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Shiyu Feng
Xi'an Jiaotong University

Xiufeng Gao
Xi'an Jiaotong University

Xiangfeng Shi
Zhuhai Landa Compressor Co.

Zhaolin Gu
Xi'an Jiaotong University

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Essential Geometric Theory of Internal-Meshing Rotary Compressor

Shiyu FENG[1*], Xiufeng GAO[1], Xiangfeng SHI[2], Zhaolin GU[3]

[1] Power and Energy Engineering School of Xi'an Jiaotong University,
Xi'an, 710049, China
Phone: +86-29-82664928, Email: shiyuf@mail.xjtu.edu.cn

[2]Zhuhai Landa Compressor Co., Ltd.,
2097 middle of juzhou road, Zhuhai City, Guangdong Province, China
Phone: +86-756-3324935, Email: lucky-casino@163.com

[3] Civil and Human Settlements Engineering School of Xi'an Jiaotong University,
Xi'an, 710049, China
Phone: +86-29-82669565, Email: zhaolingu@mail.xjtu.edu.cn

ABSTRACT

A new type of compressors named "internal-meshing rotary compressor" is proposed in this paper. This compressor could be applied in carbon dioxide trans-critical refrigerating cycle perfectly due to it simple structure, self-balanced initial mass and endurance of high pressure. However, there are few basic geometric theories to guide the design including the generation of profiles, mesh relationship of rotors and et al. In this paper, firstly, the essential equations of profiles were derived based on equidistant curve of curtate hypocycloid for inner rotor and multi-section circular arc for outer rotor. Then, the mesh relationship, position of instantaneous center, meshing range of the tooth tip arc and the modification of profile were discussed. Those basic geometric relationships could be the foundation for the application of this compressor.

1. INTRODUCTION

The traditional Freon refrigerants are damaged to the global environment, so how to solve this problem is facing to the human being urgently. Meanwhile, alternative to working substances is one of the quickest and most effective methods to prevent ruin of the ozonosphere. There are many kinds of alternative refrigerants and among them CO_2 is one of the most potential ones due to its definitely non-pollution, high latent heat, high specific volumetric refrigerating capacity, and perfect heat transfer property. Especially, the invention of the trans-critical cycle is the symbol that CO_2 can be practically applied in the commercial areas [1,2].

However, the key barrier to the application of CO_2 refrigerating system is to seek a more reliable and cheaper compressor. It is reported swing compressors, scroll compressors, two-stage rolling piston compressors and et al are being used in different air conditioners and heat pumps[3], nevertheless, till now, these compressors have disadvantages of short operating life, low efficiency or high manufacture cost and et al. Generally, the conventional trans-critical CO_2 cycle's performance coefficient is much lower than that of R22 cycle, so many methods have to be employed to reduce the input power. Among them, adding an expander to retrieve the expansion work forming a so-called "compression-expansion trans-critical CO_2 cycle" is one of the most effective methods. But, such a hybrid mechanism is not easy to be made.

The internal-meshing rotary mechanism is not a new structure and it is widely employed in internal-meshing rotary oil pumps[4-10] and planetary gear with small teeth difference[11-13]. There are few reports on its application to compress gas. In 1990, a Japanese patent[14] announced by Matsushita Electrical Co., Ltd. (Japan) mentioned a hermetic coolant pump to be applied in the domestic air-conditioners. In 1984, Sargent-Welch Scientific Company published a US patent related to the application of the similar mechanism for vacuum pumps[15]. In 2001, Yu

described the basic structure of a new rotary gear compressor[16]. In 2005, Gu and el al proposed an internal-meshing rotary compression-expansion hybrid mechanism[17] which could be adopted by CO_2 refrigerating system.

The planetary gear's purpose is mainly to transfer torques, so its theory is rather different from the fluid machinery and can not be employ to guide the design of the internal-meshing rotary compressor. Meanwhile, the internal-meshing rotary mechanism is chiefly applied in oil pumps and oil pump is not the "precious" machinery. The current study on this mechanism's essential theory is inadequate including the geometric description of profiles, the calculation of working chambers' volume, the mesh relationship of rotors, forces on rotors and et al. High quality oil pumps can be still manufactured without support of those theories, but it is impossible to produce the internal-meshing rotary compressor due to its more complicated working process, smaller clearance between rotors and higher manufacture precision. As what mentioned above, though several scholars and companies announced and describe the basic structures of the internal-meshing rotary compressor and compressor-expansion hybrid machinery, no more further reports could be obtained to detailedly explain their geometric theory.

