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Valve Dynamic Analysis of a Hermetic Reciprocating Compressor

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ABSTRACT

Valve dynamic analysis was analyzed only for structural part or fluid part individually. In this work, ADNIA, FSI was introduced to solve the valve dynamics considering heat transfer. In this study we had performed the experiments to know the characteristics of valve dynamics and can obtain the dynamic behavior of valves and pressure-volume diagram. And then, fluid structure interaction [FSI] simulation was performed. It was conformed that the fluid structure interaction can explain the realistic valve dynamic behaviors. The advantage using FSI is temperature and velocity profiles can be obtained. These data were difficult to get from the experiment, because of the inferior conditions inside the cylinder. With these obtained results one of the important factor, efficiency can be calculated. We had applied the shape optimization technique using sequential quadratic programming to get the optimal design of suction reed valve.

1. INTRODUCTION

In compressors, valve dynamic analysis is a most important part and a desired part to be analyzed to expect the dynamic characteristic of a compressor. But valve mechanism is especially the complicated part among the all parts of a compressor. Many approaches have been implemented to get the direct relationship between the radiated noise, the efficiency and the installed valve. In valve dynamic behavior analysis, even though the motion of piston is kinematically restricted as in conventional reciprocating compressors, the motion of the complicated valve can be changed. It means there are the unexpected factors during a valve motion. Until now, the various works related a compressor valve dynamics were only related to the uncoupled structure, fluid or heat transfer problem.

This work deals on the dynamic analysis of a suction valve after considering all of factors, which gives an effect to the motion of a valve mechanism. Experiment also was performed to validate not only a valve behavior but also a pressure and volume, which was an important part to know the characteristics of efficiency. Numerical analysis was implemented using fluid-structure analysis (FSI). FSI considers a fluid and structure problem at once and also considers a heat and mass transfer problem. FSI part will be more explained in later part.

2. EXPERIMENT OF A SUCTION REED AND A DISCHARGE VALVE

2.1 Overall test setup and data acquisition

At first the compressor used for the experiment was one of mass-produced compressors. But exact brand name cannot be exposed because it related the confidential of a corporation. The model and valve configuration is listed in table 2.1.

Model	Reciprocating Compressor
Suction Reed Valve	Thickness: 0.178 mm and 0.152 mm
Material	Sintered Steel, Sandvik
Operating Fluid	Air / R134a
Operating Dis. Pressure	1/ 5/ 10/ 14 kg/cm ²

Table 2.1 Experiment Configurations

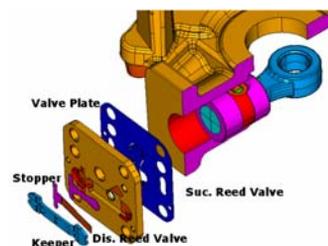


Fig. 2.1 Modeling of valve assembly

Fig. 2.1 is showing how valve is installed in a compressor. Strain gauge was used for getting the deformation voltage signal of suction and discharge valve. To know the exact deformation data of each valve during the

operation, before doing the experiment the calibration data of strain gauge was necessary. Voltage signal was obtained using strain amplifier for each forced displacement using the micrometer. For each experiment of valve displacement each strain gauge was used. Measured data was curve-fitted to know the continuous value using least mean square method like Equation 3.1

$$a = \frac{E[XY] - E[X] \cdot E[Y]}{E[XX] - E[X] \cdot E[X]} \quad (3.1)$$

Next it is necessary to know where is the origin point of motor to recognize the position of piston's top dead clearance [TDC] and bottom dead clearance [BDC]. To find the position there are two ways, an encoder attached to the shaft tip was used.

2.2 Experiment result of valve behavior in a small displacement compressor

Fig 2.2 shows the result of the behavior of suction and discharge valve in a small displacement compressor.

