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Semi-empirical Modeling of small size Radial turbines for refrigeration purpose

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ABSTRACT

Use of small size turbochargers in refrigeration (Joule/Brayton cycles), as those used by utility vehicles, demands real modeling of their behavior. This work uses the axial flow turbine model proposed by document ASHRAE TC 4.7, applying it to a Brazilian turbocharger. The model proposed by ASHRAE requires identification of several parameters as: incidence flow angle, impeller diameter, exit throttle area. The ASHRAE model was tested using real behavior curves, given by Brazilian manufactures. Our work shows that the identified parameters can not be constants when the ASHRAE model is applied to small radial turbines, and also our works show that the throttle area can not be identify using just a shock condition; it needs a mach number correction to represent with veracity the real behavior. We are proposing a new methodology to identify parameters that improves and gives good results for the HVAC component. Results obtained and physical explanations are offered for the new parameter identification methodology presented in this paper.

Keywords: Turbocharger, Joule cycle, air cycle

1. INTRODUCTION

Small radial turbines use has been increased in the last decade, because they deal with small volumetric flows, are compact and each day their efficiency is improved when compared with axial turbines. One of the most popular applications is the overfeed of engines using Otto or Diesel cycles and air cooling systems in airplanes and cryogenic systems. Normally these turbines are mounted with centrifugal compressor, named turbocompressors, Cohen et al (1996).

In radial turbines energy is transferred from the fluid to the rotor, when fluid is passing from a greater to a smaller wheel diameter. In order to develop work the product of the absolute tangential velocity by the turbine entrance rotating speed has to be bigger than the same product at exit turbine port.

Figure 1 shows a typical radial turbine configuration.

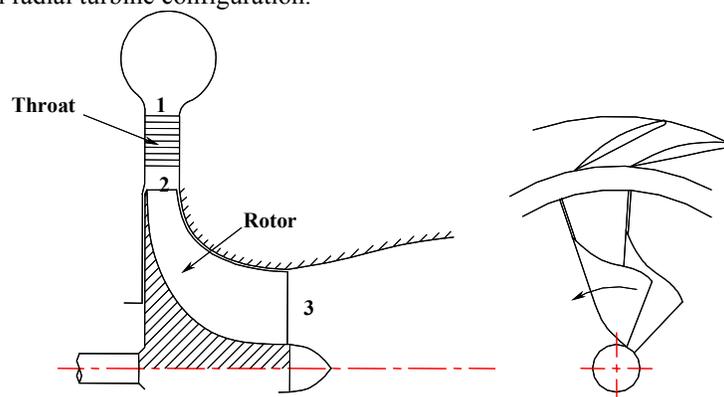


Figure 1 – Radial turbine typical configuration

Turbomachines basic operation is due to a continuous angular momentum change between the rotating element and the flow. This continuous change is responsible for the high volumetric capacity when compared with reciprocating machines. The volumetric flows obtained can go from 200 m³/hr to 3.000 m³/hr, with isentropic efficiencies up to 85% and pressure ratio per stage limited to 3, with 100.000 rpm wheel rotation, Giegel, et al. (2001).

This type of equipment is usually modeled according to Balje (1980) : using cascade data, channel arguments and velocity coefficients, Benson (1970) analysed several methods used to evaluate losses in radial turbines working outside of nominal conditions, losses are divided in throat losses including volute and rotor losses; according to Benson the throat losses are well described when compared with experimental data and the best works reported are those of Futral and Wasserbauer, Benson also shows that the flow exit angle has practically no effect on losses when compared to total losses and to flow throat angle.

Mseddi et al, (2002) developed a methodology for radial turbines based on the expansion relation in two states of the turbine ,one static and other dynamic using the isentropic efficiency and the Euler equation including sliding effects, found in the Stodola empiric relation, the methodology assumes steady state modelling ,one dimension, perfect gas and constant thermal capacity; the model uses geometric parameters and the errors found are of 3% for pressure ratio and 5,8% for efficiency.

ASHRAE TC-4.7 suggest a parametric methodology that does not require previous knowledge of geometric characteristics of axial gas turbines as follows.

