On the Optimal Design of One-Rotor Two-Stages Rotary-Vane Compressors

Roberto Valente
University of L'Aquila

Carlo Villante
University of L'Aquila

Follow this and additional works at: http://docs.lib.purdue.edu/icec
On the optimal design of one-rotor two-stages rotary-vane compressors

Roberto VALENTE1, Carlo VILLANTE1

1University of L’Aquila, DIMEG Dept.,
L’Aquila, ITALY
(Phone 0039.0862.434028, rvalente@ing.univaq.it)
(Phone 0039.0862.434319, villante@ing.univaq.it)

ABSTRACT

The efficiency increase reachable with more than one compression stage is a well known result of the thermodynamic principles, since this allows the gas cooling between consecutive stages and makes the transformation approach the isothermal limit.

In the sliding vane rotary compressors is possible to integrate two stages of compression within a single stator, so to keep down the system dimension and its weight. However, for the specific compressor type at issue, some considerations on the optimization procedure to be followed in the design phase are required. The need of maintaining a fixed pressure level at the discharge port opening, the unavoidable reduction of the swept angle useful for the compression phase in each stage, the necessity of matching vane volumes in the stages with the cooling effect on the gas specific volume and some additional constraints imposed by system design (on the possible stator geometries) offer many theoretical aspect to be deepened, in order to obtain an optimized machine design in terms of thermodynamic efficiency.

In the present paper the Authors developed a procedure which permits to define, starting from the choice of free parameters and the ideal performance expected, the optimum design in terms of ideal energy consumption. The procedure is based on a geometric and thermo-fluid dynamic modeling of the phenomena which occur inside the compressor.

1. INTRODUCTION

Sliding vane rotary compressors (SVRC) represent an interesting family of machines which shows some appreciable advantages, in comparison with both other volumetric compressors and centrifugal ones, in industrial environment. They are characterized, in fact, as volumetric devices, by an air delivered flow rate which is almost independent on the discharge pressure and linearly proportional to the speed of rotation. SVRCs, anyway, don’t suffer of significant noise and vibrations, typical of positive displacement machines; therefore they can provide an almost constant performance in terms of mass flow delivered (absence of pulsating flows), so approaching the centrifugal devices. Moreover, they present a favorable weight to power ratio and they are characterized by a very low specific energy consumption per unit mass of air induced. The latter property demonstrates to be an important aspect, greater than in the past, in industrial context. This applies toward the environment, for energy and emission saving, as well as toward the energy cost reduction, very effective to win in a global market. The overall energy consumption related to the compressed air service, in fact, can be estimated in approximately 80 TWh per year in the European Union, datum which highlights the importance of research in energy saving in compressed air systems.

The two stage compression with inter-cooling, performed with one rotor inside a two lobes-single stator (Figure 1), represent for high duty SVRC a breakthrough technology able to reduce further on the energy absorbed per unit air mass delivered. The benefits in terms of thermodynamic efficiency which approaches, thanks to the inter-cooling, the isothermal ideal process is the well known key concept. The gain in terms of weight to power ratio increases the interest of this solution, appreciated by the customers both for new installations and replacements.

The mathematical modeling of the processes inside SVRC as a support to the design stage, has not had the same attention which characterized other machines (screw, reciprocating, dynamic) in the scientific literature. Recently the Authors raised interest again, presenting a comprehensive model highly integrated between the different processes which take place inside these machines (intake and exhaust phases, interaction with the discharge line, oil circulation considering a realistic injection oil system, thermodynamic benefits of the oil injection, blade in rotor slot motion and settings, etc.), (Cipollone et al.,2005 and Cipollone et al.,2006). A strict physical consistency was used to guarantee generality and flexibility. A dynamic Two-stages compressor modeling has been also developed by the Authors (Cipollone et al., 2007)
2. THEORETICAL LIMITS FOR TWO-STAGE COMPRESSION

It is known that multiple stage compression with intercooled increases the thermodynamic efficiency of the total compression process bringing the transformation closer to the isothermal limit. This is of particular interest when the reduction of the specific energy of a compressor is important.

