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Jan Hauser
TU Dortmund

Andreas Bruemmer
TU Dortmund

Knut Kauder
TU Dortmund

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Rotor Profile Generation and Optimization of Screw Machines using NURBS

Jan HAUSER[1], Andreas BRUEMMER[2], Knut KAUDER[3]

[1]Chair of Fluidics, TU Dortmund,
Dortmund, Germany
Contact Information (Phone +49 231 755 -5725, Fax -5722, Jan.Hauser@tu-dortmund.de)

[2]Chair of Fluidics, TU Dortmund,
Dortmund, Germany
Contact Information (Phone +49 231 755 -5720, Fax -5722, Andreas.Bruemmer@tu-dortmund.de)

[3]Chair of Fluidics, TU Dortmund,
Dortmund, Germany
Contact Information (Phone +49 231 755 -5494, Fax -5722, Knut.Kauder@tu-dortmund.de)

ABSTRACT

The geometrical representation option for rotor profiles in screw machines provided by the application of Non-Uniform Rational B-Splines (NURBS), permits a display format which is suitable for both the approximation of rotor profiles and as part of an optimisation strategy for screw rotors. A comparison of different forms of representation for the crown section of rotor profiles reveals that high representational accuracy of screw rotor forms can be achieved by the use of NURBS, with relatively low information content. This type of representation can be applied both as an optimisation tool for the three-dimensional portrayal of rotor geometries and in rotor manufacturing procedures.

1. INTRODUCTION

The production-specific development of screw machines is a compromise between the minimisation of pure production/manufacturing costs and the resulting efficiency of the machine. An important area in the development of screw machines remains the construction of the screw rotors, as these are responsible for the meshing characteristics of the gearing, which determines the form and position of the gaps which border the working chambers. The utilisation of the constructed space in relation to the actual machine-specific gaps has a significant effect on the performance of the machine, e.g. its efficiency and operational safety factors. In particular there is scope for improvement by reducing the length of the intermesh clearance line by varying the rotor geometries, fig. 1. Splitting up the profile contours produces profile meshing sections which are generated by the corresponding profile areas. Varying these areas allows us to make conjectures about the type of profile geometry which would shorten the clearance line. The influence of the meshing sections on the overall performance of the machine can also be varied.

Various objectives in the area of profile development were pursued in order to further reduce the gap areas, and also to integrate improved production aspects into the form of the profiles. These measures were based on existing profiles. At present, depending on application and field of operation, established rotor geometries are normally used. The representation of screw gearing is usually carried out by means of several curved sections, which are defined partly on the secondary rotor, but also on the main rotor. These curves can be straight lines, arcs, evolvents, cycloids

and also equidistances to cycloids [Rinder, 1979]. A further profile theory for construction according to the Law of Gearing is provided by using the Profile Gradient Function method. This provides a comprehensive representation system, which includes screw rotor gearing [Steffens, 1993]. Because of the high degree of mathematical complexity and the resulting complexity of transferring this method to screw motor geometries, it is rarely deployed in this field.

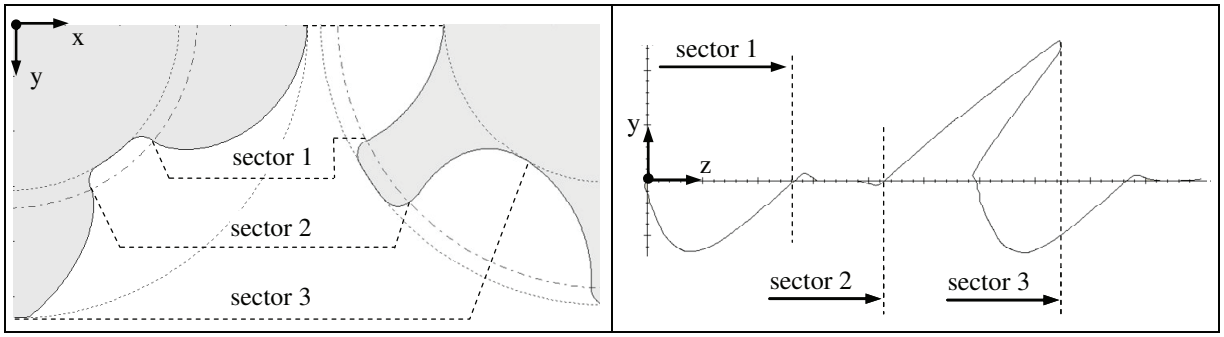

Figure 1: Interrelationship between profile geometry and meshing clearance line
a) Profile geometry area, b) Meshing clearance line area

