2010

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A Simplified Analysis of Lubrication of a Wristpin

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ABSTRACT

Long service life and reliability of reciprocating compressors depends, beside the other factors, on an acceptable wear of bearings and sliding surfaces. In order to improve COP (Coefficient of Performance), companies frequently use oil that has lower viscosity than the one that was considered in original design. While lubrication and wear of main bearings and pistons is usually designed properly and withstands use of oil with lower viscosity; good lubrication and the wear of wristpins can worsen. The load carrying capacity of the wristpin bearing is mainly due to squeeze-film action of oil film. A simplified model of relative motion of the wristpin inside the sleeve that is expressed in the terms of relative eccentricity is investigated.

1. INTRODUCTION

In the mass production of reciprocating compressors for refrigeration and air-conditioning, all parts of a compressor are machined within some tolerances. The roughness and texture of machined surfaces is also prescribed. Keeping tolerances too narrow increases the cost of production of such parts, and consequently, the cost of compressors. On the other hand, tolerances that are too loose decrease reliability and increase wear, and eventually, the noise that is generated by a running compressor. In one instance, in order to increase coefficient of performance, a compressor manufacturer used oil with lower viscosity, which resulted in sporadic random field returns that had unacceptable high wear of wristpin and its sleeve (the inside surface of small end of connecting rod). In order to understand the root cause of this phenomenon, a simplified model of lubrication of the wristpin was developed.

2. SLIDING VELOCITY AND LOAD ACTING ON A WRISTPIN

In a typical small reciprocating compressor, the sliding velocity between cylindrical surfaces of main bearings is high enough to warrant full hydrodynamic lubrication. On the other hand the velocity of sliding between the surface of the wristpin and its sleeve is oscillatory, and its maximum value is much smaller than the sliding velocity of surfaces in the main bearing. Fig. 1, and Eq. (1) visualize the situation.

$$F_G + F_A = F$$

Fig. 1: Slider-crank mechanism
The angle \( \psi = f(\varphi) \) between the centerline of connecting rod and the axis of cylinder can be found from

\[
LC \cdot \sin \psi = R \cdot \sin \varphi
\]  

(1)

By taking time derivative of Eq.(1) and rearranging it, we get the oscillating angular velocity of connecting rod \( \dot{\psi} \)

\[
\dot{\psi} = \frac{R \cdot \cos \varphi}{LC \cdot \cos \psi} = \frac{R \cdot \cos \varphi}{\sqrt{1 - \left(\frac{R}{LC} \cdot \sin \varphi\right)^2}}
\]  

(2)

The sliding velocity between the surface of the wristpin and the surface of its sleeve is

\[
u_a = ra \cdot \dot{\psi}
\]  

(3)

Where
- \( u_a \) is relative sliding velocity between the surfaces of the wristpin and its sleeve [m/s]
- \( R \) is radius of the crank (one half of the stroke) [m]
- \( L_C \) is length of connecting rod [m]
- \( ra \) is radius of the sleeve [m]
- \( \varphi \) is angle of rotation of crankshaft [Rad]
- \( \omega = \frac{d\varphi}{dt} = \dot{\varphi} \) is constant angular velocity of crankshaft [1/s]

The maximum swing angle of connecting rod is \( \psi = \pm asin(R/L_C) \). A typical small compressor has the maximum magnitude of swing angle of connecting rod about \( \pm 10 \) degrees, and the maximum angular sliding velocity \( u_a \) is about \( \pm 0.6 \) m/s. A small swing angle and low oscillatory sliding velocity do not create any significant oil wedge that can carry the load. Thus, the oil-wedge effect is small, and therefore, it can be neglected.

The forces acting on the piston and the wristpin reach maximum just before top dead center, during compression, and its minimum before the bottom dead center, during suction. At positions between both dead centers, where the sliding velocity is close or nearly equal to zero, the forces acting on the piston and the wrispin are not negligible. Because of the absence of relative sliding velocity between the wristpin and its sleeve (small end of connecting rod), the oil wedge does not develop, and therefore, only squeeze-film action has to carry that force. Fig. 2 shows simulated force acting on the piston. This force is the sum of the gas-force and inertia force of the piston.