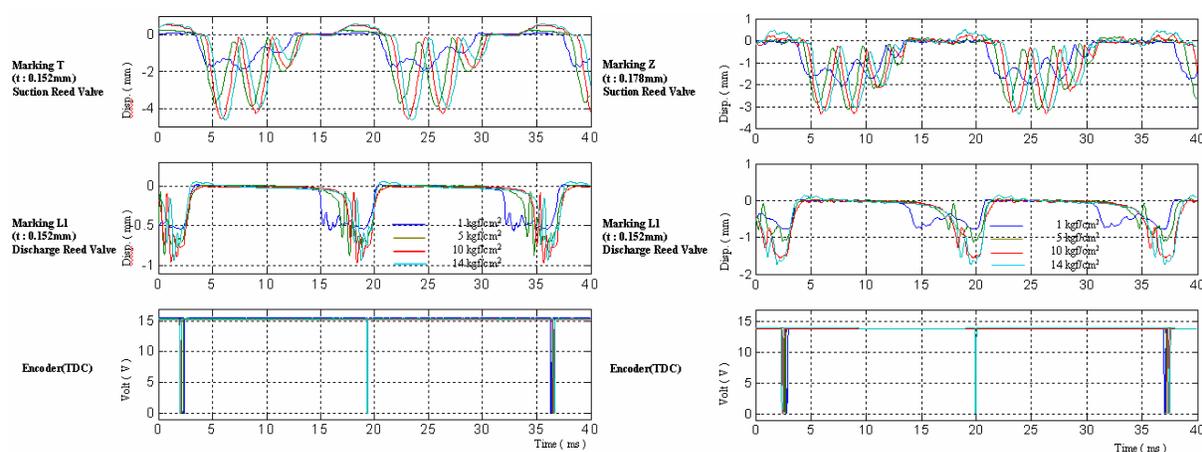


Fig 2.3 Valve dynamic behavior and position results in a small displacement compressor
[Suction reed valve thickness (a) 0.152mm (b) 0.178mm]

In suction reed valve case about 3 fluctuations were shown and in discharge reed valve 1 or 2 fluctuations were shown. The pulse in encoder represents the top dead clearance of the piston. This result is quite reasonable. The deformation of suction reed valve of the thickness of 0.152mm was about 4 mm. On the other side, in 0.178mm thickness case of the suction reed valve the maximum displacement was about 3mm. It means if the stiffness of valve increases the maximum deformation decreases. Except the deformation of suction reed valve the behavior of valves are almost same.

Experiment summary

- Dynamic behavior is strongly related to the natural frequencies of valve
- Real behavior frequencies are slightly higher than the natural frequencies
- Tendency of the increased dynamic frequencies when valve is closing, because of the stiffness of fluid
- In discharge valve case, the reason why dynamic frequency much increases than natural frequency is that the contact with stopper

These all results related to the dynamic frequencies of each valve are proofed by the modal analysis as shown in Fig. 2. 3. This explains how valve dynamic result is operating.

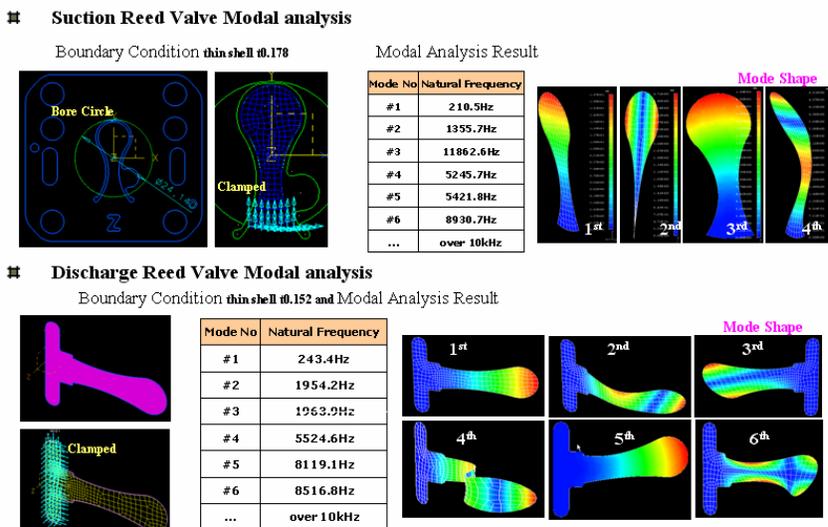


Fig. 2.3 Modal analysis result of each valve

2.3 Experiment result of valve behavior in a big displacement compressor

Fig. 2.4 shows the experiment result of the valve behavior of suction and discharge and the cylinder pressure in a refrigerant condition.

