Final Report

TRADE-OFF ANALYSIS METHODOLOGY FOR ASSET MANAGEMENT

Qiang Bai
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December 2008
FINAL REPORT

FHWA/IN/JTRP-2008/31

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Joint Transportation Research Program
Project No. C-36-64R
File No. 3-5-8
SPR-3110

Prepared in Cooperation with the
Indiana Department of Transportation and
The U.S. Department of Transportation
Federal Highway Administration

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Purdue University
West Lafayette, Indiana, 47907
December 2008
<table>
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<tr>
<th>1. Report No.</th>
<th>FHWA/IN/JTRP-2008/31</th>
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<td>2. Government Accession No.</td>
<td></td>
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<tr>
<td>3. Recipient's Catalog No.</td>
<td></td>
</tr>
<tr>
<td>4. Title and Subtitle</td>
<td>Trade-off Analysis Methodology for Asset Management</td>
</tr>
<tr>
<td>5. Report Date</td>
<td>December 2008</td>
</tr>
<tr>
<td>6. Performing Organization Code</td>
<td></td>
</tr>
<tr>
<td>7. Author(s)</td>
<td>Qiang Bai, Samuel Labi, Zongzhi Li</td>
</tr>
<tr>
<td>9. Performing Organization Name and Address</td>
<td>Joint Transportation Research Program 1284 Civil Engineering Building 550 Stadium Mall Drive, Purdue University West Lafayette, IN 47907-1284</td>
</tr>
<tr>
<td>10. Work Unit No.</td>
<td></td>
</tr>
<tr>
<td>11. Contract or Grant No.</td>
<td>SPR-3110</td>
</tr>
<tr>
<td>12. Sponsoring Agency Name and Address</td>
<td>Indiana Department of Transportation State Office Building, 100 North Senate Avenue Indianapolis, IN 46204</td>
</tr>
<tr>
<td>13. Type of Report and Period Covered</td>
<td>Final Report</td>
</tr>
<tr>
<td>15. Supplementary Notes</td>
<td>Prepared in cooperation with the Indiana Department of Transportation and Federal Highway Administration.</td>
</tr>
<tr>
<td>16. Abstract</td>
<td>In an era that is characterized by funding limitations, increased stakeholder participation, and the need for increased accountability and transparency, transportation agencies seek to ensure that comprehensive evaluation processes are identified and used for decision-making. Consistent with such processes is the incorporation of multiple performance criteria from different program areas, optimization of decisions under constrained budgets, and investigation of trade-offs between program areas, performance measures, budgetary levels, risk levels, and performance thresholds. To help INDOT carry out these processes, this study developed theoretical constructs for scaling and amalgamation of the different performance measures, and for analyzing the different kinds of trade-offs. The scaling of performance measures yields a consistent or dimensionless unit to make them comparable. Amalgamation combines the weighted and scaled performance measures to yield a single utility value that represents the overall desirability of a candidate project. This report documents, with examples, a number of alternative methods for scaling and amalgamation. Also, recognizing that project outcomes are not always known with certainty, the analysis was done for the deterministic (certainty) as well as the probabilistic (uncertainty) scenarios. For the uncertainty scenario, the report addressed two cases: the risk case, where the project outcomes (in terms of the performance measures) have a known probability distribution; and the pure uncertainty case, where the probability distributions of project outcomes are unknown. For risk case, the report presents a method that utilizes the mathematical expectation of the project impacts derived from the probability distribution of the performance measures. For the uncertainty case, the report describes, with numerical examples, Shackle’s model that can be used in addressing the problem. Finally, the report describes how INDOT can carry out an investigation of trade-offs such as changing the performance threshold and shifting budgets from one program area to another. To facilitate implementation, the report includes a set of spreadsheets that are based on hypothetical project data.</td>
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<td>18. Distribution Statement</td>
<td>No restrictions. This document is available to the public through the National Technical Information Service, Springfield, VA 22161</td>
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<td>19. Security Classif. (of this report)</td>
<td>Unclassified</td>
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<td>20. Security Classif. (of this page)</td>
<td>Unclassified</td>
</tr>
<tr>
<td>21. No. of Pages</td>
<td>153</td>
</tr>
<tr>
<td>22. Price</td>
<td></td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENTS

The authors hereby acknowledge the constant support provided by David Holtz of INDOT’s Integrated Transportation Planning Division, Brad Steckler of INDOT’s Engineering Programs Division, and Samy Noureldin of INDOT Research Office throughout the execution of the study. Also, Khalil Dughaish, Dwane Myers, and David Andrewski of INDOT, and David Unkefer and Daniel Keefer of FHWA, served on the study advisory committee (SAC) and made valuable contributions at various stages of the project. Also, the perpetual support of John Weaver of INDOT in establishing asset management initiatives at INDOT in the 1990’s, is herein recognized. William Flora, Jay Mitchell, Deborah Thomas, and Audra Butts of INDOT played important roles by providing general support and readily providing general information or preliminary data. The contributions of research engineers Kumares Sinha and Thomas Morin of Purdue University and Bei Zhou at IIT are herein acknowledged. Finally, we herein recognize the efforts of Amanda Cope and M. Hafizur Arman of Purdue University at various stages of the study.
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CHAPTER 1 INTRODUCTION

1.1 Background Information

1.1.1 Asset Management at the Indiana Department of Transportation (INDOT)

Efficient and safe transportation is critical to a society in meeting its goals of economic competitiveness, social welfare, national defense, domestic security, emergency preparedness, and better quality of life. This is particularly important in the United States where transportation facilities constitute one of the most valuable assets and account for a major share of public sector investment. Investments made in transportation serve to build, operate, and preserve the physical infrastructure thus facilitating the realization of transportation agency goals. The physical condition and operational performance of transportation facilities are key overall factors in the assessment of transportation systems from the perspective of the agency as well as the facility users and the affected community as a whole.