The gas-force acting on the wristpin was simulated by Bukac (2002) in his model of valve dynamics, Eq.(3).

\[
F_G = p(\varphi) \cdot A
\]  

(4)

Where
- \( F_G \) is force of gas acting on the piston [N]
- \( p(\varphi) \) is pressure [Pa]

The inertia force is simply acceleration of the piston multiplied by the mass of piston and the wristpin.

\[
F_A = m_P \cdot \ddot{x}
\]  

(5)

Where
- \( F_A \) is force of inertia of the piston [N]
- \( m_P \) is mass if the piston and then wristpin [kg]
- \( \ddot{x} \) is acceleration of the piston (point B in Fig. 1) [m.s\(^{-2}\)]
The acceleration of the piston $\ddot{x}$ is found by integrating twice position of the wristpin, $x$, which is given by

$$x = R \cdot \cos \varphi + L_C \cdot \cos \psi $$

(6)

$$\dot{x} = - \left( R \cdot \phi \cdot \sin \varphi + L_C \cdot \psi \cdot \sin \psi \right) $$

(7)

$$\ddot{x} = - \omega^2 \cdot R \cdot \cos \varphi - L_C \cdot \left( \ddot{\psi} \cdot \sin \psi + \dot{\psi} \cdot \cos \psi \right) $$

(8)

Where $\ddot{\psi}$ is

$$\ddot{\psi} = \dot{\psi}^2 \cdot \frac{R}{L_C} \cdot \frac{\sin \varphi}{\sqrt{1 - \left( \frac{R \cdot \sin \varphi}{L_C} \right)^2 \left( \frac{\cos \varphi}{\sqrt{1 - \left( \frac{R}{L_C} \cdot \sin \varphi \right)^2}} - 1 \right)}} $$

(9)

The total force $F$ acting on the piston in the $x$-direction is modeled in the Fig. 2, and it is

$$F = F_G + F_A $$

(10)

This force is then in the equilibrium with force $F_C$ in the connecting rod and with the force $F_N$ in $y$-direction that is perpendicular to the direction of motion of the piston. Since forces $F_G$ and $F_A$ are known, the force $F_N$ is

$$F_N = F \cdot \tan \psi $$

(11)

The forces of dry and viscous friction are small in comparison to force $F$, and therefore they are neglected.

Fig.2: Stepwise approximation of resulting force acting on the piston
3. THE REYNOLDS EQUATION OF HYDRODYNAMIC LUBRICATION

The equation of hydrodynamic lubrication can be derived from the Navier-Stokes equations and the equation of continuity (equation of conservation of mass). Such a derivation can be found in any textbook on hydrodynamic lubrication. For example, Hamrock (1994), p.153, presents full equation of hydrodynamic lubrication. This equation is commonly known as the general Reynolds equation. It describes motion of two parallel plates that carry a force that is perpendicular to the motion of plates. If there is no motion in the y-direction, and if the flow of lubricant in y-direction (the side leakage) is neglected, The Reynolds equation is

$$\frac{\partial}{\partial x} \left( \frac{\rho \cdot h^3}{12 \cdot \eta} \cdot \frac{\partial p}{\partial x} \right) = \frac{\partial}{\partial x} \left[ \frac{\rho \cdot h \cdot (u_a + u_b)}{2} \right] + \rho \left( v_a - v_b - u_a \cdot \frac{\partial h}{\partial x} \right) + h \cdot \frac{\partial p}{\partial t}$$

(12)

Where

- \(x\) is length in the direction of motion [m]
- \(\rho\) is density of lubricant [kg/m\(^3\)=N.s\(^2\)/m\(^4\)]
- \(h\) is distance between plates [m]
- \(p\) is pressure [Pa = N/m\(^2\)]
- \(\eta\) is absolute viscosity [Pa.s = N.s/m\(^2\)]
- \(u_a, u_b\) is velocity of each plate in the x-direction [m/s]
- \(v_a, v_b\) is velocity with which plates approach [m/s]
- \(t\) is time [s]

The dimension of each term in Eq. (12) is N.s/m\(^3\), which is the units of pressure divided by units of velocity (N/m\(^2\))/(m/s).

The Reynolds equation is in the matter of fact a summation of several terms that describe different physical effects. On the left side of Eq. (12) there is the Poiseuille term. On the right side there are Couette term, squeeze term and local expansion term. If we expand the Couette’s term by performing the derivative of the expression inside the brackets of Eq. (12), we get

$$\frac{\partial}{\partial x} \left[ \frac{\rho \cdot h \cdot (u_a + u_b)}{2} \right] = \frac{h (u_a + u_b)}{2} \cdot \frac{\partial \rho}{\partial x} + \frac{\rho \cdot h}{2} \cdot \frac{\partial}{\partial x} (u_a + u_b) + \frac{\rho (u_a + u_b)}{2} \cdot \frac{\partial h}{\partial x}$$

(13)

The expressions on the right side of Eq. (13) are density wedge term, stretch term and physical wedge term.