An internal-meshing rotary compressor is mainly comprised of two eccentrically placed rotors shown in fig.1. Generally, the shaft is connected with the inner rotor and it could drive the outer rotor, so the former is called "drive rotor" and the later is "driven rotor". Certainly, the outer rotor could also drive the inner rotor if it is bind with the shaft. Several variant hermetic working chambers come into being because two rotors' profiles will contact each others forming so-called "meshing points". When the drive rotor is rotated, the volume of the chamber changes from the maximum to minimum. Firstly, the fluid is sucked into the maximal chamber, then, the fluid is compressed in this chamber until it is discharged. Two rotors rotate along their own centers and there is no initial force, so it is unnecessary to set any balance mass like what have to be done for the scroll, rolling piston and other rotary compressors. From fig.1, it also can be seen, those chambers will not be interfered with each other and the chamber will be isolated after suction process, so this compressor could be operated normally even without suction and discharge valves. As well, it can be found, both rotors are able to suffer high pressure and pressure difference. It is reported that a one-stage internal-meshing rotary oil pump could compress oil from 0.1MPa to 25MPa easily. Therefore, this compressor can be satisfied with CO_2 trans-critical cycle because the maximal pressure of this cycle is below 15MPa. The most attractive advantage of this structure is several pair of rotors can be put into one shell and all inner rotors share one shaft. Hereby, this mechanism can be extended to build a compression-expansion hybrid machine easily.

A compressor's structure is quite similar to an expander, hence, in this paper, only the compressor will be studied.
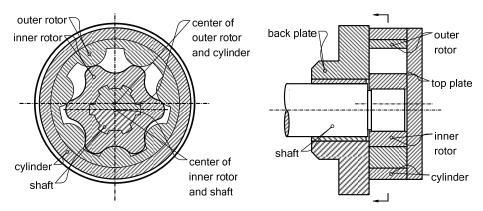

Figure 1: The schematic structure of the internal-meshing rotary compressor

According to the basic theory of differential geometry, any curve could obtain a conjugated one, but not all curves are fit for the internal-meshing rotary compressor. The potential curves for the inner rotor are: multi-section circular arc, multi-section elliptical arc, equidistant curve of curtate hypocycloid and et al. Generally, equidistant curve of curtate hypocycloid is most suitable. In this paper, the essential geometric theory adopting this kind of curves will be derived step by step.

2. ESTABLISHMENT OF BASIC PROFILES
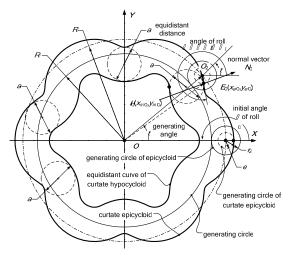
2.1 Profile of inner rotor

Figure 2 The generation of the inner rotor's profile

A circle (base circle) of radius R_1 is located in the center of the coordinate system. When another circle (generating circle) of radius r_2 rolls along this circle, the standard epicycloid is generated. Apparently, a concentric circle (eccentric circle) of radius e ($e<r_2$) rolling at a same angular velocity of the generating circle generates curtate epicycloid. $K=e/r_2$ is defined as short width coefficient. If $K=1$, curtate epicycloid decayed into epicycloid. Observe one point E_0 on curtate epicycloid, there is

$$\overrightarrow{E_0O} = \overrightarrow{O_2O} + \overrightarrow{E_0O_2} = \left[(R_1+r_2)\cos t - e\cos(\varphi+t+\varphi_0)\right] + \mathrm{i}\left[(R_1+r_2)\sin t - e\sin(\varphi+t+\varphi_0)\right] \tag{1}$$