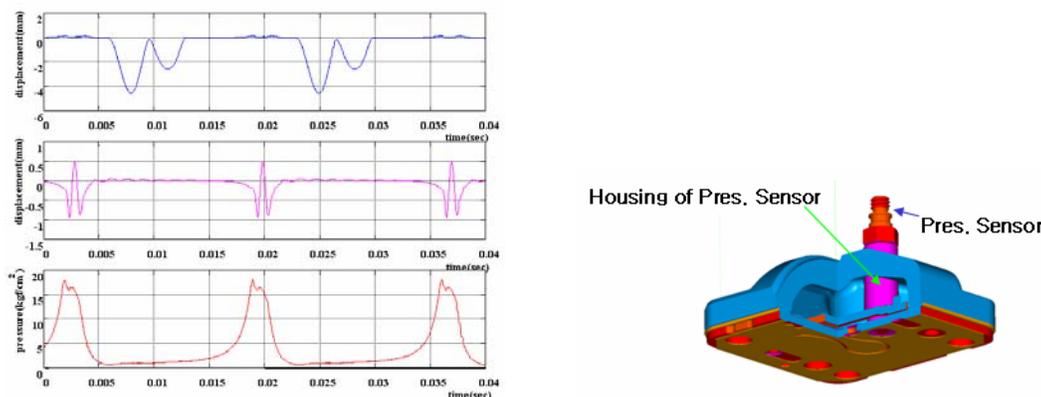


Fig. 2.4 Behavior of suction and discharge reed valve and cylinder pressure in a refrigerant condition

Fig. 2.5 Experiment setup of a cylinder internal pressure

To obtain the experiment result of a cylinder pressure, the pressure gauge was used which plugged into the cylinder head. Experiment setup is shown in Fig. 2.5

Data acquisition was obtained using TDS3054 oscilloscope and the encoder Z due to easy to eliminate a random EM noise. Refrigerant inflows into cylinder passing through the suction port of valve and the inflowing refrigerant is compressing during the compression step, then in the case when the cylinder pressure is bigger than the discharge pressure the compression gas outflows through the discharge port of valve. The motion of piston changes the volume of cylinder and these volume changes are forcing the motion of displacement of valve. So to understand the valve behavior during the operation is very important.

3. FLUID STRUCTURE INTERACTION [FSI] ANALYSIS

As mentioned in the beginning FSI is the coupled work solving the structure and fluid part at once. Until now it was usual to be solved individually. But with the development of finite element and finite volume method, it became to solve at once even if it takes a lot of time. In this part FSI analysis for a compressor valve will be introduced.

3.1 Numerical model

Analyzed numerical model for FSI analysis was a big displacement compressor of about 8cc. Before showing the numerical model it is prerequisite to introduce the analysis procedure. As mentioned formulation of fluid Navier-Stokes equation of conservation of mass, momentum and energy and Convective-Diffusive equation for modeling turbulence will be used. Formulation of Fluid-Structure Interaction [FSI] is being accomplishing by mapping operation between fluid and solid meshes. On interface between them mechanical coupling will be used as shown Fig. 3.1

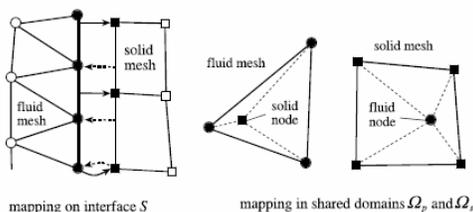


Fig.3.1 Meshes mapping between solid and fluid domain

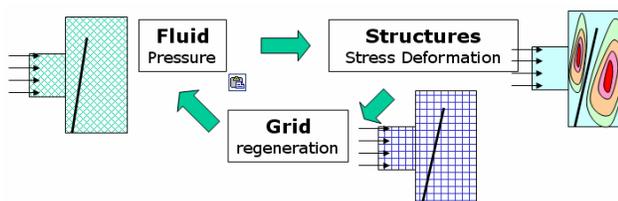


Fig. 3.2 Simple solution procedure for FSI

Simple solution procedure for valve dynamics will be shown in Fig. 3.2. As shown at first fluid problem will be solved then by the velocity or acceleration from fluid structure will be deformed at that time the constructed mesh will be rearranged. These procedures will also be done iteratively until reaching the set criteria.

3.2 Boundary condition

To solve the problem using FSI method, the special boundary conditions have to be imposed. In this part they will be introduced and how they are imposed on each faces of a domain. To explain the imposed boundary condition for suction reed valve, Fig. 3.3 is shown

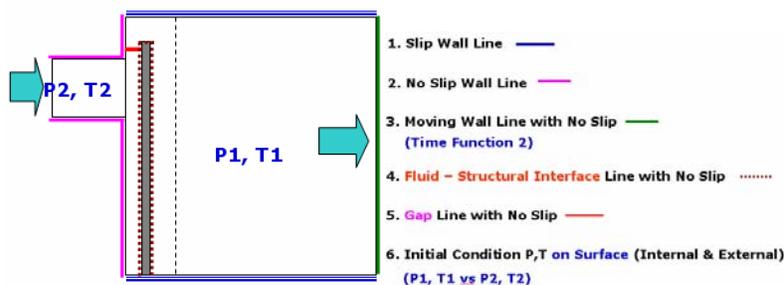


Fig. 3.3 2D fluid domain for the boundary condition

The implemented analysis was a transient with the movement of piston backward and forward transformed by the rotating motion of motor. To impose the moving wall condition on the face of a piston, it is necessary to calculate the piston movement profile. The equation (3.1) is showing the displacement of piston and the used notation can be recognized in Fig. 3.4

$$z(t) = e \left\{ 1 - \cos \theta(t) + \frac{l}{e} \left(1 - \sqrt{1 - \left(\frac{e}{l} \right)^2 \sin^2 \theta(t)} \right) \right\} \quad (3.1)$$

