before the discovery of the quantum Hall effect, and later it was predicted that a CDW should be the ground state for partially occupied high Landau levels (LL) [2,3]. Experimentally, anisotropic magnetoresistance (AMR) was observed in 2D electron [4,5] and hole [6,7] gases. The majority of experiments were conducted on samples grown on (001) GaAs. Unexpectedly, the CDW was found to be consistently oriented along the [110] crystallographic direction despite an isotropic nature of the wave functions on the high-symmetry (001) surface.

The search for the physical origin of the broken symmetry and the observed preferential orientation of stripes was actively pursued over the past decade. Reduced symmetry of the interface was suggested [8] to introduce an anisotropy of the effective mass [9] or of the cyclotron motion [10]. However, these single-particle effects seem unlikely to be responsible for the large magnitude and strong temperature dependence of the resistance [4–7]. Later work showed [11] that the precise symmetry of the 2D gas confining potential is unimportant, and micron-scale surface roughness does not correlate with the stripe orientation. In theoretical works [12–14] anisotropy of electron-electron interactions due to macroscopic elastic effects was analyzed, but no nonequivalence of [110] and [1$\bar{1}$0] emerged. The effect of the in-plane magnetic field that influences orientation of stripes [15,16] was explained by the field-induced anisotropy of the exchange potential [17,18]. Thus, naturally existing preference for the [110] orientation of the CDW in perpendicular field, the same for electron and hole samples, remained unresolved.

Here, we show experimentally and theoretically that strain breaks the symmetry of electron-electron interactions in magnetic field and results in a preferred orientation of the CDW. Our experiment shows that externally applied shear strain can enhance or reduce anisotropy of the resistance and switch low and high resistance axes. Our theory shows that spin-orbit interaction induced anisotropy of spectrum at zero strain results in equivalent preferential CDW directions [110] or [1$\bar{1}$0]; in-plane shear strain breaks the symmetry between these two directions, and depending on the sign of strain, the Hartree-Fock (HF) energy is minimized when the CDW is oriented along [110] or [1$\bar{1}$0]. We suggest that shear strains from internal electric fields in the growth direction due to midgap Fermi level pinning at the surface are responsible for the observed preferred orientation of the CDW in pristine electron and hole samples. Finally, we find that strain induces a stripe phase at a filling factor $\nu = 5/2$, with a CDW winning over other QH states.

Samples were fabricated in the van der Pauw geometry from carbon doped GaAs quantum well (QW) grown on (001) GaAs [7,19]. From the low field Shubnikov–de Haas oscillations, the hole density is 2.25×10^{11} cm^{-2}, and the mobility 0.8×10^{6} cm^2/V s at the base temperature 10 mK. Some samples were thinned to 150 μm and glued on a multilayer PZT (lead zirconate titanate) ceramic actuator with the samples' [110] or [1$\bar{1}$0] axis aligned with the polarization axis of the PZT. Application of voltage V_p to the actuator induces in-plane shear strain in the sample $\varepsilon_p/V_p = 2.8 \times 10^{-7}$ V^{-1} and small uniform biaxial strain, measured with a biaxial strain gauge [20]. The total shear strain $\varepsilon = \varepsilon_{\text{th}} + \varepsilon_p$ includes a residual strain ε_{th} due to anisotropic

thermal coefficient of the actuator, which depends on the V_p during cooldown. To ensure that V_p on the actuator does not induce charge modulation in the sample we insert a thin metal foil between the sample and the PZT. The foil was also used as a backgate to adjust 2D density that has a weak dependence on strain due to different piezoelectric coefficients of GaAs and AlGaAs [21] (density changes by 3% for the maximum voltage span on the PZT).