It is therefore critical that the highway agencies such as INDOT manage their assets in a strategic way that duly recognizes the role and importance of these assets and also in a manner that accounts for any existing or anticipated funding or institutional constraints or changes. It is envisaged that such management should focus on the various business processes in a highway agency (such as resource allocation and utilization, evaluation, and decision-making) and that the decisions associated with these business processes are based on reliable information regarding the past or future (foreseen) consequences of alternative actions at the overall system level.

In any discussion of asset management, it is important to establish the domain of assets under consideration and to define what is meant by the term asset management. INDOT manages a wide range of asset types – physical transportation infrastructure (e.g., bridges) and service assets (e.g., traffic safety and mobility infrastructure) are only a few types of the overall asset holding of the agency. Other asset types include INDOT’s human resources, financial capacity, equipment and vehicle fleets, materials stocks, real estate, corporate data and information. In this project, we focus on the physical and service infrastructure only. In this context, transportation asset management has been defined as “a strategic approach to managing transportation infrastructure,” and “a systematic process of maintaining, upgrading and operating assets cost effectively.” Interesting definitions from the literature include:
• A methodology needed by those who are responsible for efficiently allocating generally insufficient funds amongst valid and competing needs [APWA-Lemer]
• A comprehensive and structured approach to the long-term management of assets as tools for the efficient delivery of community benefits [AUSROADS]
• A programmed approach to restoring, preserving and operating physical assets to meet pre-determined goals … by combining engineering and mathematical analyses with sound business practice and economic theory [NYS DOT]
• A systematic process of maintaining, upgrading, and operating physical assets cost-effectively … which combines engineering principles with sound business practices and economic theory, and provides tools to facilitate a more organized, logical approach to decision-making [FHWA]
• A methodology which efficiently and equitably allocates resources amongst valid and competing goals and objectives, and seeks to enhance the usefulness of individual management systems and use their output to provide sound investment data that has been subjected to rigorous analysis [APWA].

As implied from the above definitions, the Asset Manager bears a heavy fiduciary duty to protect the billions of taxpayer dollars already invested in transportation infrastructure and to ensure that the system is being operated and preserved in the most cost-effective and transparent manner. Transportation asset management is still a growing discipline. A number of state agencies have proactively adopted asset management as an overall departmental initiative. The New York DOT, for example, has had since 1998, an active asset management program that is focused on system preservation. Over a decade ago, forward-looking transportation agencies such as DOTs of Arizona, Indiana, and Pennsylvania had started developing asset management plans and have undertaken initiatives that conform to good asset management practice.

1.1.2 Problem Definition

In a continuing bid to enhance its ability to diagnose existing and potential problems throughout the entire highway network, and to evaluate and prioritize alternative strategies for preservation and operations in each program area, INDOT has developed a number of program area systems including:

• Pavement Asset System (PAS)
• Bridge Asset System (BAS)
• Traffic Congestion (Mobility) Asset System (CAS)
• Traffic Safety Asset System (SAS)

These asset systems relate directly to the different “program areas”. PAS, which refers to the whole process for managing pavement assets, utilizes the systemwide Pavement Management model, the automated tool for select pavement management functions which now is the DTIMS pavement module at INDOT. A model is an important component of but does not alone make up the asset management system. The pavement Management model supports the overall Pavement Asset System; the terms or functions are not synonymous, however. This relationship is true among the other asset classes as well. In order to avoid confusion about the terminology, we herein use the term “asset system” to mean the overall broad function of managing an asset category. Also, in this report, the terms “asset system” model, and “management system” are used interchangeably, as are the terms “program area”, “functional area”, and “asset class.

The asset (management) systems were developed initially in response to requirements by the 1991 ISTEA legislation. The development of these systems was dropped in subsequent legislation but was nevertheless continued by most states. PAS and BAS are oriented towards the physical state of the highway assets, as their primary purpose is to inventory, track, and upgrade the condition of the various components of the highway network and assist in establishing cost-effective strategies to sustain an acceptable condition of such facilities. On the other hand, the SAS and CAS (even though they also involve some physical assets such as roadway safety hardware, for example), are geared, to a greater extent, toward the operational characteristics and performance of the transportation network, and thus can be described as “service assets”. At most transportation agencies, highway asset management systems, as an overarching, integrated decision-support mechanism, are still in their nascent stages of development. However, the various component management systems that will ultimately comprise an integrated highway asset management system are fairly well developed in most states.

The highway Asset Manager (AM) represents individuals not by specific position titles, but by generally assigned duties. As illustrated in Figure 1.1, the typical problem faced by the AM is that every year or programming period, managers at the different asset systems, after carrying out life-cycle costing and other analyses, generate their list of needs or potential projects (herein after called candidate projects). They then send a list of these candidate projects to the Asset Manager. In the ideal world, the AM is adequately resourced to carry out all these projects. However, due to budgetary constraints, the AM can only carry out a selected subset of these candidate projects. This selected subset (that is, the optimal solution) is one that yields maximum
returns to the AM. The issue that arises is that the returns must be expressed in a single value that duly reflects the different performance measures used by the different asset systems.

In certain cases, besides the overall asset budget, there could exist funding restrictions (or budgets) for each asset system (i.e., program area). Also, policy changes may necessitate the raising of budget of one program area and subsequent lowering of another’s (this is equivalent to full or partial transfer of funds from one program area to another). The second issue, therefore, is that the AM may wish to know the effect of such funding shifts on overall network performance in terms of the different performance measures. For example, what will be the impact (in terms of increased crashes and increased mobility) of lowering the safety budget and increasing the congestion budget, or for example, transferring $5M from a safety program to a congestion program? In other words, how many crashes is the AM prepared to trade off for a specific increase in speed? Quantifying such trade-offs is a vital aspect of the work of the Asset Manager.