In addition to these approaches parameterised curve types, so-called second degree Bézier segments have also been used for the representation of a rotor flank [Helpertz, 2003]. Using this form of representation can be justified through the deployment of an automated, optimised design for displacement rotation machines. In this case the energy conversion efficiency of a screw machine can be classified according to a previously defined evaluation of operating figures (also designated as physical abstractions). These operating figures make possible a comparative assessment of different front section geometries. Solving such optimisation problems, which involve a large number of technical criteria, even infinitesimal deviations from the normal characteristics of a second grade Bézier segment, compared with an analytical curve, can lead to serious theoretical meshing faults on the rotor flank in question.

Rotor profiles which are not in accordance with the Law of Gearing were excluded from the optimisation system, and when applied using a simple spline-type, they seriously detracted from an optimised solution. The application of the computer-supported optimisation method for screw rotor geometries can nevertheless be regarded as successful in spite of these limitations, and it shows that a manipulation of the rotor geometry directed towards application-orientated requirements is definitely possible and worth the effort.

2. PRESENTATION FORMS OF ROTOR PROFILES

In order to optimise rotor profiles with the help of key operating figures, it is necessary to select a suitable approach for an improved representation of this type of profile. Reducing the informational content while retaining the best possible quality of the representation makes it possible to carry out a more effective computer-supported optimisation process. The form of representation must continue to support the rotor kinematics according to the Gearing Law, which requires information relating to the orientation of a point on the plane, i.e. at the basis level.

2.1 Representational options for profiles

At present there are a number of representational forms which make possible an exact portrayal of the front section of a rotor profile. These forms are widely used in CAD. The rotor symmetry derived from the tooth division angle, which depends on the number of teeth, only requires the addition of profile flank dimensions to be able to depict a complete rotor profile.

The suitability of these forms of representation of the rotor flank results from the quality of the portrayal and the necessary informational content. In addition, this research into the representation of rotor geometry also takes into account the degree of freedom available for curve manipulation and the calculation requirements for validity checks on rotor profiles. The examination of different types of curves includes discrete contour points and line segments

respectively, second and third grad Bézier-polynomials, uniform non-rational basis-splines and also non-uniform rational basis-splines (NURBS), see fig. 2. It should be mentioned here that all forms of representation are considered within a rotor-compatible system of coordinates. To ensure that the types of curve in question are comparable with one another, the selected contour characteristics are those of a three-toothed rotor profile.

The representation of rotor profiles by means of contour control-points is a common method. The quality of the model depends on the number of points or line segments used. As a rule, depending on the number of teeth, 150-600 points are necessary for the depiction of a rotor flank. In addition, explicit information on the orientation of the discrete control- points to the secondary rotor calculation is necessary. The fact that this information is necessary is a serious disadvantage of this form of representation.