All terms that contain derivative of density, \(\partial \rho / \partial x\), disappear if the density of lubricant is constant. The terms that contain velocity of plates in x-direction disappear too, if there is \(u_a = u_b = 0\), which is the case of a wristpin. Therefore, Eq. (12) will reduce to the Poiseuille term and the reduced squeeze term (\(u_a = 0\)).

$$\frac{\partial}{\partial x} \left( \frac{\rho \cdot h^3}{12 \cdot \eta} \cdot \frac{\partial p}{\partial x} \right) = \rho \cdot (v_a - v_b)$$

(14)

### 3.1 Pressure Distribution in Squeeze Film Bearing

The main load carrying capacity of the wristpin bearing is due to squeeze film action. On the assumption of constant oil density \(\rho\), constant viscosity \(\eta\), and no side leakage (infinitely long bearing), we can rearrange Eq. (14) to fit wristpin geometry.

Fig.3 depicts geometry of a wristpin bearing. Under the action of force F the journal and the sleeve approach each other. This forces the oil to flow from the underneath of the journal to its top, as it is indicated by two symmetrical arrows. This provides desired cushioning effect. The rectilinear coordinate \(x\) in the Eq. (14) is in the direction of the flow of oil. But the flow of oil in the wristpin bearing follows a circular path. Therefore we have to replace \(x\) in the Eq. (14) by \(x = r \cdot \varphi\) and the velocity of approach by \(v = -\varphi h / \partial t\). While the Eq. (14) assumes the distance between
two parallel plates h to be independent from x, the distance h between the two approaching cylindrical surfaces in Fig.3 varies with the angle \( \phi \). The normal distance h varies according to

\[
h = c(1 - e \cdot \cos \varphi)
\]  

(15)

Where

- \( h \) is thickness of oil film [m]
- \( c = r_a - r_b \) is bearing clearance [m]
- \( r_a \) is radius of sleeve [m]
- \( r_b \) is radius of journal [m]
- \( e = v/c \) is relative eccentricity [
- \( e \) is eccentricity [m]

Regardless of the magnitude of relative eccentricity \( 0 \leq e \leq 1 \), it is evident from Eq. (15) that the minimum thickness of oil film, \( h_{\text{min}} \), occurs at \( \varphi = 0 \), and that it reaches maximum at \( \varphi = \pi \).

If we substitute all the above into Eq. (14) we get

\[
\frac{d}{d\varphi}(h^3 \cdot \frac{dp}{d\varphi}) = -12 \cdot \eta \cdot r_a^2 \cdot v \cdot \cos \varphi
\]  

(16)

Where

- \( v \) is velocity with which both surfaces approach [m/s]
Integrating with respect to $\varphi$ gives

$$\frac{dp}{d\varphi} = -12 \cdot \frac{\eta \cdot v^2 \cdot v \cdot \sin \varphi}{h^3} + \frac{A}{h^3}$$  \hspace{1cm} (17)$$

Because of the symmetry with respect to vertical axis, there will be $dp/d\varphi = 0$ when $\varphi = 0$, therefore constant of integration has to be $A = 0$. If we substitute for $h$ from Eq. (15) into Eq. (17), we get

$$\frac{dp}{d\varphi} = -12 \cdot \frac{\eta \cdot r_s^2 \cdot v \cdot \sin \varphi}{c^3 \cdot (1 - \varepsilon \cdot \cos \varphi)^3}$$  \hspace{1cm} (18)$$

The integration of Eq. (18) yields

$$p = \frac{6 \cdot \eta \cdot r_s^2 \cdot v}{\varepsilon \cdot c^3 \cdot (1 - \varepsilon \cos \varphi)^2} + B$$  \hspace{1cm} (19)$$

The pressure $p$ has to have some finite value even when $\varepsilon \to 0$. This condition will be satisfied when constant of integration $B$ has the value

$$B = \frac{6 \cdot \eta \cdot r_s^2 \cdot v}{\varepsilon \cdot c^3}$$  \hspace{1cm} (20)$$

When we substitute for $B$ from Eq. (20) into Eq. (19) and rearrange, we get final expression for distribution of pressure $p$ in the bearing clearance

$$p = \frac{6 \cdot \eta \cdot r_s^2 \cdot v}{c^3} \cdot \frac{(2 - \varepsilon \cdot \cos \varphi) \cdot \cos \varphi}{(1 - \varepsilon \cdot \cos \varphi)^2}$$  \hspace{1cm} (21)$$