Figure 1.1: The Problem (Mechanism of Asset Program Development) (adapted from D. Holtz Presentation, 06/06/08)
1.1.3 Further Discussion on Trade-off Analysis

In a more general context, a “trade-off”, or barter, refers to the sacrifice of a physical entity of quality in return for gaining another. It implies that a decision to is being made with full comprehension of both merits and demerits of any particular choice. In transportation asset management, trade-offs can be done at the project level or the network (or system) level. In this study, it is assumed that the project-level trade-off has already been carried out by the managers at the various individual program areas; thus the Asset Manager is interested in trade-offs only at the overall system or program level. There are many types of trade-off, as seen in the following cases:

(1) Trade-off between two alternative individual projects. This involves the comparison of two competing candidate projects and identification of the superior one, and is the implicit mechanism that forms the building block of any project selection algorithm. The merits of a project in this context, is a function of the efficiency and effectiveness of the countermeasure proposal, in addition to the condition of the asset to start with. There are two sub-cases for this situation:
   - Projects are all in the same management system (this occurs at the level of the managers of the program areas);
   - Projects are in different management systems (this occurs at the level of the Asset Manager).

(2) Trade-off between two alternative groups of candidate projects. Analysis of this trade-off is more difficult (in both conceptual and mathematical formulations) than that between two projects, because the constituent projects within a group may have beneficial and/or adverse effects on each other – this is referred to as the “inter-project effect” or “intra-group effect”. Again, the two groups of projects may or may not be from the same management system: in the former case, trade-off occurs at the level of the management systems; in the latter case, it occurs at the level of the Asset Manager. Analysis of these trade-offs is outside the scope of the present study.

(3) Trade-off between two non-cost performance measures. The AM is interested in this type of trade-off particularly where the problem involves multiple (often conflicting) performance measures. The question here, for example, is “how much of performance measure A can be bought for a given level of performance B”. This could be at the project level or the entire network level (the Asset Manager is more interested in the latter). So for example, one could ask:
How much of additional mobility can be earned (and how much of system preservation can be lost) if the Asset Manager transfers $5M from the pavement budget to the congestion budget?

(4) Trade-off between cost performance and levels of one non-cost performance measure. This type of trade-off is of interest where the issue of budget is of concern. So for example, the AM could be seeking answers to the following questions:

- How much of safety enhancement can be traded off for a $1M safety investment?
- What is the elasticity of system preservation to budget? Do the benefits taper off after a certain level of funding?

(5) Trade-off between budgets and thresholds of the various performance measures. For example, what is the minimum budget needed to ensure that certain minimum performance thresholds are attained for the overall network.

(6) Trade-off between the uncertainty (or variability) associated with the performance measures and the levels of the performance measures. All else being the same, the AM prefers projects that yield highest level of performance and smallest variabilities, in other words, we want to be certain that we will achieve superior performance. However, in some cases, a project may have expected high performance that has high uncertainty (with performance levels ranging from, say, 40 to 100 with average 60); a rival project may have relatively low performance (which is bad) with low uncertainty (which is good), with performance ranging from, say, 55 to 60 with average of 58. As such, it is possible to investigate the trade-offs between these two conflicting statistical parameters in the manner in which they relate to project outcomes.

Clearly, when the trade-off analysis is being carried out at the level of the Asset Manager, there exist several different management systems (each with its set of performance measure that may be unique or overlapping with those of other management systems). In this case, the challenge is to express all the different performance impacts of the different candidate projects on a common scale so that comparison and selection is made possible. Therefore, of the three cases of trade-off analysis presented above, Cases 3-7 are what typically interest the Asset Manager.

Finally, and pursuant to the above point, it is necessary to note a terminological issue here: even though project selection implicitly involves trade-offs between alternative projects in a manner that is pairwise and is seen more clearly in a mathematical context, this report presents
trade-off analysis as a step subsequent to (and not as a synonym for) project selection. Thus, this report considers only Cases 3-7 for its discussions of the trade-off analysis concept.

Summing up, the focus of this research is consistent with the function of the Asset Manager in carrying out analysis of trade-offs between projects from the different management systems. The research does not consider the inter-project effect due to the paucity of models that explain these phenomena. In problem statements such as this where there are multiple performance measures involved, the most effective tool to conduct trade-offs is to use techniques of Multi-Criteria Decision Making (MCDM). We discuss this further in the following section.

1.1.4 Impetus for Multiple Criteria Methods for Project Selection and Trade-off Analysis

In the trend towards integrated asset management, agencies are gradually finding that evaluation and decision-making need to be based not only on a single criterion but on a variety of criteria because (i) the different management systems (or program areas) have their own unique dominant performance criteria, (ii) projects in each program area may have additional impact types besides the dominant performance criterion for that program area. For example, a lane-addition project may have impacts not only on congestion mitigation but also on safety. Furthermore, the inclusion of multiple criteria in making investment decisions can help agencies evaluate and select projects in a fashion that duly incorporates the perspectives of multiple stakeholders and transportation functional areas. Specifically, a decision-making mechanism based on multiple criteria can (i) help structure an agency’s decision-making process in a clear, rational, well-defined, documentable, comprehensive, and defensible manner; (ii) help the agency to carry out “what-if” analyses and to investigate trade-offs between competing performance measures, program areas, risk levels, performance thresholds, or funding levels.

This report examines the application of multiple criteria decision-making (MCDM) in project selection and trade-off analysis, and focuses on two aspects of MCDM, namely, scaling and amalgamation, and presents examples to illustrate the application of these aspects. The report also shows how the amalgamated criteria could be used in investigating the different kinds of trade-offs.
1.2 Scope of this Study

1.2.1 Domain of Assets

In general, assets for DOTs include many aspects: human resources, equipment and vehicle fleets, real estate, etc. In this study, only physical assets and service assets directly associated with highway operations, such as pavements, bridges, and safety and mobility infrastructure, are considered. The concepts have been developed for state highway facilities, but could be easily applied to facilities on local systems.