2. SEMI-EMPIRICAL MODEL

The ASHRAE toolkit model proposed, says that centrifugal axial turbines can be operated in a similar way to the shocked D'Laval wheel, as shown in Fig. (2).

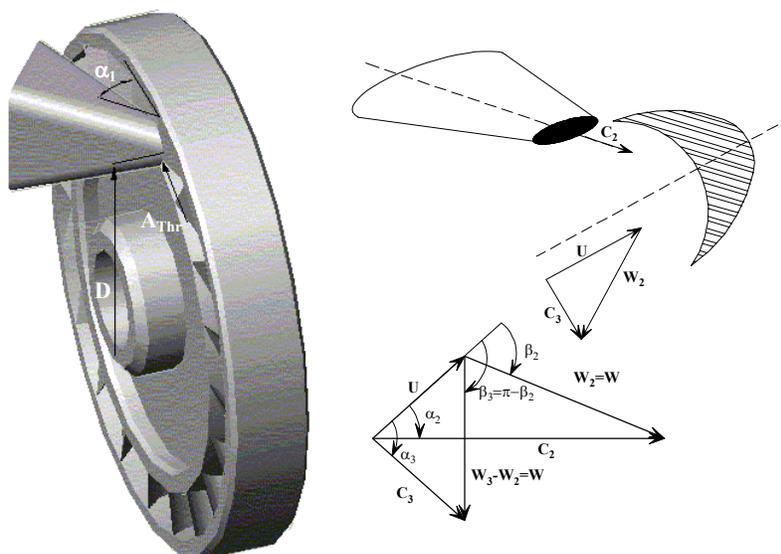


Figure 2: D'Laval model schematic representation

The work performed in this Laval stage, according to Euler theorem,

$$W_{Sh} = C_{U1} \cdot U_1 - C_{U2} \cdot U_2 = U \cdot (C_{41} \cos(\alpha) - C_{42} \cos(\alpha)) \quad (1)$$

Where:

$$U = \pi \cdot D \cdot N_2 \quad (2)$$

D -expander Diameter

N -Wheel rotate speed

From velocity triangle shown in Fig. (2)

$$C_{41} \cdot \cos(\alpha_1) = U + W \cdot \cos(\beta_1) \quad (3)$$

where

W - relative velocity speed

β_1 - inlet blade angle

$\beta_2 = \pi - \beta_1$ - outlet blade angle

from where

$$W_{Sh} = 2 \cdot U \cdot W \cdot \cos(\beta_1) = 2 \cdot U \cdot (C_{41} \cdot \cos(\alpha_1) - U) \quad (4)$$

Velocity C_{41} shown in fig (2), can be calculated from thermodynamic relationship for isentropic expansion

$$C_{41} = \sqrt{2 \cdot C_p \cdot T_{su} \cdot \left[1 - \left(\frac{P_{ex}}{P_{su}} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad (5)$$

Where

C_p - constant pressure specific heat

T_{su} - Supply temperature

P_{ex}/P_{su} - pressure ratio

γ - specific heat relation $\gamma = \frac{C_p}{C_v}$

The isentropic effectiveness (η) can be calculated as follows: efficiency

$$\eta = \frac{W_{ad}}{W_{sh}} = \frac{2 \cdot U \cdot (C_{41} \cdot \cos(\alpha_1) - U)}{\frac{1}{2} \cdot C_{41}^2} \quad (6)$$

The mass flow rate is defined as function of nozzle throat area (A_{thr}), using compressible flow analysis and nozzle flow rate area (A_{Thr})

$$\dot{M}Ar \cdot \frac{\sqrt{T_{su}}}{P_{su}} = \frac{A_{Thr}}{\sqrt{r}} \cdot \sqrt{\frac{2 \cdot \gamma}{\gamma-1}} \cdot (\pi_{E2})^{\frac{1}{\gamma}} \cdot \sqrt{1 - (\pi_{E2})^{\frac{\gamma-1}{\gamma}}} \quad (7)$$

Where

P_{su} - Supply pressure

r - gas constant

π_{E2} - pressure ratio given by; $\pi_{E2} = \text{Max} \left(\frac{P_{ex}}{P_{su}}; \pi_{critical} \right)$

where the critical pressure ratio is given by ; $\pi_{critical} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}}$

