UPDATING, UPGRAADING, REFINING, CALIBRATION AND IMPLEMENTATION OF TRADE-OFF ANALYSIS METHODOLOGY DEVELOPED FOR INDOT

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Updating, Upgrading, Refining, Calibration and Implementation of Trade-Off Analysis Methodology Developed for INDOT

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As part of the ongoing evolution towards integrated highway asset management, the Indiana Department of Transportation (INDOT), through SPR studies in 2004 and 2010, sponsored research that developed an overall framework for asset management. This was intended to foster decision support for alternative investments across the program areas on the basis of a broad range of performance measures and against the background of the various alternative actions or spending amounts that could be applied to the several different asset types in the different program areas. The 2010 study also developed theoretical constructs for scaling and amalgamating the different performance measures, and for analyzing the different kinds of trade-offs. The research products from the present study include this technical report which shows how theoretical underpinnings of the methodology developed for INDOT in 2010 have been updated, upgraded, and refined. The report also includes a case study that shows how the trade-off analysis framework has been calibrated using available data. Supplemental to the report is Trade-IN Version 1.0, a set of flexible and easy-to-use spreadsheets that implement the tradeoff framework. With this framework and using data at the current time or in the future, INDOT’s asset managers are placed in a better position to quantify and comprehend the relationships between budget levels and system-wide performance, the relationships between different pairs of conflicting or non-conflicting performance measures under a given budget limit, and the consequences, in terms of system-wide performance, of funding shifts across the management systems or program areas.
EXECUTIVE SUMMARY

UPDATING, UPGRAADING, REFINING, CALIBRATION AND IMPLEMENTATION OF TRADE-OFF ANALYSIS METHODOLOGY DEVELOPED FOR INDOT

Introduction

To ensure optimal use of available funds while addressing the goals and perspectives of its stakeholders, the Indiana Department of Transportation (INDOT) continues to evaluate and prioritize alternative strategies for preservation and operations in each program area or management system. To combine all these systems to yield an overarching, integrated decision-support and evaluation mechanism, INDOT has started to develop an asset management system (AMS). As part of this effort, INDOT sponsored studies in 2004 and 2010 that culminated in the development of an overall framework for asset management, identification and description of the various trade-offs faced by the asset manager, and the mathematical constructs for quantifying these trade-offs.

As identified in the 2010 study, the network-level trade-off types include trade-off between two alternative individual projects; trade-off between two alternative groups of projects; trade-off between cost performance and levels of one non-cost performance measure; minimum budget level requirement analysis; shifting budget analysis; and trade-off between two non-cost performance measures.

As a follow-up to these studies, INDOT identified the need to update and implement the trade-off analysis methodology developed for INDOT in 2010. Thus, the present study commenced to carry out the upgrading and refinements, and also to calibrate and implement the framework by developing an analytical, flexible and interactive tool. The analytical tool, Trade-IN Version 1.0, was intended to be flexible so as to accommodate future changes in default input values to reflect future INDOT perspectives, or to yield new trade-off functions under circumstances different from those under which the present study was carried out.

Findings

This project demonstrates that it is feasible to develop and implement a framework and tool for analyzing trade-offs in asset management. The research products from the present study include this technical report, which shows how theoretical underpinnings of the methodology developed for INDOT in 2010 have been updated, upgraded, and refined. The report also includes a case study that shows how the trade-off analysis framework has been calibrated using available data. Supplemental to the report is a set of flexible and easy-to-use spreadsheets that implement the trade-off framework. With this framework and using data at the current time or in the future, INDOT’s asset managers are placed in a better position to quantify and comprehend the relationships between budget levels and system-wide performance; the relationships between different pairs of conflicting or non-conflicting performance measures under a given budget limit; and the consequences, in terms of system-wide performance, of funding shifts across the management systems or program areas.

Implementation

The research product from this study can be used by INDOT’s asset managers at the central office or the districts. After collecting the relevant data needed for the analysis, the asset manager can use the spreadsheets submitted with this report to carry out the trade-off analysis. Implementing the study product is expected to enhance decision making at INDOT as the agency continually seeks to make transparent and comprehensive evaluations to yield cost-effective and balanced investments. By providing methodologies to incorporate multiple performance criteria from different program areas for optimization of decisions under constrained budgets, and for investigating performance and budgetary trade-offs across the program areas, this study product is poised to help address these issues.

A core group of persons at INDOT under the advisement of the Federal Highway Administration (FHWA) will further define and select implementation strategies relative to agency practices of trade-offs among asset programs. This steering group is represented by INDOT’s central office and district planning divisions and its research office. Its principal mission is to advance and institutionalize the most practicable methods outlined in the research report.
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1. INTRODUCTION

1.1 Study Background

The Indiana Department of Transportation (INDOT) seeks to manage its assets in a strategic way that duly recognizes the role and importance of their assets and also in a manner that accounts for any existing or anticipated funding or institutional constraints or changes. As such, the agency pays particular attention to key business processes, such as resource allocation and utilization, evaluation, and decision making, and seeks to routinely make decisions that are based on reliable information regarding the future consequences of alternative actions at the overall system level.

Persons in positions that are consistent with the responsibilities of an asset manager, at INDOT Central Office and INDOT’s highway administrative districts, are entrusted with a fiduciary responsibility to protect the billions of taxpayer dollars already invested in transportation infrastructure and to ensure that the system is operated and preserved in the most cost-effective and transparent manner. They seek to do this using the concepts of asset management. Transportation asset management (TAM) is still a growing discipline. As part of the ongoing evolution towards overall highway asset management, INDOT, through a 2004 SPR study (2384), sponsored the development of an overall framework for asset management, which was intended to foster decision support for alternative investments across the program areas on the basis of a broad range of performance measures and against the background of the various alternative actions or spending amounts that could be applied to the several different asset types in each program area. A subsequent SPR study (3110) in 2008, developed theoretical constructs for scaling and amalgamation of the different performance measures and for analyzing the different kinds of trade-offs. The scaling of performance measures yields a consistent or dimensionless unit to make them comparable. Amalgamation combines the weighted and scaled performance measures to yield a single utility value that represents the overall desirability of a candidate project. The report documents, with examples, a number of alternative methods for scaling and amalgamation. The implementation report for SPR 3110 stated the need to implement the trade-off techniques and framework on an INDOT-compatible computing platform such as Microsoft Excel. In March 2010, INDOT, through its Asset Management focus group, expressed the desire to commission an SPR study to carry out such implementation. These efforts culminated in the approval of the current study, SPR 3500, that is enhancing the theoretical framework developed in SPR 3110, translating the framework into a set of flexible and simple spreadsheets, and collecting data to demonstrate the use of the framework and spreadsheets. The ultimate intent is to provide the INDOT asset manager with a means of better comprehending the consequences of various top-level actions consistent with trade-off scenarios.

1.2 Problem Definition

By the inherent nature of their work, asset managers typically deal with a variety of asset types in different program areas to arrive at an investment evaluation or decision at the system level. The overall problem in transportation asset management can be viewed as a multiobjective optimization project selection problem as presented in Figure 1.1. First, in each program area, the needs for new construction, preservation, and maintenance are evaluated, from which some candidate projects are proposed. Next, these proposed projects constitute a mixed-type candidate project pool that contains various types of projects, such as pavement

![Figure 1.1 Typical structure of asset program development.](image-url)
projects, bridge projects, and safety projects; and asset managers ultimately select which of these projects should be implemented. Such a selection process needs to consider multiple performance measures, budget constraints, and performance constraints. In seeking investment options that yield the best possible system-level outcome in terms of several performance measures, asset managers face an analytical challenge that is similar to the classic multiobjective optimization problem in operations research. In such optimization problems, trade-off analysis is often carried out explicitly or implicitly to examine the consequences of different optimal solutions under different funding and performance scenarios. When equipped with an appropriate tool to solve this problem, the asset manager is able to achieve a reasonable balance between the different performance-related objectives while satisfying the budgetary and performance constraints.

The asset manager refers to a person at an INDOT state or district level who is responsible for making top-level decisions consistent with asset management principles. In their top-level functions of project selection and prioritization, the asset manager often encounters the need to analyze trade-offs. A trade-off is defined as “a balancing of factors, all of which are not attainable at the same time; a giving up of one thing in return for another” (Merriam-Webster 2002), and is also defined as “a balance achieved between two desirable but incompatible features; a sacrifice made in one area to obtain benefits in another; a bargain, a compromise” (Simpson and Weiner, 1989). Trade-off implies that a decision is being made with full comprehension of both the merits and demerits of any particular choice. In asset management, trade-offs can be carried out at the project level or the network level (also referred to as the system level or program level). At the project level, the asset manager may seek the trade-off between two projects within one program area (e.g., two pavement projects); or (e.g., one pavement project and one bridge project); or (e.g., two pavement projects); or (e.g., two pavement projects); or (e.g., two pavement projects).

Thus, the asset manager is interested in trade-offs at both the overall network level and the project level. As identified in the SPR 3110 project, the possible types of trade-offs, at a minimum, which are needed to be analyzed and quantified by INDOT, are:

A. Trade-off between two alternatives. This type of trade-off involves the comparison of two competing alternatives and identification of the superior one. The two alternatives could be:

1. Two individual projects in the same program area (e.g., two pavement projects);
2. Two individual projects from different program areas (e.g., one pavement project and one bridge project); or
3. Two project portfolios that include projects from various program areas.

B. Trade-offs involving performance measures. This type of trade-off contains two sub-types:

1. Trade-off between cost and performance measures. This type of trade-off is of interest where the issue of budget is of concern. It helps asset managers to investigate the relationships between the cost levels and some “benefit” performance, such as the asset’s condition, safety, or durability. For this type of trade-off analysis, the asset manager can conduct the following types of analyses:

   a. Check the relationship between the cost and a non-cost performance measure. This type of trade-off helps the asset manager investigate the relationship between the budget and some “benefit” performance. For instance, the asset manager could be seeking an answer to the following question: What level of pavement condition can be achieved if $100M is invested to conduct pavement rehabilitation and maintenance?

   b. Minimum budget level requirement analysis. This type of trade-off is to determine the minimum budget required to meet certain predefined (or changes in) performance standards. For example, what is the minimum budget needed to ensure that a certain minimum average crash rate and/or freeway delay are attained for the overall network.

   c. Shifting budget analysis. This type of trade-off enables the asset manager to examine the impacts (changes in levels of some specified performance measures) if a certain funding amount is transferred from one program area to another. Also, policy changes may necessitate increasing the budget of one program area and subsequently lowering the budget of another, which is equivalent to full or partial transfer of funds from one program area to another. The second issue, therefore, is that the asset manager may wish to know the effect of such funding shifts on overall network performance in terms of the different performance measures. For example, what will be the impact (in terms of increased crashes and increased mobility) of lowering the safety budget and increasing the congestion budget or transferring $5,000,000 from a safety program to a congestion program? In other words, how many crashes is the asset manager prepared to trade off for a specific increase in mobility?

2. Trade-off between two performance measures. In this case, the asset manager is interested in the extent to which one performance measure, such as mobility,
can be bartered for another, such as safety. For example, when the total budget is fixed, how much pavement condition improvement must be forfeited in order to gain a certain additional amount of bridge condition improvement?

Quantifying such trade-offs is a vital aspect of the work of the asset manager. As such, there is a need to make any needed enhancements to the theoretical framework developed in SPR 3110, translate the framework into a set of flexible and simple spreadsheets, and collect data to demonstrate the use of the framework and spreadsheets. This work would provide the INDOT asset manager with a tool to conduct various trade-off analyses.

1.3 Study Objectives

The primary objective of this research, therefore, is to update and refine the existing trade-off analysis methodologies and develop an analytical tool for translating the theoretical trade-off concepts into a flexible and interactive tool for quickly examining the trade-offs associated with INDOT projects at the state and district levels. This analytical tool will be flexible enough to accommodate future changes in default input values to reflect future INDOT perspectives and will enable asset managers to conduct two types of trade-off analyses: (1) trade-offs between alternatives and (2) trade-offs involving performance measures.

1.4 Organization of This Study

The remainder of this report is organized as follows. Chapter 2 provides the basic theory and methods for trade-off analysis between alternatives. Chapter 3 discusses the trade-off analysis process between performance measures. Chapter 4 presents a case study conducted to demonstrate the application of the proposed methodologies.

Accompanying this report is an Excel-based analytical tool and a user manual to help and guide asset managers through the process of conducting various trade-off analyses in the decision-making tasks that are associated with transportation asset management.