1.2.2 Aspects of Multi-Criteria Decision-Making to be Included

Figure 1.2 illustrates the entire process of solving the multiple-criteria decision making and trade-off analysis problem. This uses several performance criteria to evaluate each candidate project (or “alternative”) and finally make a decision based on these criteria or performance measures. Multi-criteria decision making involves many steps: identifying performance measures, weighting, scaling, amalgamation, etc. The study focuses mostly on two aspects of multiple criteria analysis – scaling and amalgamation; however, in a bid to illustrate these concepts and thus to clarify how INDOT’s Asset Manager can apply these concepts, the report also includes discussions on how the Asset Manager could carry out optimization and trade-off analysis. The report also includes a set of spreadsheets in which hypothetical data are used to illustrate the application of these theories and concepts.

(a) Scaling (Normalizing or Standardizing) the Performance Measures.

As the multiple performance measures have different units, an effort is herein made to make them (and their different units) comparable by normalizing them to a certain scale (e.g., 0 to 100). Scaling renders the performance measures onto a dimensionless scale this making it easy to compare the different impacts and to amalgamate them (i.e., to yield an overall combined impact or desirability for each alternative project).

(b) Amalgamating the Performance Measures to Conduct Trade-off Analysis.

Another major scope of the study is the amalgamation or combination of the scaled performance measures to develop trade-off analysis. The basic idea of amalgamation is to combine the performance measures of a (candidate) project to form a single value which reflects the overall impact of that project. This report presents a number of methods for amalgamation.
(c) Optimization Techniques and Analysis of Trade-offs.

The report shows how INDOT’s Asset Manager, after scaling and amalgamation, could select projects from the overall portfolio of candidate projects for the “knapsack” (that is, only those which can be funded) either through prioritization or by optimization. This takes due cognizance of funding or political constraints. Also, the Research Team went beyond the revised work scope to provide methodologies to quickly investigate the impacts of shifting funds across the management systems and other trade-offs that arise in the course of network-level asset management.

It is worth mentioning that there are other key aspects of multiple criteria decision-making, namely identification of performance measures to suit a specific decision-making problem, and establishing weights of the performance to reflect their relative importance to the decision-maker, as seen in Figure 1.2. These are outside the revised scope of this research. At the time of reporting, INDOT is in the process of revising and finalizing individual performance measures, standards of acceptable performance, and relative “weights” as an internal exercise. The Appendix to this report presents a number of alternative techniques for developing the weights for a selected set of performance measures in an impartial and defensible manner.

1.2.3 Scenarios involving “Benefit” Performance Measures (for each Program Area)

The problem can be thought of as comprising the following alternative scenarios:

- Each asset (management) system or program area has only one “benefit” performance measure. So, for example, for any pavement project, the only benefit is pavement preservation. In other words, this scenario assumes that a pavement project has no impact on the other benefit performance measures (safety enhancement and congestion mitigation). For this scenario, relatively few of the discussed MCDM methods of scaling and amalgamation can be used for the analysis.

- Each management system or program area has an array of “benefit” performance measures. So, for example, a pavement project is one that is initiated principally to address pavement performance and/or sponsored/supported by a particular funding program having pavement as its focus; however, the benefits of such a project could be not only an improvement in pavement condition, but also safety enhancement or congestion mitigation. For this scenario, all the discussed MCDM methods of scaling and amalgamation can be used for the analysis.
Figure 1.2: The Solution (Steps in Typical Multi-Criteria Decision-Making) (Sinha and Labi, 2007)

(Shaded Area represents the Main Focus of the Present Study)
1.2.4 Certainty and Uncertainty Considerations

Project outcomes are not always known with certainty. For example, the reduction in IRI or final IRI may hover around a certain average value but is not expected to be the same even for all similar projects. Thus, INDOT needs methodologies to carry out optimization and trade-off analysis not only for the deterministic (certainty) but also for the probabilistic (uncertainty) scenarios. In classical literature, and indeed in real life such as INDOT practice, there are two subcases for the uncertainty scenario: the risk case, where the project outcomes (in terms of the performance measures) have a known probability distribution; and the pure uncertainty case, where the probability distributions of project outcomes are unknown. It is useful for INDOT to have the capability for carrying out the analysis under all these cases and subcases.

1.3 Contents of this Report

This report first provides a brief background to the study, including the study scope and objectives. Then Chapter 2 of the report presents and explains, with examples, the standard methods to “normalize” the different performance measures so that a common scale can be established to account duly for the different units of the performance measures – that way, “apples and oranges” can be compared for purposes of project selection and trade-off analysis. After scaling has been carried out, there is a need to determine the overall impact of a given project on the basis of its scaled or weighted-and-scaled performance measures; thus Chapter 3 presents the techniques for amalgamation. Knowing the amalgamated value of impacts associated with each project (or multiple alternatives within a project), a smaller subset is chosen from the overall population of candidate projects in such a manner that maximizes the Asset Manager’s benefits and yet minimizes his/her costs, under given funding limitations or budget. Thus, Chapter 4 presents the techniques for developing this smaller subset or asset program. Chapter 5 provides some methods to deal with the uncertainty situation where the project outcomes in terms of performance measure are not known with certainty. For trade-offs, Chapter 6 presents a few scenarios for trade-off analysis and shows how they could be addressed. Chapter 7 summarizes and concludes the report.
CHAPTER 2: SCALING METHODS

2.1 Introduction

In attempting to make decisions on the basis of multiple criteria, the Asset Manager is faced with an array of performance measures that reflect the performance (various costs and benefits) of each candidate project. These multiple performance measures have different units or metrics, for example, safety enhancement is often measured as a reduction in fatal and serious personal-injury crashes; improved mobility (congestion relief or accessibility/connectivity) is often expressed in terms of reduction in delay, enhanced level of service (LOS), decrease in travel time, or reduced volume-to-capacity ratio; pavement system preservation can be measured as a reduction in International Roughness Index (IRI), extension in pavement remaining life for friction or other pavement attributes, etc.; bridge system preservation is often measured as an increase in NBI condition rating, reduction in earthquake vulnerability, and at a network level, the decrease in number or percentage of structurally deficient or functionally obsolete bridges, etc. These are typically referred to as the benefit performance measures because they reflect some benefit to INDOT or facility users. Often included in the multiple performance measures also are the cost performance measures, which often refer to the agency cost of project implementation. Unlike the benefit performance measures, cost performance measures are applicable to all projects irrespective of program area. User costs may be considered a benefit or cost performance measure depending on the wishes of the Asset Manager (AM). If the AM wishes to express user cost as a cost performance measure, then it must be used in the analysis in its absolute terms or raw cost values; if, on the other hand, the AM wishes to express user cost as a benefit PM, then it must be calculated as the reduction in user cost relative to a base case (such as the do-nothing alternative).