If there are Z_1 teeth on the inner rotor, $Z_1=R_1/r_2$, so the parametric equation of curtate epicycloids is

$$\begin{bmatrix} x_{e,0}(t) \\ y_{e,0}(t) \end{bmatrix} = \begin{bmatrix} (R_1+r_2)\cos(t) - e\cos\left(\dfrac{R_1t}{r_2}+t+\varphi_0\right) \\ (R_1+r_2)\sin(t) - e\sin\left(\dfrac{R_1t}{r_2}+t+\varphi_0\right) \end{bmatrix} = \begin{bmatrix} R\cos(t) - e\cos(Z_2t+\varphi_0) \\ R\sin(t) - e\sin(Z_2t+\varphi_0) \end{bmatrix} \tag{2}$$

In order to place the maximal axis of curtate epicycloid on X axis, let $\varphi_0=\pi$. The normal unit vector of curtate epicycloids is

$$\overrightarrow{N}_0(t) = \begin{bmatrix} \dfrac{R\cos(t)+eZ_2\cos(Z_2t)}{\sqrt{R^2-2eZ_2R\cos(Z_2t+t)+e^2Z_2^2}} \\ \dfrac{R\sin(t)-eZ_2\sin(Z_2t)}{\sqrt{R^2-2eZ_2R\cos(Z_2t+t)+e^2Z_2^2}} \end{bmatrix} \tag{3}$$

The profile of inner rotor is equidistant curve of curtate hypocycloid, so its parametric equation is

$$\begin{bmatrix} x_{i,0}(t) \\ y_{i,0}(t) \end{bmatrix} = \begin{bmatrix} x_{e,0}(t) \\ y_{e,0}(t) \end{bmatrix} - a\begin{bmatrix} N_{x,0} \\ N_{y,0} \end{bmatrix} = \begin{bmatrix} R\cos(t)+e\cos(Z_2t) - \dfrac{a\left[R\cos(t)+eZ_2\cos(Z_2t)\right]}{\sqrt{R^2+2eZ_2R\cos(t-Z_2t)+e^2Z_2^2}} \\ R\sin(t)+e\sin(Z_2t) - \dfrac{a\left[R\sin(t)+eZ_2\sin(Z_2t)\right]}{\sqrt{R^2+2eZ_2R\cos(t-Z_2t)+e^2Z_2^2}} \end{bmatrix} \tag{4}$$

Where, a is the equidistant distance, t is the generating angle and $t\in[0,2\pi]$. When the inner rotor rotates θ_l degree, the equation is

$$\begin{bmatrix} x_i(t,\theta_i) \\ y_i(t,\theta_i) \end{bmatrix} = \overrightarrow{M}(\theta_i) \begin{bmatrix} x_{i,0}(t) \\ y_{i,0}(t) \end{bmatrix} \qquad (5)$$

Where, $\overrightarrow{M}(\theta_i)$ is spin matrix

$$\overrightarrow{M}(\theta_i) = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) \end{bmatrix} \qquad (6)$$

2.2 Profile of outer rotor

There are Z_2 ($Z_2=Z_1+1$) teeth on the outer rotor and each teeth contains a tip circular arc and a root circular arc shown in fig.3. The center of the outer rotor is located on X axis, but there is an eccentricity e between two centers of rotors. In order to simplify the analysis, move the outer rotor to the origin of the coordinate system. All centers of tip circular arcs are on a circle (center circle) of radius R ($R=R_1+r_2$) and the tip circular arc's radius is a; the center of root circular arc is as same as the origin and the root circular arc's radius is R_3, there is

$$R_3 = R + 2e - a = R_1 + r_2 + 2e - a \qquad (7)$$

The parametric equation of the tip circular arc is

$$\begin{bmatrix} x_{o,tip,0}(j,\phi) \\ y_{o,tip,0}(j,\phi) \end{bmatrix} = \overrightarrow{M}[\alpha(j)] \begin{bmatrix} R + a\cos(\phi) \\ a\sin(\phi) \end{bmatrix} \qquad (8)$$

Where

$$\overrightarrow{M}[\alpha(j)] = \begin{bmatrix} \cos[\alpha(j)] & -\sin[\alpha(j)] \\ \sin[\alpha(j)] & \cos[\alpha(j)] \end{bmatrix} \qquad (9)$$