2. TRADE-OFF ANALYSIS BETWEEN ALTERNATIVES

2.1 Introduction

In transportation asset management, there is always a need to compare different alternatives to identify the optimal approach; and this process could also be viewed as a type of trade-off analysis (1). Due to the integration of different program areas in transportation asset management, this type of trade-off analysis could be conducted between two alternative individual projects that originate in same or different program areas and between two alternative project portfolios that could contain several projects from the same or different program areas.

In transportation asset management, an ongoing trend is the increasing number of stakeholder categories. Different stakeholders, who represent widely diverse views, want their concerns to be considered during the decision-making process and also tend to call for more transparency and accountability in the project evaluation and selection process. For instance, stakeholders may seek that highway agencies, within funding constraints, provide the best possible service to system users and also to create more jobs for the community; highway users may demand superior riding condition, enhanced freeway mobility, greater accessibility to local roads, and safer travel; and environmental groups may advocate for sustaining the quality of the environment such as reduced emissions, lower noise, and minimal damage to the ecology. These concerns translate into a wide array of highway performance measures for decision making. To incorporate these multiple concerns in the decision-making process, a variety of performance measures need to be considered that generate multiple objectives at the time of the decision making. Also, in transportation asset management, the integration of various program areas also requires asset managers to apply multiple performance measures in the decision-making process. Because the projects from different program areas (i.e., different types of projects) need different performance measures to conduct the evaluation. For instance, to evaluate the condition of pavements, the International Roughness Index (IRI) could be used or to evaluate the condition of a bridge, various bridge condition rating may be applied. In a nutshell, in transportation asset management, there is a vital need to use multiple performance measures.

Therefore, in the trade-off analysis between two alternatives, each alternative may have several performance measures, which makes this a multicriteria decision-making (MCDM) problem. In the final report for SPR 3110 (2), the proposed process to conduct MCDM analysis is: (1) scale the various performance measures with different units to a same or dimensionless unit; then, (2) conduct amalgamation to combine the scaled values of the various performance measures to form a single value to represent the importance or the benefit of the implementation of each alternative project or project portfolio; and finally, (3) conduct a comparison to identify the alternative with the better amalgamated value as the solution. For example, as shown in Figure 2.1, there are \( m \) performance measures for two alternatives. To conduct trade-off between them, first, the value of each performance measure for each alternative \( p_{ij} \) is obtained as presented in the upper-left table in Figure 2.1. A different \( p_{ij} \) may have a different unit. Then, scaling techniques can be used to transform all \( p_{ij} \) into \( s_{ij} \). After the scaling, all \( s_{ij} \) should have the same unit as presented in the lower-left table in Figure 2.1. Next, amalgamation techniques are used to combine the scaled values to form a single value \( (A_i) \) for each project. Last, by comparing \( A_1 \) and \( A_2 \), the better alternative can be identified. Thus, it can be seen that,
in such types of trade-off analysis, the most important processes are scaling and amalgamation. In the following sections of this chapter, the scaling and amalgamation methods will be discussed in detail, which are derived from the final report of SPR 3110 (2).

2.2 Scaling Methods

In attempting to make decisions on the basis of multiple performance measures, these multiple performance measures may have different units or metrics. For example, safety enhancement is often measured as a reduction in fatal and serious personal-injury crashes; improved mobility is often expressed in terms of reduction in delay, enhanced level of service (LOS), decrease in travel time, or reduced volume-to-capacity ratio; pavement system preservation can be measured as a reduction in IRI, extension in pavement remaining life for friction or other pavement attributes, etc.; bridge system preservation is often measured as an increase in its NBI condition rating, reduction in earthquake vulnerability, and at a network level, the decrease in number or percentage of structurally deficient or functionally obsolete bridges, etc. These are typically referred to as the benefit performance measures because they reflect some benefit to INDOT or facility users. Often included also in the multiple performance measures are the cost performance measures, which refer to the agency cost of project implementation. Unlike the benefit performance measures, cost performance measures are applicable to all projects irrespective of their program areas. User costs may be considered a benefit or cost performance measure depending on the wishes of the asset manager. If the asset manager wishes to express user cost as a cost performance measure, then it must be used in the analysis in its absolute terms or raw cost values; if, on the other hand, the asset manager wishes to express the user cost as a benefit performance measure, then it must be calculated as the reduction in user cost relative to a base case (such as the do-nothing alternative).

This section discusses a number of alternative techniques that could be used to render all of the different performance measures onto the same scale, dimension, or unit. Based on the scaling methods identified in the final report of SPR 3110 (2), some of the main scaling methods presented in Figure 2.2 will be discussed in detail in this report and incorporated in the Excel tool. However, the Excel tool also provides the flexibility of using other types of scaling methods. From Figure 2.2, it can be seen that scaling methods can be categorized as so-called “objective” methods and preference-based methods. In each method, scaling is carried out separately for each performance measure. The results of the scaling procedure yield a value that represents the worth or desirability of the different levels of the performance measure. In the simplest case, the least preferred level of the performance measure is assigned a value of one (or 100%) and the worst a value of zero. This way, one can assign a scaled unit to represent the impact of any project in terms of any performance measure.

The objective methods include linear scaling and monetization. The preference-based methods are considered by some schools of thought as being subjective because they are developed on the basis of expert opinion, through surveys. Scaling functions developed using preference-based methods can be categorized into the value functions and utility functions. A utility function is considered a more general form of a value function. Similar to value functions, utility functions incorporate the innate values that the asset manager attaches to the different levels of the performance measure. Unlike value functions, however, utility functions incorporate the asset manager’s attitudes toward risk (i.e., whether the asset manager is risk prone, risk neutral, or risk averse).

2.2.1 Linear Scaling Method

The linear scaling method is utilized to derive a scaling function that is assumed to be linear. This
technique can be used when the asset manager has no data that can help him/her develop a scaling function. Thus, a linear scaling function can be considered as the default for all scaling functions. The scale of linear scaling function often ranges from 0 to 1, 0 to 10, or 0 to 100, depending on the wishes of the asset manager. There are at least four shapes of the linear scaling function: monotonically increasing, monotonically decreasing, upward V, and downward V.

In monotonically increasing linear scaling functions, higher values of the performance measure are more desirable to the asset manager, such as bridge condition rating and facility remaining service life. On a 0–1 scale, Equation (2.1) and Figure 2.3 represent a general form of this type of scaling function for performance measure $x$. In Equation (2.1), $x_0$ is the minimum value, or the minimum acceptable value of $x$. For example, in the scaling of bridge condition rating using the NBI rating scale, Rating 3 is the common minimum acceptable value, then the $x_0$ could be 3. Thus, any rating equal to or less than 3 can be assigned a scaled value of 0; but for the scaling of the facility remaining service life, the minimum value 0 could be used for $x_0$.

In Equation (2.1), $x_1$ is the maximum value, or the fully satisfied value of $x$. For instance, in the scaling for bridge condition rating using the NBI rating, the maximum value is Rating 9, which can be used for $x_1$.

$$s(x) = \begin{cases} 0 & x \geq x_1 \\ \frac{x-x_0}{x_1-x_0} & x_0 \leq x \leq x_1 \\ 1 & x \geq x_1 \end{cases}$$  \hspace{1cm} (2.1)

In monotonically decreasing linear scaling functions, higher values of the performance measure are typically less desirable to the asset manager, such as agency cost, IRI, crash rate, and delay. On a 0–1 scale, Equation (2.2) and Figure 2.4 can be used for this type of scaling procedure. Similarly, $x_0$ is the maximum value, or the maximum acceptable value of performance measures $x$; $x_1$ is the minimum value, or the fully satisfied value, of $x$.

$$s(x) = \begin{cases} 0 & x_1 \leq x \\ \frac{x_1-x}{x_1-x_0} & x_0 \leq x \leq x_1 \\ 1 & x \leq x_0 \end{cases}$$  \hspace{1cm} (2.2)

In some cases, the linear scaling function first increases up to a point and then monotonically decreases thereafter or monotonically increases to a point and then monotonically increases thereafter. This is the case when the asset manager prefers a performance measure that is not too small or too large or where the asset manager desires that the performance measure is desirable only when it is lower than some threshold or when it exceeds some threshold. For instance, for the travel speed performance measure, it is often desired that speed should not be too low or too high because either extreme is associated with higher fuel consumption.

On a 0–1 scale, the linear concave non-monotonic scaling function can be represented as:

$$s(x) = \begin{cases} 0 & x \leq x_0 \\ \frac{x-x_0}{x^*-x_0} & x_0 \leq x \leq x^* \\ 1 & x \geq x_1 \end{cases}$$  \hspace{1cm} (2.3)

This function can be illustrated as Figure 2.5. $x_0$ and $x_1$ are the least desired values of performance measures $x$; $x^*$ is the most desired value of $x$.

On a scale of 0–1, the linear convex non-monotonic scaling function can be represented as:
Figure 2.3 Scaling function for linearly monotonically increasing performance measures.

Figure 2.4 Scaling function for linearly monotonically decreasing performance measures.

Figure 2.5 Scaling function for non-monotonic performance measures (concave).
This function can be illustrated as Figure 2.6. \( x_0 \) and \( x_1 \) are the most desired values of performance measures \( x \); \( x^* \) is the least desired value of \( x \).

An example is presented here to illustrate the linear scaling method. On a highway with a speed limit 50 mph, average travel speed \( (X) \) can be used as a performance measure to evaluate mobility. Thus, the theoretical range of \( X \) is \([0, 50]\), and its scaling function can be shown in Figure 2.7. For instance, if the actual average travel speed after a project implementation is 36 mph, then the scaled value of that project impact is \((36-0)/(50-0) = 0.72\).

### 2.2.2 Monetization

In highway asset management, there are relatively few performance measures that have monetary units; these include the agency cost and user cost of a project. Then there are those that are **intrinsically monetary**, that is, they are typically not expressed in monetary units but could be expressed in such units using appropriate relationships established through research. Consider safety and pavement surface performance for example. Safety performance can be measured in terms of a reduction in crash rate (e.g., 50 crashes per 100 million VMT); pavement performance can be measured in terms of the International Roughness Index (IRI) in inches/mile. Transforming all of these different performance measures into their monetary equivalents or dollar units is thus a special type of scaling that is appropriately termed “monetization.” As a simple example of intrinsically monetary performance measures, consider a highway project that is expected to yield a reduction of 20 crashes/100 million VMT. If the project is expected to serve a demand of 50 million VMT at the time of project completion, then the annual benefit is a reduction of 10 fatal crashes. If the cost of a fatal crash is $1 million, then the monetized benefit (or the scaled value of safety performance), assuming constant demand, is 10*1 = $10 million.

Even though it is often not recognized explicitly as a scaling technique, monetization is a common method for bringing different performance measures to the same dimension or scale. In most transportation project evaluations, decisions are made on the basis of the monetized values of the relevant performance measures while non-monetized performance measures are often relegated to the background of mere conceptual (and often, inconsequential) discussion. As only a relatively few measures can be quantified in their monetary values,
monetization severely limits the number of performance measures that can be considered in evaluation. For example, ecological damage that accompanies the construction and operations of freeway systems in rural areas is difficult to satisfactorily measure in terms of its monetary equivalent as there are no universally accepted models for doing so. Following are various models from the literature that could be used to monetize a number of commonly used performance measures.

A. Conversion of Travel Time Reduction into Monetary Units. Table 2.1 shows how the asset manager could scale the performance benefits of travel time reduction (in hours) into a dollar value. In the simplest case, only one vehicle class is used and no clocking status is considered. In a more comprehensive analysis, however, it is useful to consider such nuances in travel time estimation and valuation. On-the-clock travel time, which represents work-related travel, are based on costs to the employer such as wages and fringe benefits, costs related to vehicle productivity, inventory-carrying costs, and spoilage costs. Off-the-clock trips include trips for commuting to and from work, personal business, and leisure activity. Heavy trucks are assumed to be used only for work, so the value of time equals the on-the-clock value. Table 2.1 summarizes the estimates of the major cost components of the value of travel time by vehicle type, on the basis of FHWA’s HERS software (3). For a future congestion mitigation project in the asset program, if the travel time reduction is known for each of the indicated categories (On-the-Clock and Off-the-Clock), then the indicated values can be used to find the equivalent dollar value of the congestion mitigation performance of the project.

B. Conversion of Safety Benefits into Monetary Units. When safety benefits are expressed as the number of reduced crashes per VMT, the corresponding monetary cost savings is determined as the product of the crash reduction per VMT and the unit monetary crash cost to yield the dollars saved per VMT. The two commonly used sources for the unit dollar value estimates are the annual publication of the National Safety Council Estimates and the 1988 FHWA memorandum. Also, the cost of road crashes can be based on a weighted injury scale by using indices for the level of severity of the road crash. The 2005 unit costs of each crash severity type are available for injury scales such as the KABCO rating scale (5) and the Abbreviated Injury Scale (6). Table 2.2 shows the unit crash cost values for the KABCO scale, updated from NSC (5) using the consumer price indices from the U.S. Department of Labor.

