Higher Moments of Net Proton Multiplicity Distributions at RHIC

We report the first measurements of the kurtosis ($\kappa$), skewness ($S$), and variance ($\sigma^2$) of net-proton multiplicity ($N_p - N_\bar{p}$) distributions at midrapidity for Au + Au collisions at $\sqrt{s_{NN}} = 19.6, 62.4$, and
One of the major goals of the heavy-ion collision program is to explore the QCD phase diagram [1]. Finite temperature lattice QCD calculations [2] at baryon chemical potential $\mu_B = 0$ suggest a crossover above a critical temperature ($T_c \approx 170–190$ MeV [3]) from a system with hadronic degrees of freedom to a system where the relevant degrees of freedom are quarks and gluons. Several QCD-based calculations (see, e.g., [4]) find the quark-hadron phase transition to be first order at large $\mu_B$. The point in the QCD phase plane ($T$ vs $\mu_B$) where the first order phase transition ends is the QCD critical point (CP) [5,6]. Attempts are being made to locate the CP both experimentally and theoretically [7]. Current theoretical calculations are highly uncertain about the location of the CP. Lattice QCD calculations at finite $\mu_B$ face numerical challenges in computing. The experimental plan is to vary the center of mass energy ($\sqrt{s_{NN}}$) of heavy-ion collisions to scan the phase plane [8] and, at each energy, search for signatures of the CP that could survive the time evolution of the system [9].

In a static, infinite medium, the correlation length ($\xi$) diverges at the CP. $\xi$ is related to various moments of the distributions of conserved quantities such as net baryons, net charge, and net strangeness [10]. Typically variances ($\sigma^2 = \langle (\Delta N)^2 \rangle$; $\Delta N = N - M$; $M$ is the mean) of these distributions are related to $\xi$ as $\sigma^2 \sim \xi^2$ [11]. Finite size and time effects in heavy-ion collisions put constraints on the values of $\xi$. A theoretical calculation suggests $\xi = 2–3$ fm for heavy-ion collisions [12]. It was recently shown that higher moments of distributions of conserved quantities, measuring deviations from a Gaussian, have a sensitivity to CP fluctuations that is better than that of $\sigma^2$, due to a stronger dependence on $\xi$ [13]. The numerators in skewness ($S = \langle (\Delta N)^3 \rangle / \langle (\Delta N)^2 \rangle$) go as $\xi^{4.5}$ and kurtosis ($\kappa = \langle (\Delta N)^4 \rangle / \langle (\Delta N)^2 \rangle^2 - 3$) go as $\xi^7$. A crossing of the phase boundary can manifest itself by a change of sign of $S$ as a function of energy density [13,14].

Lattice calculations and QCD-based models show that moments of net-baryon distributions are related to baryon number ($\Delta N_B$) susceptibilities ($\chi_B = \langle (\Delta N_B)^2 \rangle / V$; $V$ is the volume) [15]. The product $\kappa \sigma^2$, related to the ratio of fourth order ($\chi_B^{(4)}$) to second order ($\chi_B^{(2)}$) susceptibilities, shows a large deviation from unity near the CP [15]. Experimentally measuring event-by-event net-baryon numbers is difficult. However, the net-proton multiplicity ($N_p - \bar{N}_p = \Delta N_p$) distribution is measurable. Theoretical calculations have shown that $\Delta N_p$ fluctuations reflect the singularity of the charge and baryon number susceptibility as expected at the CP [16]. Non-CP model calculations (discussed later in the Letter) show that the inclusion of other baryons does not add to the sensitivity of the observable. This Letter reports the first measurement of higher moments of the $\Delta N_p$ distributions from $\text{Au} + \text{Au}$ collisions to search for signatures of the CP.