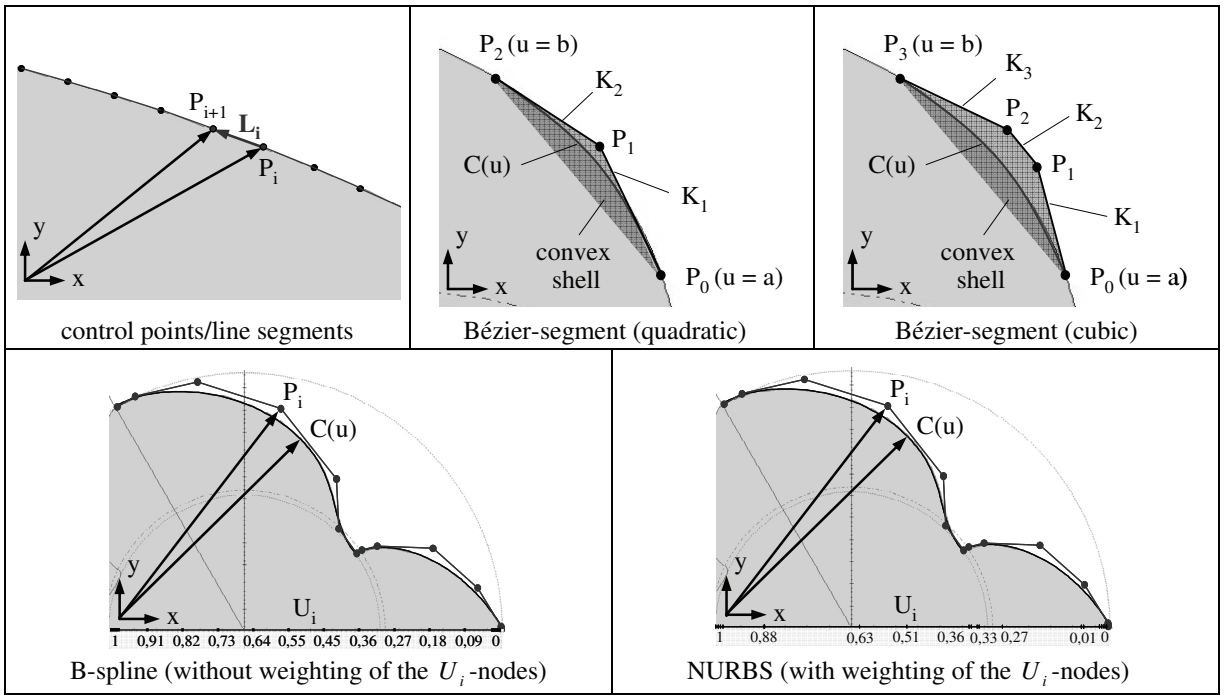

Figure 2: Representation options for rotor profiles

An alternative to discrete contour points is provided by parameterised splines, which include Bèzier segments, a more recent representation option for screw rotor geometries. They have already been used for a computer-supported optimising method [Helpertz, 2003, Berlik, 2006]. Compared with the discrete point method, this approach can portray more complex geometries while requiring considerably less informational content, but without any loss of quality. The basic advantage compared with an implicit functional representation lies in the uniqueness of the curve definition, which is achieved by means of a parameter representation which defines every single point on the curve at a specific interval. The representation of a rotor flank is generated with the help of several adjoining Bèzier segments. This approach is valid both for quadratic (p=2) and cubic (p=3) Bézier segments. An advantage of the cubic Bézier segments compared with the quadratics is the greater displacement range of the splines within the area of variation as a result of the additional control- point. A disadvantage of both forms is the global influence of the control points on curve characteristics when there is local spline manipulation. A complex consistency check at the transition points of two adjoining segments only leads to a rise in the number of non-valid profiles within the optimisation process.

However, B-splines, which consist of a combination of several polynomials, have the potential both to portray rotor flanks in general and also with reference to profile optimisation. In contrast to the approaches referred to earlier, the entire rotor profile can be represented by a single curve segment. Alongside the advantage that there is no direct interdependence between the number of control points and the polynomial degree of the B-splines, there is no need for a curve consistency check when points are adjusted. The number of points can be adapted to match the

complexity of the geometry, which enables precise manipulation of certain profile areas due to the local influence of the control points. There is a disadvantage related to the fact that the gap between nodes is constant, which leads to periodic weighting functions of the control- points. As the control- points cannot be weighted, the representation of specific geometries, e.g. a circle, by means of a segment, is not possible.

NURBS, on the other hand, include all the essential advantages of uniform, non-rational B-splines. An extension of the weighting factor for the control points broadens the scope for curve manipulation. A supplementary irregular node vector for NURBS-curves reinforces the influence of the points on the calculated form of the curve by weighting each individual point.