Magnetoresistance for pristine (i.e., not attached to PZT) sample is shown in Fig. 1(a). States at $\nu = 7/2$ and $11/2$ are highly anisotropic with low resistance direction along [110], while states at $\nu = 5/2$, $9/2$ and $13/2$ are almost isotropic, consistent with the previous study [7]. In Figs. 1(b) and 1(c) similar traces are shown for large $\varepsilon = \varepsilon_{\rm th} \gtrsim +10^{-4}$ and $\varepsilon = \varepsilon_{\rm th} \lesssim -10^{-4}$ ($\varepsilon_p = 0$). For $\varepsilon_{\rm th} > 0$ the anisotropy is enhanced compared to the unstrained sample, with states at $\nu = 5/2$, $9/2$ and $13/2$ becoming anisotropic and resistance for $I \parallel [110]$ approaching zero for half-filled LL. For $\varepsilon_{\rm th} < 0$ the low and high resistance axes are switched. Here strain also leads to strong anisotropy at $\nu = 5/2$ with high resistance axis along [110] direction.

Residual strains $\varepsilon_{\rm th}$ in Figs. 1(b) and 1(c) are larger than the *in situ* adjustable strain ε_p. In Fig. 2 we analyze AMR for a sample cooled with $V_p = -150$ V, aiming for $\varepsilon_{\rm th} \sim 0$. Resistance as a function of ε_p is plotted near $\nu = 5/2$ and $7/2$. At $V_p < 0$ resistance is highly anisotropic. For high resistance direction, $R_{I\|[110]}$ strongly depends on ε_p and decreases by a factor of 50 (4.4) at $\nu = 5/2$ (7/2) as V_p is varied from -300 V to 300 V. $R_{I\|[1\bar{1}0]}$ increases only 1.7 (1.3) times. At $V_p > 100$ V, the resistance at $\nu = 5/2$ is isotropic, with no maxima for either current direction, as in unstrained sample. Thus, for small strains, the CDW is not a ground state at $\nu = 5/2$, consistent with observations in unstrained samples. From the data we conclude that $\varepsilon < 0$ is within the adjustable range of V_p, because $R_{I\|[110]} > R_{I\|[1\bar{1}0]}$ at $\nu = 7/2$. Continuous evolution of $R_{I\|[110]}$ and $R_{I\|[1\bar{1}0]}$ is consistent with continuous change reported with in-plane magnetic field [16].

Having presented experimental results, we now develop the Hartree-Fock theory of CDW by extending theory [2,3,22] to anisotropic 2D systems. For both electrons and holes, the 3D Hamiltonian [23,24], quantized to an infinite rectangular QW in a magnetic field $\mathbf{H} = \nabla \times \mathbf{A}$ is given by $\mathcal{H}_{\rm 2D} = \mathcal{H}^{\varepsilon} + \mathcal{H}_4^{\rm an}$, where

$$\mathcal{H}^{\varepsilon} = \frac{(p_x - \frac{e}{c}A_x)^2}{2m_x} + \frac{(p_y - \frac{e}{c}A_x)^2}{2m_y}, \qquad (1)$$

includes parabolic and $\mathcal{H}_4^{\rm an} = \beta p_x^2 p_y^2$ includes fourth order terms in p. Here $x \parallel [110]$, $y \parallel [1\bar{1}0]$ are the principal axes of the reciprocal mass tensor, $m^{-1} = (m_x^{-1} + m_y^{-1})/2$ is isotropic, and $\mu^{-1} = (m_x^{-1} - m_y^{-1})/2$ the anisotropic part. In the ground subband of holes quantized along the [001] direction $m^{-1} = -(\gamma_1 + \gamma_2 + \alpha\gamma_3)/m_0$, where γ_1, γ_2, γ_3 are negative constants defining spin-orbit effects in the bulk hole spectra. The coefficient α is defined by these constants [25]. For the anisotropic part of mass induced by the shear strain $\varepsilon = \varepsilon_{xx} = -\varepsilon_{yy}$ our result is $\mu^{-1} = \gamma_3\sqrt{3}d\varepsilon/[\gamma_2(\pi\hbar/a)^2]$, where d is the deformation potential [24], and a is the QW width. For electrons, m is the 3D effective electron mass m_c, while for the anisotropic part in the third-order perturbation theory,