C. Conversion of Pavement Condition Improvement into Monetary Units. To some extent, pavement roughness, measured in terms of the Present Serviceability Rating (PSR), or the International Roughness Index (IRI), can affect the maintenance, tire wear, repair, and depreciation components of vehicle operating cost (VOC) and thus can translate into direct increases in the out-of-pocket costs of road users. This occurs because the motion of vehicle tires on a rough pavement surface is associated with greater resistance to movement, which leads to higher levels of fuel consumption compared to traveling at a similar speed on a smooth surface; and a bumpy ride which leads to increased vibration and wear-and-tear of vehicle parts. Also, an indirect effect of poor pavement condition is that road users may be forced to drive at lower speeds, leading to higher fuel consumption. Projects that improve the pavement surface, such as resurfacing, lead to reductions in unit VOCs caused by pavement roughness.

High levels of pavement condition (low roughness) increments in condition have relatively little effect on the VOC (Figure 2.8), and additional costs of vehicle operation start to accrue only when the IRI exceeds approximately 100 in/mi (3.33 m/km). For paved roads in poor condition and for gravel roads, changes in road surface condition, can lead to very drastic reductions in VOC.

Papagiannakis and Delwar (8) concluded that a unit increase in IRI (in m/km) will generally lead to an increase of $200 (or 1.67 cents per vehicle-mile, assuming 12,000 annual mileage) in vehicle maintenance and repair costs alone. Barnes and Langworthy (9) developed adjustment factors for all of the VOC

<table>
<thead>
<tr>
<th>Category</th>
<th>Small Automobile</th>
<th>Medium Automobile</th>
<th>4-Tire Truck</th>
<th>6-Tire Truck</th>
<th>3-4 Axle Truck</th>
<th>4-Axle Combination Truck</th>
<th>5-Axle Combination Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>On-the-Clock</td>
<td>$34.34</td>
<td>$34.70</td>
<td>$24.77</td>
<td>$30.61</td>
<td>$33.13</td>
<td>$38.04</td>
<td>$38.72</td>
</tr>
<tr>
<td>Off-the-Clock</td>
<td>$17.54</td>
<td>$17.58</td>
<td>$18.50</td>
<td>$30.61</td>
<td>$33.14</td>
<td>$38.04</td>
<td>$38.73</td>
</tr>
</tbody>
</table>

Source: Updated from (3) and (4).
components combined, as a function of pavement condition (Figure 2.9). They assumed a baseline of a PSI of 3.5 or better (IRI of about 85 inches/mile or 1.35 m/km) at which an increase in pavement condition would have no impact on operating costs, and then adjusted for three levels of rougher pavement as shown in Figure 2.9. This figure can be used to estimate the VOC corresponding to a given pavement state on the basis of the VOC at a baseline state of the pavement. For the depreciation component, there are relatively few studies that have explicitly shown a relationship with pavement roughness. However, it is clear that a vehicle that is operated on a rough pavement surface is likely to lose its value more quickly than one that is operated on a smooth surface pavement.

As a scaling technique, monetization has serious drawbacks. First, there has not been enough research to quantify all of the transportation impacts in their monetary equivalents. Secondly, there can be ethical issues in the attempt to assign monetary values to safety impacts. Thirdly, the use of monetary values yields a scale that is unbounded and this could cause some computation problems.

2.2.3 Preference-Based Scaling Methods

Preference-based scaling methods are those that involve a survey of asset management experts (and/or other stakeholders) so that their preferences regarding the various levels of a given performance measure can

![Figure 2.8](image1.png) Conversion of pavement condition to cost (7).

![Figure 2.9](image2.png) VOC adjustments for pavement roughness levels.
be expressed on a dimensionless scale showing the desirability of utilities at the different levels. For a given performance measure, such a scale can be established from 0-1, 0-10, or 1-100. If this is repeated for several performance measures that originally had different units, a normalized scale is attainable that can be used to compare or combine the different performance measures.

Of the preference-based scaling methods, the most popular and most widely used measure of desirability is utility theory (10). In this study, the concept of utility is used as the measure of the asset manager’s desirability, and this will be used for all of the different preference-based scaling methods herein discussed.

In utility theory, the basic element is value function or utility function, which reflects the preference structure of asset managers. In the process of decision making, if there are \( n \) performance measures \( (X_1, X_2, \ldots, X_n) \), let us assume \( (x_{i1}, x_{i2}, \ldots, x_{in}) \) and \( (x_{j1}, x_{j2}, \ldots, x_{jn}) \) are the performance measures values of any two alternatives. If one can find a scalar-valued function \( v() \) with the following property:

\[
\begin{align*}
v(x_{i1}, x_{i2}, \ldots, x_{in}) & \geq v(x_{j1}, x_{j2}, \ldots, x_{jn}) \\
\iff (x_{i1}, x_{i2}, \ldots, x_{in}) & \succeq (x_{j1}, x_{j2}, \ldots, x_{jn})
\end{align*}
\]

where the symbol \( \succeq \) means “preferred or indifferent to,” then one can call the function \( v() \) a value function or utility function (10). The process of scaling therefore yields the value function or utility function for the performance measure in question.

The difference between a utility function and a value function lies in the level of certainty of the project outcome in terms of the given performance measure. For instance, when we resurface a highway, the change in pavement performance (say, surface roughness in IRI units) is not known with certainty. Where there is more certainty than uncertainty regarding the project outcome, the resulting scaling function is referred to as a value function; in uncertainty condition, it is called a utility function. So, in a general sense, a value function is a special case of the utility function where uncertainty is zero.

As presented in Figure 2.2, there are several methods for developing a preference-based scaling function for a given performance measure. We present here two categories of these methods: the scaling methods under certainty scenario and the scaling methods under risk scenario.

### 2.2.3.1 Certainty scenario

1. Direct rating. The simplest scaling method, the Direct Rating technique (10), asks the decision-maker to indicate directly the value or desirability he/she attaches to each level of the performance measure on a scale of say, 0 to 1. This method is most appropriate where the performance measure has only a few levels and when these levels are discrete. Thus, it can be used for Present Serviceability Index (PSI) which ranges from 0 to 5; and congestion levels of service (LOS) which ranges from A to F, but is not appropriate for IRI (in/mile). The process of direct rating is described as follows:

   **Step 1:** List all possible values of the performance measure; for performance measure \( X \), its values are \( x_1, x_2, \ldots, x_n \);
   **Step 2:** Find out the least preferred value of \( X \), denote it as \( x_0 \) and define its value function as \( v(x_0) = 0 \);
   **Step 3:** Find out the most preferred value of \( X \), denote it as \( x_n \) and define its value function as \( v(x_n) = 1 \);
   **Step 4:** Directly assign intermediate values \( v(x) \) to the various values of the performance measure \( x \)'s between \( x_0 \) and \( x_n \);
   **Step 5:** List all the values of \( X \) and their corresponding scaling values.

The flow chart of this method is shown in Figure 2.10.

2. Midvalue splitting technique. The midvalue splitting method (10) is based on the identification of the concept of the midvalue point and the differentially value equivalent points. For two performance measures \( X \) and \( Y \), the pair \((x_1, x_2)\) \((x_1 < x_2)\) is said to be *differentially value-equivalent* to the pair \((x_3, x_4)\) \((x_3 < x_4)\) if the decision-maker is willing to forego the same amount of \( Y \) for the increase of \( X \) from \( x_1 \) to \( x_2 \) as for the increase from \( x_1 \) to \( x_4 \) at any point of \( Y \). Thus, for any interval \([x_1, x_2]\) of \( X \), its midvalue point \( x_3 \) is such that the pair \((x_1, x_3)\) and \((x_3, x_2)\) are *differentially value-equivalent* (10). Based on the concept of mid-value splitting, the following steps can be used to develop a value function for performance measure \( X \) (Figure 2.11).

   **Step 1:** Determine the range of \( X \), and define \( u_X(x_0) = 0 \) and \( u_X(x_1) = 1 \), where \( x_0 \) is the least preferred value and \( x_1 \) is the most preferred value;
   **Step 2:** Determine the midvalue point of \([x_0, x_1]\), denote it as \( x_{0.5} \), and let \( u_X(x_{0.5}) = 0.5 \);

   ![Figure 2.10](image-url)
Step 3: Determine the midvalue point of \([x_0, x_0.5]\), denote it as \(x_{0.25}\), and let \(u_1(x_{0.25}) = 0.25\);
Step 4: Determine the midvalue point of \([x_0.5, x_1]\), denote it as \(x_{0.75}\), and let \(u_1(x_{0.75}) = 0.75\);
Step 5: Check consistency. Determine whether the midvalue point of \([x_{0.25}, x_{0.75}]\) is \(x_{0.5}\), if not, repeat steps 2 to 4;
Step 6: Plot points \((x_i, u(x_i))\) and draw the curve using these points; the resulting curve is the value function of \(X\) (Figure 2.12).

This easy-to-use method is applicable only in the certainty condition.

3. Statistical regression to enhance the outcome of scaling. In practice, many decisions are made by a group of people, not a single decision-maker. So for each person in the decision group, the direct rating or midvalue splitting methods can be used to generate a number of observations for each level of the performance measure. Then statistical regression can be used to obtain the line of best fit through these points, thereby offering the value function that represents the preference structure of the entire decision group.

2.2.3.2 Risk scenario. The risk scenario is used when the project outcome in terms of a given performance measure is not known with certainty, but a probability distribution can be developed for the levels of that performance measure. The distribution can be developed using historical data from similar projects. Under the risk scenario, scaling functions can be developed using the direct questioning approach and the certainty equivalent approach, which are described below (10):

1. Direct questioning approach. There are two variations to this approach (10), depending on whether the variable representing the performance measure is discrete or continuous.

A. Where the performance measure is a discrete variable. In such cases, especially where the discrete levels of the performance measure are relatively few, the following direct assessment procedure can be used to develop the utility function.

Define \(u_1(x_i) = 0\) and \(u_3(x_i) = 1\).

Find the midvalue point of \([x_0, x_1]\) denote it as \(x_{0.5}\).

Find the midvalue point of \([x_{0.5}, x_1]\) denote it as \(x_{0.75}\).

Find the midvalue point of \([x_0, x_{0.5}]\) denote it as \(x_{0.25}\).

Consistency check

Calibrate the value function

Step 0: Determine all possible values of \(X\), e.g., \(x_1, x_2, \cdots, x_m\);
Step 1: Denote the least preferred value of \(X\) as \(x_0\), the most preferred value of the performance measure as \(x_m\); then define \(u(x_0) = 0\) and \(u(x_m) = 1\);
Step 2: For each \(x_i\), determine the probability \(p_i\) which render the following situations indifferent:

i. A guaranteed prospect of an outcome of \(x_i\);
ii. A risk prospect of obtaining an outcome of \(x_i\) with probability \(p_i\) and an outcome of \(x_m\) with probability \(1 - p_i\);

Step 3: Calculate the utility of \(x_i\)

\[u(x_i) = p_i u(x_i) + (1 - p_i) u(x_m) = p_i\]

Step 4: Repeat step 2 and step 3 until the utilities of all other levels of the performance measure have been determined;
Step 5: Check for consistency. Choose any three levels of the performance measure: \(x_1, x_2, \) and \(x_3\). Then consider these two situations:

i. A guaranteed prospect of an outcome of \(x_2\);
ii. A risk prospect of obtaining an outcome of as \(x_1\) with probability \(p\) and an outcome of \(x_3\) with probability \(1 - p\);

If the decision-maker considers the above two situations as indifferent, then for consistency, \(p\) should be equal to \(p = \frac{u(x_2) - u(x_1)}{u(x_3) - u(x_1)}\)

B. Where the performance measure is a continuous variable. If the performance measure is continuous, it is impossible to establish utilities for all of the infinite possible levels it could take. In such cases, a number of discrete levels are taken from the continuum to adequately represent its spread, and the utilities of these discrete values are determined using a survey. The detailed steps are as follows:

---

Figure 2.11  Midvalue splitting method.
Step 0: Determine the value range of $X$;
Step 1: Denote the least preferred value of the performance measure as $x_w$, the most preferred as $x_b$; then define $u(x_w) = 0$ and $u(x_b) = 1$;
Step 2: Compare the following situations:
  i. A guaranteed prospect of an outcome of $X = 0.5(x_b - x_w)$;
  ii. A risk prospect of obtaining an outcome of $X = x_b$ with probability $p$ and an outcome of $x_w$ with probability $(1-p)$;

This is to determine the probability $p$ which renders the above situations indifferent. Then $p = p_{0.5}$;
Step 3: Repeat step 2 by setting the guaranteed prospect as $0.25(x_b - x_w)$ and $0.75(x_b - x_w)$, and get $p_{0.25}$ and $p_{0.75}$;
Step 4: Check consistency. Compare the following situations:
  i. A guaranteed prospect of an outcome of $X = 0.5(x_b - x_w)$;
  ii. A risk prospect of obtaining an outcome of $X = x_b$ with probability $50\%$ and an outcome of $x_w$ with probability $50\%$;

Determine $X_{0.5}$ that renders the above situations indifferent;
Step 5: Plot $(X_w, 0)$, $(0.25(X_b - X_w), p_{0.25})$, $(0.5(X_b - X_w), p_{0.5})$, $(0.75(X_b - X_w), p_{0.75})$, and $(X_b, 1)$, then use statistical regression to obtain the utility function.