$\alpha(j) = 2\pi(j-1)/Z_2 \qquad j = 1,2,...,Z_2$

The central angle of tip arc $\phi \in [\phi_{min}, \phi_{max})$, and

$$\phi_{min} = \pi - \cos^{-1}\left[\left(a^2 + R^2 - R_3^2\right)/(2aR)\right] \qquad (10)$$

$$\phi_{max} = \pi + \cos^{-1}\left[\left(a^2 + R^2 - R_3^2\right)/2aR\right] \qquad (11)$$

The parametric equation of the root circular arc is

$$\begin{bmatrix} x_{o,root,0}(j,\varphi) \\ y_{o,root,0}(j,\varphi) \end{bmatrix} = \overrightarrow{M}[\alpha(j)] \begin{bmatrix} R_3\cos(\varphi) \\ R_3\sin(\varphi) \end{bmatrix} \qquad (12)$$

Where, the central angle of root arc $\varphi \in [\varphi_{min}, \varphi_{max})$, and

$$\varphi_{min} = \cos^{-1}\left[\left(R^2 + R_3^2 - a^2\right)/2RR_3\right] \qquad (13)$$

$$\varphi_{max} = 2\pi/Z_2 - \cos^{-1}\left[\left(R^2 + R_3^2 - a^2\right)/(2RR_3)\right] \qquad (14)$$

Move the outer rotor back to its actual position and it rotates θ_o degree, so the tip circular arc is

$$\begin{bmatrix} x_{o,tip}(j,\phi,\theta_o) \\ y_{o,tip}(j,\phi,\theta_o) \end{bmatrix} = \overrightarrow{M}[\alpha(j)+\theta_o] \begin{bmatrix} R + a\cos(\phi) \\ a\sin(\phi) \end{bmatrix} + \begin{bmatrix} e \\ 0 \end{bmatrix} \qquad (15)$$

The root circular arc is

$$\begin{bmatrix} x_{o,root}(j,\varphi,\theta_o) \\ y_{o,root}(j,\varphi,\theta_o) \end{bmatrix} = \overline{\boldsymbol{M}}[\alpha(j)+\theta_o] \begin{bmatrix} R_3\cos(\varphi) \\ R_3\sin(\varphi) \end{bmatrix} + \begin{bmatrix} e \\ 0 \end{bmatrix} \qquad (16)$$
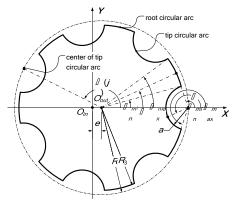

Figure 3: The generation of the outer rotor's profile

3. MESH RELATIONSHIP BETWEEN ROTORS

3.1 Angular velocity of inner and outer rotors

Two rotors contact each other at Z_2 meshing points and rotate with the certain angular velocities. However, the angular velocities of two rotors are different. In terms of the basic mesh principle of conjugate curves, these two angular velocities have to satisfy the following relationship

$$\frac{\theta_i}{\theta_o} = \frac{Z_2}{Z_1} = \frac{Z_1+1}{Z_1} \qquad (17)$$

$$\frac{\omega_i}{\omega_o} = \frac{Z_2}{Z_1} = \frac{Z_2}{Z_2-1} \qquad (18)$$

So,

$$\theta_o = \theta_i \cdot \frac{Z_1}{Z_2} \qquad (19)$$

$$\theta_i = \theta_o \cdot \frac{Z_2}{Z_1} \qquad (20)$$
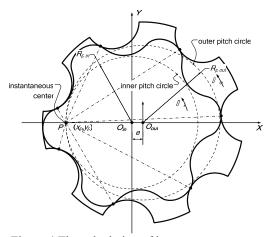
3.2 instantaneous center

Figure 4 The calculation of instantaneous center

According to Kennedy's theorem, all normal vectors of meshing points will pass through a same point and this point is called "instantaneous center"[18]. Each rotor has a unique pitch circle and two pitch circles are tangent at this

instantaneous center shown in fig.4. When rotors rotate, pitch circles rotate too and their angular velocities are as same as those of rotors.