The data presented in the Letter are obtained using the time projection chamber (TPC) of the Solenoidal Tracker at RHIC (STAR) [17]. The event-by-event proton ($N_p$) and antiproton ($\bar{N}_p$) multiplicities are measured for $\text{Au} + \text{Au}$ minimum bias events at $\sqrt{s_{NN}} = 19.6, 62.4,$ and 200 GeV for collisions occurring within 30 cm of the TPC center along the beam line. The numbers of events analyzed are $4 \times 10^4, 5 \times 10^6$, and $8 \times 10^6$ for $\sqrt{s_{NN}} = 19.6, 62.4,$ and 200 GeV, respectively. Centrality selection utilized the uncorrected charged particle multiplicity within pseudorapidity $|\eta| < 0.5$, measured by the TPC. For each centrality, the average numbers of participants ($\langle N_{\text{part}} \rangle$) are obtained by Glauber model calculations. The $\Delta N_p$ measurements are carried out at midrapidity ($|y| < 0.5$) in the range $0.4 < p_T < 0.8$ GeV/$c$. Ionization energy loss ($dE/dx$) of charged particles in the TPC was used to identify the inclusive $p(\bar{p})$ [18]. To suppress the contamination from secondary protons, we required each $p(\bar{p})$ track to have a minimum $p_T$ of 0.4 GeV/$c$ and a distance of closest approach to the primary vertex of less than 1 cm [18]. The $p_T$ range used includes approximately 35%–40% of the total $p + \bar{p}$ multiplicity at midrapidity. $\Delta N_p$ was not

\[ N_p - \bar{N}_p = \Delta N_p \]

FIG. 1 (color online). $\Delta N_p$ multiplicity distribution in $\text{Au} + \text{Au}$ collisions at $\sqrt{s_{NN}} = 200$ GeV for various collision centralities at midrapidity ($|y| < 0.5$). The statistical errors are shown.
corrected for reconstruction efficiency. Typical $\Delta N_p$ distributions from 70% to 80%, 30% to 40%, and 0% to 5% Au + Au collision centralities are shown in Fig. 1.

The four moments ($M$, $\sigma$, $S$, and $\kappa$) which describe the shape of the $\Delta N_p$ distributions at various collision energies are plotted as a function of $\langle N_{\text{part}} \rangle$ in Fig. 2. The typical statistical errors on $\sigma$, $S$, and $\kappa$ for central Au + Au collisions at 200 GeV are 0.2%, 11%, and 16%, respectively. The $M$ shows a linear variation with $\langle N_{\text{part}} \rangle$ and increases as $\sqrt{S_{NN}}$ decreases, in accordance with the energy and centrality dependence of baryon transport [8]. The variation of $M$ within a centrality bin has been taken into account in higher moment calculations. The $\sigma$ increases with $\langle N_{\text{part}} \rangle$. The values are similar for three beam energies studied. The $S$ is positive and decreases as $\langle N_{\text{part}} \rangle$ increases for a given collision energy. The values also decrease as $\sqrt{S_{NN}}$ increases. This indicates that the distributions become symmetric for more central collisions and for higher beam energies. The $\kappa$ decreases as $\langle N_{\text{part}} \rangle$ increases, but is similar for all three $\sqrt{S_{NN}}$ studied.

Experimentally it is difficult to correct such observables for the particle reconstruction efficiency on an event-by-event basis. Construction of observables independent of the efficiency, such as factorial moments, leads to loss of one-event basis. Construction of observables independent of the detector and significant difficulty in comparing to theoretical expectations. We have investigated the effects of the detector and reconstruction efficiency, such as factorial moments, leads to loss of one-event basis. Construction of observables independent of the detector and significant difficulty in comparing to theoretical expectations. To understand the evolution of centrality dependence of moments in Fig. 2, we invoke the central limit theorem (CLT) and consider the distribution at any given centrality $i$ to be a superposition of several independent source distributions. We assume the average number of the sources for a given centrality to be equal to some number $C$ times the corresponding $\langle N_{\text{part}} \rangle$, and obtain

