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Dynamic Transfer Stiffness of Suspension Springs and Discharge Tubes in Hermetic Reciprocating Compressors

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ABSTRACT

Scaling laws are presented for the dynamic transfer stiffness of slender elastic structures with specific emphasis on suspension springs and discharge tubes of hermetic compressors. Measurements of the dynamic transfer stiffness of suspension springs are reported. The calculation of point mobility of the compressor shell is discussed.

1. INTRODUCTION

In hermetic reciprocating compressors, the suction line is commonly regarded as the dominating transmission path of noise which can excite the compressor shell. However, with the introduction of acoustically optimized suction lines, other transmission paths can potentially become dominating. Therefore, in order to further lower the radiated noise from the compressor shell, these transmission paths need to be taken in account. Two obvious candidates for further lowering the transmitted noise are the suspension springs and the discharge tube. A useful quantity for describing the noise transmission through these structures is the dynamic transfer stiffness (DTS) i.e. the ratio of the blocked force output at one end and the displacement input at the other end.

Suspension springs and discharge tubes are slender elastic structures which can transmit structure-borne noise from the pump-unit to the hermetic shell. In most cases, these structures can be regarded as one-dimensional spatially curved waveguides with constant cross-sections along which a multitude of evanescent and propagating waves can exist. The theory of wave propagation in straight slender structures is well understood (Cremer and Heckl, 1988). In straight rods, for example, the propagating waves can be characterized as longitudinal, torsional and flexural waves, which are completely uncoupled. In irregularly shaped structures, however, the waves are coupled and this makes the analysis very difficult. In general, dynamic finite element (FE) methods need to be employed in these cases. Modern commercial FE software packages offer such dynamic analysis both in time- and frequency-domain. Dedicated numerical techniques for analyzing wave propagation in one-dimensional waveguides have been developed. Some of these techniques are wave FE (Mace *et al.*, 2005) and spectral FE (Gavric, 1995).

To some extent compressor suspension springs can be regarded as helical springs except at their ends, where they can have close-wound coils and a rather ill defined contact with the spring retainers. Wave propagation in helical springs has been comprehensively treated by Wittrick (1966) by using Timoshenko beam theory. He obtained a set of 12 linear coupled partial differential equations, which describe the beam deformations. Over the years there have been various attempts to simplify the theory developed by Wittrick (1966). Sorokin (2009) assesses the validity range of these theories by use of dispersion curves. Lee and Thomsen (2001) used a dynamic stiffness formulation for helical springs from which they obtained the DTS.

Studies focusing on suspension springs and discharge tubes of compressors have been published. Simmons and Soedel (1996) investigated surging effects in springs coupled with sub-systems. The springs and sub-systems were treated as one-dimensional and a receptance technique was used to predict the response of the system. Kelly and Knight (1992) used a dynamic FE model of an assembled compressor in which the suspension springs and discharge tube were described by a series of beam elements and those of the compressor housing by shell elements. In an early work by Bernhard and Seidel (1986), beam elements were also used to model the dynamic motion and stresses of discharge tubes. Design optimization of discharge tubes with respect to noise transmission has been done by Silva *et al.* (2004) and Wang *et al.* (2004) using FE and response surface methods, respectively.

The aim of this paper is to derive scaling laws for the DTS of slender structures by performing numerical experiments rather than deriving them from first principles. The thus obtained scaling laws can be used as a simple rule of thumb for the design engineer. The paper is organized as follows: in Section 2 a simplified expression for structure borne-noise is derived. In Section 3, the methodology for determining scaling laws is described and in Section 4 specific scaling laws for suspension springs and discharge tubes are derived. In Section 5 measurements of the DTS are reported. Finally, in Section 6 the calculation of the point mobility is discussed.

2. SIMPLIFIED EXPRESSION FOR STRUCTURE-BORNE NOISE

In this section a simplified expression for the power injected into the shell through a suspension spring or a discharge tube is derived. Figure 1(a) shows a household compressor with four suspension springs and a discharge tube. Figure 1(b) shows a magnified view of a suspension spring and the surrounding structures. In Figure 1(b) the quantities from which the transmitted power is calculated are shown on the right. These are the displacement vector, x , the point mobility matrix, Y , and the DTS matrix, K . The arrows indicate the position at which x and Y is defined.