Because

$$\frac{\omega_i}{\omega_o} = \frac{Z_2}{Z_1} = \frac{R_{p,o}}{R_{p,i}} \tag{21}$$

$$R_{p,o} - R_{p,i} = e \tag{22}$$
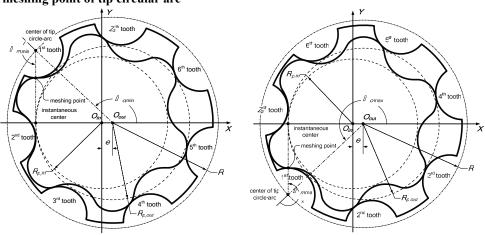
Then,

$$R_{p,i} = Z_1 \cdot e \tag{23}$$

$$R_{p,o} = Z_2 \cdot e \tag{24}$$

So, the coordinate of instantaneous center $\vec{P}$ is

$$\vec{P} = \begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} -Z_1 \cdot e \\ 0 \end{bmatrix} \tag{25}$$

3.3 Meshing range of tip circular arc
(1) Limiting meshing point of tip circular arc

(a) upper limiting meshing point　　　　(b) lower limiting meshing point
Figure 5 Limiting meshing positions of tip circular arcs

First, from Figure 4, it can be seen that all root circular arcs will not participate into the meshing process. Though tip circular arcs will be meshed with the profile of the inner rotor, not the whole tip circular arc is useful. Two limiting meshing points of the 1st tooth are shown in Figure 5 (other teeth are similar) and are called "upper limiting meshing point" and "lower limiting meshing point" respectively. Obviously, the connecting line between the limiting meshing point and the instantaneous center is vertical to X axis.

The minimal meshing angle of the tip circular arc is

$$\phi_{m,\min} = \pi - \sin^{-1}(R_{p,out}/R) \tag{26}$$

The maximal meshing angle of the tip circular arc is

$$\phi_{m,\max} = \pi + \sin^{-1}(R_{p,out}/R) \tag{27}$$

When the outer rotor rotates to the upper limiting meshing point, the rotate angle of the outer rotor is

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$$\theta_{o,\min} = \pi - \cos^{-1}(R_{p,out}/R) \tag{28}$$

When the outer rotor rotates to the lower limiting meshing point, the rotate angle is

$$\theta_{o,\max} = \pi + \cos^{-1}(R_{p,out}/R) \tag{29}$$

(2) Relationship between rotating angle of outer rotor θ_o and central angle of tip arc ϕ
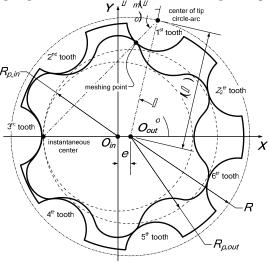

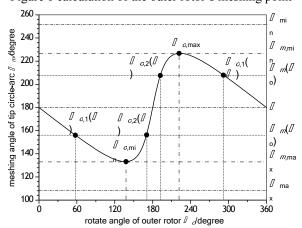Figure 6 calculation of the outer rotor's meshing point

Figure 7 The relationship between rotate angle and meshing angle of the 1st tip circular arc

The distance between the center of the outer rotor and the center of the 1st tip circular arc is

$$l(\theta_o) = \sqrt{R^2 + R_{p,out}^2 - 2RR_{p,out}\cos(\pi - \theta_o)} \tag{30}$$

So, the meshing angle of the 1st tip circular arc related to the rotate angle of the outer rotor is

$$\phi_m(\theta_o) = \pi - \sin^{-1}\left[\frac{-R_{p,out}\sin\theta_o}{\sqrt{R^2 + R_{p,out}^2 + 2RR_{p,out}\cos\theta_o}}\right] \tag{31}$$

The meshing angle of the jth angle is

$$\phi_m(j,\theta_o) = \pi - \sin^{-1}\left[\frac{-R_{p,out}\sin(\theta_o + \alpha(j))}{\sqrt{R^2 + R_{p,out}^2 + 2RR_{p,out}\cos(\theta_o + \alpha(j))}}\right] \tag{32}$$

The relationship between the meshing angle of the 1st tip circular arc and the rotate angle of the outré rotor is shown in Figure 7 and it can be seen that though the range of the tip circular arc is $\phi \in [\phi_{min},\ \phi_{max}]$, the effective range is $\phi \in [\phi_{m,min},\ \phi_{m,max}]$.