For multiple survey respondents, further regression can be used to obtain the line of best fit for all observations, thus enhancing the scaling function further.

2. Certainty equivalent approach. From the literature (10), this technique appears to be the most popular approach for developing utility functions under the risk situation. To develop the utility function for a performance measure $X$, the following steps are used:

Step 1: Define the worst level of the performance measure $X$ as $X_w$, the best level of $X$ as $X_b$, then define $u(x_w) = 0$ and $u(x_b) = 1$;
Step 2: Compare the following two situations:
  i. A guaranteed prospect of an outcome of $X_{0.5}$;
  ii. A risk prospect of obtaining an outcome of as $X_w$ with probability $50\%$ and an outcome of $X_b$ with probability $50\%$;

Determine $X_{0.5}$ that renders the above situations indifferent;
Step 3: Repeat step 2 by setting the guaranteed prospects $X_{0.25}$ and $X_{0.75}$, and get final $X_{0.25}$ and $X_{0.75}$;
Step 4: Check consistency. Compare the following situations:
  i. A guaranteed prospect of an outcome of $X_{0.5}$;
  ii. A risk prospect of obtaining an outcome of as $X_{0.75}$ with probability $50\%$ and an outcome of $X_{0.25}$ with probability $50\%$;

If the decision-maker considers either situation (a) and (b) as superior to the other, then go back to step 2, until the decision-maker considers two situations as being indifferent;
Step 5: Plot $(X_w, 0)$, $(X_{0.25}, 0.25)$, $(X_{0.5}, 0.5)$, $(X_{0.75}, 0.75)$, and $(X_b, 1)$, choose the utility function form and calibrate the parameters in the function (Figure 2.13).
2.2.4 Shapes of Scaling Functions and Their Implications

2.2.4.1 Shapes of scaling functions. Irrespective of scaling method used, there generally are four major shapes that a scaling function can take: monotonically increasing, monotonically decreasing, concave, and convex.

A. Monotonically increasing scaling functions. These functions (Figure 2.14), which may be linear or non-linear, represent the performance measure for which higher values are more desirable to the decision-maker. Examples include IRI Change (but not IRI), Bridge Health Index, Bridge Sufficiency Rating, Pavement Condition Index (PCI), Present Serviceability Rating (PSR), Pavement Quality Index, reductions in roughness, reductions in crash rates, etc. So, for example, a higher PCI translates into a good condition while a lower PCI translates into a poorer condition. Also, a higher IRI change is more desirable while a lower IRI change is less desirable.

B. Monotonically decreasing scaling functions (Figure 2.15). These functions typically represent the performance measure for which higher values are less desirable to the decision-maker. The function shape may be linear or non-linear. Examples include IRI, Rutting, Bridge Corrosion Index, crash rate, delay, reduction in speed, reduction in travel time, reduction in facility health/condition, etc. So, for example, a higher IRI translates into a poor condition and has a lower value or scale while a lower IRI translates into a superior condition and has a higher value or scale.

C. Non-monotonic scaling functions. Scaling functions are not always monotonically increasing or decreasing. In some cases, the function is monotonically increasing up to a point and then monotonically decreasing thereafter. In other cases, it is monotonically decreasing up to a point and monotonically increasing thereafter, which happens where it is desired that the performance measure should not be too small or too large or where it is desired that the performance measures is desirable only when it is lower than some threshold or when it exceeds some threshold. For instance, where speed is a performance measure, it is often desired that speed should not be too low or too high as either extreme is associated with higher fuel consumption. Non-monotonic scaling functions may be linear or non-linear.

2.2.4.2 Implication of the shapes of scaling functions. A scaling function developed from the preference of asset managers can show revealing patterns of their risk-taking attitudes. The risk-taking attitude is reflected in the concavity or convexity of the scaling function. It can be proven mathematically that a risk-taking decision-maker has a strictly convex utility function, a risk-averse decision-maker has a strictly concave scaling function, and a risk-neutral decision-maker has a linear scaling function. Figure 2.16 presents the relation between the concavity and risk-taking tendency of a decision-maker.

2.3 Amalgamation Methods

In the previous section, various scaling methods, which render performance measures with different units into a unit that is commensurate across all the performance measures under consideration, were discussed. Thus, for any given candidate project, the asset manager can determine the dimensionless values of the impacts of the project separately for safety, congestion, preservation, etc. So the question that now arises is how best to combine them to get the overall impact for the project. The combination of the different impacts for each candidate project in the asset manager’s portfolio is known as amalgamation. As identified in the final report of SPR 3110, there are several types of methods that can conduct amalgamation. After careful consideration, four methods are presented in this report: the weighted sum method (or weighting method), the benefit/cost ratio method, the goal programming method, and the utility function method. Among the four methods, only the first three will be incorporated in the developed Excel tool; however, the Excel tool also provides the flexibility to incorporate other types of amalgamation methods.

2.3.1 Weighted Sum Method (WSM)

The weighted sum method is commonly used by many asset managers. It uses the additive function form to obtain the final value of an alternative. The final value of alternative \( i \) can be calculated as (II):

\[
U_i = \sum_{j=1}^{m} w_j s_{ij}
\]  

(2.5)

Where: \( w_j \) is the weight of the performance measure \( j \); \( s_{ij} \) is the scaled value of the performance measure \( j \) for alternative \( i \); \( m \) is the number of performance measures.

The alternative with the highest \( U_i \) is the best choice.

When the WSM is used, the value of the performance measures must be dimensionless or have the same units (e.g., scaled value). If the scaled values are from preference-based scaling methods, the multiple performance measures must be utility independent and
Utility independence means that each criterion’s utility function does not depend on the levels of other performance measures. Preference independence assumes that the trade-offs between two performance measures do not depend on the levels of other performance measures. In addition, in the risk condition, the expected values of performance measures are used in Equation (2.5).

To apply the weighted sum method, it is necessary to derive the relative weights among the asset performance measures. Any one of several methods could be used to conduct weighting to obtain the weights in Equation (2.5). The equal weighting approach (same weights to each objective) is simple and straightforward and easy to implement, but it does not capture the preference among different attributes. The observer-derived weights approach (12) estimates the relative weights of multiple goals by analyzing the unaided subjective evaluations of alternatives using regression analysis. For each alternative, the decision-maker is asked to assign scores to the benefits under individual goals as well as a total score on a scale of 0 to 100. A functional relationship is then established using the total score as a response variable and the scores assigned under the individual goals as explanatory variables through regression analysis. The calibrated coefficients of the model thus become the relative weights of the multiple goals. Psychologists and pollsters have shown preference for the observer-derived weighting method because it yields the weights that best predict unaided opinions. Direct weighting methods (13) ask the decision-maker to directly specify numerical values between 1 and 10 on
an interval scale for individual goals. The Analytic Hierarchy Process (AHP), which allows considering objective and subjective factors in assigning weights to multiple goals (14), is based on three principles: decomposition, comparative judgments, and synthesis of priorities. The relative weights of individual asset managers that reflect their importance are first established, and then the relative weights of individual asset managers for the multiple goals are assessed. The local priorities of the goals with respect to each decision-maker are finally synthesized to arrive at the global priorities of the goals. One criticism of this technique is the rank reversal of goals when an extra goal is introduced. The gamble method chooses a weight for one goal at a time by asking the decision-maker to compare a “sure thing” and a “gamble.” The first step is to determine which goal is most important to move from its worst to best possible level. Then, two situations are considered: 1) the most important goal is set at its best level and the other goals are set at their least desirable levels; and 2) the chance of all goals at their most desirable levels is set to $p$, and chance of $(1 - p)$ for all goals at their worst values. If the two situations are equally desirable, the weight for the most important goal will be precisely $p$. The same approach is repeated to derive the weights for the remaining goals with decreasing relative importance. The hypothetical probabilities for all goals in their best or worst cases are prone to vary for different assessors.
2.3.2 The Multiplicative Utility Function

The multiplicative utility function of alternative $A_i$ is defined as follows (10):

$$U_i = \frac{1}{k} \left( 1 + kw_1(x_{i1}) \right) \left( 1 + kw_2(x_{i2}) \right) \ldots \left( 1 + kw_m(x_{im}) \right) - 1 \quad (2.6)$$

Where: $u(x_{ij})$ is the utility of alternative $i$ on the $j$th performance measure; $w_j$ is the relative weight of performance measure $j$; $m$ is the number of performance measures; $k$ is a scaling constant that is determined from Equation (2.7)

$$1 + k = (1 + kw_1) \ast (1 + kw_2) \ast \ldots \ast (1 + kw_m) \quad (2.7)$$

The premise of using the multiplicative utility function is that all of the criteria must be mutually utility-independent. If $X_1, X_2, \ldots, X_m$ are the $m$ performance measures, we say criteria $X_i$ is utility-independent if $X_i$’s utility function does not depend on the levels of other criteria. Also $X_1, X_2, \ldots, X_m$ are mutually utility-independent if every subset of $\{ X_1, X_2, \ldots, X_m \}$ is utility-independent of its complement (10). The project alternative with the higher final utility is superior to that with the lower final utility.

2.3.3 Benefit/Cost Ratio Method

In the benefit/cost ratio method, the weighted sum of the scaled benefit performance measures is divided by the cost of the alternative. Then, typically, the larger the benefit/cost ratio is, the better the alternative is. Equation (2.8) can be used to calculate the benefit/cost ratio.

$$U_i = \frac{\sum_{j=1}^{m} w_j s_{ij}}{c_i} \quad (2.8)$$

Where: $U_i$ is the benefit/cost ratio of project $i$; $n$ is the number of performance measures; $c_i$ is the agency cost of implementing project $i$; $w_j$ is the weight of performance measure $j$; $s_{ij}$ is the scaled value of performance measure $j$ for alternative $i$.

2.3.4 The Goal Programming Method

In the goal programming method, the asset manager first establishes target levels or goals that need to be achieved. Then, for each alternative, the distance from the target levels is calculated using Equation (2.9).

$$U_i = \left( \sum_{j=1}^{m} (s_{ij} - M_j)^p \right)^{1/p} \quad (2.9)$$

Where: $U_i$ represents the sum of the deviations from the goals; $s_{ij}$ is the scaled value of performance measure $j$ for alternative $i$; $M_j$ is the target value of the $j$th performance measure; $m$ is the number of performance measures.

There are different norm metrics that can be used in the minimization of the goal programming function. The parameter ‘$p$’ is varied to determine the type of distance metric being measured. The three most commonly considered metric norms in goal programming are:

- If $p = 1$, “city block” distance
- If $p = 2$, “Euclidean” distance
- If $p = \infty$, “Minmax” distance (or infinity norm)

Figure 2.17 presents a 3-D example of how the amalgamated impacts of a project can be found on the basis of the project impact in terms of three performance measures, using goal programming.

2.4 Trade-off Analysis Methods

2.4.1 Trade-off Analysis between Two Alternative Individual Projects

To conduct trade-off analysis between two alternative individual projects, which could be from the
same program area (e.g., two pavement projects) or from different program areas (e.g., a pavement project and a bridge project), the basic processes are:

Step 1: Establish performance measures to evaluate the two projects under consideration;
Step 2: Evaluate the value of each performance measure for each project;
Step 3: Conduct scaling to scale all performance measures to a same or dimensionless unit;
Step 4: Perform amalgamation to obtain a single value to represent the importance or benefit of each project;
Step 5: Compare the amalgamated values of the two projects, and then the one with the superior value is the preferred one.

The above trade-off analysis is incorporated into the developed Excel tool to help the asset manager conduct such type of trade-off analysis. Please check the Excel tool (Trade-IN) and its User Manual for details.