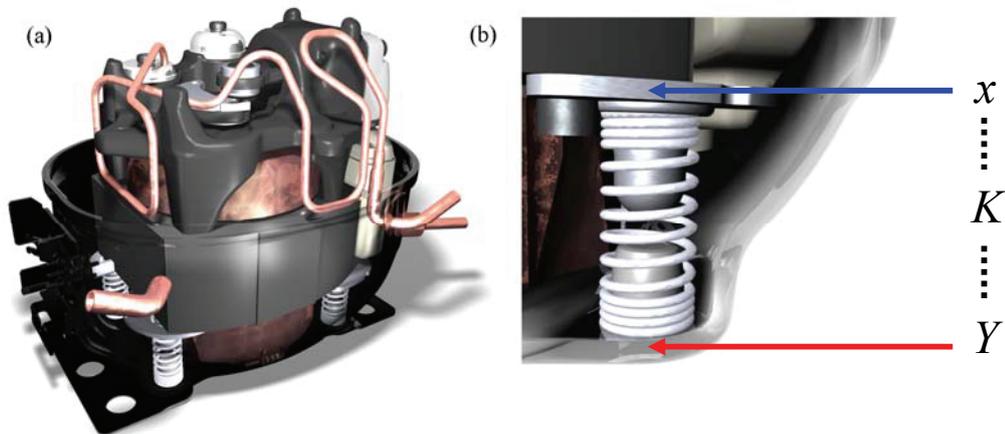


Figure 1: (a) Compressor with springs and discharge tube; (b) Quantities from which the power is derived

Assuming that the vibrations of the compressor can be described by simple time harmonic dependences, the injected time-averaged power, Π , into the shell at a single point can be described by the following expression:

$$\Pi = \frac{1}{2} \operatorname{Re}(F^* \cdot v) = \frac{1}{2} \operatorname{Re}(x^* \cdot K^* \cdot Y \cdot K \cdot x). \quad (1)$$

where F is the force vector and v is the velocity vector at the injection point on the shell. The asterisk indicates conjugate transpose. The quantities are interrelated by the equations given below:

$$v = Y \cdot F \quad , \quad F = K \cdot x. \quad (2, 3)$$

In general, the quantities can be thought of as complex Fourier coefficients of harmonic frequencies. In principle, x should include rotation and F should include torque. However, in order to keep the analysis simple, rotation and torque are not included in the forthcoming analysis. A similar expression for the transmitted power given in Equation (1) can be found in Fahy (1985) and it has been employed by Silva *et al.* (2004).

3. DETERMINATION OF SCALING LAWS

In this section scaling laws for the DTS is derived. Scaling laws can be useful in the case, where the equations which describe certain phenomena are unknown or where the equations are so complicated that it is impossible to derive analytic expressions. The latter is true for the DTS.

First of all, the quantities on which the DTS depends need to be found. For a slender elastic structure these are: Young's modulus, E , Poisson's ratio, ν , the mass per unit length, m' , the second moment of area or radius of gyration, I , the excitation frequency, ω , and a length scale, L , which somehow describes the extension of the structure in space. In this treatment the length scale is chosen to be the length of the structure. It is seen that the DTS is described by six quantities, one of which is non-dimensional. In consistency with the Π -theorem (Barenblatt, 2003) the DTS can be written in the form:

$$K_{ij} = E^1 L^1 m'^0 \Phi_{ij}(\chi_1, \chi_2, \chi_3), \quad (4)$$

where Φ is a non-dimensional shape function, which is multiplied by three dimensionally independent quantities, that are raised to a certain power in order to obtain the correct dimension. Furthermore, Φ must be dependent on three non-dimensional quantities, χ , which, for instance, could have the following form:

$$\chi_1 = \omega \sqrt{\frac{m'}{E}}, \quad \chi_2 = \frac{I}{L^4}, \quad \chi_3 = \nu. \quad (5, 6, 7)$$

The way in which Equation (4) is constructed is just one out of infinitely many, which are also in accordance with the Π -theorem. In the forthcoming analysis the following expression of the DTS will be used:

$$K_{ij} = EL\chi_2 \Phi_{ij}(\chi_1 \chi_2^{-1/2}, \chi_2^{1/4}, \chi_3). \quad (8)$$

The difference between Equations (4) and (8) merely reflects the flexibility with which Φ can be chosen.

4. DYNAMIC TRANSFER STIFFNESS

In this section, the scaling laws of the DTS for slender elastic structures will be given together with those of discharge tubes and suspension springs. The scaling laws have been obtained by performing numerical experiments using the commercial FE software package Pro/ENGINEER Mechanical. The structures were represented by use beam elements with 6 degrees of freedom at each node. The calculations were performed in the frequency domain using the modal superposition method without internal damping present in the structure.