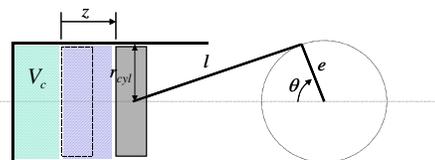


Fig. 3.4 Schematic diagram of the cylinder

Next special boundary condition is fluid structure interface imposed on the suction reed valve. This condition makes it is possible to calculate fluid and structure interactively. Only fluid or structure problem it is unnecessary but all FSI problems it is prerequisite.

And the special and crucial boundary condition is a gap one. This is necessary for a valve problem. Initial state of a compressor there is very small gap between valve plate and suction reed valve due to the gasket. It may be about 200µ m. During the operation of a compressor the valve will be opened due the pressure difference between suction port and the internal of cylinder. At that time from when the program recognizes the valve is open the separated

fluid domains are merged into one. So some value has to be input, which the start point will be recognized by program.

First closed condition is the fluid cannot flow across the interface and if a gap is open, which the fluid can flow across the interface without involving any boundary condition. The face is imposed no slip wall condition and zero heat flux condition.

Last one is the initial condition about pressure and temperature on the domain.

Next structure case will be introduced. As FSI is the fluid and structure problem, structure has to be modeled individually. Later part is explained based on Fig. 3.5

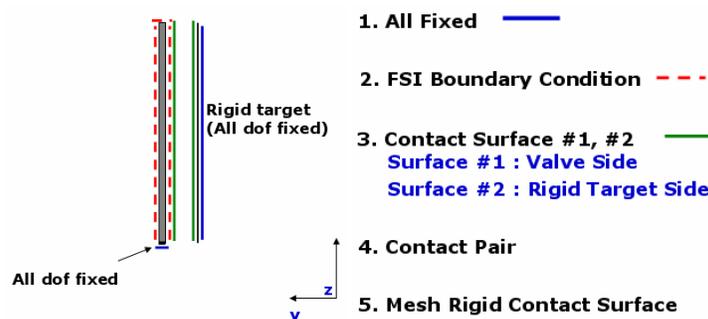


Fig. 3.5 2D structure domain for the boundary condition

In structure case, contact condition is newly introduced. During the operation of a compressor, the valve is fluctuating and repeating the contact with valve plate. Assumption of a contact problem is frictionless and the used contact algorithm is rigid target method.

3.3 FSI numerical analysis result

To solve the problem using FSI, 2D domain was used using ADINA. ADINA was made by Bathe, who is the professor in MIT University. In this analysis part, heat transfer was considered. Until now there had been a lot of studies but there was not the case about considering heat transfer on the fluid-structure problem. Also non-linear term such as impact, oil stiction and gas pulsation were considered to raise the exactness and reliability of FSI analysis. Absolutely the obtained results were compared to the experiment result of the valve dynamic.

Before looking through deeply the results, the atmosphere analysis was implemented because the same experiment was already performed. Fig. 3.6 shows the comparison graph. As shown there are 4 oscillations during there suction process. And the period comparison is also reasonable except that the 2nd period displacement is slightly different between them. The reason was why the analysis was implemented in 2D whereas experiment was real 3D system.

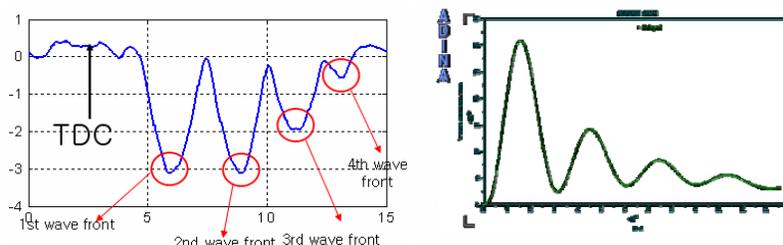


Fig. 3.6 FSI result of suction reed valve in atmosphere

Important one is the comparison under the operating condition of in the refrigerant gas case. As already mentioned in problem definition Fig. 3.3 was showing the 2D cylinder, suction port and suction reed valve. Fig. 3.7 is the result of the temperature of the internal of cylinder and suction port during the suction process and the right one is showing the particle movements also during the same process. Particle movements are good way to understand how the fluid is flowing. It is based in Lagrangian coordinate in ADINA. At the suction process the valve is opened due to the pressure difference between the cylinder and suction port. Pressure will be shown later, but temperature is also very important factor because the high temperature during the operation weakens the valve and affects the fatigue to the suction reed valve. The significant results of FSI analysis give us the temperature

