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Estimation of Motor Startup Speed Profile Using Low-Resolution Timing Signals and Motor Speed-Torque Curve

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ESTIMATION OF MOTOR STARTUP SPEED PROFILE USING LOW-RESOLUTION TIMING SIGNALS AND MOTOR SPEED-TORQUE CURVE

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ABSTRACT

Motor startup speed profile of motor rotational speed versus rotational angle can often be very useful during product developments, such as for detailed kinematic or dynamic component analyses. In compressor tests, a sensor, usually a proximity probe or an optical pickup, is commonly used to generate low-resolution timing signals. The low-resolution timing signals are sufficient to detect the angular position of crankshaft for stable operating conditions. However, the low-resolution timing signals can not be directly used to estimate the motor startup speed profile. A numerical calculation method is thus developed to estimate the motor startup speed profile from the low-resolution timing signals and the motor speed-torque curve (or torque-speed curve).

An iterative numerical solution procedure using the explicit finite-difference method that employs a time marching algorithm to integrate the differential equations over time is presented in this paper. The finite-difference calculations marches the solutions in time. The solutions calculated by the finite difference method are iterated to estimate the mass moment of inertia according to the low-resolution timing signals. The numerical calculation method described in this paper can also be used to predict other motor dynamic behaviors of a motor-load system.

1. INTRODUCTION

In many industrial applications, the estimation of motor speed profile during a startup becomes necessary. For example, significant events often occur during a compressor startup, which study may be of high importance. To estimate a component velocity during a startup for any in-depth kinematic or dynamic analysis, the motor startup speed profile of rotational speed versus rotational angle is required.

The timing sensor generates a voltage pulse at a specific crank angle to provide timing reference for other test measurements. The resolution of timing signals (the number of timing signals per revolution) is usually low in compressor tests. It is a very common test setup in which the timing sensor only observes a once-per-revolution event marker on a rotating component. Installing a low-resolution timing sensor is very convenient and cheap because it requires no modification or minimal modifications to the internal components of a compressor. Besides, the low-resolution timing signals are adequate for stable operating conditions based on the fact that motor speed is nearly constant. Because the motor startup speed increases rapidly from zero to full speed, the low-resolution timing sensor can only provide a very limited number of timing signals during a startup. Therefore, it is impossible to directly estimate the startup speed profile from the low-resolution timing signals with enough accuracy. High-resolution timing devices, such as optical encoder or resolver, can be used to define the motor startup speed profile. However, it is often very difficult to incorporate them into a running compressor.

The dynamics of a motor-load system is governed by the moment equation. Compressor startup is a transient process, in which the motor rotational speed accelerates rapidly, and the effect of mass moment of inertia is significant. In fact, finding the motor startup speed profile is essentially an initial value problem. In the method described in this paper, the motor is modeled by the speed-torque curve that defines the torque developed by the motor as a function of motor rotational speed. The motor speed-torque curve is generally nonlinear. The load torque applied to the motor shaft during a startup can be modeled by nonlinear functions of rotational speed and/or angle. Furthermore, the moment of inertia is usually unknown. As a result, the analytical solutions to the moment equation are not readily available. An iterative numerical calculation procedure using explicit the finite-difference method is thus developed to estimate the motor startup speed profile. The finite-difference calculations integrate the differential equations forward through time, starting from the initial conditions. The solutions calculated by finite difference method are iterated using the low-resolution timing signals to calibrate the calculations for the unknown mass moment of inertia.

The calculation results using once-per-turn timing signals are presented to demonstrate the application of the numerical method. The moment of inertia is estimated during a startup because of nonzero rotational acceleration. After the determination of mass moment of inertia from startup calculations, the numerical calculation method described in this paper can also be used to predict other dynamic behaviors of a motor-load system. As an example, the calculations are finally performed to predict motor dynamic responses to a load torque surge.