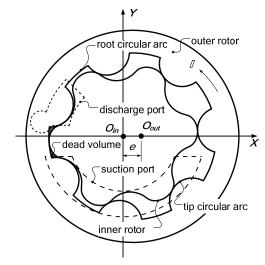## 4. MODIFICATION OF PROFILE FOR OUTER ROTOR

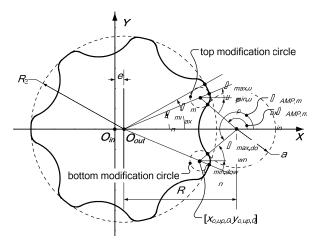Figure 8: schematic diagram of a unmodified profiles

Figure 9: Arc modification of profile

4.1 Purpose of modification
It has been discussed in section 2.2, the original profile of outer rotor is consist of tip and root circular arcs, so there exists two serious problem: 1) When the working chamber reaches to the minimum, the volume is not zero, so there is dead volume shown in Figure   8. The high pressure working fluid in the dead volume will expand when it connects with the suction port and the effective swept volume reduces sharply; 2) A sharp angle is formed at the point of junction between the tip and root circular arc. Hence, conventional machining method can not be applied to make the outer rotor. Obviously, the cost will increase.

In order to overcome these shortcomings, the profile of outer rotor should be modified to reduce the dead volume and remove the sharp angle. There are many modification methods and the most precious one is called PMP

modification (Perfect Modification of Profile). According to the mesh principle, under a certain transmission ratio, any profile of the outer rotor could obtain a perfect enveloping curve and this curve is the theoretical limiting modification profile. However, PMP modification is too complicated to be actually applied into the design of the inter-meshing rotary compressor, so in the present paper, another simple and practical modification method called AMP modification (Arc Modification of Profile) will be proposed.

4.2 AMP modification

Arc is one of the easiest and most simple curves which can be manufactured by the conventional machining method. As what has been mentioned above, only part of the tip circular arc will be participated into the meshing process and the whole root circular arc is useless in this process, so another arc of radius r_4 called "modification arc" is employed to modify the profile shown in Figure 9. This modification arc is the common tangent arc of the tip and root circular arc so the sharp angle between the tip and root circular arc is removed. As well, the medication arc will intrude into the dead volume so the dead volume will reduce also.

One tooth of the outer rotor needs two modification arcs and they are called "top modification arc" and "bottom modification arc" respectively. Hence, compared with the original profile shown in Figure 3, totally, $2 \times Z_2$ sections of modification arcs are required. Additionally, the range of the tip and root circular arc's central angle is changed.

The new central angle of the tip circular arc is $\phi \in [\phi_{AMP,\min}, \phi_{AMP,\max})$, where

$$\phi_{AMP,\min} = \pi - \cos^{-1}\left[\frac{(a+r_4)^2 + R^2 - (R_3 - r_4)^2}{2 \cdot (a+r_4) \cdot R}\right] \tag{33}$$

$$\phi_{AMP,\max} = \pi + \cos^{-1}\left[\frac{(a+r_4)^2 + R^2 - (R_3 - r_4)^2}{2 \cdot (a+r_4) \cdot R}\right] \tag{34}$$

Also, the new central angle of the root circular arc is $\varphi \in [\varphi_{AMP,\min}, \varphi_{AMP,\max})$

$$\varphi_{AMP,\min} = \cos^{-1}\left[\frac{R^2 + (R_3 - r_4)^2 - (a+r_4)^2}{2R(R_3 - r_4)}\right] \tag{35}$$

$$\varphi_{AMP,\max} = \frac{2\pi}{Z_2} - \varphi_{AMP,\min} \tag{36}$$

In order to simplify the study, let the rotate angle $\theta_o = 0$ and move the outer rotor to the origin of the coordinate system. The parametric equation of the 1st top modification arc can be derived

$$\begin{bmatrix} x_{o,up,0}(\gamma) \\ y_{o,up,0}(\gamma) \end{bmatrix} = \begin{bmatrix} x_{o,up,o} + r_4 \cdot \cos(\gamma) \\ y_{o,up,o} + r_4 \cdot \sin(\gamma) \end{bmatrix} \tag{37}$$