In Equation 1 the ratio \( \eta_{bi} \) between the polytropic (\( m \) exponent) specific work needed by a two-stage device (\( L_{bi} \)) and a one-stage reference machine (\( L_{mono} \)), \( L_{bi} \) is calculated considering a further isochoric compression when the overall compression ratio of the two stages (\( \beta_i \cdot \beta_{II} \)) is lower than the reference one (\( \beta_{tot} \)). This case can occur in volumetric machines if limits exist on the death volume, as it is in the devices object of this paper (Figure 1). \( \eta_{bi} \) may be interpreted as the ratio between the two-stages efficiency and that of the reference one-stage machine.

The degree of intercooling (considered as ideally isobaric) is defined through a temperature ratio (\( \chi_{tot} \), Equation 2) between the actual cooling (\( \chi_{I, tot} - \chi_{II, tot} \)) and its maximum limit (\( \chi_{I, tot} - \chi_{II, tot} \)). Two non-dimensional variables \( \eta_{opt} \) and \( \eta_{II, opt} \) varying from 0 to 1, are introduced in Equation 2. When \( \eta_{II, opt} = \frac{1}{\eta_{tot, opt}} \) and \( \eta_{opt} = 1 \), the one-stage machine is represented. The hyperbole \( \eta_{tot, opt} = \frac{1}{\eta_{II, opt}} \) represents all the devices which shows no isochoric compression: in this condition the lowest \( \eta_{bi} \) is obtained for each given \( \eta_{II, opt} \). On this curve, Equation 1 can be minimized obtaining the optimal non-dimensional compression ratios (\( \chi_{I, tot, opt} \cdot \chi_{II, tot, opt} \)) and the optimized \( \eta_{bi, opt} \) reported in Equation 2. As seen, the optimal compression ratios are equal and independent on the inter-cooling degree.

Anyway, \( \chi_{I, tot} \) and \( \chi_{II, tot} \) don’t completely define the machine, being \( k_{IC} \) influent on the desired geometric ratio \( \lambda_{des} \) between the two stages. A matching is needed, in fact, to avoid an isochoric transformation between the stages. \( \lambda_{des} \) can be easily calculated through Equation 3. \( \lambda_{des} \) represents respectively a volume ratio or a flux area ratio in volumetric and dynamic machines.

\[
\eta_{bi} = \frac{L_{bi}}{L_{mono}} = \frac{m-1}{\beta_i^{\frac{m}{m-1}} - 1} + \left( k_{IC} + (1-k_{IC}) \beta_i^{\frac{m-1}{m}} \right) \frac{m-1}{\beta_i^{\frac{m}{m-1}} - 1} \frac{m-1}{m-1} + \frac{m-1}{m-1} k_{IC} \left( \beta_i^{-1} \beta_{II}^{\frac{m-1}{m}} \beta_{tot}^{-\frac{m}{m-1}} \right)
\]

\[
k_{IC} = \frac{T_{OUT,I} - T_{IN,J}}{T_{OUT,J} - T_{IN,J}} ; \quad \chi_{I(J), tot} = \frac{\beta_i^{\frac{m}{m-1}} - 1}{\beta_{tot}^{\frac{m}{m-1}} - 1} ; \quad \chi_{II, tot, opt} = \frac{1}{\beta_{II, opt}^{\frac{m}{m-1}}} ; \quad \eta_{bi, opt} = 1 - k_{IC} \frac{m-1}{m-1} \beta_{tot}^{-\frac{m}{m-1}} - 1
\]

\[
\lambda_{des} = k_{IC} \cdot \beta_i'^{1-\frac{m}{m}}
\]

Figure 2 shows the obtained value of \( \eta_{bi} \), for \( k_{IC} = 1, m=1.4 \) and for two different value of \( \beta_{tot} \). The blue curve is the locus of configurations showing a null isochoric compression. The red dot in figure represents the optimal design.
2. MATHEMATICAL MODEL OF A ONE-ROTOR TWO-STAGES COMPRESSOR

2.1 Modeling of a stage

The geometrical model of a stage starts from the hypothesis of a circular stator, eccentric in respect of the rotor. Depending on stator dimension a different wrapping angle ($\gamma$) is obtainable. Two of this stages can be matched to obtain the device like the one in Figure 1. The option of tilting the blades (of an angle $\psi$ with respect to the radial direction) has also been taken into account. In the model derivation the Authors made also use of some preliminary results on one-stage compressors available in scientific literature (Tramschek and Mkumbwa, 1996a, Tramschek and Mkumbwa, 1996b).