Prior to scaling, it may be necessary to modify the actual values of the performance measures to account for differences in project size or traffic volume in order to avoid bias. This is necessary if the bias issue is not already addressed in the manner the performance measures themselves are defined to avoid such bias. To illustrate the bias issue, consider two projects that have the same reduction in crash rate but one serves a higher traffic compared to the other, or one is a longer segment than another; a traditional way to reduce these values for the analysis is to express the performance measure as a value per traffic volume, per mile, or per vehicle-miles of travel. In some texts, this may be referred to as “scaling or “normalization” but it should be noted
that while such “scale’ adjustments are necessary to obviate bias, they are different from the scaling that is addressed in this chapter.

The Asset Manager can choose the best projects that satisfy his/her limited asset program budget only after he/she has expressed all the different benefit and costs performance measures, for each candidate project, in terms of a single representative overall performance measure or “desirability”: the candidate projects yielding the highest value of that overall desirability are chosen successively until the budget is exhausted. This chapter discusses a number of alternative techniques that could be used to render all the different performance measures onto the same scale, dimension, or unit. The next chapter (Chapter 3) presents how the Asset Manager can then amalgamate (or, combine) the different performance measures (additive benefits, benefits less cost, benefit/cost, etc.). Clearly, some types of amalgamation can proceed without carrying out scaling. Chapter 4 presents techniques on how to choose the best projects after they have been scaled and amalgamated.

As seen in Figure 2.1, scaling techniques may be categorized as follows: so-called “objective” methods and preference-based methods. In each method, scaling is carried out separately for each performance measure. As implied in earlier sections of this chapter, the results of the scaling procedure yield a function that represents the worth or desirability of the different
levels of the performance measure. In the simplest case, the least preferred level of the performance measure is assigned a value of one (or 100%) and the worst a value of zero. This way, one can assign a scaled unit to represent the impact of any project in terms of any performance measure.

The objective methods include linear scaling, probability distributions, and monetization. The preference-based methods are considered by some schools-of-thought as being subjective because they are developed on the basis of expert opinion, through surveys. Scaling functions developed using preference-based methods can be categorized into the value functions and utility functions. A utility function is considered a more general form of a value function: like value functions, utility functions incorporate the innate values that the Asset Manager attaches to the different levels of the performance measure; unlike value functions, utility functions incorporate the Asset Manager’s attitudes toward risk (i.e., whether the Asset Manager is risk prone, risk neutral, or risk averse).

2.2 Objective Scaling Methods

2.2.1 Linear Methods

These are used to derive a scaling function that is assumed to be linear. This technique can be used when the Asset Manager has no data that can help him/her develop a scaling function. Thus, a linear scaling function can be considered as the default for all scaling functions. The linear scaling function often ranges from 0 to 1, 0 to 10, or 0 to 100, depending on the wishes of the Asset Manager. There are at least four shapes of the linear scaling function: monotonically-increasing, monotonically-decreasing, upward V, and downward V.

For monotonically-increasing linear scaling functions (where higher values of the performance measure are more desirable to the Asset Manager) such as bridge sufficiency and pavement condition, Equation (2.1) and Figure 2.2 represent the scaling function.

\[
r(x) = \begin{cases} 
0 & x \leq x_0 \\
\frac{x - x_0}{x_1 - x_0} & x_0 \leq x \leq x_1 \\
1 & x \geq x_1 
\end{cases}
\]  

(2.1)
Figure 2.2: Scaling Function for Linearly Monotonically-Increasing Performance Measures

For monotonically decreasing linear scaling functions (where higher values of the performance measure are less desirable to the Asset Manager), such as agency cost, IRI, crash rate, and delay, Equation (2.2) and Figure 2.3 can be used for the scaling procedure.

\begin{equation}
    r(x) = \begin{cases} 
        0 & x \geq x_1 \\
        \frac{x_1 - x}{x_1 - x_0} & x_0 \leq x \leq x_1 \\
        1 & x \leq x_0 
    \end{cases}
\end{equation} 

(2.2)

Figure 2.3: Scaling Function for Linearly Monotonically Decreasing Performance Measures

In some cases, the linear scaling function is monotonically increasing up to a point and then monotonically decreasing thereafter or monotonically decreasing up to a point and then monotonically increasing thereafter. This is the case when the Asset Manager prefers that the performance measure should not be too small or too large, or where the AM desires that the performance measure is desirable only when it is lower than some threshold or when it exceeds some threshold. For instance, for the travel speed performance measure, it is often desired that
speed should not be too low or too high because either extreme is associated with higher fuel consumption.

On a 0-1 scale, the linear concave non-monotonic scaling function can be represented as:

$$r(x) = \begin{cases} 
L & x \leq x_0 \\
\frac{x - x_0}{x^* - x_0} & x_0 \leq x \leq x^* \\
H & x^* \\
\frac{x_1 - x}{x_1 - x^*} & x^* \leq x \leq x_1 \\
L & x \geq x_1 
\end{cases} 
$$

(2.3)

When $L = 0$ and $H = 1$, this function can be illustrated as Figure 2.4.