4.1 Dynamic Transfer Stiffness of Slender Elastic Structures

According to the preceding discussion, the DTS can be written in the following way:

$$K_{ij} = \frac{EI}{L^3} \Phi_{ij}(\Omega, \xi, \nu), \quad (9)$$

where the first two non-dimensional arguments in Φ are given by

$$\Omega = \omega L^2 \sqrt{\frac{m'}{EI}}, \quad \xi = \frac{I^{1/4}}{L} \quad (10, 11)$$

Here Ω can be regarded as a frequency parameter and ξ is the ratio between the radius of gyration of the cross-section and the length of the structure. For slender structures the condition $\xi \ll 1$ must be satisfied. The form of Ω in Equation (10) has been chosen so that the modal frequencies in terms of Ω are invariant for self-similar geometries i.e. the line traced out in space by the structure can always be brought to cover it-self by scaling isotropically in space. In other words, all the modal frequencies scale with $\sqrt{EI/m'}/L^2$ as long as $\xi \ll 1$ is valid. Furthermore, it

should be noted, that the boundary conditions are implicitly contained in Φ . For example, Φ changes when changing one rotational degree of freedom from clamped to free. Such changes, however, still preserve the validity of the obtained scaling laws.

4.2 Dynamic Transfer Stiffness of Discharge Tubes and Suspension Springs

Below formulae are given for the second moment of area and the mass per unit length:

$$I = \pi(b^4 - a^4)/4 \quad , \quad m' = \rho\pi(b^2 - a^2), \quad (12, 13)$$

where a and b is the inner and outer radius of the discharge tube. For a spring wire a is equal to zero. For a helical spring, Φ will depend on two additional non-dimensional parameters; these could be the pitch angle, θ , and the number of turns, N . In this case, the DTS becomes:

$$K_{ij} = \frac{EI}{L^3} \Phi_{ij}(\Omega, \xi, \nu, \theta, N). \quad (14)$$

Figure 2 shows an example of the DTS versus frequency of a 6 turn suspension spring of an N-series compressor, see Figure 1. Three DTS curves are seen with the displacement along the vertical (up-down), radial and tangential axes of the spring. For all three curves the resulting force is sampled in the vertical direction. It is seen, that the vertical part is the dominating up to 1.5k Hz and thereafter neither of the components are dominating. The curves show both troughs and peaks, where the later are caused by structural resonances of the spring.

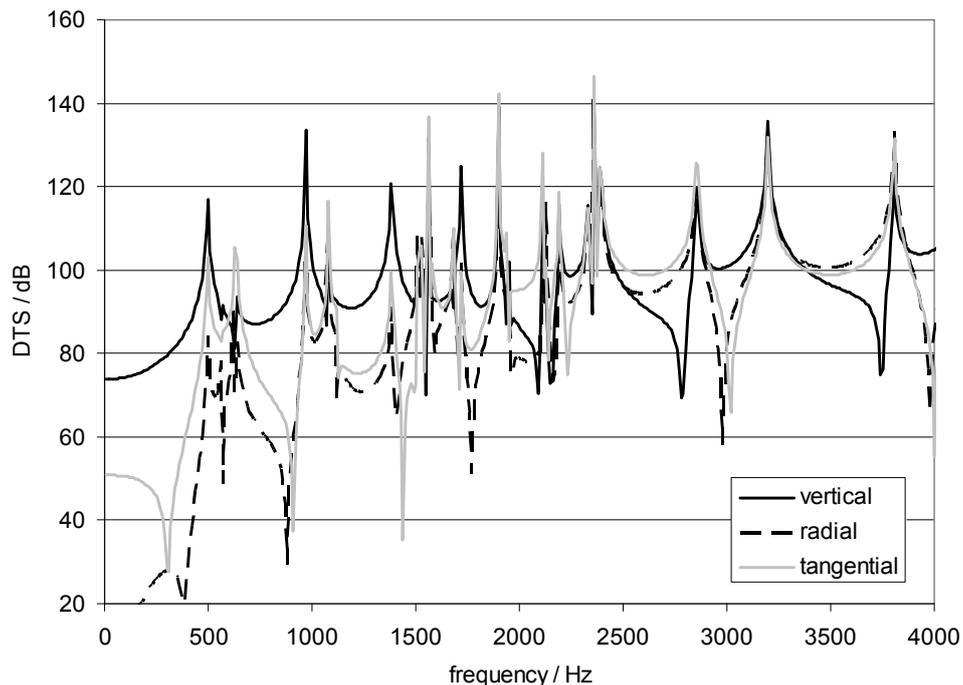


Figure 2: DTS components with excitation in vertical, radial and tangential directions.