In Equation 4 the relationship among rotor radius ($R_{\text{rot}}$), stator radius ($R_{\text{st}}$) and eccentricity ($e$) is reported. Equation 5 and 6 defines the maximum possible eccentricity ($e_{\text{max}}$), considering that the slots can’t intersect each other (therefore having a maximum length $L$) and the blade outcome can’t exceed its length. A non-dimensional parameter ($k_{\alpha}$) defines the actual eccentricity with respect to $e_{\text{max}}$. For every angular position $\theta$, the vane volume is calculated by Equations 6 – 8: this volume has been calculated as the difference between the volumes defined by the two blades (referenced with + and – suffixes) and the $\theta=0$ reference axis. A detailed derivation of these equations, together with the description of more general stator shapes, is not here reported for space constraints, but will be published soon in a separate dedicated paper.

Figure 2: Theoretical efficiency ratios

Figure 3: Single-stage geometric representation
\[ R_{st} = \sqrt{e^2 - 2eR_{rot} \cos \gamma + R_{rot}^2} \]  

\[ k_e = \frac{e}{e_{max}}; \quad e_{max} = R_{rot} \left( \sqrt{\left( 2 \cos \psi + \sin \psi \tan \frac{\alpha}{2} \right)^2 + \sin^2 \psi} - R_{st\ max} \right) \]  

\[ \frac{R_{st\ max}}{R_{rot}} = \frac{1 + \left( 2 \cos \psi + \sin \psi \tan \frac{\alpha}{2} \right)^2 + \sin^2 \psi - 2 \cos \gamma \sqrt{\left( 2 \cos \psi + \sin \psi \tan \frac{\alpha}{2} \right)^2 + \sin^2 \psi}}{2 \left( \sqrt{\left( 2 \cos \psi + \sin \psi \tan \frac{\alpha}{2} \right)^2 + \sin^2 \psi} - \cos \gamma \right)} \]  

\[ V = V_+ - V_-; \quad V_+ = \frac{\delta_+ \cdot R_{st}^2 + e \cdot R_{st} \cdot \sin \delta_+ - R_{rot} \cdot \sin \psi \cdot (L_+ - \cos \psi) - \theta_+ \cdot R_{rot}^2}{2} \]  

\[ \delta_+ = \sin^{-1} \left[ \frac{R_{rot} \sin \psi \sin \theta_+ + L_+ \cos \theta_+}{R_{st}} \right]; \quad L_+ = -e \sin \theta_+ + \sqrt{R_{st}^2 - (R_{rot} \sin \psi - e \cos \theta_+)^2} \]  

\[ \theta_+ \equiv \theta + \frac{\alpha}{2} = \theta - \frac{\pi}{2} \pm \frac{\alpha}{2}; \quad \delta = \delta_+ - \delta_- \]  

Equation 10 defines how the inlet and outlet volumes \((V_{IN}, V_{OUT})\) of the compression stage are calculated. An ideal behaviour hypothesis is made on the breathing processes, so that \(V_{IN}\) and \(V_{OUT}\) are respectively the maximum and the minimum vane volume. The compression ratio \(\beta\) is afterwards defined in the assumption of a polytropic transformation. At the same way (Equation 11), reference values for the inlet and outlet volumes \((V_{IN,0}, V_{OUT,0})\) and for the compression ratio \(\beta_0\) are calculated when \(\gamma = \pi\) and \(\psi = 0\) (one-stage radial-blades reference machine, at the left side of Figure 1).

\[ V_{IN} \equiv \max \{V\}_\theta; \quad V_{OUT} \equiv V(\theta = \gamma = -\frac{\alpha}{2}); \quad \beta = \left( \frac{V_{IN}}{V_{OUT}} \right)^m \]  

\[ V_{IN,0} \equiv V(\theta = 0, \gamma = \pi, \psi = 0); \quad V_{OUT,0} \equiv V(\theta = \pi - \frac{\alpha}{2}, \gamma = \pi, \psi = 0); \quad \beta_0 = \left( \frac{V_{IN,0}}{V_{OUT,0}} \right)^m \]  