profile during the operation. Actually there were many efforts to know the temperature profile due the above reasons, but it had been very difficult to know because internal part is high pressure and there are no enough room for the installation for temperature sensor. Needless to say to decide the phenomena of temperature profile it is prerequisite to compare the valve dynamic behavior and the pressure experiment results.

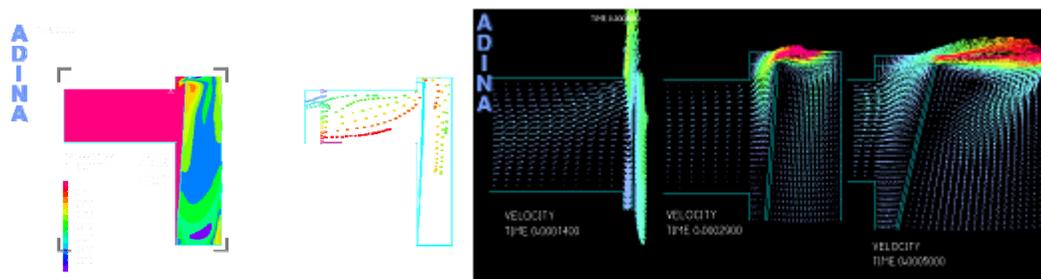


Fig.3.7 Temperature, fluid particle movement' and velocity plot

Also the velocity vectors' profile was shown in fig. 3.7 during the suction process. First one is at the time of $0.14\mu\text{sec}$, second is at the time of $0.29\mu\text{sec}$ and last one is at the time of $0.5\mu\text{sec}$. This problem was solved using turbulence method. So at the tip of the suction reed valve the velocity was changed dramatically. These flowing patterns are quite affecting to the suction reed valve and an efficiency of a compressor. And Fig. 3.8 is showing the graph the velocity profile at the entrance of suction port, at first suction part the velocity is increasing up to 9 m/s and after the half cycle the velocity is having the negative value, which it means the refrigerants go back ward to the suction part due to the valve oscillation motion. And it means the 2nd oscillation starts when the velocity value again goes into positive value.

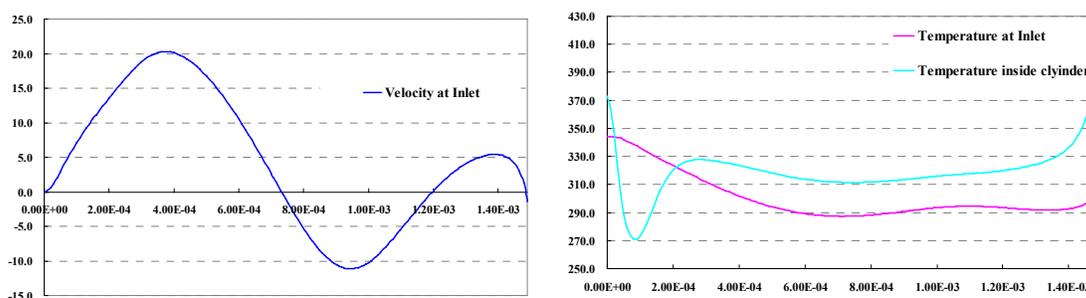


Fig.3.8 Velocity and temperature graph at the entrance of suction port

Fig. 3.8 is showing the temperature graph during the suction process. At first time when the valve is opened the internal pressure and temperature was decreasing at some extent range due the pressure inside cylinder goes into the vacuum condition. And during the opening time of suction reed valve the temperatures between the cylinder and the suction port parts will be same; means going into the saturation state. And last the starting point of compression. The temperature will go into very high. But to decide the temperature profile is right or not, the experiment of dynamics temperature profile has to be performed. But generally a compressor usually operates at 50 or 60 Hz which is 3000 or 3600 RPM. So even if a very precision temperature sensor used it is hard to get the dynamic temperature profile. So it has to be utilized to know the tendency of temperature inside cylinder.