2.4.2 Trade-off Analysis between Two Alternative Project Portfolios

To conduct trade-off analysis between two alternative project portfolios, which could contain projects from the same program area or from various different program areas, the basic processes are:

Step 1: Establish performance measures to evaluate each project in the two project portfolios;
Step 2: Evaluate the value of each performance measure for each project in the two project portfolios;
Step 3: Conduct scaling to scale all performance measures to a same unit or dimensionless unit;
Step 4: Perform amalgamation to obtain a single value to represent the importance or benefit of each project;
Step 5: Compare the amalgamated values of the two portfolios, and then the one with the superior value is the preferred project portfolio.

The above trade-off analysis is incorporated into the developed Excel tool to help the asset manager conduct a trade-off analysis between project portfolios. Please check the Excel tool and the User’s Manual for details.

2.5 Chapter Summary

This chapter first discussed the importance to conduct trade-off analysis between alternative individual projects or between alternative project portfolios in transportation asset management. The basic processes for such types of trade-off analysis are: (1) scale various performance measures with different units to the same or dimensionless unit; then (2) conduct amalgamation to combine the scaled values of various performance measures to form a single value to represent the importance or benefit of the implementation of each alternative project or project portfolio; (3) conduct comparisons to identify the alternative with a better amalgamated value as the solution. Next, several scaling methods and amalgamation methods were presented. Finally, the detailed steps to conduct trade-off analysis between alternative individual projects or between alternative project portfolios in transportation asset management were discussed.

3. TRADE-OFF ANALYSES INVOLVING PERFORMANCE MEASURES

3.1 Introduction

Trade-off analyses involving performance measures are very important for asset managers in the decision-making process of transportation asset management.
The trade-off between cost and performance measures assists asset managers in their investigation of the relationship between cost and performance measures and conducting minimum budget analysis and shifting budget analysis. The trade-off between performance measures examines the relationship between performance measures under certain constraints to achieve a reasonable balance in the decision-making process. To conduct these analyses, it is necessary to start from the basic problem in the decision making of transportation asset management. In the following sections of this chapter, the decision framework and the mathematical formulation of the project selection problem in transportation asset management is first presented, followed by a discussion of the methodology for each type trade-off analysis.

3.2 Decision Framework and Formulations for Transportation Asset Management

3.2.1 Decision-Making Framework for Transportation Asset Management

Based on the characteristics of transportation asset management, a decision-making framework for the project selection problem in transportation asset management is proposed in Figure 3.1. In this framework, possible candidate project portfolios, or project selection sets, are first identified. Each project portfolio contains one or more projects, which may be from the same program area or from different program areas. The evaluation is carried out on the basis of the performance impacts of each portfolio. Also, the performance impacts are indicated in terms of the raw network-level performance measures, such as the average network crash rate. Therefore, for each portfolio, the impact of implementing each constituent project is determined in terms of the performance measures \( p_n \). Then, the overall network-level performance \( P:\text{NPM}_{i0} \) of each portfolio can be expressed in terms of some simple statistics of the performance measures, such as the simple mean, the percentage of assets whose performance exceeds some specified threshold. A multiobjective optimization follows in order to identify the optimal project portfolio with the best network performance under given constraints. It has been shown that this framework appropriately incorporates the characteristics of transportation asset management and provides a general process for the project selection problem in transportation asset management.

3.2.2 Multiobjective Optimization Formulation for Transportation Asset Management

3.2.2.1 General formulation. Mathematically, the project selection problem in a typical transportation asset management decision-making context can be described as follows:

There are \( n \) candidate projects in a pool of highway projects comprising \( k \) asset types (pavements, bridges, safety assets, mobility assets, etc.). There is a budgetary constraint, \( B \), for all these “candidate” projects so only a subset of the projects can be implemented. Each program area budget \( b_j \) may have a lower bound \( b_j^L \), an upper bound \( b_j^U \), or both. A program area refers to a specific asset type, subarea, or management system such as pavement, bridges, safety, or congestion/mobility. There are \( s \) performance measures that are used to evaluate the broad range of impacts or benefits of implementing the selected projects. On the basis of the \( s \) performance measures, \( m \) objectives are formulated for selecting projects (\( s \) may or may not equal \( m \)). For each objective, some performance threshold constraints may exist. The asset managers seek the best combination of projects that yields the best possible levels of each objective. This problem can be generally formulated as follows:

**Objective Functions**

\[
\begin{align*}
\text{min (or max) } f_1(x) \\
\text{min (or max) } f_2(x) \\
& \ldots \\
\text{min (or max) } f_m(x)
\end{align*}
\]

(3.1)

**Constraints**

Total budget constraint: \( \sum_{i=1}^{n} x_i c_i \leq B \)

(3.2)

Program area budget constraints:

\[
b_j^L \leq \sum_{i=1}^{n} x_{ij} c_i \leq b_j^U \quad j = 1, 2, \ldots, k
\]

(3.3)

Performance constraints: \( f_i^{\text{min}} \leq f_i(x) \leq f_i^{\text{max}} \) \( l = 1, 2, \ldots, m \) (3.4)

Where:

- \( x \) is a vector of the decision variables \( (x_1, x_2, \ldots, x_n) \), where \( x_i \) \( i = 1, 2, \ldots, n \) is a binary variable used to indicate whether a project is selected or not; \( x_i = 1 \) indicates the candidate project \( i \) is selected; \( x_i = 0 \) means the candidate project \( i \) is not selected;
- \( y_j = 0 \) (project \( i \) is not associated with the program area or asset type \( j \) ) or 1 (project \( i \) is associated with the program area or asset type \( j \));
- \( c_i \) is the cost of project \( i \);
- \( B \) is the total budget; and
- \( f_j(x) \) is the decision-makers’ \( j \)th objective.

It can be seen that the problem is basically a multiobjective optimization problem. Each objective function \( f_j(x) \) in Formulation (3.1) is a network-level performance measure (NPM) in Figure 3.1. Formulations (3.1) through (3.4) constitute a general formulation that incorporates most of the practical situations and may spawn some variations, depending on the decision-making context for a particular problem or the culture of decision making that exists in a particular agency. For example, instead of
considering the total budget as a constraint, the total cost could be viewed as a performance measure and be placed in the objective function. Also, the agency may or may not impose constraints on the program area budgets or on the average network-level performance. In addition, the project selection process at the network level in asset management could be conducted for a long-term plan, a short-term plan, or even a one-year plan, depending on the actual needs of the agency. For instance, the planning period of a long-term strategy could exceed 20 years; a Statewide Transportation Improvement Program (STIP) typically covers four years. In practice, project selection may take place every year in order to determine the projects that are going to be implemented in the following year based on the following year’s budget. Then, the performance measure could be for a specific time after the project implementation or for the average performance during a period, depending on the actual decision-making context. Furthermore, the problem formulation presented in Formulations (3.3) and (3.4) affords asset managers the flexibility to specify the thresholds for specific performance measures, as well as any upper or lower budgetary constraints for each program area.

### 3.2.2.2 Network-level performance formulation

As evidenced by Formulation (3.1), this study uses network-level performance measures to measure the overall performance of the outcomes of the project selection. To investigate the effects of different combinations of projects on the network-level performance measures, there is a need to incorporate the decision variables into the objective functions; in other words, to express the objectives as functions of decision variables. In this study, an “asset” refers to a physical facility in the highway system, such as a segment of pavement, a bridge, a traffic signal, or a traffic sign. A candidate “project” is a planned action to construct, to renew, or to maintain a highway asset, such as the construction of a new road or a new bridge, the resurfacing of a segment of pavement, the rehabilitation of a bridge, the installation of a traffic sign/signal, or the improvement of an intersection. The performance outcomes arising from the implementation of a candidate project are typically reflected by the change in the performance of the recipient highway asset(s). Thus, the network-level performance measures herein are actually the network-level performance outcomes of the highway assets, which can incorporate the effect of implementing the candidate project. The basic steps for incorporating decision variables in the project selection problem in highway asset management are as follows:

**Step 1:** For each asset, establish the current project-level performance measures related to that asset, which can be obtained from routine inspections or can be calculated from relevant transportation characteristics. For instance, a routine inspection can provide data on pavement conditions. The current levels of the performance measures of each asset often serve as a basis for the prediction of the future levels of the performance outcomes of each asset.
Step 2: Estimate the levels of the project-level performance measures for each asset for the WITH and WITHOUT project implementation scenarios. Figure 3.2 illustrates the change pattern for the performance measure (IRI) for a pavement resurfacing project. In the WITHOUT project scenario where no project is implemented, the IRI follows the deterioration trend (pre-treatment performance curve). In the WITH project scenario where a project is implemented, the IRI level suddenly decreases (performance jump) after project implementation and then follows a new deterioration trend (post-treatment performance curve). Thus, in the year as shown in Figure 3.2, the IRIs in the WITH and the WITHOUT scenarios are different and can be evaluated based on the deterioration curves.

Step 3: Formulate each network-level performance measure as a function of the decision variables \(x_1, x_2, \ldots, x_n\) and the estimated project-level performance measures of each project under the WITH and WITHOUT scenarios. More specifically, when \(x_i = 1\), the project-level performance measures of project \(i\) in the WITH scenario are used; and when \(x_i = 0\), the project-level performance measures of project \(i\) in the WITHOUT scenario are used.

The commonly used network-level performance measures in transportation asset management, which can be expressed for all assets in the entire network, include:

i. An average of the performance of all relevant assets, such as the average crash rate in the highway network, in terms of the number of crashes per 100 million VMT;

ii. A percentage of assets whose performance satisfy some specified threshold, such as the percentage of structurally deficient bridges and the percentage of pavement in good condition;

iii. A sum of the performance of all relevant assets, such as the total number of jobs created and the total tonnage pollutant emissions.

Clearly, the final value of network-level performance depends on which projects are selected. The expressions derived for the above three types of network-level performance measures are presented in Appendix A.

In practice, most of the commonly used network performance measures can be expressed in the above three forms, as a function of the decision variables \((x_i)\). There are also some performance measures that are too complex to be expressed as a mathematical formulation of the decision variable, particularly for economic development and accessibility performance measures. For example, the average cost per trip is related to investment decision making, but they are very complex and cannot be simply expressed as a mathematical function of the decision variables. In these situations, the problem cannot be simply formulated as a multiobjective optimization using pure mathematical formulation that can be solved using traditional multiobjective optimization techniques; there is a need to apply simulations in the multiobjective optimization to evaluate the network-level performance measures.

It is also worth mentioning, from a practical standpoint during project selection, that asset managers may seek to incorporate a weight assigned to each project to reflect its importance. For instance, asset managers may consider a bridge with a traffic volume of 10,000 vehicles/day as being more important compared to a bridge with 500 vehicles/day. Thus, there is often a need to assign a relative weight to each bridge in the calculation of the network average performance measures. To do this analysis, instead of using the actual values of the performance measures of bridges in all the above formulations, the weighted, scaled, or weighted scaled values of the performance measures can be used for each project.

3.3 TRADE-OFF ANALYSIS METHODOLOGY

3.3.1 Trade-off Analysis Formulations

In Formulations (3.1) through (3.4), the project selection problem in transportation asset management
was formulated as a multiobjective optimization problem with network-level performance measures as the objectives. To conduct trade-off analysis in the multiobjective optimization problem, mathematical formulations are needed. The following section presents the formulation for each type of trade-off analyses identified in Section 1.2.

### 3.3.1.1 Formulations for trade-off analysis between cost and performance measures

The trade-off analysis between the cost and performance measures seeks to ascertain the level of the performance measures obtained under different budget levels. For this type of trade-off analysis for the multiobjective optimization problem defined in Formulations (3.1) through (3.4), the following formulation can be used:

**Objective Functions**

\[
\begin{align*}
\text{min} \quad & \sum_{i=1}^{n} x_i c_i \\
\text{min} (\text{or max}) \quad & f_j(x)
\end{align*}
\]  

(3.5)

Where the \(f_j(x)\) is the \(j\)th objective; and \(\sum_{i=1}^{n} x_i c_i\) is the total cost; other variables have the same meanings as explained for Formulations (3.1) through (3.4).

There could also be some constraints in Formulation (3.5), such as the program area budget constraints and performance constraints, depending on the actual problem in practice.

### 3.3.1.2 Formulations for minimum budget level requirement analysis

Minimum budget level requirement analysis is to determine the minimum budget required to meet certain predefined performance standards/thresholds. To conduct this analysis, the following formulation can be used:

**Objective Functions**

\[
\begin{align*}
\text{min} \quad & \sum_{i=1}^{n} x_i c_i \\
\text{min} (\text{or max}) \quad & f_j(x)
\end{align*}
\]  

(3.6)

**Constraints**

Performance constraints:

\[
f_j^{\text{min}} \leq f_j(x) \leq f_j^{\text{max}} \quad l = 1, 2, \ldots, m
\]  

(3.7)

Where all variables have the same meanings as explained for Formulations (3.1) through (3.4).