2.2 Parametric analysis of a stage

In Equation 12, some non-dimensional variables are introduced to better put in evidence the behaviour of two-stage compressors, with respect to the reference one-stage machine: \(\phi\) is a non-dimensional wrapping angle (varying from 0, when \(\gamma = \alpha/2\), to 1, when \(\gamma = \pi\); \(\chi_0\) is the non-dimensional compression ratio (which is 0, when \(\gamma = \alpha/2\), equal to 1, when \(\gamma = \pi\) and \(\psi \neq 0\), and may become greater than 1 when the blades are tilted \(\psi \neq 0\); \(\Omega\) is a geometric scale-factor which must be applied to the device to make its inlet volume equal to that of the reference machine.

\[ \phi = \frac{2\gamma - \alpha}{2\pi - \alpha}; \quad \chi_0 = \frac{\beta - 1}{\beta_0 - 1}; \quad \Omega = \sqrt{\frac{V_{IN,0}}{V_{IN}}} = \frac{R_{rot,0}}{R_{rot}} \]  

\[ \psi_{\text{min}} = \cos^{-1} \left[ \frac{R_{rot}^2 - R_{st}^2 - e^2}{2e \cdot R_{st}} \right] = \gamma - \frac{\pi}{2} = \cos^{-1} \left[ \frac{R_{rot}^2 - R_{st}^2 - e^2}{2e \cdot R_{st}} \right] = \left[ \phi \pi + (1 - \phi) \frac{\alpha}{2} \right] - \frac{\pi}{2} \]  

Equation 13 express the minimum value \((\psi_{\text{min}})\) required for the blade tilt-rotation angle to permit a complete income of the blades at the end of the stage. Figures 4 gives a parametric representation of the performance indicator \(\chi_0\) of two families of compressors respectively with 5 (left side) and 9 (right side) blades. The blue curves are relative to \(\psi_{\text{min}}\): the unusable region widens as the number of blades increases. The results show that \(\chi_0\) increases both with \(\phi\) and \(\psi\). The slope of the contour lines
enlightens that the effect of blades tilt angle becomes much greater as the wrap angle decreases, so permitting, in part, to preserve the performance of the stage: this consideration may be important if more than one stage has to be coupled on the same rotor.

For the same machines, Figure 5 is relative to the geometric scale factors required to assure the same flow rate of the reference device. Clearly the contour lines are symmetric with respect to the $\psi = 0$ line. Depending on the specific design case, the rotor dimension may be greater or smaller than the reference one: anyway, the $\Omega$ values are near to unity for the most of the devices, with a greater range of variation as the number of blades increases.

3. ONE-STAGE NON-RADIAL BLADES COMPRESSOR OPTIMIZATION

The one-stage behavior can be derived from Figures 4 and 5, considering $\phi = 1$. Figure 4 in fact shows that the performance in terms of $\chi_0$ can be increased if a positive blade tilt angle is used. Moreover, it can be observed that, in comparison to the radial blades design, a reduction of $\Omega$ can be achieved by acting on the tilt blade angle (Figure 5). In this way, an optimal one-stage design can be defined, following the criterion of minimizing the value of $\Omega$. Figure 6 reports the result of a parametric analysis carried on by defining the optimal one-stage designs, with different number of blades: left side reports the optimal values of $\psi$ (which decrease with the number of blades) and the correspondently
achieved value of $\Omega$ (which increase with the number of blades). The so-calculated minimum values for $\Omega$ will be used as reference in the optimization of two-stages non-radial blades compressor optimization described in paragraph 5.2.

Figure 7 reports the simplified sketches relative to 3 of the optimized machines, respectively with 5, 8 and 11 blades.

4. MODELING THE MATCHING OF THE STAGES

The cooling process between two compression stages (ideally isobaric) introduces a reduction in fluid specific volume: the inlet volume of the second stage has to be suitable to receive the mass discharged from the first stage, in order to avoid isochoric expansions or compressions between the stages.