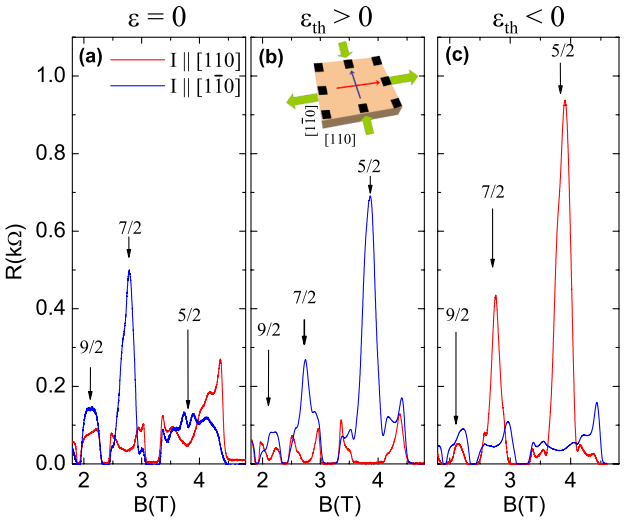

FIG. 1 (color online). Magnetoresistance plotted as a function of **B** for $I \parallel [110]$ (red) and $I \parallel [1\bar{1}0]$ (blue) directions in strained and unstrained samples. In (a) $\varepsilon_p = 0$, (b) thermally induced tensile strain is along [110], $\varepsilon_p = 0$ and (c) along $[1\bar{1}0]$, $\varepsilon_p = 0$. Inset shows sample schematic, red and blue arrows show current, and green arrows show strain.

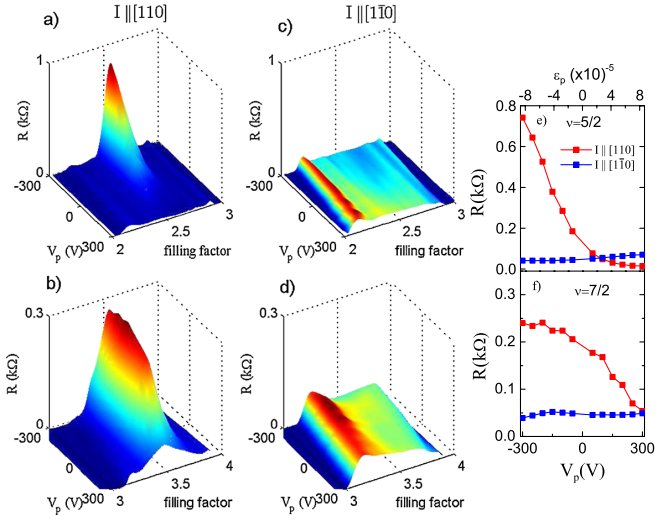

FIG. 2 (color online). Strain dependence of the anisotropic magnetoresistance. (a–d) Magnetoresistance in the vicinity of $\nu = 7/2$ and $5/2$ for $I \parallel [110]$ and $I \parallel [1\bar{1}0]$ as a function of voltage on the piezoelectric actuator V_p. In (e)–(f) magnetoresistance at $\nu = 7/2$ and $5/2$ is extracted. On the top axis, V_p is converted to the piezo-induced shear strain in the heterostructure; actual strain includes thermally induced offset.

we get $\hbar^2/2\mu = -P^2 d\varepsilon/\sqrt{3}E_g^2$, where P is the Kane band coupling parameter [26] and E_g is the band gap. The sign of the strain-induced term for electrons is opposite of that for holes. Anisotropic terms $\mathcal{H}_4^{\rm an}$ do not break C_4 symmetry and do not result in the inequivalence of [110] and [1$\bar{1}$0] directions. Thus, it is instructive to treat the parabolic part separately and add the effect of strain-independent anisotropy, for which we find $\beta = 3(\gamma_3^2 - \gamma_2^2)a^2/2\pi^2\hbar^2 m_0\gamma_2$, to final results by using perturbation theory.