Figure 3 shows a comparison between the DTS of a suspension spring and that of a discharge tube of an N-series compressor. Both DTS have roughly of the same order of magnitude up to 3k Hz, but from then on the DTS of the discharge tube becomes dominant. It should be noted, however, that no damping effects from the damping spring of the discharge tube has been included. This would of course significantly lower the peaks around the structural resonances.

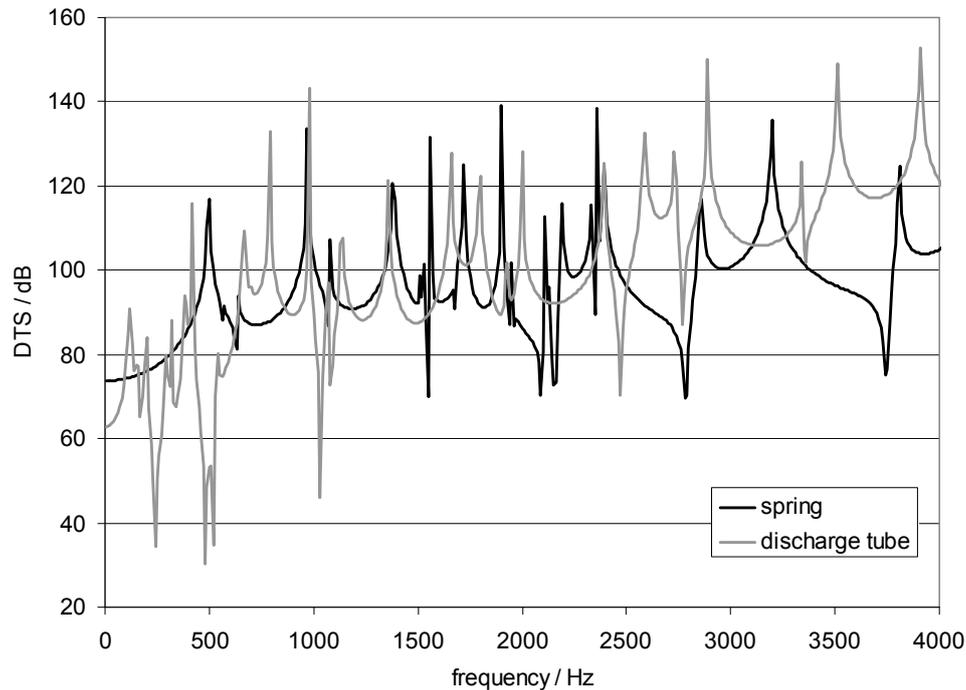


Figure 3: Comparison between DTS of a suspension spring and a discharge tube

The DTS curves shown in this section are for idealized structures with well defined boundary conditions and where all parts of the structure can vibrate freely. For real suspension springs this is hardly the case. The next section shows measurements of the DTS, where large variations in the DTS can be seen.

5. MEASUREMENTS OF DYNAMIC TRANSFER STIFFNESS

The DTS of the suspension springs have been measured by means of the experimental set-up shown in Figure 4. The top part of the set-up consists of a mass, m , with an accelerometer, a_1 , by which the force exerted on the spring can be measured. The vibration excitation is obtained by use of an electro-dynamic shaker below the spring. The displacement of the spring is measured with an accelerometer, a_2 . Brüel & Kjær accelerometers type 4393V and a PULSE signal acquisition front-end have been used. The shaker was driven by a stepped sine and the signals were sampled at the driving frequency in order to obtain a high signal-to-noise ratio of the upper signal a_1 .

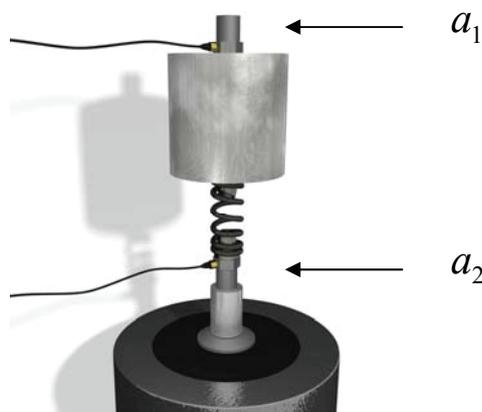


Figure 4: Experimental set-up for measuring DTS using two accelerometers and a shaker.