Table 1: Evaluation of the presentation forms (based on a flank profile)

Characteristics	**Line segments**	**Bézier** (quadratic)	**Bézier** (cubic)	**B-Spline**	**NURBS**
Representation quality	high	high	high	high	**high**
Number of segments per flank	530	36	25	1	**1**
Difficulty in moving points	high	high	high	low	**low**
Weighting of points	-	-	-	no	**yes**
Manipulating nodes	-	-	-	no	**yes**

The characteristics of the forms of presentation under examination are summarised in Table 1 above. It is fair to say that all the curve types referred to can depict a rotor profile with sufficient accuracy. The quality of a rotor flank portrayal with up to 530 line segments can be kept constant by using second or third degree Bézier segments with 36 or 25 segments respectively. A further improvement can be achieved by using B-splines or NURBS-curves, which enables a single curve segment to represent the whole profile flank. The ability to depict a constant curve progression, and the fact that validity checks after points have been moved are no longer necessary, confirms the high potential of these two forms of representation. In contrast to uniform, non-rational B-splines, NURBS-curves add further characteristics such as weighting of the control points, the ability to influence the nodes and alter the polynomial level without changing the number of control points.

Taken together, the advantages of the NURBS presentation form enable a high quality representation of profile geometry with low information content. NURBS can also be deployed as a tool for the optimising of screw rotor profiles.

2.2 Validation of NURBS via the optimising method

The automatic computer-supported optimisation of screw machine profiles will be carried out by means of evolutionary optimisation analyses (stochastic procedure). The applicability of NURBS-curves as an optimisation tool within this procedure is to be examined by means of a trivial solution, maximising the scoop area, compared with the Bézier method.

To this end, the mathematical construction of the NURBS- curve trajectory, as defined by the parametric form $\vec{C}(u)$ with the following equations (1 - 4), has been transferred to the application of rotor profiles [Piegl, 1997, Salomon, 2006].

$$\vec{C}(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u,\vec{U}) \cdot w_i \cdot \vec{P}_i}{\sum_{i=0}^{n} N_{i,p}(u,\vec{U}) \cdot w_i} \tag{1}$$

derives from the basis function $N_{i,p}(u,\vec{U})$

$$N_{i,p}(u) = \frac{u - \bar{u}_i}{\bar{u}_{i+p} - \bar{u}_i} \cdot N_{i,p-1}(u) + \frac{\bar{u}_{i+p+1} - u}{\bar{u}_{i+p+1} - \bar{u}_{i+1}} \cdot N_{i+1,p-1}(u) \tag{2}$$

whereas

$$N_{i,0}(u) = \begin{cases} 1 & ,if\ \bar{u}_i \le u \le \bar{u}_{i+1}, \\ 0 & else \end{cases} \tag{3}$$

depending on the polynomial level p and the node vector $\vec{U}$ together.

$$\vec{U} = \left(\bar{u}_0 = \ldots = \bar{u}_p = a,\ \bar{u}_{p+1}, \ldots, \bar{u}_n,\ \bar{u}_{n+1} = \ldots = \bar{u}_{n+p+1} = b\right) \tag{4}$$

When the scoop area is increased by $(C_1)_{2D} = Max\ (A_{scoopArea})$ there is a trivial maximisation problem, in that the number of teeth ($z_{male/female} = 3$) on the two start-rotors is the same. To deal with this problem, the entire profile area will in the following be subjected to the optimisation process. In the course of optimisation, convergence with a Roots profile is to be expected, independent of the choice of presentation type and a randomly-generated start-profile. The main characteristic of a Roots profile is the symmetrical form of the two rotors, whereby the pitch circle has the same clearance to crown and base circles. Figure 3 shows the progression of criterion $(C_1)_{2D}$ in logarithmic form via the optimisation steps *N,* for Bézier and NURBS representations.