To find the single-particle wave functions of Eq. (1) we define new coordinates, $x' = x\sqrt{m_x/m'}$, $y' = y\sqrt{m_y/m'}$, where $m' = \sqrt{m_x m_y}$. In these coordinates, $\mathcal{H}_{\rm 2D} = \frac{1}{2m'} \times (\mathbf{p}' - \frac{e}{c}\mathbf{A}')^2$ is isotropic, and the wave functions are LLs wave functions with degeneracy in the guiding center coordinate $X' = k_y' l^2$, where $l = (\hbar c/eH)^{1/2}$ is the magnetic length. According to Aleiner and Glazman [22], the electron-electron interactions in a partially filled high topmost LL can be considered as the Coulomb interactions with an effective dielectric constant defined by the electrons of the fully filled LLs. In the deformed coordinates, for $N \gg r_s^{-1} \gg 1$, where N is the LL index, $r_s = (\pi n a_B^2)^{-1}$, n is the 2D carrier density, and $a_B = \hbar^2\kappa_0/me^2$ is the Bohr radius, the low energy physics of the 2D electron liquid in weak magnetic field is described by $\mathcal{H}_{\rm eff} = \frac{1}{L_x L_y}\sum_{\mathbf{q}'} \rho(\mathbf{q}')v(\mathbf{q}')\rho(-\mathbf{q}')$. Here L_x, L_y are the size of the sample,

$$v(\mathbf{q}') = \frac{2\pi e^2}{\kappa_0 \kappa(\mathbf{q}')\sqrt{(q_x'^2)\frac{m_x}{m} + (q_y'^2)\frac{m_y}{m}}} \qquad (2)$$

is the Fourier transform (FT) of the renormalized interaction potential, κ_0 is the background dielectric constant, and at $1/(\sqrt{2N+1}l) < q' < k_F$, k_F is the Fermi wave vector, $\kappa(\mathbf{q}') = 1 + 2/qa_B$ [22,27]. The $\kappa(\mathbf{q}')$ is the effective dielectric constant isotropic in the original coordinates but anisotropic in the deformed ones. Finally, $\rho(\mathbf{q}')$ is the FT of charge density of the partially filled LL N, $\rho(\mathbf{q}') = \sum_{X'} \alpha_N(\mathbf{q}') e^{-iq_x'(X' - q_y' l^2/2)} a_{X'}^\dagger a_{X' - q_y' l^2}$, $\alpha_N(\mathbf{q}') = L_N^0(\frac{\mathbf{q}'^2 l^2}{2}) \times \exp(-\frac{\mathbf{q}'^2 l^2}{4})$, $L_N^0(x)$ is the Laguerre polynomial, and $a_{X'}^\dagger$ ($a_{X'}$) is the creation (annihilation) operator for a hole with guiding center at X' in the topmost LL. Defining the CDW order parameter $\Delta(\mathbf{q}') = \frac{2\pi l^2}{L_x L_y}\sum_{X'} a_{X' + q_y' l^2/2}^\dagger a_{X' - q_y' l^2/2}$, we find the HF energy

$$E_{\rm HF} = \frac{1}{2\pi l^2 \nu_N}\sum_{\mathbf{q}' \neq 0} [u_H(\mathbf{q}') - u_{\rm ex}(\mathbf{q}')]\Delta(\mathbf{q}')\Delta(-\mathbf{q}'), \quad (3)$$

where $0 \leq \nu_N \leq 1$ is the filling of the topmost LL. The Fourier transforms of the Hartree and exchange potentials are, respectively, $u_H(\mathbf{q}') = v(\mathbf{q}')[\alpha_N(\mathbf{q}')]^2$ and

$$u_{\rm ex}(\mathbf{q}') = \frac{2\pi l^2}{L_x L_y}\sum_{\mathbf{Q}'} u_H(\mathbf{Q}') \exp[il^2(-Q_x' q_y' + Q_y' q_x')]. \quad (4)$$

The two potentials are related by FT but the arguments in u_H are transposed relative to the arguments of $u_{\rm ex}$. The transposition is not important in the isotropic case [2], but taking it into account here is crucial for finding the preferred orientation of the CDW.