2. THE ITERATIVE NUMERICAL CALCULATIONS OF FINITE DIFFERENT

The rotation of a rigid body around a fixed axis can be governed by the moment equation as follows:

$$I\frac{dV}{dt} = T_M - T_L, \quad \& \quad V = \frac{d\theta}{dt} \tag{1}$$

Where V is the rotational speed, θ is the rotational angle, I is the moment of inertia including the mass of motor rotor, shaft, counterweights, and any other masses rotating with the shaft. T_M is the torque developed by the motor that drives the compressor. T_L is the load torque including the torque required to compress refrigerant, the friction torque, and the windage torque. In this analysis, the torque required to do compression work is considered as a predominant part of motor load torque. The motor speed-torque curve that is available from motor manufacturer defines the motor torque as a function of motor rotational speed:

$$T_M = f(V) \tag{2}$$

During the transient period of a startup, the load torque applied to the motor shaft can be functions of rotational speed and/or rotational angle based on the physical interpretation of how the load torque is built up. The general form of the function for T_L can be expressed as:

$$T_L = g(\theta, V) \tag{3}$$

With finite-difference algorithm, the numerical method is very flexible to easily incorporate almost any nonlinear functions into the calculations. The initial conditions for startup calculations are:

$$\theta = 0, \quad V = 0, \quad T_L = g(V = 0, \theta = 0) \quad \& \quad T_M = f(V = 0) \quad @ \quad t = 0 \tag{4}$$

To apply the finite-difference method, all derivatives in differential equations are replaced by approximating finite difference formulations. The equations for the iterative numerical calculation scheme using finite-difference method are given below. In the following equations, the superscript j is the iteration number, and superscript i is the time step number of time integration with time step Δt.

$$t^{j,\, i+1} = t^{j,\, i} + \Delta t \tag{5}$$

$$V^{j,\, i+1} = V^{j,\, i} + \frac{T_M^{j,\, i} + T_L^{j,\, i}}{I^j}\Delta t \tag{6}$$

$$\theta^{j,\, i+1} = \theta^{j,i} + \frac{V^{j,\, i} + V^{j,\, i+1}}{2}\Delta t \tag{7}$$

$$T_L^{j,\, i+1} = g(\theta^{j,\, i+1}, V^{j,\, i+1}) \tag{8}$$

$$T_M^{j,\, i+1} = f(V^{j,\, i+1}) \tag{9}$$

The timing data is given in the format of:

$$t_{TM,\ k},\quad k = 1, 2, \ldots m \tag{10}$$

where $t_{TM,k}$ is the time measured from the beginning of a startup (zero speed) to the peak of time signals. The corresponding rotational angles at ($t_{TM,k}$, $k=1, 2, \ldots, m$) can be estimated using linear interpolations as follows:

$$\theta^{j}_{TM,k} = \theta^{j,i} + \frac{\theta^{j,i+1} - \theta^{j,i}}{t^{j,i+1} - t^{j,i}}\ (t_{TM,\ k} - t^{j,i})\quad when\ \ t_{TM,\ k} \geq t^{j,i}\ \ \&\ \ t_{TM,\ k} \leq t^{j,i+1}, k = 1, 2, \ldots m \tag{11}$$

The actual rotational angles between two adjacent timing signals are known according to the hardware setup. The actual rotational angles between two adjacent timing signals are denoted by:

$$(\varphi_{TM,k+1} - \varphi_{TM,k}),\quad k = 1, 2, \ldots m-1 \tag{12}$$

From Equation (1), one can obtain:

$$I \cdot d\theta = \left[\int_0^t (T_M - T_L)\, dt\right] dt \tag{13}$$

Therefore, using the timing data, the moment of inertia can be updated for the next iteration by:

$$I^{j+1} = \frac{1}{m-1} \sum_{k=1}^{m-1} I^{j} \cdot \frac{\theta^{j}_{TM,k+1} - \theta^{j}_{TM,k}}{\varphi_{TM,k+1} - \varphi_{TM,k}} \tag{14}$$