Equation 14 defines the desired volume ratio ($\lambda_{des}$), written as function of $\chi_l$, for a known degree of inter-cooling, and the actual volume ratio $\lambda$, calculated through the geometric model described before and applied to both the stages (evaluating by Equation 10 $V_{IN,I}$ and $V_{IN,II}$). As underlined, for a correctly matched device, it should be: $\lambda = \lambda_{des}$.

$$\lambda \equiv \frac{V_{IN,II}}{V_{OUT,II}}, \quad \lambda_{des} = k_{IC} \cdot [1 + \chi_l \cdot (\beta_0 - 1)]^{1-m} / m$$

(14)

For each given $\phi_I$, in order to minimize overall machine dimension for a fixed flow rate delivered, it’s convenient to impose the maximum possible eccentricity to the first compression stage ($k_{e,I} = 1$). For the same reasons, it was also chosen to wrap completely the rotor by the two stators, so that $\gamma_{II} + \gamma_I = \pi$ (Equation 15).

$$\phi_{II} = \frac{2\pi - 2\alpha}{2\pi - \alpha} - \phi_I$$

(15)

In the hypotheses of Number of blades equal to 5, $m=1.4$, $k_{IC}=1$, and radial blades (which doesn’t affect the generality of the described procedure), Figure 8 explains the matching procedure used with a graphical approach.

For any chosen $\phi_I$, in fact, the chart in Figure 4 (a) permits the evaluation of $\chi_{I,0}$. 

---

International Compressor Engineering Conference at Purdue, July 14-17, 2008
Plot (a) in Figure 8 is, then, relative to the second part of Equation 14 and defines the desired value for \( \lambda (\lambda_{des}) \). Plot (b), making use of the first part of Equation 14, defines the actual value for \( \lambda \), including the hypothesis on \( \phi_{II} \) stated in Equation 15. Imposing \( \lambda = \lambda_{des} \), \( k_{e,II} \) may be evaluated, so completing the definition of the second stage. Then, the maximum performance, in terms of \( \chi_{II,0} \), obtainable by the second stage is evaluable by Plot (c) which is relative to Equation 12. The resulting value can be used as input in Plot (d) to verify if \( \chi_{I,0} \) and \( \chi_{II,0} \) are sufficient to reach the desired overall target value (\( \beta_{tot} \)).

![Figure 8: Matching Procedure](image)

Plot (d) represents Equation 16, which shows a saturation of \( \beta_{II} \) when \( \beta_{tot} \) is reached: this corresponds to an advance in outlet port opening to avoid a non-convenient continuing of the compression and the subsequent isochoric expansion.

\[
\beta_{II} = \min \left\{ 1 + \chi_{II} \cdot (\beta_0 - 1), \frac{\beta_{tot}}{\beta_1} \right\}
\]  

(16)

The so-obtained values for \( \beta_I \) and \( \beta_{II} \), are then introduced in Equation 1 to get the value of efficiency ratio (\( \eta_{bl} \)) of the two-stage device under analysis.

## 5. TWO-STAGES COMPRESSOR OPTIMIZATION

### 5.1 Radial-blades compressors

Applying the previously described matching procedure for a varying wrap angle (and therefore first stage compression ratio) the performances of radial-blades two stage compressors can be calculated as a function of \( \chi_{I,0} \), if a desired overall compression ratio is fixed (Figure 9, relative to \( \beta_{tot} = 10 \)). The optimal machine can be so identified, as that having the lower \( \eta_{bl} \). If a family of devices are characterized by the same minimum \( \eta_{bl} \), among those the one having the minimum \( \Phi \) is chosen.
Repeating this analysis for various values of $\beta_{tot}$ and number of blades, the charts in Figure 10 can be obtained, which are relative to the design parameters (upper part), and the consequent performances (lower part) of all the possible optimized machines.

5.2 Non-radial-blades compressors

As already showed for one-stage compressors in paragraph 3, the rotation of the blades can offer permit further optimization, both in terms of efficiency ratio, and in terms of geometric ratio. For non-radial blades machines two geometric ratios $\Omega$ and $\Omega'$ may be used (according to Equation 17), the first using as reference the one-stage radial-blades machines, the second using as reference the optimized one-stage non radial-blades compressors identified paragraph 3. 

$$V_{IN,0,\text{max}} = \max\{V(\theta = \vartheta_{IN}, \gamma = \pi)\}\psi; \quad \Omega' = \frac{V_{IN,0,\text{max}}}{V_{IN}} = \frac{\Omega}{\min\{\Omega(\gamma = \pi)\}_\psi}$$ (17)