Where, the central angle of the top modification angle is $\gamma \in [\gamma_{up,\min}, \gamma_{up,\max}]$, where

$$\gamma_{up,\min} = \phi_{AMP,\min} - \pi \tag{38}$$

$$\gamma_{up,\max} = \varphi_{AMP,\min} \tag{39}$$

The center of the 1st top modification is

$$\begin{bmatrix} x_{o,up,o} \\ y_{o,up,o} \end{bmatrix} = \begin{bmatrix} \dfrac{R^2 + (R_3 - r_4)^2 - (a+r_4)^2}{2R} \\ \sqrt{(R_3 - r_4)^2 - \dfrac{[R^2 - (a+r_4)^2 + (R_3 - r_4)^2]^2}{4R^2}} \end{bmatrix} \tag{40}$$

The equation of the 1st bottom modification arc is

$$\begin{bmatrix} x_{o,down,0}(\gamma) \\ y_{o,down,0}(\gamma) \end{bmatrix} = \begin{bmatrix} x_{o,down,o} + r_4 \cdot \cos(\gamma) \\ y_{o,down,o} + r_4 \cdot \sin(\gamma) \end{bmatrix} \tag{41}$$

Where, $\gamma \in [\gamma_{down,\min}, \gamma_{down,\max}]$

$$\gamma_{down,\min} = -\varphi_{AMP,\min} \tag{42}$$

$$\gamma_{down,\max} = \pi - \phi_{AMP,\min} \tag{43}$$

The center is

$$\begin{bmatrix} x_{o,down,o} \\ y_{o,down,o} \end{bmatrix} = \begin{bmatrix} \dfrac{R^2 + (R_3 - r_4)^2 - (a + r_4)^2}{2R} \\ -\sqrt{(R_3 - r_4)^2 - \dfrac{[R^2 - (a + r_4)^2 + (R_3 - r_4)^2]^2}{4R^2}} \end{bmatrix} \tag{44}$$

Move the outer rotor back to the original position, so the parametric equations of the jth top and bottom modification arc are

$$\begin{bmatrix} x_{o,up}(j,\gamma,\theta_o) \\ y_{o,up}(j,\gamma,\theta_o) \end{bmatrix} = \overline{\boldsymbol{M}} \left[\alpha(j) + \theta_o\right] \begin{bmatrix} x_{o,up,0}(\gamma) \\ y_{o,up,0}(\gamma) \end{bmatrix} + \begin{bmatrix} e \\ 0 \end{bmatrix} \tag{45}$$

$$\begin{bmatrix} x_{o,down}(j,\gamma,\theta_o) \\ y_{o,down}(j,\gamma,\theta_o) \end{bmatrix} = \overline{\boldsymbol{M}} \left[\alpha(j) + \theta_o\right] \begin{bmatrix} x_{o,down,0}(\gamma) \\ y_{o,down,0}(\gamma) \end{bmatrix} + \begin{bmatrix} e \\ 0 \end{bmatrix} \tag{46}$$

4. CONCLUSION

1) The basic structure and operational principle of the internal-meshing rotary compressor are discussed in the paper. It concludes that the study of its essential geometric theory is quite necessary.

2) Based on a proper coordinate system, equidistant curve of curtate hypocycloid and multi-section arc are adopted to establish the parametric equations of profiles for the inner and outer rotor respectively.

3) The mesh relationship, the coordinate of instantaneous center and the meshing range of the tip circular arc are derived.

4) A simple and practical modification method called "Arc Modification of Profile" is studied in this paper and via the modification, the dead volume can reduce, the sharp angle can be removed and the conventional machining tool is able to be employed to manufacture the outer rotor.

5) More geometric and mechanical theories including the volume of working chambers, the modification of profiles, the relative speed between meshing points, forces on rotors and et al will be introduced in the future.

NOMENCLATURE

a	equidistant distance	**Subscripts**
e	eccentricity	1, *in*, *i* inner rotor
i	transmission ratio	2, *out*, *o* outer rotor
K	short width coefficient	
R_1	radius of base circle	
r_2	radius of generating circle	
R_3	radius of center circle	
r_4	radius of modification circle	
t	generating angle of cycloid	
x, y	coordinate	

Z	number of teeth
γ	central angle of modification arc
φ	central angle of root arc
θ	rotate angle
ϕ	central angle of tip arc
ω	angular velocity

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