In this type of analysis, asset managers need to specify the threshold for at least one performance measure, then conduct optimization using Formulations (3.6) and (3.7). In Formulation (3.7), if higher values of the performance measure are more desirable to asset managers, \(f_j^{\text{min}} \leq f_j(x)\) should be applied; if lower values of the performance measure are more desirable to asset managers, \(f_j(x) \leq f_j^{\text{max}}\) should be used.

### 3.3.1.3 Formulations for shifting budget analysis

Shifting budget analysis is used to examine the impacts if a certain funding amount is transferred from one program area to another. Thus, in such an analysis, a base scenario of budget allocation is first proposed. In the base scenario, each program area is assigned a certain budget amount. Then, budget shifting is conducted between program areas to establish various budget allocation scenarios. The total budget for each scenario should remain constant. Finally, optimization is conducted for each scenario and the resulting performance is checked. For each scenario, the following formulation could be used:

\[
\begin{align*}
\text{max} \quad & \sum_{i=1}^{n} x_i U_i
\end{align*}
\]  

(3.8)

**Constraints**

Program area budget constraints:

\[
\sum_{i=1}^{n} x_i y_i c_i \leq b_j^U \quad j = 1, 2, \ldots, k
\]  

(3.9)

Where the \(U_i\) is the amalgamated value for project \(i\) obtained from Equations (2.5) through (2.9); other variables have the same meanings as explained for Formulations (3.1) through (3.4).

### 3.3.1.4 Formulations for trade-off analysis between performance measures

The trade-off between performance measures seeks to estimate the relationship between the objective \(f_j\) and the objective \(f_{j^*}\), or between \(f_{j^*}\) and two or more other objectives, in the general formulation presented in Formulations (3.1) through (3.4). For conducting the trade-off between \(f^j\)th and \(f^{j^*}\)th objectives, the following formulation can be used:

**Objective Functions**

\[
\begin{align*}
\text{min} (\text{or max}) \quad & f_j(x) \\
\text{min} (\text{or max}) \quad & f_{j^*}(x)
\end{align*}
\]  

(3.10)

**Constraints**

\[
\sum_{i=1}^{n} x_i c_i \leq B
\]  

(3.11)

\[
b_j^l \leq \sum_{i=1}^{n} x_i y_i c_i \leq b_j^U \quad j = 1, 2, \ldots, k
\]  

The variables have the same meanings as in Formulations (3.1) through (3.4).

On the basis of Formulations (3.5) through (3.11), the trade-off analyses involving performance measures identified in Section 1.2 can be conducted.

### 3.3.2 Trade-off Analysis Algorithm Design

In this section, the detailed algorithms used for the trade-off analyses in the Excel tool are discussed. In addition to the four types of trade-off analyses discussed in Section 3.2.1, other types of analysis to check the levels of performance measures under a given
budget, are also discussed and incorporated into the Excel tool (Trade-IN).

### 3.3.2.1 Algorithm design for trade-off analysis between cost and performance measures

As presented in Formulation (3.5), the problem for the trade-off between the cost and performance measures is a bi-objective optimization problem. Figure 3.3 presents the feasible solutions of a bi-objective optimization problem with the minimization of each objective of \( f_1 \) and \( f_2 \). As seen in Figure 3.3, there are several feasible solutions, each of which is represented by a dot. In multiobjective optimization, if a solution \( a \) is superior than or at least equal to another solution \( b \) at all of the objectives and Solution \( a \) is superior than Solution \( b \) for at least one objective, then we say Solution \( a \) dominates Solution \( b \) (16). As an example in Figure 3.3, Solution \( A \) has a better value than Solution \( B \) at both objective \( f_1 \) and \( f_2 \), thus Solution \( A \) dominates Solution \( B \). It can be seen that all of the solutions on the curve are not dominated by any other solution; these solutions are described as Pareto-optimal solutions (17,18). All Pareto solutions collectively constitute a Pareto frontier. Theoretically, a trade-off analysis can be conducted between any two solutions (19). For instance, if asset managers choose Solution \( B \) instead of \( A \), then an increase of \( f_2 \) is traded off for a decrease of \( f_1 \). However, this trade-off is not meaningful in practice because the two solutions are not Pareto solutions; in other words, they are dominated by other solutions. In a rational decision-making process, neither of these two solutions is expected to be chosen by asset managers because we can find better feasible solutions that dominate these two solutions. In other words, a meaningful trade-off analysis should be conducted only between solutions that are Pareto-optimal (i.e., those lying on the curve in Figure 3.3). With the Pareto frontier shown in Figure 3.3, the relationship between different objectives can be investigated. Thus, to conduct trade-off analysis in multiobjective optimization, there is a need to generate Pareto frontiers.

In general, Pareto frontier generation methods can be categorized into two groups: classical methods and evolutionary methods (20). Classical methods for Pareto frontier generation mainly include the weighting method, the \( \varepsilon \)-constraint method, the weighted metric method, the normal boundary intersection (NBI) method, the normal constraint (NC) method, and the physical programming (PP) method (15). All of the classical methods seek to generate real Pareto solutions (or at least weak Pareto solutions) to produce Pareto frontiers (21). Evolutionary methods, also referred to as Pareto frontier approximation methods, have been widely used in practical multiobjective optimization problems. The main advantages of evolutionary methods are (22): (1) evolutionary methods generate simultaneously a set of Pareto solutions in a single run because they apply the concept of “population” in the Pareto solution generation process and (2) evolutionary methods are more robust to accommodate different shapes of the Pareto frontier.

There are several evolutionary methods for the Pareto frontier generation in the multiobjective optimization, such as the Vector Evaluation Genetic Algorithm (VEGA) developed by Schaffer (23); Pareto Archived Evolution Strategy (PAES) (24); Pareto Envelope-based Selection Algorithm (PESA) (25); Niched Pareto Genetic Algorithm 2 (NPGA 2) (26); Strength Pareto Evolutionary Algorithm 2 (SPEA2) (27); and Non-dominated Sorting Genetic Algorithm II (NSGA-II) (28). Due to the practical needs, we needed to develop an analytical tool for trade-off analysis using the platform of Microsoft Excel in this study. When the NSGA II was programmed into Microsoft Excel using Visual Basic for Applications (VBA), the speed was very slow, requiring a few hours possibly to generate one Pareto frontier. Thus, it cannot meet the practical needs for decision making in transportation asset management. Therefore, we applied the classical method, \( \varepsilon \)-constraint method (29), to generate Pareto frontiers, using heuristic methods to solve each single objective optimization in the \( \varepsilon \)-constraint method to save computational time.

For the trade-off analysis in Formulation (3.5), the \( \varepsilon \)-constraint method keeps one objective (the performance objective \( f_j(x) \)) as the objective and transforms the other (the cost objective) into a constraint by adding a parameter \( \varepsilon \) as presented in Formulation (3.12).

\[
\min (\text{or max}) \quad f_j(x) \\
\text{s.t.} \\
\sum_{i=1}^{n} x_i c_i \leq \varepsilon
\]

Where the \( f_j(x) \) is the \( j \)-th objective; \( \sum_{i=1}^{n} x_i c_i \) is the total cost; \( \varepsilon \) is a specified value for the total cost; other
variables have the same meanings as explained for Formulation (3.5).

By using different values of \( \varepsilon \) (i.e., different budget levels), we can obtain a number of Pareto solutions. For instance, in Figure 3.4, we want to generate Pareto frontiers for the trade-off analysis between average pavement IRI and cost. If we can generate Pareto solutions A, B, C, D, and E, then we can produce an approximation of the Pareto frontier. Generally, if adequate Pareto solutions are generated, very good approximation of the Pareto frontier is possible.

Therefore, if we want to generate \( q \) number of Pareto solutions, we need to conduct \( q \) times of optimization as presented in Formulation (3.12). Each of the optimization problems presented in Formulation (3.12) is a project selection problem. If there are \( n \) candidate projects, then there will be \( 2^n \) possible project portfolios, which renders the problem NP-Hard (30). In addition, in the practical project selection task in transportation asset management, hundreds, if not thousands, of projects exist in a typical problem setting. Thus, it is laborious and very difficult to use an exact algorithm to achieve the optimal solution. In this case, the common approach is to apply a heuristic algorithm to find the solution. In this study, a heuristic algorithm that is similar to the Incremental Utility-Cost (IUC) Ratio algorithm in multiobjective bridge management system is used (31). It has been tested that the IUC can produce excellent results in the optimization problem as shown in Formulation (3.12) using VBA in Microsoft Excel. The basic steps for the algorithm used to solve the problem in Formulation (3.12) are:

**Step 1:** Set the budget level as \( \varepsilon \).
**Step 2:** Calculate the incremental benefit-cost ratio for each project based on the performance measure changes in the WITH and WITHOUT cases presented in Figure 3.2. The incremental benefit-cost ratio for project \( i \) based on a specific performance measure, \( boc_i \), can be calculated as:

\[
 boc_i = \frac{(p_1^i - p_0^i) \cdot f_i}{c_i} \quad \text{or} \quad boc_i = \frac{(p_1^i - p_0^i) \cdot f_i}{c_i}
\]

(3.13)

where \( p_0^i \) is the level of the performance measure of project \( i \) if it is not implemented; \( p_1^i \) = the level of the performance measure of project \( i \) if it is implemented; \( f_i \) = a usage-related variable associated with the performance measure for project \( i \) (see Table A.1); and \( c_i \) is the cost of project \( i \).

In Formulation (3.13), different types of performance measures use different formulations. If higher values of the performance measure are more desirable, \( boc_i = \frac{(p_1^i - p_0^i) \cdot f_i}{c_i} \) should be used; if lower values of the performance measure are more desirable \( boc_i = \frac{(p_1^i - p_0^i) \cdot f_i}{c_i} \) should be used;

**Step 3:** Rank all the projects based on their \( boc_i \) value. The project with the highest \( boc_i \) ranks first;
**Step 4:** Select the projects from the highest to the lowest ranking until the specified budget is used up;
**Step 5:** Based on the selected projects, calculate the actual cost and the final value of the performance objective in Formulation (3.12).

The entire process of Pareto frontier generation for trade-off analysis between the cost and the performance measures is presented in Figure 3.5.

3.3.2.2 Algorithm design for checking levels of all performance measures under a given budget. In decision making, asset managers may want to know what level of network performance could be earned with a given total budget. Even though this is not exactly a trade-off analysis, it is very important and useful for the decision analysis. Therefore, this function is incorporated in our Trade-IN Excel tool. The formulation for this problem context is:

![Figure 3.4](image.png)

Figure 3.4  Exact Pareto solutions distribution in \( \varepsilon \)-constraint method.
Objective Functions

\[ \text{Max} \sum_{i=1}^{n} x_i U_i \]  
(3.14)

Constraints

\[ \sum_{i=1}^{n} x_i c_i \leq B \]  
(3.15)

Where \( x_i = 1 \) indicates the candidate project \( i \) is selected; \( x_i = 0 \) means the candidate project \( i \) is not selected; \( U_i \) is the amalgamated value for project \( i \); \( c_i \) is the cost of project \( i \); and \( B \) is the total budget.

The basic steps for this analysis are:

Step 1: Set total budget information;
Step 2: For each project, conduct scaling and amalgamation using the techniques presented in Chapter 2 to generate amalgamated values for the project in the WITH and WITHOUT cases presented in Figure 3.2. The incremental utility-cost ratio for project \( i \) can be calculated as:

\[ uoc_i = \frac{(U_i^1 - U_i^0)}{c_i} \]  
(3.16)

where \( U_i^0 \) is the amalgamated value of project \( i \) if it is not implemented; \( U_i^1 \) is the amalgamated value of project \( i \) if it is implemented; and \( c_i \) is the cost of project \( i \).

Step 3: Rank all of the projects based on their amalgamated value \( uoc_i \). The project with the highest \( uoc_i \) is assigned the highest rank;
Step 4: Select the projects from the highest to the lowest ranking until the budget is used up;
Step 5: Based on the selected projects, calculate the level of each performance measure.

The entire process is presented in Figure 3.6.