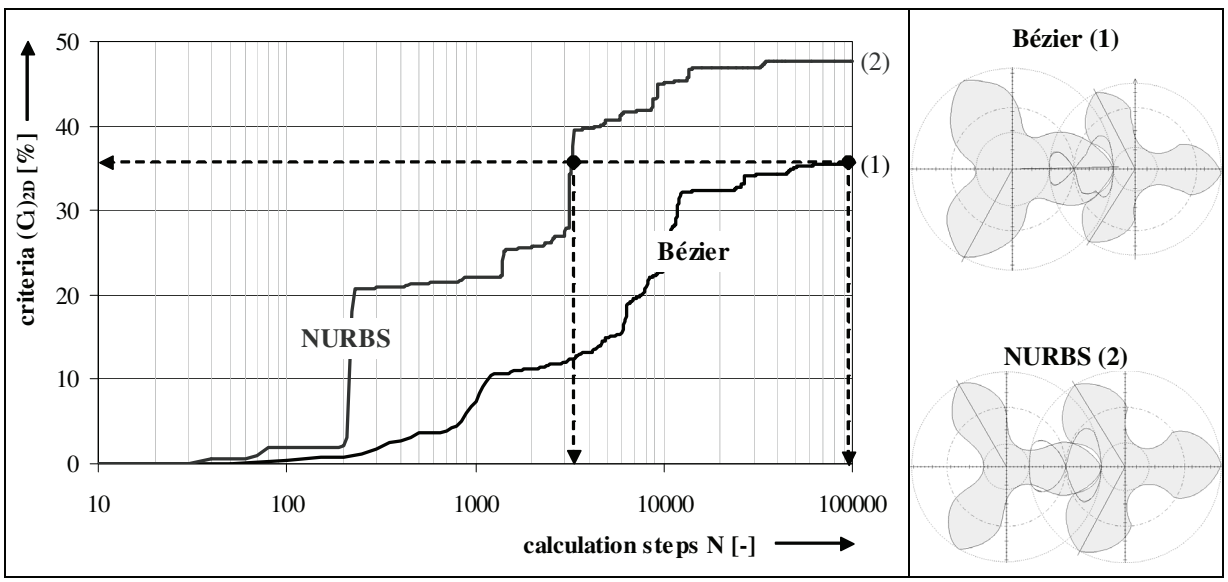

Figure 3: Scoop area maximisation with Bézier and NURBS

A comparison between two forms shows that the NURBS-curve with a polynomial level $p = 3$ achieves a higher optimisation speed. The optimisation state for the Bézier segments at $N = 10^5$ steps is achieved by the NURBS-curve after only $N = 3{,}2 \cdot 10^3$ steps. The convergence boundary for criterion C_1 is established for the NURBS-curve after about $4 \cdot 10^4$ steps, which corresponds with fulfilling the criterion to almost 50%. The Bézier-segment profile, on the other hand, achieves only 37%. We may conclude from this, that the optimisation method is significantly improved by the presentation and handling characteristics of the NURBS-curve used as an optimisation tool.

3. APPLYING THE OPTIMISING STRATEGY

In addition to a trivial task such as optimising scoop area, there are further influencing variables to optimise, some of them with opposing characteristics, with a view to improving both the operating performance of screw machines and the production of screw rotors [Weckes, 1994, Kauder, 2007].

3.1 Minimising of intermesh clearance

In the field of geometrical optimisation, the intermesh clearance and the blow hole should be mentioned as major factors influencing the performance of a machine along with the actual geometrical parameters. Attempts to select a typical intermesh clearance or blowhole in the crown section as assessment criteria led to contradictory results during optimisation. Due to the relatively powerful influence of rotor length and wrap angle on the intermesh clearance in three-dimensional rotors, a mere examination of the crown section is no longer acceptable. Three-dimensional gap progression is shown in figure 4a.

The length of the chamber curve as such is described, in compliance with the Law of Gearing, in equation (5) below, as characteristic $(C_2)_{3D}$.

$$(C_2)_{3D} = Min(l_{IMC}) = \int_{u=a}^{u=b} \sqrt{\left(x'(u)\right)^2 + \left(y'(u)\right)^2 + \left(\frac{l}{\Phi}\alpha'(u)\right)^2}\, du \qquad (5)$$

So that the three-dimensional intermesh clearance is kept to a minimum during optimisation, a localised optimisation is carried out. The profile area in question, which determines the area of the blowhole, is excluded from the optimisation. Thus optimisation takes place not along the whole of the profile flank, but only in the area of selected parameter boundaries of the curve.