$$M_i = CM_{x}\langle N_{\text{part}} \rangle_{i},$$

$$\sigma_{i}^{2} = C\sigma_{x}^{2}\langle N_{\text{part}} \rangle_{i},$$

$$S_i = S_{x}/[\sqrt{C\langle N_{\text{part}} \rangle_{i}}],$$

and

$$\kappa_i = \kappa_{x}/[C\langle N_{\text{part}} \rangle_{i}].$$

The various moments of the parent distribution $M_{x}$, $\sigma_{x}$, $S_{x}$, $\kappa_{x}$, and constant $C$ have been determined from fits to data. The dashed lines in Fig. 2 show the expectations from the CLT. The $\chi^2$/ndf between the CLT expectations and data are <1.5 for all the moments presented. If collision centrality reflects the system volume, then the results in Fig. 2 which approximate baryon number susceptibilities suggest that the susceptibilities do not change with the volume [2]. Deviations from $\langle N_{\text{part}} \rangle$ scaling could indicate new physics such as might result from the CP.

To get a microscopic view, we present two observables, $S\sigma$ and $\kappa\sigma^2$, which can be used to search for the CP. These products will be constants as per the CLT and other likely non-CP scenarios, as seen from the dependences on $\langle N_{\text{part}} \rangle$ discussed above. These observables are related to the ratio of baryon number susceptibilities ($\chi_B$) at a given temperature ($T$) computed in QCD models as $S\sigma = (\chi_B^{(3)}/T)/(\chi_B^{(2)}/T^2)$ and $\kappa\sigma^2 = \chi_B^{(4)}/(\chi_B^{(2)}/T^2)$ [6]. Close to the CP, models predict the net-baryon number distributions to be non-Gaussian and susceptibilities to diverge causing $S\sigma$ and $\kappa\sigma^2$ to deviate from being constants and have large values. Figure 3 shows that $S\sigma$ and $\kappa\sigma^2$ for

![FIG. 2 (color online). Centrality dependence of moments of $\Delta N_p$ distributions for Au + Au collisions at $\sqrt{s_{NN}} = 19.6$, 62.4, and 200 GeV. The lines are the expected values from the central limit theorem. Error bars are statistical and caps are systematic errors.](attachment:fig2.png)
Au + Au collisions at $\sqrt{s_{NN}} = 19.6, 62.4,$ and 200 GeV are constants as a function of $\langle N_{\text{part}} \rangle$.

In Fig. 3(a), lattice QCD results on $S\sigma$ for net baryons in central collisions are found to agree with the measurements. Near the CP, the system will deviate from equilibrium [12], and results from lattice QCD, which assumes equilibrium, should not be consistent with the data. These lattice calculations, which predict a CP around equilibrium, should not be consistent with the data. These measurements for a CP in the region of the phase plane studied, we conclude that there is no indication from our measurements for a CP in the region of the phase plane with $\mu_B < 200$ MeV. It is difficult to rule out the existence of CP for the entire $\mu_B$ region below 200 MeV. The extent

To understand the various non-CP physics background contribution to these observables, in Fig. 3 we also present the results for the net-proton distribution as a function of $\langle N_{\text{part}} \rangle$ from URQMD (ver.2.3) [23], HIJING [20], AMPT (ver.1.11) [24], and THERMINATOR (ver.1.0) [25] models. The measurements are consistent with results from various non-CP models studied. In Fig. 3(c), several model calculations from Au + Au collisions at 200 GeV are presented to explain the effect of the following on our observable: with (W) and without (W/O) resonance decays, inclusion of all baryons (both studied using URQMD), jet production (HIJING), coalescence mechanism of particle production (AMPT String Melting, ver.2.11), thermal particle production (THERMINATOR), and rescattering (URQMD and AMPT). All model calculations are done using default versions and with the same kinematic coverage as for data. The $\kappa\sigma^2$ [Fig. 3(b)] and $S\sigma$ [Fig. 3(a)] are found to be constant for all the cases as a function of $\langle N_{\text{part}} \rangle$. This constant value can act as a baseline for the CP search. QCD model calculations with CP predict a nonmonotonic dependence of these observables with $\langle N_{\text{part}} \rangle$ and $\sqrt{s_{NN}}$ [13].