From equation (6) we can observe that maximum efficiency is

$$\eta_{max} = \cos^2(\alpha_1) \quad (8)$$

Use of above model requires the ID of several parameters as: Laval wheel diameter (D), nozzle throat (A_{Thr}) area, incident angle α_1 . This can be accomplished through the appropriate use of curves given by manufacturers, as shown below

1.1 ID of parameters

- The incident angle α_1 is obtained with the maximum efficiency curve, which is considered the nominal or design condition and using Eq. (8)
- The throat area is obtained, accepting that the nozzle works in shock condition (Mach=1.0), in this case Eq. (7) is valid
- The D'Laval diameter is found using the speed and Eq.(1) and Eq. (2)

The model above-described is recommended for axial turbines; although in this work, this model with some small changes can be easily used for small radial turbines.

1.2 Radial turbine model

It was observed that the model proposed for axial turbines when applied to radial turbines showed a lack of precision. Parameters were not constants. Near to the shock region the behavior was unstable when the nozzle throat area was considered constant.

The corrections proposed are based on nozzle mass flow rate relationship as shown below:

$$\dot{M}_{Ar} \cdot \frac{\sqrt{\gamma \cdot R_{AR} \cdot T_{SU}}}{A_{Thr} \cdot P_{SU} \cdot \gamma} = M_2 \cdot \left[1 + \frac{\gamma - 1}{2} \cdot M_2^2 \right]^{\frac{1}{2} \frac{\gamma + 1}{\gamma - 1}} \cdot \left[1 - \frac{\gamma - 1}{2} \cdot \zeta_{Thr} \cdot M_2^2 \right]^{\frac{\gamma}{\gamma - 1}} \quad (9)$$

Where

\dot{M}_{AR} - Mass flow rate

M_2 - Nozzle outlet Mach number

ζ_{Thr} - Nozzle Loss coefficient

The nozzle coefficient was defined by Whitfield and Baines (1976):

$$\zeta_{Thr} = \frac{h_{EX} - ih_{EX}}{\frac{1}{2} \cdot C_2^2} \quad (10)$$

This work propose the use of multi-variable minimum square methodology to identify parameters ,the application of this methodology is explained below:

$$\sum_{i=1}^n (a + b \cdot X_i - Y_i)^2 \rightarrow \text{minimum} \quad (11)$$

Where:

- a, b, c - Adjust coefficients
- X_i -Dependent Variável
- Y_i -Independent Variável

3. RESULTS AND DISCUSSIONS

Initially the model proposed by ASHRAE for axial turbines was evaluated; results are shown in Fig (3) (a) mass flow