Usually, the earlier timing signals during a startup could be more important to estimate the moment of inertia due to higher accelerations; thus, a weighting factor w_k is introduced into Equation (14), which yields an enhanced version of Equation (14):

$$I^{j+1} = \frac{1}{\sum_{k=1}^{m-1} w_k} \sum_{k=1}^{m-1} w_k \cdot I^{j} \cdot \frac{\theta^{j}_{TM,k+1} - \theta^{j}_{TM,k}}{\varphi_{TM,k+1} - \varphi_{TM,k}} \tag{15}$$

The convergence criterion for terminating the iteration may be set as:

$$\left|\frac{I^{j+1} - I^{j}}{I^{j}}\right| \leq \varepsilon \tag{16}$$

where ε is a user specified tolerance. Equations (1) though (16) give a complete description of the iterative numerical calculation scheme. The examples of calculations are given below to demonstrate the application of the numerical method.

3. EXAMPLES TO DEMONSTRATE THE APPLICATION

The inputs of the calculations are the motor speed-torque curve, the timing signals, the stable rotational speed and the motor torque at the stable rotational speed. The motor speed-torque curve of a three-phase induction motor is given in Figure 1. The once-per-turn timing signals are used in the calculations, as shown in Figure 2. Timing data used in the calculation are also listed in Table 1. For numerical calculations, the motor speed-torque curve is saved into a data file in ASCII format as:

$$(T_{M,k},\ V_k),\quad k = 1, 2, \ldots n \tag{17}$$

Linear interpolation is used to determine T_M for a known V:

$$T_M = T_{M,k} + \frac{T_{M,\ k+1} - T_{M,\ k}}{V_{k+1} - V_k}\ (V - V_k)\quad when\ \ V \geq V_k\ \ \&\ \ V \leq V_{k+1} \tag{18}$$

In this analysis, the load torque during a startup is considered to be predominately from the pressure built-up of compressor compression. Since the load torque during a startup is gradually increased with compressor pressure built-up, the fan-type or centrifugal pump-type load torque is used; it can be modeled mathematically by a quadratic function:

$$T_L = a + b\,V + c\,V^2 \quad (Nonlinear\ \ load - speed\ \ relation) \tag{19}$$

Equation (19) can also include friction torque to a certain extent. Besides, two more relationships between load torque and rotational speed during a startup are also considered to examine how the load torque affects the startup speed profile:

$$T_L = 0 \qquad (Zero\ \ load) \tag{20}$$

$$T_L = \frac{V}{V_S} T_S \quad (Linear\ \ load - speed\ \ relation) \tag{21}$$

where V_s and T_s are the motor stable rotational speed and the torque at the stable rotational speed respectively. The motor stable rotational speed is estimated from the timing signals when the motor reaches full speed after a startup. The torque at the stable rotational speed is obtained from the motor speed-torque curve. The moment of inertia is unknown initially, and is arbitrary set to unit before the numerical iterations.

The predicted motor dynamic behaviors during a startup are given in Figure 3. The results shown in Figure 3 indicate that the differences in startup speed profiles between three load torque-speed relationships defined in Equations (19) to (21) are insignificant for general industrial applications. Therefore, when the load torque is fairly lower than the torque developed by the motor during the whole period of a startup, the accuracy of estimating motor load torque during a startup is not very important. The comparison of the calculated angle and the nominal angle between two adjacent timing signals are listed in Table 2, indicating the agreement between the two angles is very good. Therefore, without knowing exactly the motor load torque during a startup, the method gives very reasonable estimation of motor startup speed profile with low-resolution timing signals.

The numerical method converges very quickly and stably without using the successive relaxation technique for iterative scheme. However, to obtain a converged calculation using the finite-difference method requires that time step be chosen sufficiently small. Different sizes of time step should be tried to make sure that a converged solution with sufficiently small numerical error is obtained.