Next the valve dynamic behavior will be discussed. Valve dynamic behavior affects a lot to the efficiency and radiated sound pressure level. The comparing data will be used the obtained experiment data. Fig. 3.9 shows the comparison graph between the FSI analysis and experiment about the suction dynamic behavior. As shown, the opening timing of suction reed valve during the suction process because the oil stiction which on the surface valve plate. This oil stiction force delayed the opening timing of suction reed valve compared to the FSI analysis result.

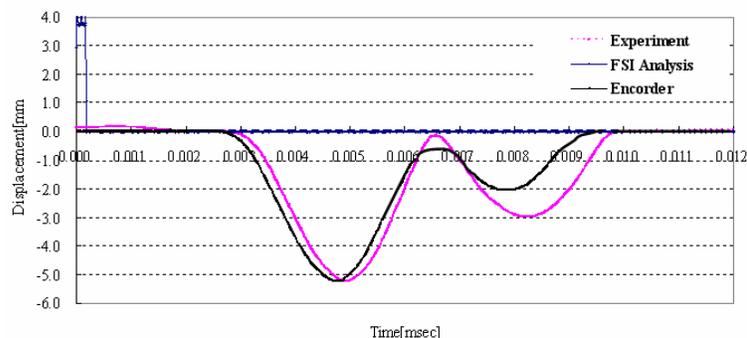


Fig. 3.9 Comparison result of displacement behavior of suction valve

With the comparison result plot, because the FSI analysis was performed in 2 D, the period and 2nd opening displacement showed a slightly difference. In experiment result plot the data was obtained in real phenomena. To solve in this problem 3 D FSI analysis result needs to be implemented.

4. COMPARISONS AND MODEL UPDEATE

Comparisons were done in previous chapter, and it gives quite reasonable result, which was shown in Fig. 3.9. In experiment case the strain gauge and pressure sensor was used to get the valve dynamic behavior and pressure data. And in FSI analysis, with the same result in experiment the other result such as temperature and velocity profiles were obtained and these results actually were impossible to be obtained in experiment. In this reason model updated FSI result gives more understanding phenomena about the valve. And as future work 3 D FSI analysis needs to be implemented.

5. SHARPE OPTIMIZATION OF SUCTION REED VALVE

In this part the shape optimization for suction reed valve will be implemented. Shape optimization will be performed without a fluid problem because the subjective and objective function for optimization is only related to the structure problem. If FSI problem is need, it can be implemented FSI on an optimization problem. Anyway only structure part for shape will be introduced after that the initial and the optimized will be solved using ADINA for the contact.

5.1 Numerical model and problem definition for shape optimization

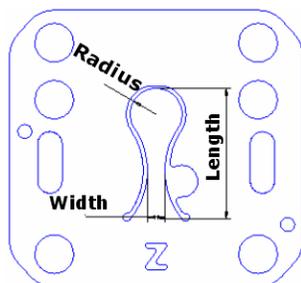


Fig. 5.1 Design variables of Suction reed valve for Shape optimization

Optimization problem was defined the minimum function was the vonmises stress and the constraint was subject to the deflection and the volume. Lastly the design variables were the length, radius, width and thickness as shown in Fig. 5.1

The reason why the design variables were set to the length, radius, width and thickness was they were key parameters to decide the objective function and constraints. And vonmises stress was chosen as objective function because fatigue stress was the main reason of valve breakdown. In constraint function selection, valve deflection was the factor to increase the cooling efficiency through increasing mass flow rate and volume was the factor to

reduce the material cost. In design variables it very important to set the appropriate bound to get the reasonable optimization result.

5.2 Response surface method

To construct response surface method the used design of experiment [DOE] was chosen as Central Composite Design [CCD].

Next selection of approximation domain will be considered. At the first stage of constructing response surface model it is very difficult to set the reasonable design variable bound, so briefly selected the bounds like table 5.1

Variables	Lower Bound	Upper Bound	Deviation
Radius	4.00	5	ΔR=1.0
Length	17.30	18.3	ΔL=1.0
Width	2.20	3.2	ΔW=1.0
Thickness	0.15	0.25	ΔT=0.1

Table 5.1 Initial selections of the design variable bounds

The bounds of design variables at the first time were only considered such the factor as modeling setup, interference with cylinder and leakage to suction hole. So the optimization result was poor that test criteria $R^2 = 0.5 \sim 0.7$ and $R^2_{Adj} = 0.1 \sim 0.3$. R^2 and R^2_{Adj} are the test criteria value as equation (5.1)

$$R^2 = 1 - \frac{SS_E}{S_{yy}}, \quad R^2_{adj} = 1 - \frac{SS_E / (n - p)}{S_{yy} / (n - 1)} = 1 - \left(\frac{n - 1}{n - p} \right) (1 - R^2) \tag{5.1}$$

With the reselection design variables' bounds response surface method was implemented. The results became better as $R^2 = 0.97 \sim 0.99$ and $R^2_{Adj} = 0.95 \sim 0.99$. The method to get the curve fitting was Least Square Method and quadratic model. Design variables priority check was done to know which design variable was most effective to the performance of the objective and the constraint functions.