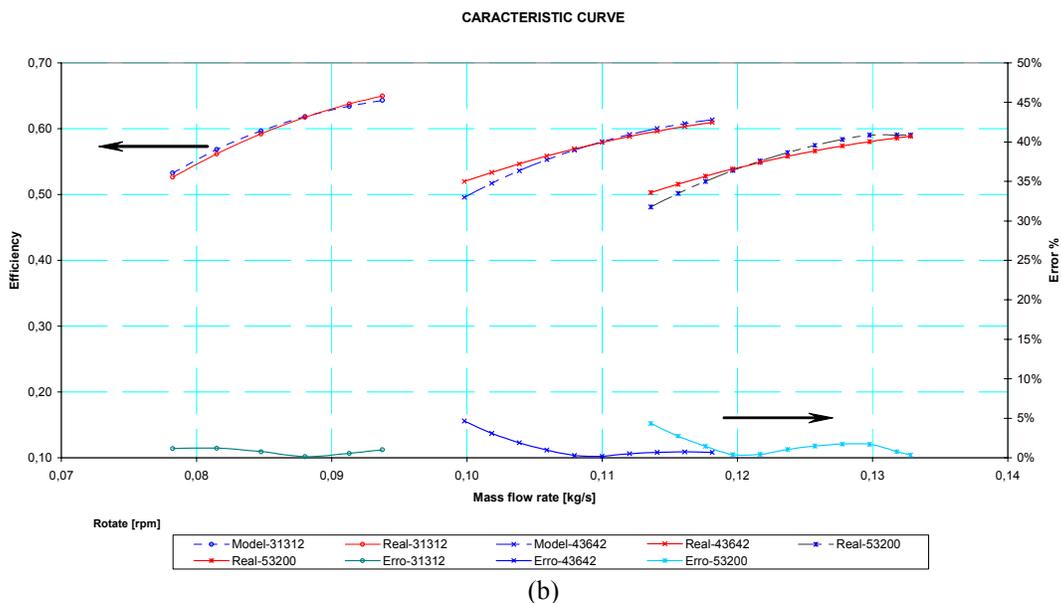
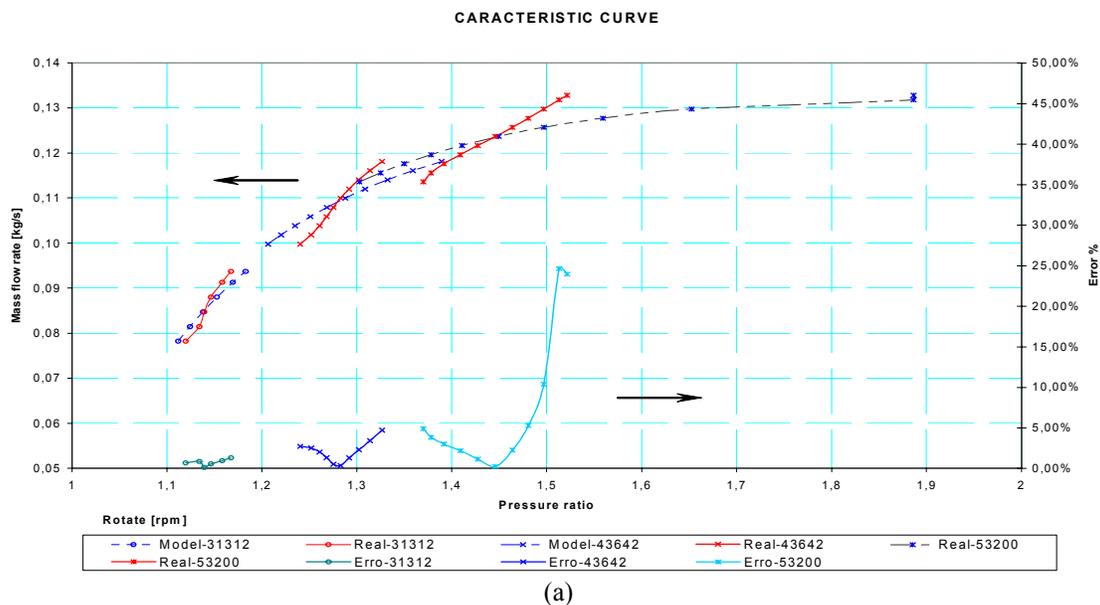
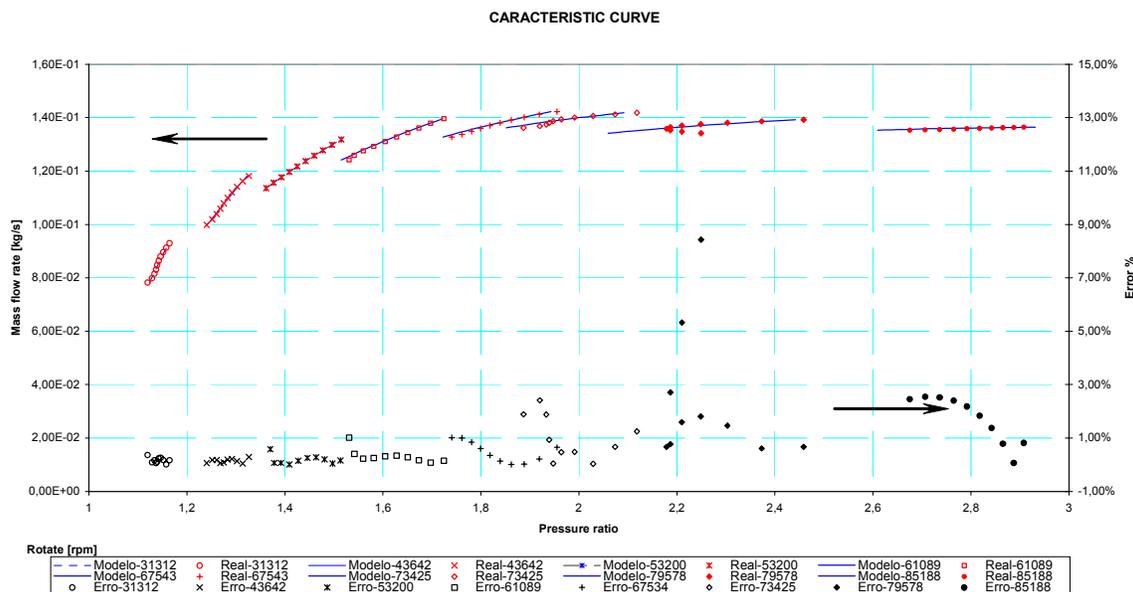