The moment of inertia needs to be estimated during a startup because the rotational acceleration is not zero. After the determination of moment of inertia from startup calculations, the numerical method presented in this paper can also be used to predict other dynamic behaviors of a motor-load system, not just limited to motor startups. Finally, the calculations are performed to estimate motor dynamic responses to a load torque surge. The results are shown in Figure 4. Note that the accuracy of calculated moment of inertia depends on the motor speed-torque curve and the load torque used in the startup calculations.

4. DISCUSSIONS AND SUMMARY

An iterative numerical calculation procedure using explicit finite-difference method that integrates the differential equations over time is presented in this paper to estimate motor startup speed profile with low-resolution timing signals. This iterative numerical method has been successfully used to estimate the impact velocity for impact-induced failure analysis using once-per-turn timing signals.

The implementation of the numerical calculations reveals that the convergence of the numerical procedure is very stable and fast. The moment of inertia estimated by this numerical method and the principal of the numerical method can be used to predict motor behaviors for other applications, such as motor dynamic behaviors under changing load torque. Since the finite difference algorithm is used, the method is quite robust to easily incorporate almost any nonlinear functions for the load torque into the calculations.

To obtain accuracy results, a sufficiently small time step Δt is required. Several trials with different sizes of Δt are needed to determine the proper Δt. The motor speed-torque curve (i.e., the shape of motor speed-torque curve) can significantly affect the accuracy of calculations. It thus becomes important to obtain the proper motor speed-torque curve. Increasing the resolution of timing signals can improve accuracy. Therefore, the timing signals with a resolution higher than once-per-turn are favorable. High–resolution timing data can be readily used in the method. Besides, multiple startup timing data of low-resolution timing signals can be used to investigate the variability of the startup process and to improve the accuracy of numerical calculations.

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Table 1 The input timing signal data from Figure 2
(The rotational angle between two adjacent timing signals = 360^0)

No.	Timing data @ timing signals (second)	Weighting factor
1	1.313E-2	3
2	3.626E-2	2
3	5.304E-2	1
4	7.032E-2	1
5	8.716E-2	1
6	0.10431	1

Table 2 Comparison of the calculated angles and the nominal angles between two adjacent timing signals
(The motor speed-torque curve in Figure 1 and the timing signals in Figure 2 and Table 1 are used)

Rotational angle between two adjacent timing signals	Calculated			Nominal
	Zero load	Linear load-torque relation	Nonlinear load-torque relation	
Between timing signals #1 and #2	361.1^0	361.2^0	361.2^0	360^0
Between timing signals #2 and #3	355.4^0	355.3^0	355.3^0	360^0
Between timing signals #3 and #4	366.0^0	366.0^0	366.0^0	360^0
Between timing signals #4 and #5	356.7^0	356.7^0	356.7^0	360^0
Between timing signals #5 and #6	363.2^0	363.2^0	363.2^0	360^0