Figure 4 shows the energy dependence of $\kappa\sigma^2$ for $\Delta N_p$, compared to several model calculations that do not include a CP. The experimental values plotted are average values for the centrality range studied; they are found to be consistent with unity. Also shown at the top of Fig. 4 are the $\mu_B$ values corresponding to the various $\sqrt{s_{NN}}$ [18,22]. We observe no nonmonotonic dependence with $\sqrt{s_{NN}}$. The results from non-CP models are constants as a function of $\sqrt{s_{NN}}$ and have values between 1 and 2. The result from the thermal model is exactly unity. Within the ambit of the models studied, the observable changes little with change in non-CP physics (such as due to a change in $\mu_B$, collective expansion, and particle production) at the various energies studied. From comparisons to models and the lack of nonmonotonic dependence of $\kappa\sigma^2$ on $\sqrt{s_{NN}}$ studied, we conclude that there is no indication from our measurements for a CP in the region of the phase plane with $\mu_B < 200$ MeV. It is difficult to rule out the existence of CP for the entire $\mu_B$ region below 200 MeV. The extent

FIG. 3 (color online). Centrality dependence of (a) $S\sigma$ and (b) $\kappa\sigma^2$ for $\Delta N_p$ in Au + Au collisions at $\sqrt{s_{NN}} = 19.6, 62.4,$ and 200 GeV compared to various model calculations. The shaded band for $S\sigma$ and $\kappa\sigma^2$ reflects contributions from the detector effects. (c) shows the model expectations for $\kappa\sigma^2$ from various physical effects in Au + Au collisions at 200 GeV. The lattice QCD results are for net baryons corresponding to central collisions [6]. See text for more details.

FIG. 4 (color online). $\sqrt{s_{NN}}$ dependence of $\kappa\sigma^2$ for net-proton distributions measured at RHIC. The results are compared to non-CP model calculations (slightly shifted in $\sqrt{s_{NN}}$). The left-right arrow at the bottom indicates the energy range for the CP search at RHIC.
to which these results can do that is guided by the following theoretical work. One QCD-based model including a CP ($\xi = 3$ fm) predicts the value of $\kappa \sigma^2$ to be at least a factor of 2 higher than the measurements presented ($\kappa \sigma^2 \sim 2.5, 35, 3700$ for the CP at $\sqrt{s_{NN}} = 200, 62.4$, and $19.6$ GeV, respectively) [13]. In addition, the expectation of the extent of the critical region in $\mu_B$ is thought to be about 100 MeV [6,26].

In summary, the first measurements of the higher moments of the net-proton distributions at midrapidity ($|y| < 0.5$) within $0.4 < p_T < 0.8$ GeV/c in Au + Au collisions at $\sqrt{s_{NN}} = 19.6, 62.4$, and $200$ GeV have been presented. New observables $\tilde{S}^r$ and $\kappa \sigma^2$ derived from the $\Delta N_{\text{p}}$ distribution to search for the CP in heavy-ion collisions are discussed. These observables are found to be constant as a function of $\langle N_{\text{part}} \rangle$ for all collisions’ energies studied. This is consistent with expectations from the central limit theorem and in general agreement with results from various models without the CP. The measured $\tilde{S}^r$ in central collisions are consistent with lattice QCD calculations of the ratio of third order to second order baryon number susceptibilities. Within the uncertainties, $\kappa \sigma^2$ is found to be constant as a function of $\sqrt{s_{NN}}$ studied. This trend is consistent with models without a CP and in sharp contrast to models [13] which include a CP in this $\mu_B$ range. Our measurements show no evidence for a CP to be located at $\mu_B$ values $\leq 200$ MeV in the QCD phase plane. The RHIC beam energy ($100 < \mu_B < 550$ MeV) scan will look for nonmonotonic variation of $\kappa \sigma^2$ for net protons as a function of $\sqrt{s_{NN}}$ to locate the CP.

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[19] Factorial moments: $F_2 = \frac{\langle N \rangle}{\langle N^2 \rangle} = \text{Func}(M, \sigma^2)$; $F_4 = \frac{\langle N^2 \rangle}{\langle N^2 \rangle}$ $\leq 200$ MeV = $\text{Func}(M, \sigma, \Delta, \sigma)$.