The optimisation directed towards reducing the length of the three-dimensional intermesh clearance resulted in an improvement in the desired operating figures, and in the resulting new intermesh clearance, Figure 4b. The start profile for minimising characteristic C_2 has a long intermesh clearance (1), which is accounted for by a reversed progression in the area of negative coordinate values. The optimisation shows that when the size of the blowhole is kept constant, the intermesh clearance is modified by the profile form with earlier meshing, which means that the length can be significantly reduced (2).

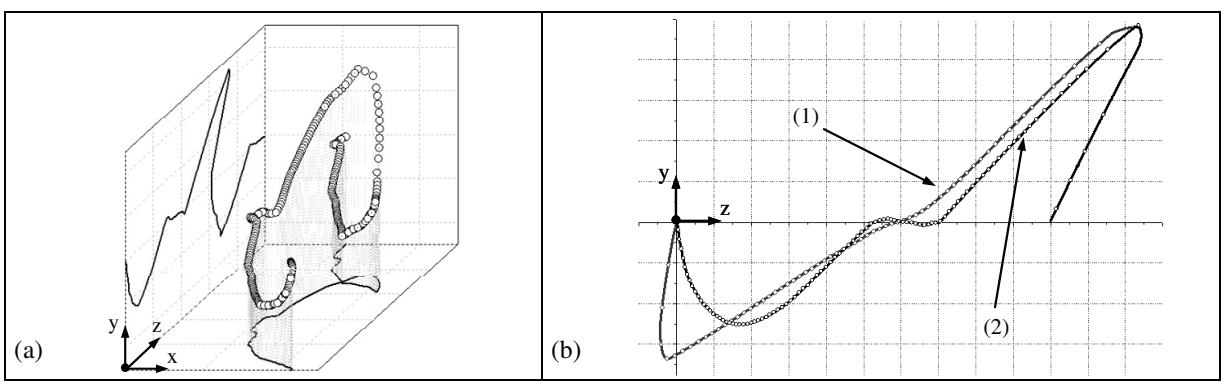

Figure 4: (a) Spatial curve of the intermesh clearance with projection levels (x-z, y-z) where z = length of rotor
(b) Clearance line of the start-profile (1), clearance line after minimising C_2 (2)

3.2 Reduction of undercut

In addition to a purely geometrical optimisation, operational figures suitable for production can also be generated, taking into account the aspect of easier manufacture. If the undercut on the secondary rotor is reduced, this results, for example, in lower stress conditions for suitable profile forms. The marked flank area of the secondary rotor in Figure 5a illustrates an example of this kind of undercut. A reduced undercut can be achieved by representing the contour progression of the tooth in the marked area by a regular straight form. Depending on the manufacturing processes, the maximum extent of the undercut can be laid down in advance, and described in terms of equation (6).

$$(C_3)_{2D} = Min(K_{curve}) = \left| \frac{1}{R_{curve}} \right| \tag{6}$$

The undercut problematic can be portrayed via curvature K of a curve in the area concerned. The curvature radius R of a curve at point P derived from this is the reciprocal of the value of the curvature. A limited amount of undercutting is therefore the same thing as a minimisation of the curvature at the point of the curve in question, leading to an increase in the curvature radius. The curvature characteristics equivalent to a straight line can be given as $K = 0$ and $R = \infty$.

The geometrical results provide the desired operational figures for optimising the undercut for different degrees of curvature, Figure 5. The same procedures apply as for the previous optimisation process.