Applying an analysis similar to that in [2], we see that when the $L_N^0(x) = 0$ in $\alpha_N(\mathbf{q}')$, $u_H(\mathbf{q}') = 0$ also, so that $E_{\rm HF} < 0$, as given by the exchange contribution. The system becomes unstable with respect to the formation of the CDW with a wave vector close to $\mathbf{q}'$. In contrast to the isotropic system, in which the direction of the CDW is chosen spontaneously, in the presence of $\mathcal{H}_4{}^{an}$ quartic anisotropy exchange is minimal either in [110] or in [1$\bar{1}$0] directions; furthermore in the presence of strain, the smallest $\mathbf{q}'$, corresponding to the direction with the largest mass, gives the largest value of the exchange and the lowest HF energy. In Fig. 3(a), we plot the ratio of the anisotropic and isotropic parts of the hole exchange potential at $d = -5.4$ eV, $\gamma_1 = -6.8$, $\gamma_2 = -2.4$, $\gamma_3 = -2.9$, and $\alpha = 0.4$. The difference in exchange energy between CDW in [110] or in [1$\bar{1}$0] directions reaches 5% for strains of 10^{-4}. The CDW near the half filling of the Nth LL results in stripes with alternating $\nu_N = 0$ and $\nu_N = 1$, which, in turn, translates into low resistance direction along the stripes and high resistance direction perpendicular to the stripes. Thus, for $\varepsilon \neq 0$, the CDW has a preferential direction defined by the sign of the strain, consistent with the experiment. While the theory is valid, strictly speaking, in high LL, it describes experiments well even at $N = 1$, the lowest LL for which $L_N^0(x)$ has a zero.

The preceding analysis suggests that internal strain may be responsible for the observed orientation of stripes in pristine samples, with no external strain. GaAs is a piezoelectric material and electric field E_z results in an in-plane shear strain $\varepsilon = d_{14}E_z$, where $d_{14} = -2.7 \times 10^{-10}$ cm/V. A calculated band diagram for our samples is shown in Fig. 3(b). Inside the QW $E_z < 2 \times 10^4$ V/cm and results in strain too small to orient the stripes. E_z on both sides of the QW, caused by doping, is also small and, in our samples, odd in z. However, in all GaAs samples there is a large field near the surface of the wafer due to the pinning of the

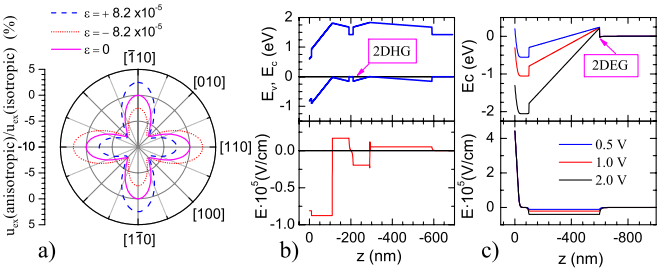

FIG. 3 (color online). (a) The ratio of isotropic and anisotropic parts of the hole exchange potential for three values of shear strain ε. Strain-dependent [Eq. (4)] and strain-independent quartic effects on the exchange are included. (b) Self-consistent calculations of the band profile and internal electric fields in the studied wafer. (c) Modeling of a HIGFET structure from [28].