Term Name	Response1	Response2	Response3
1	3.9376e+000	4.5025e+005	2.5894e+001
x1	3.3479e-002	6.2854e+002	3.1752e-001
x2	6.2042e-002	3.5161e+003	2.4747e-002
x3	-2.1181e-001	-2.9220e+004	4.6778e-001
x4	-6.0899e-001	-4.9243e+004	1.3562e+000
x1*x1	1.7554e-002	-2.2821e+002	9.7720e-002
x2*x1	5.3247e-003	9.6345e+002	-2.9266e-001
x2*x2	-4.4184e-003	-2.5035e+002	-1.3896e-001
x3*x1	2.4036e-003	8.9907e+002	-3.8951e-001
x3*x2	-8.0595e-003	-1.3887e+003	1.5796e-002
x3*x3	6.2816e-003	1.6925e+003	-3.4159e-002
x4*x1	-7.4911e-003	-1.0478e+002	2.5067e-002
x4*x2	-8.8343e-003	-3.6113e+002	3.7357e-003
x4*x3	3.0948e-002	3.1806e+003	2.1920e-002
x4*x4	5.4580e-002	3.7883e+003	-3.7788e-002

Table 5.2 RSM result table with respect to the design variables

As shown above table the thickness among the design variables was most sensitive to the objective and the constraint functions. In table response 1 represents maximum deflection, response 2 represents stress and response 3 represents volume respectively.

After the response surface method, response surface function will be obtained. It can let us know how the design variables can relate to the responses.

5.3 Optimization by means of response surface method

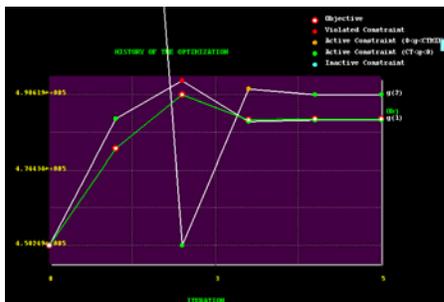
Optimization was implemented using sequential quadratic programming [SQP] and central finite difference method for gradient. Equation (5.2) shows the optimization formulation of SQP that is modified method of feasible direction.

$$\text{Minimize } Q(S) = F^0 + \nabla F^T S + \frac{1}{2} S^T B S$$

Subject to;

$$(\nabla g_j)^T S + g_j^0 \leq 0 \quad j=1,M \quad \text{where, } Q : \text{Hessian Matrix} \quad (5.2)$$

Fig. 5.2 is the optimization history graph, which means iteration end at 2 active constraints.



DV	Initial Design	Lower B	Upper B	Opt. Value	Deviation
Radius	ref1	ref1*A	ref1*E	ref5	-2.0%
Length	ref2	ref2*B	ref2*F	ref6	0.0%
Width	ref3	ref3*C	ref3*G	ref7	-18.8%
Thickness	ref4	ref4*D	ref4*H	ref8	4.8%

Comparison	Initial Analysis	Estimated by RSM	True RSM	Error	Improvement Rate
Max. Deformation	4.376	4.239	4.204	-0.8%	-4.1%
Max. Stress	480000	490783	489249	0.3%	1.9%
Volume	24.3	24.0	23.9	0%	-2%

Fig. 5.2 Shape optimization history plot

Table 5.3 Design variable and response result of optimization

It calls 62 functions and zero gradients during iterating 5. Optimization result is shown in table 5.3

5.4 Contact simulation using ADINA

From the initial design of valve the optimized value was obtained. So to compare how the contact force is improved the simulation was implemented using ADINA. From table 5.3 we utilized the initial and optimum design of suction reed valve. As mentioned the key problem on valve was the crack and breakdown caused by contact and impact with valve plate. For contact problem frictionless contact and coulomb friction was assumed and use the rigid target method used. Besides the rigid target method, there are another ways of segment method and constraint-function method.

After implementing the contact analysis using ADINA, the graph in Fig.5.3 and table 5.4 was obtained.