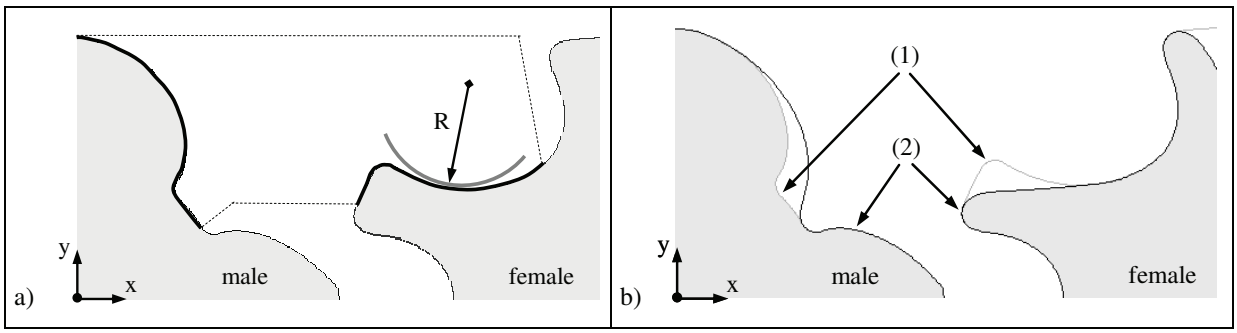

Figure 5: (a) Representation of a profile with undercut, curvature radius as reference code
(b) Reduction of the undercut with start-profile (1), end-profile (2)

In order to achieve the best possible outcome and the consequent high-grade operational figures for this optimisation, it is absolutely essential to set precise limits for the area of the profile flank on the secondary rotor. This can be applied for both the front and rear flanks of the rotor. Basically, the optimisation shows that presetting smaller degrees of curvature leads to a continuous reduction of the undercut. The smaller the curvature, the straighter the profile (2). However, this inevitably results in the tooth crest of the secondary rotor becoming progressively narrower. Very narrow teeth, in turn, tend to become distorted during machining, and are inefficient at dissipating heat generated during the manufacturing process. Consequently there are other specific requirements which need to be addressed in the operating codes. The development of appropriate geometry for production will therefore always be a compromise between a number of practical requirements and theoretical operating figures.

The results achieved for the optimisation are significant in this case, and also demonstrate the potential for this method, which enables both whole flanks and also limited areas to be evaluated and optimised by means of theoretical operating figures.

4. CONCLUSIONS

The NURBS representation is capable of improving the optimisation options for the complex geometry of screw machines, including profile variation. Compared with the previously-employed Bézier type of representation, NURBS has considerable improvement potential.

This is achieved both with regard to higher optimisation speeds for similar results, and in the higher degree to which the performance criteria are met, compared with Bézier segments. These claims can be substantiated by the increase in valid rotor profiles generated by the optimisation process, during which a continuous progression of the tooth flank is postulated, and the laborious checks for constant curvature which are required when the control points of two Bézier segments are moved can be dispensed with. In addition to the high quality of the representation and the relatively low information content, NURBS-curves used as an optimisation tool also possess a wide spectrum of manipulation options within the optimisation method.

However, the quality of the optimisation, i.e. arriving at the best solution, depends not only on the type of representation, but also, to a high degree, on the generation of the best possible theoretical operating codes. More precisely-defined user requirements, and their representation by means of a really meaningful operating code will make possible an improved optimisation of rotor geometries. An extension of the method to the representation of a three-dimensional rotor form and its associated gap properties will in future make possible the generation of further operating codes, resulting in an even better match between rotor geometry and specific practical applications.

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NOMENCLATURE

α	Angle between control point and associated contour point on the plane $z = 0$	$A_{scoopArea}$	Scoop area/surface
$\vec{C}(u)$	Parameterised curve	C_i	Operating figures/code
K_i	Control polygon	K_{curve}	Curvature of a curve
l	Rotor length	L_i	Line segment
l_{IMC}	Length of the three-dimensional intermesh clearance	$N_{i,p}(u,\vec{U})$	Basis function of degree p
$\vec{P}_i$	Control point of the curve	P_i	(Control) point
p	Degree of curvature	R_{curve}	Curvature radius at point P
u	Curve parameter with $u \in [a,b]$	$\vec{U}$	Node vector
$\bar{u}_{0...n+p+1}$	Parameter of the individual nodes of the node vector with $\bar{u}_i < \bar{u}_{i+1}$	Φ	Wrap angle
w_i	Weighting factor	x, y, z	Cartesian coordinates
2D, 3D	Two/three-dimensional		

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