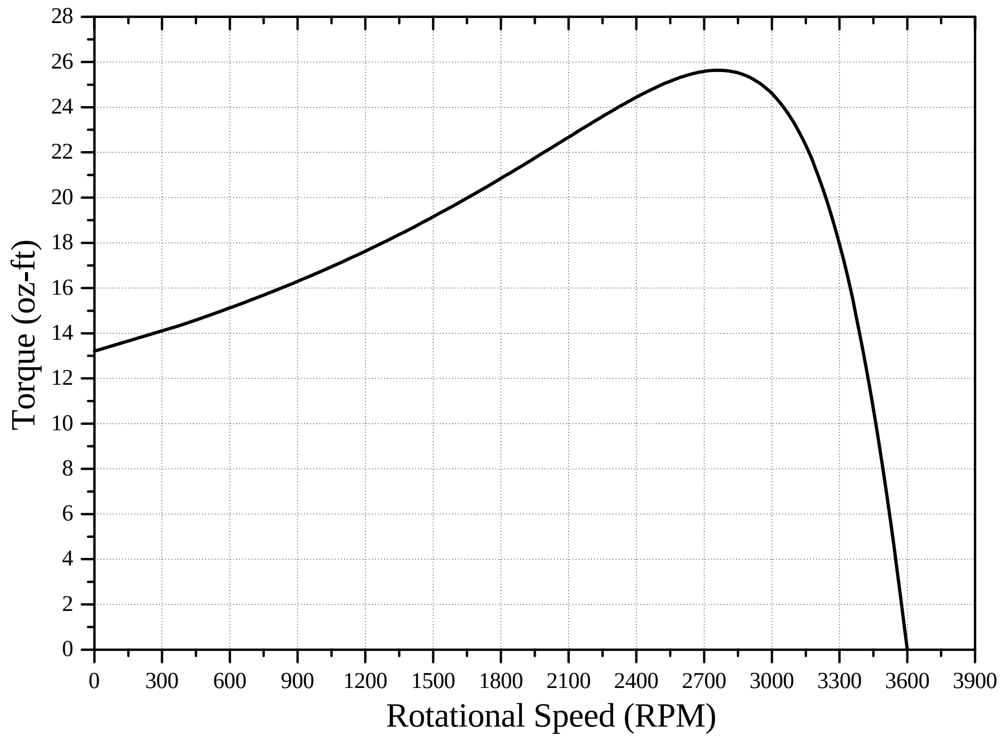

Figure 1 Motor speed-torque curve of a three-phase induction motor (1 oz-ft = 0.08474 Newton-Meter, RPM = Revolutions Per Minute)

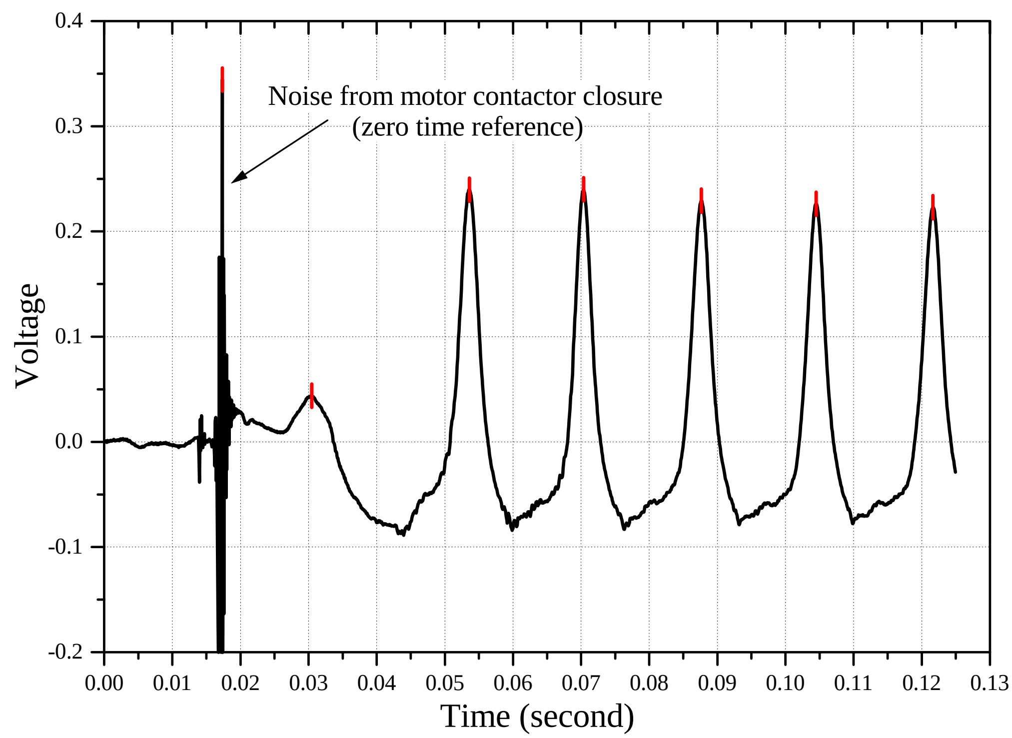

Figure 2 The once-per-turn timing signals. The rotational angle between two adjacent timing signals is 360^0.