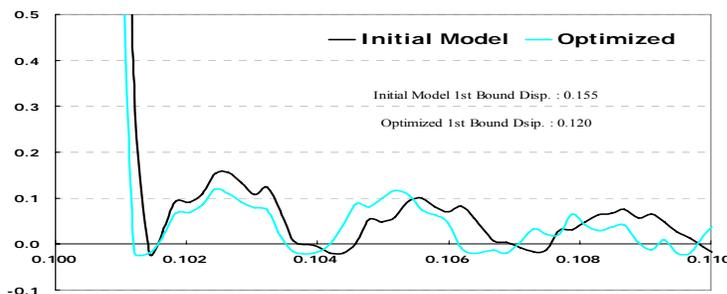


Fig. 5.3 Contact comparison result between the initial and optimized one

Table 5.4 is the summary of the contact simulation result between the initial and optimized design of valve.

Comparison	Initial Model	Optimized Model	Deviation
1st Contact Time [sec]	0.1016	0.1012	-0.0004
1st Bounded Disp. [mm]	0.155	0.120	-23%
Max. Contact Force [mN]	16.50	14.06	-15%

Table 5.4 Comparison contact force between the initial and optimized one

As shown in the above table the optimized contact force was reduced about 15 % compared to the initial model of valve.

6. CONCLUSION

Valve dynamic analysis for suction reed valve and discharge reed valve, and shape optimization for only suction reed valve were implemented. To get valve dynamic behavior, experiment was done using strain gauge, encoder and pressure sensor. As result of experiment we could know the valve dynamic behavior in suction and discharge reed valve and pressure-volume data was also obtained. And fluid-structure interaction analysis was moreover implemented to validate the experiment. Fluid-structure interaction analysis considers the fluid and structure at once and also heat transfer problem. But 2-dimension analysis was only done due to the complexity of 3-dimension. The result of FSI was reasonable. And after validating the valve dynamic behavior, shape optimization for suction reed valve was performed. Response surface method was especially used to get the response function with respect to the each design variables. Thickness of valve was the most sensitivity part to the all responses such as stress, volume and deflection. Using linear least square method and sequential linear programming finally shape optimization was implemented. And optimum value was obtained. With the initial and optimized value of suction reed valve, contact analysis was performed using ADINA. As previously shown with the optimum design the contact force was reduced about 15 % compared to the initial design. It was sufficient result.

NOMENCLATURE

F	Force matrix	G	Influence matrix in BEM
G	Green function in 3D free field	H	Influence matrix in BEM
J	Jacobian Matrix	K	Stiffness matrix
k	Thermal conductivity	k_{ini}	Initial thermal conductivity
l	Face Length of element	n	Unit normal vector
P	Sound pressure	$p(P)$	Sound pressure at a specific point P
P_e	Sound pressure at field point	T	Temperature

REFERENCES

- J.P. Zolesio, 1981 "The material derivative method for shape optimization; In optimization of distributed parameter structures (Edited by E.J. Haug and J. Cea)," *Sijthoff & Noordhoff, Alphen aan den Rijn, Netherlands*,
- Costagiola, M. 1950. "The theory of spring-loaded valves for reciprocating compressor," *ASME Journal of Applied Mechanics*, Vol.17 No.4. 415-420
- John F.T. MacLaren, 1972 "A review of simple mathematical models of valves in reciprocating compressor," *Proceedings of the Purdue compressor technology conference*
- Soedel, W., 1984,"Design and Mechanics of compressor valves," Purdue University
- Josef Brablik, 1972, "Gas pulsation as factor affecting operating of automatic valves in reciprocating compressor" *Proceedings of the Purdue compressor technology conference*
- MacLaren J.F.T. and Anthony B. Tramsch, 1972, "{rediction of Vavle behavior with Pulsation flow in reciprocating compressor", *Proceedings of the Purdue compressor technology conference*
- T.J. Trella, 1974 "Effect of valve port gas inertia on valve dynamics" *Proceedings of the Purdue compressor technology conference*
- Rajendra Prakash 1974, "Mathematical Modeling and simulation of refrigerating compressor" *Proceedings of the Purdue compressor technology conference*
- Dernis D. Schwerzler and James. F. Hamilton, 1974 "An analytical method for determining effective flow and force areas for refrigerating compressor valve system" *Proceedings of the Purdue compressor technology conference*
- Papastergiou, S., Brawn, J., and MacLaren J.F.T., 1982 "The dynamics behavior of Half-annular valve reeds in reciprocating compressor", *Proceedings of the Purdue compressor technology conference*
- S. Rao, "Mechanical Vibrations," 1995, Addison-Wesley , 3rd Ed.,
- D. Logan, 1993, "A first course in the finite element method," PWS, Inc., 2nd Ed.,