The optimization procedure applied is reported in Figure 11, and follows the same approach used for radial machines (Figure 9) repeated for various values of the tilt rotation angle $\psi$. If $\psi$ is sufficiently increased, non isochoric compression is needed any more, so reaching the minimum theoretical value of $\eta_{bl}$ calculated in Equation 2.
Figure 11 is relative to $\beta_{tot} = 10$ and number of blades equal to 5. Repeating this analysis for various values of $\beta_{tot}$ and number of blades, the charts in Figure 11 can be obtained, which are relative to the design parameters (upper part), and the consequent performances (lower part) of all the possible optimized machines.

5.3 Comparison among two-stage optimized compressor configurations

Figure 12 reports the sketches of some of the two-stages optimized machines, both with radial (upper part), and with non-radial (lower part) blades. Moving from left to right, number of blades is increased from 5, to 8 and 11. All the machines are relative to an overall desired compression ratio $\beta_{tot} = 10$.

Figure shows that for the higher numbers of blades (8 and 11) no great advantages are obtained using non-radial blades. The same efficiency gain is in fact obtained, and only a slighter decrease in $\pi$ (around 3-4%) is obtained. A completely different situation is encountered when choosing low numbers of blades (5 blades machines in the left side of Figure 12). A further 3.5% reduction in energy saving is in fact obtainable by the non-radial device, with a not-negligible 8% reduction in the rotor radius.
An optimization procedure has been developed to define the best two-stages inter-cooled rotary vane compressor design, in terms of ideal energy consumption; to this aim, a geometric model and a fluid dynamic model have been conceived and the results of the model have been compared with theoretical limits.

The geometric model considers a compressor characterized by a circular stator wrapping a portion of the rotor. By this approach, every two-stages compressor obtained by two consecutive circular lobes stators may be represented. In order to minimize overall machine dimension for a fixed flow rate delivered, it’s convenient to impose the maximum possible eccentricity to the first compression stage. For the same reasons, it was also chosen to wrap completely the rotor by the two stators.

A detailed derivation of the model, together with the description of more general stator shapes, is not here reported for space constraints, but will be published soon in a separate dedicated paper.

The presence of non-radial blades and the necessity of volume matching (not to produce isochoric transformation) between the two stages have been taken into account.

The optimization criteria were the lower efficiency ratio $\eta_{bi}$ design and the minimum geometric scale factor $\Omega$ (this latter being used if a family of devices are characterized by the same minimum $\eta_{bi}$ value).

The results consist in the definition of the optimal geometrical configurations for a varying overall compression ratio and number of blades. For the optimal devices, the procedure evaluates the foreseen performance, in terms of efficiency ratio and geometric scale factor.

The analysis has been previously conducted for radial blades compressors, showing that, for some design choices (low number of blades) and/or operative condition (high desired overall compression ratios) theoretical limits, in terms of energy saving cannot be reached (leading to a loss of up 1/3 of the potential benefit). The paper shows that this occurrence is due to the incapability of providing the needed compression ratio, so making it necessary to complete the transformation by means of a further isochoric compression at the second stage discharge port opening. The last part of the present work demonstrates that this limit can be overcome properly tilting the blades (forward in the direction of motion from 20 to 45 degrees).

Using non-radial blades, in fact, allows to increase the obtained compression ratio and, in addition, the inlet volume, so reducing the geometric scale factor of the machine (which is proportional to the rotor dimension for a given delivered flow rate) up to 8%. The choice between increasing the number or tilting the blades is then a matter of production costs.

In a further paper the same Authors will report also on the different behavior of the compressors in off-design conditions and for various user profile and regulation strategies.
REFERENCES

Cipollone, R., Contaldi, G., Villante, C., Tufano, R., 2006, A theoretical model and experimental validation of a sliding vane rotary compressor, 18th International Compressor Engineering Conference at Purdue.

ACKNOWLEDGEMENT

The Authors would like to give an acknowledgement to the Ing. Enea Mattei S.p.A in the person of Dr. Giulio Contaldi and Professor Roberto Cipollone (University of L’Aquila) for the sponsorship, the industrial know-how and the precious advices provided about model derivation.