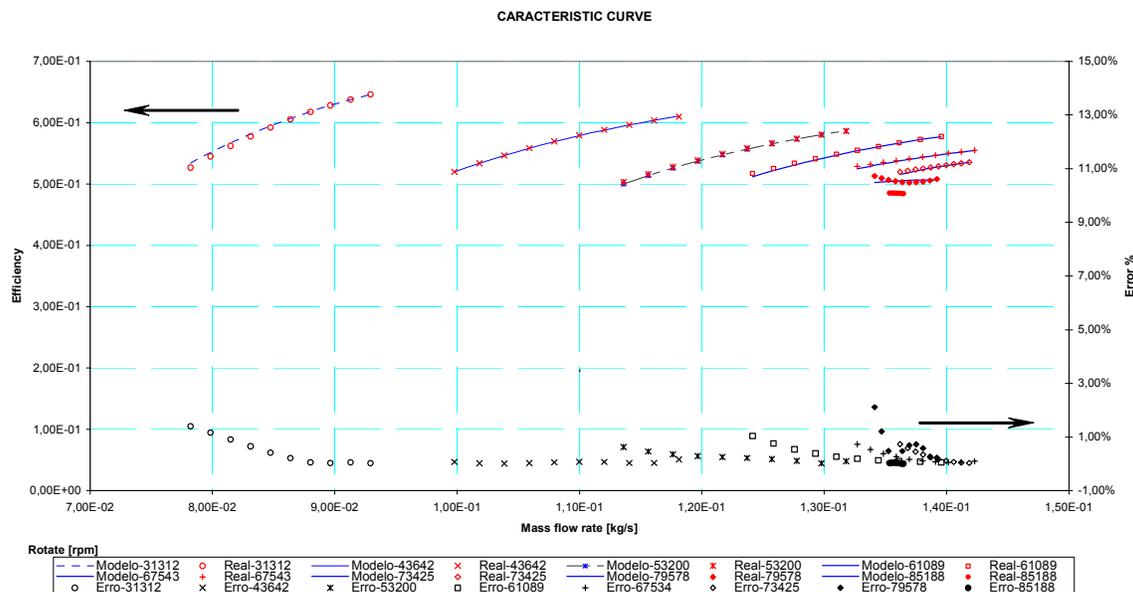
Figure 3: Model behavior without corrections

It is observed in Figure (3) (a) that at 31312 rpm, the difference in pressure ratio is less than 3%. When the rotation is increased to 53200 rpm, this difference goes up to 25%. It was observed that around 53200 rpm, the pressure ratio was close to the shock pressure ratio.

In order to get better results, was introduced a nozzle loss coefficient (ζ_{Thr}) as a new parameter, results obtained are shown in Fig. (4)



(a)



(b)

Figure 4. Model behavior with corrections

As seen in figures (4) (a),(4) (b) the differences in pressure ratio estimation observed with the corrected model are always below 4% and when efficiency is estimated the differences observed are less than 2.5% in the whole range of rotation velocities.

4. CONCLUSIONS

The ASHRAE model, seems to be adequate for axial turbines, using pressure ratios bigger than 3.0. The throat area correction recommended in this work gives good results for small radial turbines working with pressure ratios less than 3.0

The radial turbine model proposed is very simple to implement. The parameter ID procedure is not difficult to follow and we strongly recommend the use of this model to explore applications of small radial turbines in non conventional air cooling systems using Brayton/Joule reverse Cycles.

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