On a scale of 0-1, the linear convex non-monotonic scaling function can be represented as:

$$r(x) = \begin{cases} 
H & x \leq x_0 \\
\frac{x^* - x}{x^* - x_0} & x_0 \leq x \leq x^* \\
L & x^* \\
\frac{x - x^*}{x_1 - x^*} & x^* \leq x \leq x_1 \\
H & x \geq x_1 
\end{cases} 
$$

(2.4)

When $L = 0$ and $H = 1$, this function can be illustrated as Figure 2.5.

![Figure 2.4: Scaling Function for Non-monotonic Performance Measures (Concave)](image-url)
Example: On a highway with speed limit 50 mph, average travel speed (X) can be used as a performance measure to evaluate mobility. Thus, the theoretical range of X is [0, 50], and its scaling function can be shown in Figure 2.6. So, for example, if the actual average travel speed after a project implementation is 36mph, then the scaled value of that project impact is \((36-0)/(50-0) = 0.72\).
2.2.2 Scaling Methods based on Probability Distributions

This method utilizes a relative frequency distribution of the outcomes of all similar projects. It is rather easy to construct if the data on expected outcomes (in terms of the performance measure) is available. It is important to note that the use of probability distributions for scaling should not be confused with the issue of uncertainty or risk of project performance outcomes. In this case, the probability distributions are used solely as a measure of relative standing of project impacts relative to others (which is consistent with the purpose and intent of scaling).

There is a large number of distribution types whose cumulative probably functions could be used for scaling. These include the normal distribution, the standard normal distribution, the exponential distribution, the Erlang distribution, the beta or gamma distribution, etc. For each of these distribution types, there are two methods that could be used for scaling:

(a) Using the probability distribution function

In this method, the outcome of the project (in terms of the performance measure), say $X$ is standardized by subtracting from it the mean value of all outcomes and dividing it by the standard deviation of all outcomes, as follows:

$$ Z = \frac{X - \mu}{\sigma} $$

$X$ is a raw score or observation (or in our case, project outcome in terms of some performance measure); $\mu$ is the population mean; $\sigma$ is the population standard deviation.

It can be seen that the units cancel out and thus the outcome is rendered into a dimensionless or unitless number: this is the essence of any scaling procedure.

Thus, the standard score, or $Z$ score, is a dimensionless quantity derived by subtracting the population mean from an individual raw score and then dividing the difference by the population standard deviation. In many statistics texts, this conversion process is called standardizing or normalizing. Synonyms for $Z$-score include normal score and standardized variable, or in the context of asset management, standard outcome. The $Z$ score indicates how many standard deviations an observation is above or below the mean. $Z$ may be negative or positive, and where the direction of deviation is not of concern to the decision-maker, its absolute value or squared value is taken to remove the signs, in such cases, it is referred to as a $Z$-square score. It allows comparison of observations from different normal distributions (or in the case of asset management, different distributions from the different management systems).
Paradoxically, the merits of the Z-score technique also give rise to its limitations. First, not all the project outcomes follow a normal distribution, as can be seen when one plots the data. Secondly, the Z score may be biased because it is greatly influenced by the variability in the data. Thus, a highly beneficial project in one management system may get a low Z score only because the benefits of its sister projects are highly variable. On the other hand, a project with little benefits may get a high Z-score only because its benefits indicate little variability with those of its sister projects.

A second limitation of this method is that before we can calculate Z, we need to know the population mean and the population standard deviation, not the mean or standard deviation of a sample drawn from the population of interest. But knowing the true standard deviation of a population is often unrealistic except in cases such as standardized testing where the entire population is measured. In cases where it is impossible to measure every member of a population, the standard deviation may be estimated using a random sample, but if the sample is not truly random, this could give rise to statistical issues of bias and consequently, imperfect predictions and evaluations.

Example:
Assume that the change in International Roughness Index (IRI) (inches/mile) of a certain type of pavement improvement follows normal distribution with mean 100 inches/mi and standard deviation 51 inches/mi (Figure 2.7). So, for example, for a pavement section that received that treatment, if the change in IRI 118 inches/mile, find the scaled value of the project impact.

![International Roughness Index Distribution Function](image)

Figure 2.7: Distribution Function for the Change in International Roughness Index
Then the Z-score for the IRI change of the pavement segment is:
\[ Z = \frac{118 - 100}{51} = 0.353 \]

If higher values of the performance measure are more desirable, such as change in IRI or travel speed, then a higher Z score indicates a more desirable impact; if lower values of the performance measure are more desirable, such as IRI or air pollution, then a higher Z score indicates a less desirable impact.

(b) Using the cumulative probability distribution function
This method of scaling uses not the probability distribution but its corresponding cumulative function of the performance measure to derive the scaled value of the project impact in terms of that performance measure.

The cumulative probability of a performance measure represents the probability that the value of performance measure is lower than a certain value. Let \( x \) be a value of performance measure \( X \), then the cumulative probability of \( x \) can be viewed as the relative position of \( x \) in the whole population of the performance measure \( X \). In fact, monotonically increasing/decreasing linear scaling can also be viewed as a special case of the cumulative probability scaling method where the distribution of performance measure is uniform distribution all across its range. In practice, however, few, if any, performance measures are uniformly distributed.

The cumulative probability distribution function method can be only used when performance measures are monotonically increasing/decreasing “desirableness” (where higher values of the performance measure are more/less desirable to the Asset Manager). For monotonically-increasing desirableness performance measures, the scaled value can be calculated as:
\[ S(x) = F(x) \]
Where \( S(x) \) is the scaled value of \( x \);
\( F(x) \) is the cumulative probability function of performance measure \( X \).