3.3.2.3 Algorithm design for minimum budget analysis.

The basic objective of minimum budget analysis is to check the minimum budget needed to reach a certain level for each performance measure. The steps for the algorithm used to solve the problem in Formulation (3.12) are:

Step 1: Set thresholds for performance measures;
Step 2: Select a performance measure \( j \), calculate the incremental benefit-cost ratio for each project based on the performance measure changes in the WITH and WITHOUT cases presented in Figure 3.2. The incremental benefit-cost ratio for project \( i \) based on performance measure \( j \) can be calculated as:

\[ boc_{ij} = \frac{(p_i^1 - p_i^0) * f_j}{c_i} \]  
(3.17)

where \( p_i^0 \) is the level of the performance measure \( j \) of project \( i \) if it is not implemented; \( p_i^1 \) = the level of the
performance measure $j$ of project $i$ if it is implemented; $f_{ij}$ = a usage-related variable associated with the performance measure $j$ for project $i$ (see Table A.1); and $c_i$ is the cost of project $I$;
Step 3: Rank all of the projects based on their $boc_{ij}$ value. The project with the highest $boc_{ij}$ ranks first;
Step 4: Select projects from the highest to the lowest ranking until the specified performance is reached;
Step 5: Check if all the performance thresholds are reached. If "Yes," go to Step 6; otherwise, select a performance measure whose threshold has not been reached and go to Step 2;
Step 6: Based on the selected project, calculate the total cost as the minimum cost.

The entire process of minimum budget analysis is presented in Figure 3.7.

3.3.2.4 Algorithm design for budget shifting analysis.
In budget shifting analysis, asset managers want to investigate the impacts of shifting the budget allocations among program areas while keeping the total budget fixed. The basic steps for this analysis are:

Step 1: Set the total budget and provide a base scenario for budget allocation, where the total budget is distributed to the program areas under consideration;
Step 2: For each project, conduct scaling and amalgamation using the techniques developed in Chapter 2 to generate amalgamated values for the project in the WITH and WITHOUT cases presented in Figure 3.2. The incremental utility-cost ratio for project $i$ can be calculated as:

$$ uoc_i = \frac{(U^j_i - U^0_i)}{c_i} \quad (3.16) $$

Figure 3.6 Procedure for checking levels of performance measures under a given budget.

Figure 3.7 Procedure for minimum budget analysis.
Where: $U_i^0$ is the amalgamated value of project $i$ if it is not implemented; $U_i^1$ is the amalgamated value of project $i$ if it is implemented; and $c_i$ is the cost of project $i$.

**Step 3:** Select one program area, rank all the projects in this program area based on their amalgamated value $uoc_i$. The project with the highest $uoc_i$ ranks first;

**Step 4:** Select projects from the highest to the lowest ranking until the budget for this program area is used up;

**Step 5:** Check if all the program areas have been considered. If "Yes," go to Step 6; if "No," select a program that has not be considered and go the Step 3;

**Step 6:** Calculate the network-level performance measures on the basis of the performance to be earned from the selected projects;

**Step 7:** Shift budget allocations among program areas to generate a new budget allocation scenario, and then go to Step 3;

**Step 8:** Check if asset managers still want another budget shifting analysis. If "No," go to Step 9; if "Yes," generate a new budget allocation scenario and then go the Step 3;

**Step 9:** Output the performance measures for all budget allocation scenarios and conduct the analysis.

The entire process of budget shifting analysis is presented in Figure 3.8.

### 3.3.2.5 Algorithm design for trade-off analysis between performance measures

In the trade-off analysis between performance measures under a given budget, similar to the trade-off analysis between the cost and performance measures, there is a need to generate Pareto frontiers. Here we also apply the $\varepsilon$-constraint method. The

---

**Figure 3.8** Procedure for budget shifting analysis.
optimization problem in Formulation (3.10) and (3.11) can be transformed to:

**Objective Functions**

\[
\text{min} \ (\text{or } \text{max}) \ f_j(x) \quad (3.17)
\]

**Constraints**

\[
f_j(x) \leq (\geq) \epsilon
\]

\[
\sum_{i=1}^{n} x_i c_i \leq B \quad (3.18)
\]

\[
b_j^l \leq \sum_{i=1}^{n} x_i y_i c_i \leq b_j^u \quad j = 1, 2, \ldots, k
\]

Where: \( \epsilon \) is a specified value based on the range of \( f_j \), all other variables have the same meanings as in Formulation (3.10) and (3.11).

By using different values for \( \epsilon \), we can obtain a number of Pareto solutions as presented in Figure 3.4. In order to ensure that the generated Pareto solutions can be well-distributed on the Pareto frontier, in this study, a bi-direction \( \epsilon \)-constraint method is used. In this method, first, we use Formulations (3.17) and (3.18) to generate a set of Pareto solutions. Then, we shift \( f_j \) Formulations (3.17) and (3.18) (i.e., keep \( f_j \) as an objective and transform \( f_j \) (x) to a constraint). Then we generate another set of Pareto solutions. The two sets of Pareto solutions together constitute the final Pareto solution set.

The basic steps for the algorithm used to conduct trade-off analysis between two performance measures under budget constraints are:

**Step 1:** Select two performance measures that need to be traded off; set the total budget and program area budget information;

**Step 2:** Using Formulations (3.17) and (3.18), select one performance measure \( f_j \) as the objective and transform the other \( f_j \) to a constraint. Then conduct \( \epsilon \)-constraint method to generate some Pareto solutions;

**Step 3:** In Formulations (3.17) and (3.18), use performance measures \( f_j \) as the objective and transform the other \( f_j \) so that it becomes a constraint. Then conduct \( \epsilon \)-constraint method to generate a number of Pareto solutions;

**Step 4:** Combine the Pareto solutions in Steps 2 and 3 to form the final performance measures and then conduct the related trade-off analyses;

For the optimization in the \( \epsilon \)-constraint method, to generate one Pareto solution, a heuristic method similar to the Incremental Utility-Cost (IUC) Ratio algorithm in multiobjective bridge management system is used (31). For solving an optimization problem in Formulations (3.17) and (3.18), the steps are:

**Step 1:** Select two performance measures that need to be traded off; set the total budget and program area budget information;

**Step 2:** For each project, conduct scaling and amalgamation based on performance measures \( f_j \) using the techniques developed in Chapter 2 to generate amalgamated values for the project in the WITH and WITHOUT cases presented in Figure 3.2. The incremental benefit-cost ratio for project \( i \) can be calculated as:

\[
boc_i = \frac{(p_i^1 - p_i^0) * f_i}{c_i} \quad \text{or} \quad boc_i = \frac{(p_i^0 - p_i^1) * f_i}{c_i}
\]

**Step 3:** Rank all the projects based on their \( boc_i \) value. The project with the highest \( boc_i \) ranks first;

**Step 4:** In each program area, select projects from the highest to the lowest ranking until the program area budget is used up;

**Step 5:** Is the \( \epsilon \)-constraint is met? If "Yes," go to Step 6; otherwise, continue to select projects from the highest to the lowest ranking until the \( \epsilon \)-constraint is met;

**Step 6:** Is there any budget left? If "No," go to Step 9; if "Yes," for each project, conduct scaling and amalgamation based on performance measures \( f_j \) using the techniques developed in Chapter 2 to generate amalgamated values for the project in the WITH and WITHOUT cases presented in Figure 3.2. The incremental benefit-cost ratio for project \( i \) can be calculated as:

\[
boc_i = \frac{(p_i^1 - p_i^0) * f_i}{c_i} \quad \text{or} \quad boc_i = \frac{(p_i^0 - p_i^1) * f_i}{c_i}
\]

**Step 7:** Rank all the projects based on their new \( boc_i \) value. The project with the highest \( boc_i \) ranks first;

**Step 8:** Select projects from the highest to the lowest ranking until the total budget is used up;

**Step 9:** Calculate the values of the performance measures.

The entire process is presented in Figure 3.9.

### 3.4 Chapter Summary

This chapter first presented a framework for the project selection problem in transportation asset management using network-level performance measures to evaluate the performance of project selection. Next, based on the framework, mathematical formulations were developed to describe the project selection problem as a multiobjective optimization problem. In the formulation, each objective is a network-level performance measure that can be formulated as an average, percentage, or sum form. Based on the concept of Pareto solutions, it was found that, to conduct trade-off analysis between cost and performance measures or between different performance measures, there is a need to generate a Pareto frontier. Then the formulations for each type of trade-off analysis were presented. Finally, the designed algorithms for the different types of trade-off analysis were developed and presented.
4. CASE STUDY

4.1 Introduction

This chapter presents a case study to demonstrate various trade-off analyses in transportation asset management and the application of the developed Excel tool for trade-off analyses. In this case study, a mixed-asset candidate project pool, developed using 2012–2014 transportation improvement plan data from the Indiana Department of Transportation (INDOT), was used to demonstrate the proposed method for trade-off analyses. The basic project information is presented in Table 4.1. For each type of project, the performance measures shown in Table 4.2 were adopted to evaluate the consequences of the project implementation.

It is worth mentioning that the performance measures presented here are for illustration purposes only, and asset managers can choose other performance measures according to the practicality of the situation context, exigencies, or agency policy. For the overall impact of project implementation, five network-level performance measures were used to evaluate the final decision: average IRI, average BCR, average RSL of safety assets, average travel speed, and average crash rate.

Correspondingly, there are five objectives in this problem:

1. Minimize average IRI
2. Maximize average BCR
3. Maximize average RSL
4. Maximize average travel speed
5. Minimize average crash rate

Based on the above information, the objective functions of the optimization problem were formulated as follows:

\[
\begin{align*}
\min & f_1 - \text{average IRI}(x) \\
\max & f_2 - \text{average BCR}(x) \\
\max & f_3 - \text{average RSL}(x) \\
\min & f_4 - \text{average speed}(x) \\
\max & f_5 - \text{average CR}(x)
\end{align*}
\]

Where: \( x \) is a decision variable vector \((x_1, x_2, \ldots, x_n)\), each \( x_i \) can be 0 or 1 (1 means that project \( i \) is selected and 0 otherwise), and \( n \) is the number of candidate projects;

<table>
<thead>
<tr>
<th>Project Type</th>
<th>Number of Projects</th>
<th>Total Cost (SM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pavement Projects</td>
<td>190</td>
<td>681.02</td>
</tr>
<tr>
<td>Bridge Projects</td>
<td>400</td>
<td>701.60</td>
</tr>
<tr>
<td>Safety Projects</td>
<td>107</td>
<td>59.83</td>
</tr>
<tr>
<td>Mobility Projects</td>
<td>102</td>
<td>894.01</td>
</tr>
<tr>
<td>Total</td>
<td>799</td>
<td>2336.46</td>
</tr>
</tbody>
</table>

TABLE 4.1 Basic Project Information for the Case Study
\( f_i(x) \) is the \( i \)th network-level performance measure (objective).

### 4.2 Trade-off Analysis between Alternatives

#### 4.2.1 Trade-off Analysis between Alternative Individual Projects

Trade-off between two projects is a very common type of trade-off analysis in practice. The two projects could be the same types of projects (e.g., both are pavement projects) or they could be different types of projects (e.g., one is a pavement project and the other is a bridge project). For this type of trade-off analysis, the steps presented in Section 2.4.1 should be used. For example, to conduct trade-off analysis between a Pavement Project A and a bridge project B, after scaling and amalgamation, the overall importance for Pavement Project A and Bridge Project B are 0.12 and 0.1, respectively (Figure 4.1). Therefore, Pavement Project A should be superior to Bridge Project B.

#### 4.2.2 Trade-off Analysis between Two Alternative Projects Portfolios

This is similar to the trade-off type discussed above but involves multiple projects, not just one in each evaluation group. Again, the two groups of projects may or may not be from the same program area. For this type of trade-off analysis, the steps presented in Section 2.4.2 should be used. For Example, both Portfolio A and Portfolio B contain 64 projects from different program areas. After scaling, amalgamation, and summation, the total value for Portfolio A and Portfolio B are 19.61 and 19.36, respectively, as shown in Figure 4.2. Thus, project Portfolio B is superior.

### 4.3 Trade-off Analyses Involving Performance Measures

#### 4.3.1 Trade-off Analysis between Cost and Performance Measures

In analyzing investment alternatives, asset managers often seek to know the quantitative relationship between the cost and each performance measure, such as what level of funding is needed to achieve a certain level of average pavement surface roughness (IRI) or average crash rate. Figure 4.3 presents the trade-off between the total cost and individual performance measures.

It can be seen that with increasing spending (cost), there are decreases in the average pavement surface roughness (IRI) and the average crash rate, and increases in the average bridge condition, the average RSL, and the average travel speed. These results are consistent with expectations. In Figure 4.3 (a), the absolute value of the slope of the trade-off curve decreases when the cost increases; that is, as the spending increases, the pavement surface roughness (IRI) decreases rapidly at the beginning and then decreases at a slower rate at higher spending levels. In other words, when the budget is small, a unit increase in budget can provide a greater decrease in surface roughness than at higher spending levels.