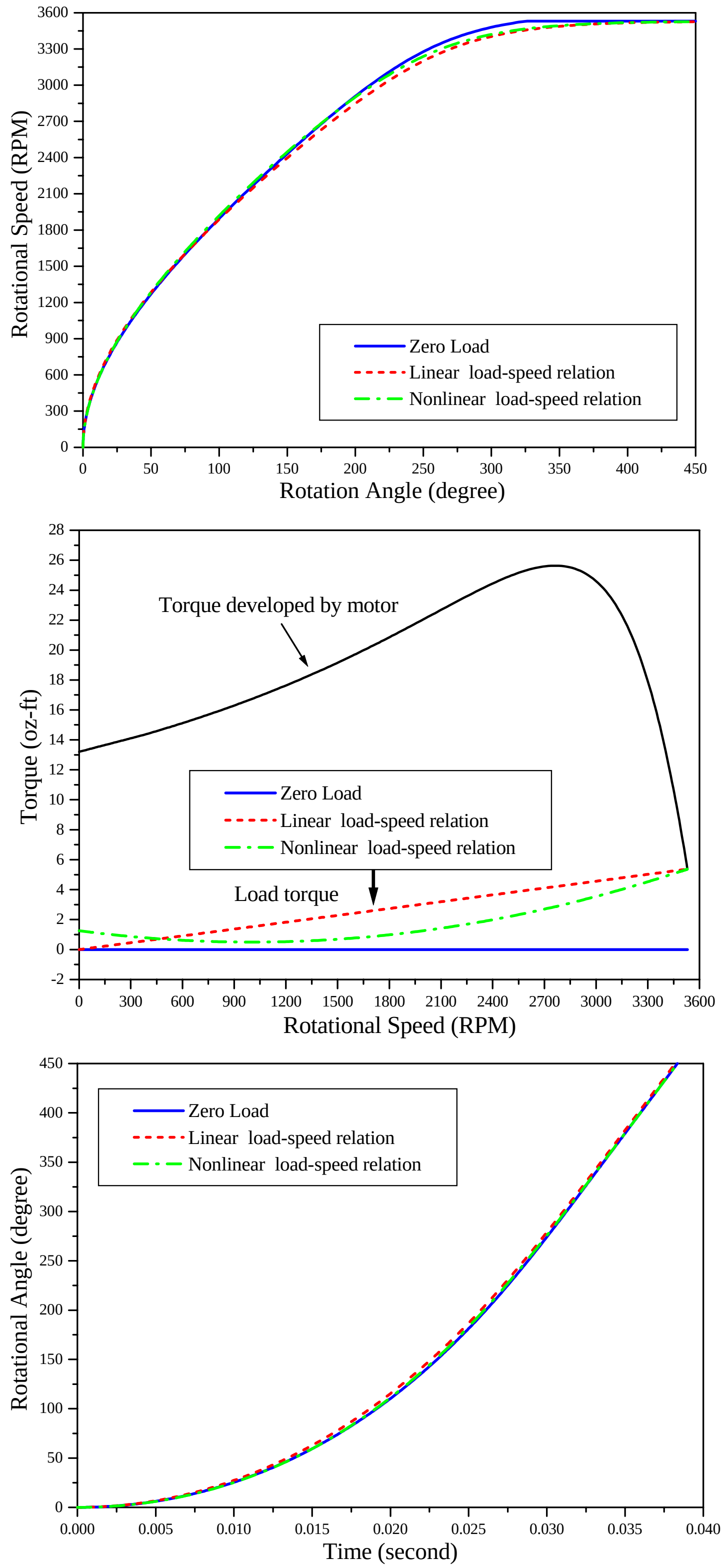

Figure 3 Startup calculation results using the motor speed-torque curve in Figure 1 and the timing signals in Figure 2 and Table 1. Three different functions of load torque T_L versus rotational speed V are used in the calculations. (1 oz-ft = 0.08474 Newton-Meter, RPM = Revolutions Per Minute).