For performance measures that are monotonically-decreasing, such as IRI or crash rate, the scaled value can be calculated as
\[ S(x) = 1 - F(x) \]

Figure 2.8 and Figure 2.9 show examples of the cumulative probability scaling method for a performance measure with monotonically-increasing desirableness.
We herein present examples of how the Asset Manager could use the cumulative probability distribution method to scale a given set of outcomes of a performance measure. This can be applied to any probability distribution which has a continuous-variable outcome (that is where the outcome is not discrete categorical, etc.): the examples below are for the relatively common cases where the system performance outcome is normally-distributed or beta-distributed.

**Example involving the Normal distribution**

Assume the average unit expenditure \( X \) of a specific type of bridge rehabilitation follows the normal distribution. \( X \sim \mathcal{N}(150, 60^2) \), the unit of \( X \) is $/ft^2. Then the scaling function of \( X \) is

\[
S(x) = 1 - F(x)
\]

Where \( F(x) \) is the cumulative normal distribution function.

The graph of the function is shown below (Figure 2.10).

For example, assume that a specific intended application of that type of bridge rehabilitation is estimated to have a unit expenditure of $200/ft^2. Determine the corresponding scaled value of this cost performance.
Solution: If the unit expenditure of bridge rehabilitation is 200 dollars/ft$^2$, then the scaled value is 0.202.

**Example involving the Beta distribution**

The average fatality collision rate $X$ (Number of fatality collisions per Million VMT) of highway network follows the Beta distribution: $B(2.4, 2.37)$. Then the scaling function is:

$$S(x) = 1 - F(x)$$

Where $F(x)$ is the cumulative beta distribution function.

The graph of this function is shown as Figure 2.11.

For example, State Road 555 has a fatality collision rate of 3.8 fatality collisions per Million VMT. Assuming this performance measure follows a Beta distribution with mean 2.4 and standard deviation 2.37, determine the scaled value of the safety performance of that highway.
Solution: Since the fatality collision rate of the highway is 3.8 fatality collisions /Million VMT, its scaled value can be determined using the equation or figure as 0.125.

2.2.3 Monetization

In highway asset management, there are relatively few performance measures whose units are monetary – these are a project’s agency cost and user cost. Then there are those that are intrinsically monetary, that is, they are not expressed in monetary units but could be expressed in such units using appropriate relationships established through research. For example, safety performance can be measured in terms of a reduction in crash rate, for example, 50 crashes per 100 million VMT; pavement performance can be measured in terms of the International Roughness Index (IRI) in inches/mile. Transforming all these different performance measures into their monetary equivalents or dollar units is thus a special type of scaling that is appropriately termed “monetization”. Intrinsically non-monetary performance measures are those that cannot be expressed in monetary equivalent because it is considered impractical or perhaps, even unethical to do so.

As a simple example of intrinsically monetary performance measures, consider a highway project that is expected to yield a reduction of 20 crashes/100 million VMT. If the project is expected to serve a demand of 50 million VMT at the time of project completion, then the annual benefits is 10 fatal crashes. If the cost of a fatal crash is $1M, then the monetized benefit (or the scaled value of safety performance), assuming constant demand, is 10*1 = $10M.
For bringing different performance measures to the same dimension or scale, monetization is a common method, even though it is often not recognized explicitly as a scaling technique. In most transportation project evaluations, decisions are made on the basis of monetized values of the relevant performance measures while non-monetized performance measures are often relegated to the background of mere conceptual (and often, inconsequential) discussion. As only relatively few measures can be quantified in their monetary values, monetization severely limits the number of performance measures that can be considered in evaluation. For example, ecological damage that accompanies the construction and operations of freeway systems in rural areas cannot be satisfactorily measured in its monetary equivalent as there are not universally accepted models for doing so. Below, we present some models from the literature that could be used to monetize a number of performance measures.

(a) Conversion of Travel Time Reduction into Monetary Units

Table 2.1 shows how the Asset Manager could scale the performance benefits of travel time reduction (in hours) into a dollar value. In the simplest case, only one vehicle class is used and no clocking status is considered. In a more comprehensive analysis, however, it is useful to consider such nuances in travel time estimation and valuation. On-the-clock travel time, which represents work-related travel, are based on costs to the employer such as wages and fringe benefits, costs related to vehicle productivity, inventory-carrying costs, and spoilage costs. Off-the-clock trips include trips for commuting to and from work, personal business, and leisure activity. Heavy trucks are assumed to be used only for work, so the value of time equals the on-the-clock value. Table 2.1 summarizes the estimates of major cost components of the value of travel time by vehicle type, on the basis of FHWA’s HERS software (FHWA, 1999). For a future congestion mitigation project in the Asset program, if the travel time reduction is known for each of the indicated categories (On-the-Clock and Off-the-Clock), then the indicated values can be used to find the equivalent dollar value of the congestion-mitigation performance of the project.

Table 2.1: Distribution of Hourly Travel Time Values by Vehicle Class (2005$)

<table>
<thead>
<tr>
<th>Category</th>
<th>Small Automobile</th>
<th>Medium Automobile</th>
<th>4-Tire Truck</th>
<th>6-Tire Truck</th>
<th>3-4 Axle Truck</th>
<th>4-Axle Combination Truck</th>
<th>5-Axle Combination Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>On-the-Clock</td>
<td>$34.34</td>
<td>$34.70</td>
<td>$24.77</td>
<td>$30.61</td>
<td>$33.13</td>
<td>$38.04</td>
<td>$38.72</td>
</tr>
<tr>
<td>Off-the-Clock</td>
<td>$17.54</td>
<td>$17.58</td>
<td>$18.50</td>
<td>$30.61</td>
<td>$33.14</td>
<td>$38.04</td>
<td>$38.73</td>
</tr>
</tbody>
</table>

(b) Conversion of Safety Benefits into Monetary Units

When safety benefits are expressed as the number of reduced crashes per VMT, the corresponding monetary cost savings is determined as the product of the crash reduction per VMT and the unit monetary crash cost to yield the dollars saved per VMT. The two commonly used sources for the unit dollar value estimates are the annual publication of the National Safety Council Estimates and the 1988 FHWA memorandum. Also, the cost of road crashes can be based on a weighted injury scale by using indices to the level of severity of the road crash. The 2005 unit costs of each crash severity type are available for injury scales such as the KABCO rating scale (NSC, 2001), the Abbreviated Injury Scale (Blincoe et al., 2002). Table 2.2 shows the unit crash cost values for the KABCO scale, updated from NSC (2001) using consumer price indices from the US Department of Labor (USDL, 2005).