<table>
<thead>
<tr>
<th>Project Type</th>
<th>Preservation</th>
<th>Safety</th>
<th>Mobility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pavement Projects</td>
<td>International Roughness Index (IRI)</td>
<td>Crash Rate</td>
<td>Average Travel Speed</td>
</tr>
<tr>
<td>Bridge Projects</td>
<td>Bridge Condition Index (BCR)</td>
<td>Crash Rate</td>
<td>Average Travel Speed</td>
</tr>
<tr>
<td>Safety Projects</td>
<td>Remaining Service Life (RSL)</td>
<td>Crash Rate</td>
<td>Average Travel Speed*</td>
</tr>
<tr>
<td>Mobility Projects</td>
<td>RSL*/IRI*</td>
<td>Crash Rate</td>
<td>Average Travel Speed</td>
</tr>
</tbody>
</table>

*This performance measure is only used for some projects because other projects may not have an effect on this performance measure.
roughness (IRI) than when the budget is large, which is consistent with the intuition that the marginal rate of substitution (between performance and budget) decreases as the spending level increases.

The trade-off analysis between cost and performance at the network level can help asset managers analyze the relationships between the total cost and the network performance and therefore can help an agency establish reasonable goals for investment. For instance, in Figure 4.3 (a), if the network-level goal is to reduce the average IRI to a certain level (80 inches/mile), then asset managers can ascertain the minimum budget level needed ($245.81M) to achieve this goal, and then seek adequate budget from funding resources to achieve the network level performance goal. Alternatively, with a certain amount of available budget (e.g., $400M), asset managers can determine the level of performance that can be achieved, that is 75.84 inches/mile for pavement roughness in the case study.

On the basis of the generated Pareto frontier, in some cases, regression can be applied to yield a trade-off function for the Pareto frontier. For instance, by adding a trend line in Figure 4.3 (b), the relationship between cost (y) and average BCR(x) can be expressed as a function in Equation (4.2).
On the basis of Equation (4.2), asset managers can specify a performance level on the average BCR and then ascertain the approximate minimum budget needed to achieve that level of performance. Further, based on the inverse of Equation (4.2), asset managers can investigate the approximate level of performance on the average BCR that could be attained under a given budget. It may be noticed that a large number of decimal digits were kept in each coefficient in Equation (4.2) because Equation (4.2) is a 6-degree polynomial function, where a very small variation in the coefficient could have a huge effect on the dependent variable y. Therefore, a large number of decimal digits were kept in each coefficient to ensure the accuracy of the estimation using Equation (4.2). It is worth mentioning that the relationships between the cost and performance measures depend on the projects in the candidate project pool and therefore are not necessarily transferable from one project selection problem to another or from one programming period to another. However, the proposed framework and Pareto frontier generation method can be applied to other problems to generate relationships between the cost and performance measures for those problem settings.

4.3.2 Budget Shifting Analysis

One of the most important types of trade-off analysis is “budget shifting” analysis. An asset manager’s candidate projects are typically generated and “housed” or sponsored in select program areas, such as a pavement program, a bridge program, etc. The competition can be severe between different program areas for the limited funding. Changes in agency policy and mission or the desire to address public concerns in a particular program area, sudden disaster, and other circumstances can lead to shifts of substantial funds from one program area to another. To address trade-off problems of this nature, the procedure illustrated in the flowchart (Figure 4.4) can be used.

Table 4.3 presents an example of budget shifting among the four program areas with a total budget of $800M. Figure 4.5 presents the results of the five performance measures in each budget case.

4.3.3 Trade-off between Performance Measures under Budget Constraint

In practice, a budget constraint usually exists for the project selection problem in transportation asset management; thus, asset managers seek to arrive at a compromise between different objectives. Figure 4.6 presents the trade-off analysis between the five performance measures under a given budget level of $600M different budget levels.

From Figure 4.6, it is easy to see the relationships between performance measures under the budget constraints. For instance, in Figure 4.6 (a), when the average BCR increases, the average IRI also increases because, for a fixed total budget, if the asset managers seek an increase in the BCR, then more money needs to be spent on bridge projects and thus less money will be spent on pavement projects. As a result, the average IRI for pavements increases. It is also noticed that the slope in Figure 4.6 (a) is very steep at the beginning, then decreases, and finally becomes very small. This result indicates that, when the average BCR is small, giving up in one unit of decrease in average IRI will yield a drastic increase in the average BCR. However, after the average BCR reaches a certain level, giving up one unit of decrease in the average IRI will yield only a minimal increase in the average BCR. This knowledge will assist asset managers in determining the optimal choice to balance the two network objectives.

The relationship between performance measures can be expressed approximately using smoothing functions.
For instance, in Figure 4.6 (a), the relationship between the average bridge condition rating ($y$) and the average IRI ($x$) can be expressed by the following function using the software *Eureqa* (32):

$$y = 7.908 - 0.2269/(x - 6.935)$$

The value for the $R^2$ in Equation (4.3) is 0.994.

The function in Equation (4.3) can be used as an approximation to the Pareto frontier shown in Figure 4.6 (a). In practice, various functions, such as exponential or linear functions can be investigated and then the best-fit functional form can be selected to describe the Pareto frontier. The $dy/dx$ of the function at a certain point is the slope of the curve at that point, which also provides an indication of the marginal rate of substitution at that point. Thus, the marginal rate of substitution function can be derived from Equation (4.3) and is presented as follows:

$$y' = 0.2269/(x - 6.935)^2$$

For Equation (4.4), it can be seen that the marginal rate of substitution value depends on the value of $x$ (average IRI). The value of the marginal rate of substitution means the amount of increase in the average BCR must be given up in order to obtain a unit of decrease in average IRI. Similarly, a mathematical function can be applied to describe the relationship between each pair of performance measures. It is worth mentioning that the relationship between each pair of performance measures in Figure 4.6 is based on the data in the case study of this report from the perspective of the effects of the investment at the network level. It is an estimation used in the decision-making process at the planning level and at the programming stage.

In the Pareto frontier generation process, the relationships between the same pair of performance measures under different budget levels are different. An example is shown in Figure 4.7, which presents the Pareto frontiers between the average IRI and the average BCR at the budget level $500M, $600M, and $800M. It can be seen that even though the three Pareto frontiers have similar trends, they are not parallel. Also, the slopes at the same level of average speed are different. Thus, the trade-off relationships are also different. Therefore, when asset managers conduct trade-off analyses, there is a need to generate a trade-off curve for each different budget level.

### 4.4 Chapter Summary

This chapter presented a case study to demonstrate various trade-off analyses in transportation asset management using the developed Excel tool (*Trade-IN*). From the results, it was found that, for the trade-off

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**TABLE 4.3**

<table>
<thead>
<tr>
<th>Bridging Budget Analysis</th>
<th>Safety Program Area Budget (SM)</th>
<th>Pavement Program Area Budget (SM)</th>
<th>Mobility Program Area Budget (SM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>360</td>
<td>20</td>
<td>220</td>
</tr>
<tr>
<td>Case 2</td>
<td>150</td>
<td>40</td>
<td>360</td>
</tr>
<tr>
<td>Case 3</td>
<td>300</td>
<td>50</td>
<td>300</td>
</tr>
<tr>
<td>Case 4</td>
<td>250</td>
<td>30</td>
<td>270</td>
</tr>
<tr>
<td><strong>Total Budget (SM)</strong></td>
<td><strong>800</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.6 Trade-off analyses between performance measures.
analysis between the cost and performance measures, typically, high performance requires high costs, which is consistent with intuition. At high performance levels, the marginal cost is higher compared to that at low performance levels. It was also found that, for the trade-off analysis between performance measures, at different budget levels, the trade-off relationships between performance measures are different.

REFERENCES

Figure 4.7 Trade-off analyses between performance measures under different budget levels.


APPENDIX. FORMULATION OF EXPRESSIONS FOR NETWORK-LEVEL PERFORMANCE

AVERAGE FORM

In certain cases, transportation agencies express the overall performance of their systems as an average, such as the average crash rate in the highway network, in terms of the number of crashes per 100 million vehicle miles traveled (VMT). The general formula for calculating the average form of the network-level performance measures is:

$$P_{\text{Avg}} = \frac{\sum_{i=1}^{n} [(1-x_i)p_i^0 e_i + x_i p_i^1]}{\sum_{i=1}^{n} [(f_i e_i + (1-e_i)x_i) f_i]}$$

(A.1)

Where:

- $n$ = number of projects in the candidate project pool;
- $e_i$ = 1 (if project $i$ is recommended for an existing asset) or 0 (if project $i$ is a new construction project that yields a new asset);
- $p_i^0$ = the level of the performance measure of project $i$ if it is not implemented, for new construction project, the level can be set to 0; for existing assets, $p_i^0$ represents the total VMT after all recommended projects have been implemented. Thus, the total number of crashes divided by the total VMT gives the average crash rate in the network. For the calculation of the average BCR, $f_i = 1$. Thus, the average BCR equals the sum of the ratings of all bridges divided by the total number of bridges.
- $p_i^1$ = the level of the performance measure of project $i$ if it is implemented;
- $f_i$ = a usage-related variable associated with the performance measure for project $i$ (see Table A.1); and
- $x_i$ = 1 (project $i$ is selected) or 0 (otherwise).

For some commonly used performance measures, the meanings of the parameters are presented in Table A.1.

For example, in the calculation of average crash rate, $\sum_{i=1}^{n} [(1-x_i)p_i^0 e_i + x_i p_i^1]$ represents the number of crashes at the assets that are in the candidate project pool in the network. The denominator, $\sum_{i=1}^{n} [(f_i e_i + (1-e_i)x_i) f_i]$, represents the total VMT after all recommended projects have been implemented. Thus, the total number of crashes divided by the total VMT gives the average crash rate in the network. For the calculation of the average BCR, $f_i = 1$. Thus, the average BCR equals the sum of the ratings of all bridges divided by the total number of bridges.

PERCENTAGE FORM

In practice, certain agencies are more interested in the percentage of assets in their jurisdiction that satisfy some specified thresholds. For instance, the Colorado DOT, at a certain time, sought to optimally choose projects such that the percentage of structurally deficient bridges would not exceed 25%. A general formula for calculating the percentage form of network-level performance is:

$$P_{\%} = \frac{\sum_{i=1}^{n} [(1-x_i)p_i^0 e_i + x_i p_i^1]}{\sum_{i=1}^{n} [(1-e_i)x_i + e_i]}$$

(A.2)

Where:

- $y_i^p = 1$ if the performance measure of asset $i$ achieves a certain level if project $i$ is not implemented, 0 otherwise; for new construction project, $y_i^p$ can be set to 0;
- $y_i^e = 1$ if the performance measure of asset $i$ achieves a certain pre-specified level if project $i$ is implemented; 0 otherwise.

For example, consider a policy where it is specified that the percentage of bridges with BCR > 4 must exceed some threshold. In calculating the percentage of bridges that satisfy this requirement, $\sum_{i=1}^{n} [(1-x_i)p_i^0 e_i + x_i p_i^1]$ is the number of bridges in the candidate project pool that will have condition ratings greater than 4 after the project implementation (note that all newly constructed bridges will be expected to have condition ratings greater than 4). The denominator $\sum_{i=1}^{n} [(1-e_i)x_i + e_i]$ represents the total number of bridges in the network after implementing the projects.

SUM FORM

There are certain performance measures that are best expressed as a sum and not as an average or a percentage. Examples include calculating the percentage of bridges that satisfy a certain requirement, the number of bridges that exceed a certain threshold. The general formula for the additive form of network-level performance measures is:

$$P_{\text{sum}} = \sum_{i=1}^{n} [(1-x_i)p_i^0 e_i + x_i p_i^1]$$

(A.3)

where the variables have the same meanings as those in Equation (A.1).

<table>
<thead>
<tr>
<th>TABLE A.1</th>
<th>Explanations of Parameters for Average Form of Example Performance Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Network-Level Performance Measures</strong></td>
<td>$p_i$</td>
</tr>
<tr>
<td>Average IRI</td>
<td>IRI of road segment $i$</td>
</tr>
<tr>
<td>Average Crash Rate</td>
<td>Crash rate at road segment $i$</td>
</tr>
<tr>
<td>Average Travel Speed</td>
<td>Average travel speed at road segment $i$</td>
</tr>
<tr>
<td>Average BCR</td>
<td>BCR of bridge $i$</td>
</tr>
</tbody>
</table>