### 3.2 The Normal Load-Carrying Capacity

We can see in the Fig.3 that the pressure distribution is symmetrical about vertical axis. Therefore, we can find normal load–carrying capacity of a squeeze film bearing as twice the product of one half of bearing’s area and the pressure $p$ ($\varphi = 0 \to \pi$).

$$F = 2 \cdot b \cdot r_s \cdot \int_0^\pi p \cdot \cos \varphi \cdot d\varphi$$  \hspace{1cm} (22)$$

Where

- $F$ is constant load-carrying force [N]
- $b$ is the width of bearing [m]

After we substitute for pressure $p$ from Eq. (21) into Eq. (22), we get normal load-carrying force as

$$F = \frac{12 \cdot \eta \cdot b \cdot r_s^3}{c^3} \cdot \frac{v}{\pi} \cdot \frac{(2 - \varepsilon \cdot \cos \varphi) \cdot \cos^2 \varphi \cdot \cos \varphi}{(1 - \varepsilon \cdot \cos \varphi)^2} \cdot d\varphi$$  \hspace{1cm} (23)$$

If we carry out the integral in Eq. (23) and omit expressions with higher orders, we can reduce the result of integration to
3.3 Motion of the Sleeve Relative to Journal

Fig. 3 shows that the minimum thickness of oil film, \( h_{\text{min}} \), occurs at the angle \( \varphi = 0 \). For this reason Eq. (15) can be written as

\[
h_{\text{min}} = c \cdot (1 - e)
\]  

(25)

Since we are primarily interested in how the minimum thickness of oil film changes with the load the bearing needs to carry, we can take time derivative of Eq. (25). The derivative of Eq. (25) is

\[
\frac{dh_{\text{min}}}{dt} = -c \frac{d\varepsilon}{dt}
\]  

(26)

If we assume the compressor spins with constant angular velocity \( \Omega \), we can write

\[
d\theta = \omega \cdot dt \quad \Rightarrow \quad dt = \frac{d\theta}{\omega}
\]  

(27)

Where

- \( \theta \) is angle of rotation of compressor’s crankshaft [rad]
- \( \omega \) is constant angular velocity of rotation of crankshaft [1/s]

But the time change of minimum thickness of oil film \( dh_{\text{min}}/dt \) in Eq. (26) is velocity of approach, \( v \). Therefore the substitution of Eq. (27) into Eq. (26) gives

\[
v = -\frac{dh_{\text{min}}}{d\theta} \cdot \omega = c \cdot \frac{d\varepsilon}{d\theta} \cdot \omega
\]  

(28)

If we substitute \( v \) from Eq. (28) into Eq. (24) and rearrange, we get a differential equation of first order that can be solved by separation of variables.

\[
\frac{d\varepsilon}{\left(1 - e^2\right)^{3/2}} = \frac{c^2}{12 \pi \eta \cdot \omega \cdot b \cdot r_e^3} F(\theta) \cdot d\theta
\]  

(29)

Where

- \( F(\theta) \) is force expressed as a function of the position of crankshaft as it may be seen in Fig. 3 [N]

The Eq. (29) could be solved by separation of variables if it was possible to find integral \( \int_0^{2\pi} F(\theta) \cdot d\theta \).

Unfortunately, force \( F(\theta) \) is rather complicated function of angle \( \theta \), and therefore, it is not easy to find its integral \( \int_0^{2\pi} F(\theta) \cdot d\theta \). Thus, the numerical integration is the easiest way to solve Eq. (29).

Fig. 4 shows result of numerical integration of Eq. (29) for original viscosity of oil for, which the wristpin bearing was designed, and for the viscosity that is one half of the original one. Each curve in Fig. 4 is actually a trajectory of the geometrical center of sleeve with respect to fixed geometrical center of the journal that is expressed in terms of relative eccentricity \( e \).
The trajectory is modeled as a superposition of two simultaneous perpendicular motions. One motion is in the x-direction and the second one is in y-direction. Thus the Eq. (29) is solved twice in each step of angle $\theta$. First time for the force $F(\theta) = F_c(\theta) + F_a(\theta)$, and then for the force $F(\theta) = F_n(\theta)$.

4. CONCLUSION

The result of simulation shows that the wristpin can still safely operate with the oil that has one half of original viscosity for which the bearing was designed. Nevertheless, the minimum thickness of the oil film can be too small to provide enough space for microscopic debris to pass through the bearing. Microscopic particles that are product of electric arc welding are common in hermetic compressors, and it might be the reason for occasional scratches and excessive wear of the wristpin.

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