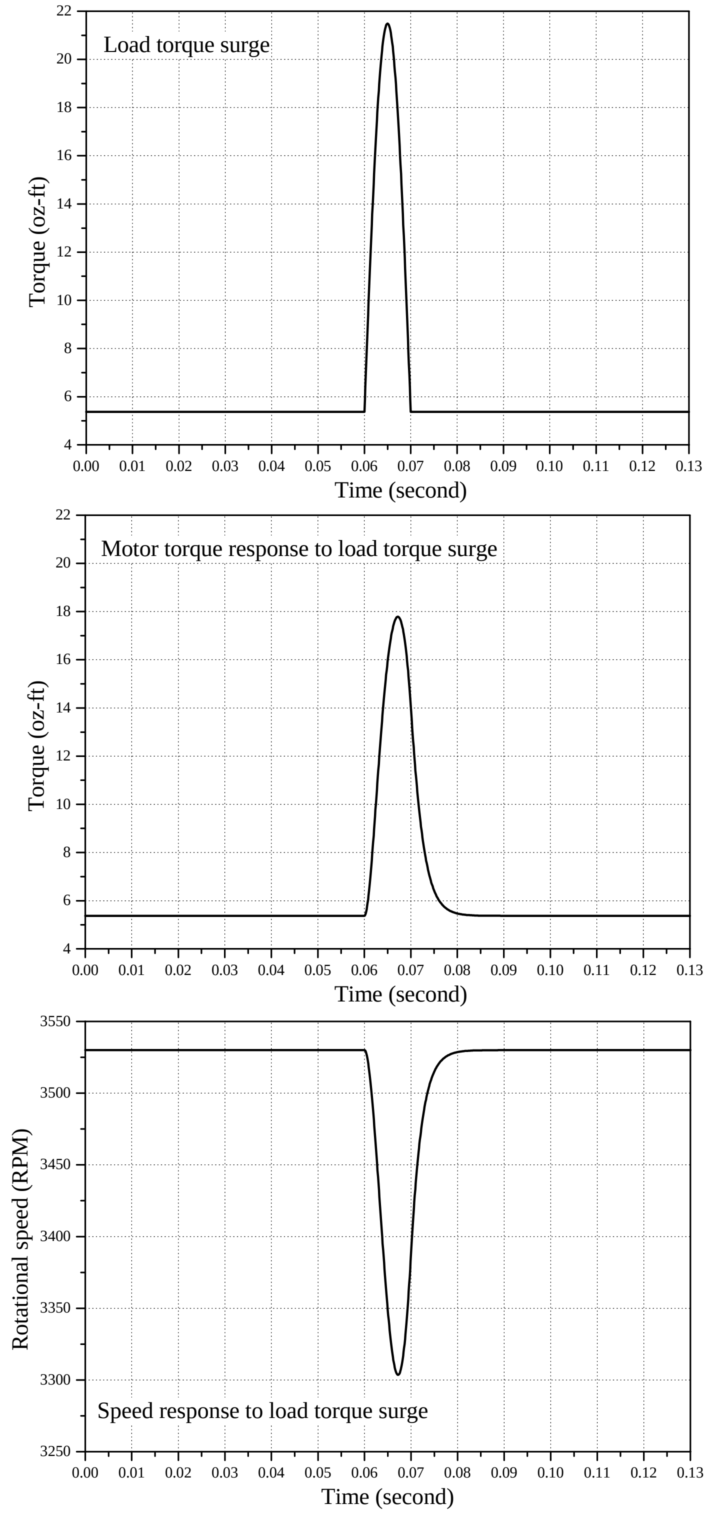

Figure 4 After the determination of the mass moment of inertia from startup calculations, the numerical calculations described in this paper can be extended to predict other dynamic behaviors of a motor-load system. The figures show the calculated motor dynamic responses to a load torque surge. Rigid body motions without dumping are assumed. Linear load-speed relation is used in the startup calculations. (1 oz-ft = 0.08474 Newton-Meter, RPM = Revolutions Per Minute).