Assuming that the displacement of the mass is much smaller than that of the shaker, the DTS, K , can be expressed in terms of a frequency response function (FRF), H_1 , between the upper and lower accelerometer signals:

$$K = m\omega^2 H_1, \quad H_1 = \frac{a_1 a_2^*}{a_2 a_1^*}. \quad (15, 16)$$

The FRF in Equation (16) is given in terms of the cross-spectrum of the two signals divided by the auto-spectrum of the lower signal. This FRF is well suited for reducing the influence of noise contained in the upper signal a_1 .

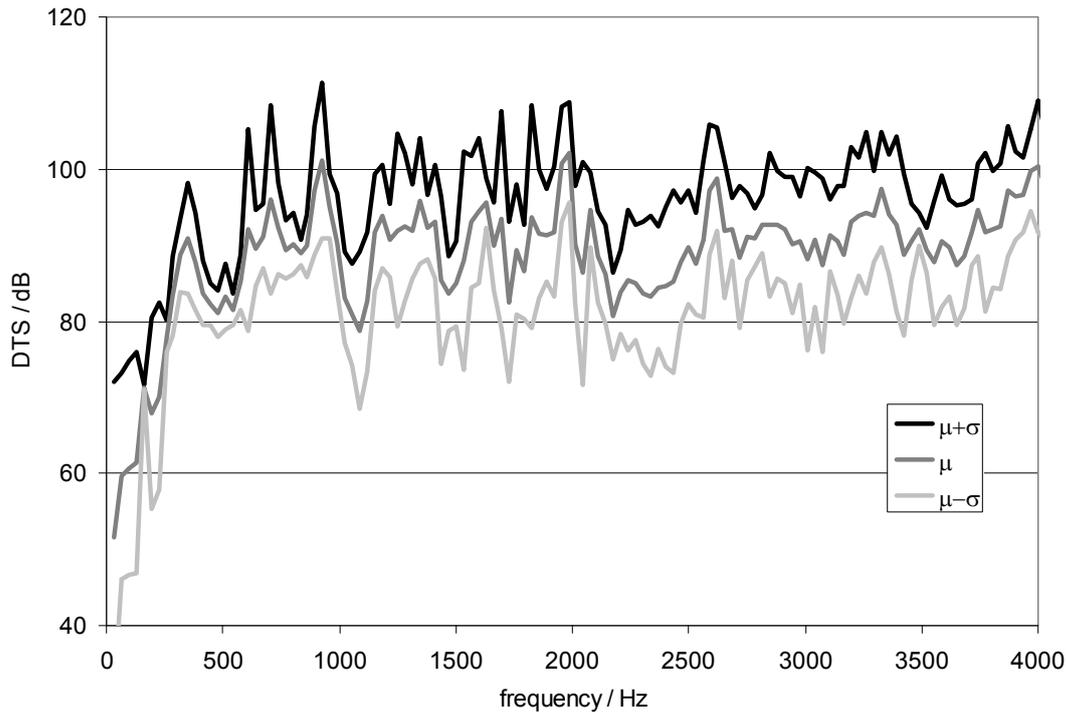


Figure 5: Standard deviation of the measured DTS when rotating the spring

Figure 5 shows the average, μ , and standard deviation, σ , of the measured DTS level for a suspension spring of a T-series compressor. Ten measurements were performed and the variations were obtained by carefully turning the spring by 36° . The variations are attributed to small changes in the boundary conditions seen by the spring ends and small misalignments of the spring. The influence on the DTS of these small perturbations is very strong. The same level of variations will probably also be observed for suspension springs mounted in a compressor.

In Equation (1) it is seen that the transmitted power is not only dependent on the DTS but also on the displacement of the pump-unit and the point mobility of the shell. The point mobility will be discussed in the next section.

6. POINT MOBILITY

In the one-dimensional case, the expression for the transmitted power in Equation (1) reduces to the following expression:

$$\Pi = \frac{1}{2} x^2 |K|^2 \operatorname{Re}(Y), \quad (17)$$

where x can be considered real in this case. It is seen that the transmitted power is proportional at the real part of the point mobility of the shell. In the case, where no energy dissipation mechanisms in or around the shell are present, the real part will be zero; and hence no power can be transmitted. It is a complicated task to include dissipation mechanisms explicitly. It is far easier to include damping in each of the structural modes of the shell.