Fermi energy near midgap. This surface charge-induced field $\sim -10^6$ V/cm produces $\varepsilon \sim 3 \times 10^{-5}$. If transmitted to the QW region, this strain has the correct sign and magnitude to explain the observed orientation of stripes in pristine samples. To show that transmission of strain does indeed occur, we consider the minimal model in which this effect is present, with the free energy of the model, given by $\mathcal{F} = \mathcal{F}_{\rm el} + 2\beta E_z \varepsilon$, where the elastic free energy is $\mathcal{F}_{\rm el} = \frac{1}{2}\lambda[(\partial_x u_y + \partial_y u_x)^2 + (\partial_z u_x)^2 + (\partial_z u_y)^2]$, u_x and u_y are displacements, strain $\varepsilon = \frac{1}{2}(\partial_x u_y + \partial_y u_x)$, x, y, z now correspond to the [100], [010], [001] directions, β is the piezoelectric constant, and λ is the elastic one. In realistic devices, $E_z(x, y, z)$ is nonuniform in (x, y) plane due to charge fluctuations on the surface and in the doping layer. For illustration, we consider a cylindrical sample of radius L and height d with cylindrically symmetric $E_z(r, z) = \sum_n E_n(z) J_0(q_n r)$, where $r = (x^2 + y^2)^{1/2}$, J_0 is the Bessel function, and q_n are quantized by the condition $J_0'(q_n L) = 0$. The solution for the strain is

$$\varepsilon(r, z) = \frac{\beta}{2\lambda} \sum_n \frac{q_n^2}{\partial_z^2 - q_n^2} E_n(z) J_0(q_n r) + \varepsilon_b(r, z), \qquad (5)$$

where ε_b is localized near $r = L$. If $E_z(r, z)$ varies with r at some characteristic scale R, q_n is of order $1/R$. Then, from Eq. (5), the corresponding component of ε propagates largely undiminished over distances $z \sim R$ from the region where $E_n(z)$ is large (sample surface). Thus, macroscopic regions with sizable strain exist throughout the QW region.

The presence of internal strain also explains orientation of stripes in 2D electron gases. A typical band diagram of an electron sample is similar to that shown in Fig. 3(b), but is inverted relative to the Fermi level with both the surface electric field and the shear strain changing sign. However, the term distinguishing [110] and [1$\bar{1}$0] axes in Eq. (1) also has opposite sign for electrons and holes, so that the sign of the anisotropic term in electron and hole exchange is the same. Thus, for both holes and electrons, a surface field will orient the CDW along [110], as seen in experiments. Our model also explains the reorientation of stripes as a function of density in a HIGFET (Heterojunction-Insulated Gate Field Effect Transistor) [28]. At low gate voltages (low densities), shear strain will be dominated by the surface field; see Fig. 3(c). At large gate voltages (2 V corresponds to 3×10^{11} cm^{-2}), the electric field across the AlGaAs barrier becomes large enough to change the sign of the strain in the 2D gas region, reorienting the stripes.

In summary, we have shown experimentally that orientation of CDW in the QH regime can be controlled by external strain. Theoretically, we have traced this effect to a strain-induced anisotropy of the exchange interaction and a competition between the internal and external strain. In general, any factor that brings in a crystallographic anisotropy of spectrum gives rise to a crystallographic anisotropy of the Hartree-Fock energy of the CDW state (this is the case, for example, in hole gases grown on low-symmetry (311) GaAs [6]). Here we have calculated the effect of quartic anisotropy of spectrum on exchange potential: directions [001] and [010] become unfavorable, and directions [110] and [1$\bar{1}$0] become favorable for the propagation of CDW. However, the most important effect arises because of strain, and in heterostructures grown in the high-symmetry (001) surfaces, piezoelectricity due to surface electric fields becomes the source of inequivalence of [110] and [1$\bar{1}$0] directions for CDW. We underscore that, although the anisotropy of electron Hartree-Fock energy is 2 orders of magnitude smaller than that for holes, it must still choose a preferential direction for the CDW of guiding centers. Therefore, the preferential direction of the resistance anisotropy in pristine (001) samples appears to be universally set by internal strain.

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