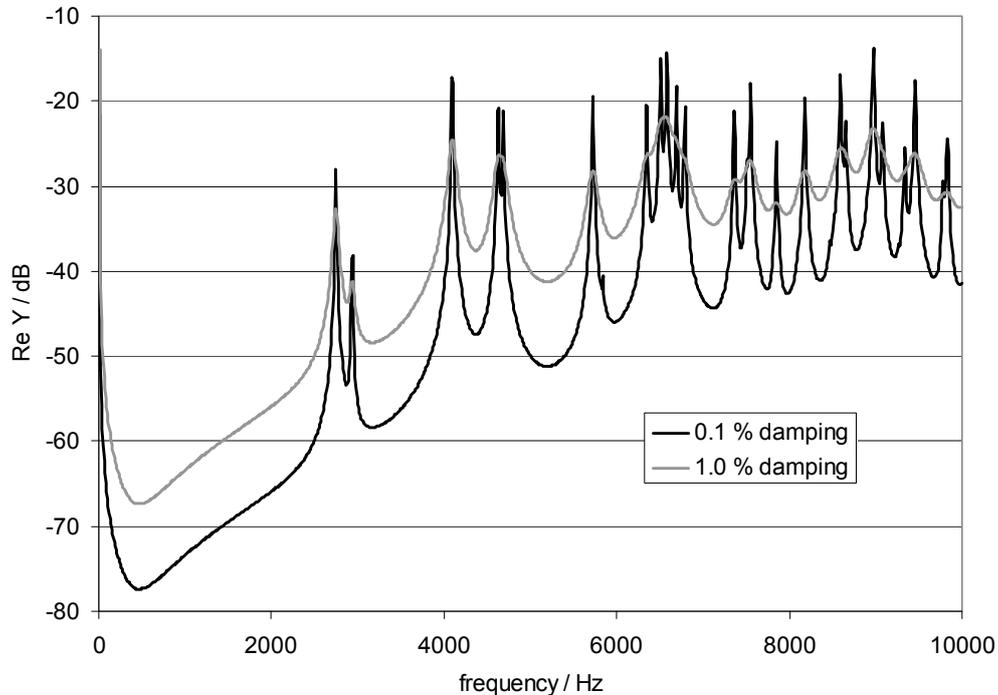


Figure 6: Real part of the point mobility of a shell with 0.1 % and 1 % of critical mode damping

Figure 6 shows the real part of the point mobility in the normal direction to the surface of an idealized compressor shell. In this calculation, shell elements have been used and a damping corresponding to 0.1 % and 1 % of the critical damping for each mode has been applied. In the case of high damping, it is seen that the power transmission is lowered around the structural resonance while it is increased in the regions between the resonances.

7. CONCLUSION

In this study the following has been reported:

- A simplified expression for the structure-borne power transmitted to the compressor shell
- Scaling laws for the dynamic transfer stiffness of slender elastic structures
- Specific scaling laws for suspension springs and discharge tubes
- Measurements of the dynamic transfer stiffness of suspension springs
- Calculation of the point mobility of an idealized compressor shell

NOMENCLATURE

| | | | |
|-----|----------------------------|--------|---|
| P | transmitted power | (W) | Definitions: DTS level = $20 \log K /K_{ref}$, where $K_{ref} = 1 \text{ N/m}$ |
| F | force | (N) | |
| v | velocity | (m/s) | |
| x | displacement | (m) | |
| K | dynamic transfer stiffness | (N/m) | |
| Y | point mobility | (s/kg) | |
| E | Young's modulus | (Pa) | |

| | | | |
|-------------|--------------------------------|-----------------------------------|---|
| L | length of structure | (m) | Data for idealized N-series spring: Height = 24 mm Number of turns = 6 Wire radius = 0.85 mm Turn radius = 7.05 mm |
| m' | mass per length | (kg/m) | |
| Φ | shape function | (-) | |
| χ, ξ | aux. parameters | (-) | |
| ω | excitation frequency | (s ⁻¹) | |
| I | second moment of area | (m ⁴) | |
| ν | Poisson's ratio | (-) | |
| Ω | frequency parameter | (-) | |
| ρ | density | (kg/m ³) | |
| a, b | inner and outer radius | (m) | |
| θ | pitch angle | (rad) | |
| N | number of turns | (-) | |
| a_i | acceleration | (m/s ²) | |
| H_1 | frequency response function | (-) | |
| $a_1 a_2^*$ | cross-spectrum of acceleration | (m ² /s ⁴) | |
| $a_2 a_2^*$ | auto-spectrum of acceleration | (m ² /s ⁴) | |
| μ | average level | (-) | |
| σ | standard deviation level | (-) | |

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