<table>
<thead>
<tr>
<th>Code</th>
<th>Severity</th>
<th>Unit Cost (2005$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>Fatal</td>
<td>$3,654,299</td>
</tr>
<tr>
<td>A</td>
<td>Critical</td>
<td>$181,276</td>
</tr>
<tr>
<td>B</td>
<td>Severe</td>
<td>$46,643</td>
</tr>
<tr>
<td>C</td>
<td>Serious</td>
<td>$22,201</td>
</tr>
<tr>
<td>PDO</td>
<td>Moderate</td>
<td>$2,116</td>
</tr>
</tbody>
</table>

1. Updated from (NSC, 2001).

(c) Conversion of Pavement Condition Improvement into Monetary Units

To some extent, pavement roughness, measured in terms of Present Serviceability Rating (PSR), or International Roughness Index (IRI), can affect maintenance, tire, repair, and depreciation components of VOC (vehicle operating cost) and thus can translate into direct increases in out-of-pocket costs of road users. This is because the motion of vehicle tires on a rough pavement surface is associated with greater resistance to movement which leads to higher levels of fuel consumption compared to traveling at a similar speed on a smooth surface; and a bumpy ride which leads to increased vibration and wear-and-tear of vehicle parts. Also, an indirect effect of a poor pavement condition is that road users may be forced to drive at lower speeds leading to higher fuel consumption. Projects such as resurfacing that improve the pavement surfaces therefore lead to reductions in unit VOCs caused by pavement roughness.
High levels of pavement condition (low roughness) increments in condition have relatively little effect on vehicle operating cost (Figure 2.12), and additional costs of vehicle operation start to accrue only when IRI exceeds at a point at approximately 100 in/mi (3.33 m/km). For paved roads in poor condition and for gravel roads, changes in road surface condition, can lead to very drastic reductions in VOC.

![Figure 2.12: Conversion of Pavement Condition to Cost (Opus, 1999)](image)

Papagiannakis and Delwar (2001) concluded that a unit increase in IRI (in m/km) will generally lead to an increase of $200 (or 1.67 cents per vehicle-mile, assuming 12,000 annual mileage) in vehicle maintenance and repair costs alone. Barnes and Langworthy (2003) developed adjustment factors for all VOC components combined, as a function of pavement condition (Figure 2.13). They assumed a baseline of PSI of 3.5 or better (IRI of about 85 inches/mile or 1.35 m/km) at which an increase in pavement condition would have no impact on operating costs, and then adjusted for three levels of rougher pavement as shown in the figure. The figure can be used to estimate the VOC corresponding to a given pavement state on the basis of the VOC at a baseline state of the pavement. For the depreciation component, there seems to be relatively few studies that have explicitly shown a relationship with pavement roughness. However, it is clear that a vehicle that is operated on a rough pavement surface is likely to lose its value faster than one that is operated on a smooth surface pavement.
Figure 2.13: VOC Adjustments for Pavement Roughness Levels

Example: A warranty HMA resurfacing project on Interstate 599 yielded a performance jump of 40 IRI (in/mi). If the base vehicle operating cost is $143 per 1000 vehicle-miles, (i) determine the change in unit VOC due to the resurfacing using the Barnes and Langworthy relationship. The IRI before the improvement was 110 in/mi. (ii) If the traffic volume is 67,500 vpd, and the section is 6.5 miles in length, determine the overall change in VOC.

Solution: (i) Before improvement: IRI = 110 in/mi., and the VOC adjustment multiplier is:

\[ m = 0.001((110-80)/10)^2 + 0.018((110-80)/10) + 0.9991 = 1.06 \]

VOC = 1.06*143 = $151.58/1000VMT

After improvement: IRI = 110 – 40 = 70 in/mi, \( m = 1.00 \) since 70 is less than 80, and therefore VOC = $143/1000VMT.

Change in unit VOC = 151.58 – 143 = $8.58/1000VMT.

(ii) Overall change in VOC = $8.58 * 67,500 * 365*6.5 = $1.37 million per year

Overall General Example for Monetized Performance
Consider projects A, B and C for which we seek to scale their performance impacts.

A: Pavement project. An interstate pavement project A will improve the pavement condition. The initial IRI is 120 in/mi. After the project is finished, the IRI will be 50 in/mi. The base vehicle operating cost is $143 per 1000 vehicle-miles, the traffic volume is 100,000 vpd, and the section is 16 miles in length. Then the monetary benefit can be calculated as follows:
(i) Before improvement:
\[ \text{VOC} = (0.001((120-80)/10))^2 + 0.018((120-80)/10) + 0.9991) \times 143 \]
\[ = 1.0871 \times 143 = \$155.455/1000 \text{VMT} \]

(ii) After improvement: \( \text{VOC} = 1 \times 143 = \$143/1000 \text{VMT} \)

Overall change in VOC = \$(155.455-143) \times 100 \times 365 \times 16 = \$7.27 \text{ million per year}

**B: Safety Project B.** A safety project B is expected to yield a reduction of 10 fatal crashes per 100 million VMT (From 15 fatal crashes per 100 million VMT to 5 fatal crashes per 100 million VMT). And the project is expected to serve a demand of 20 million VMT at the time of project completion, so the annual benefit will be 2 fatal crashes. The cost of a fatal crash is $3M. So the monetized benefit is $6M. Assume constant demand across the years.

**C: Congestion Project.** A congestion project C can reduce the On-the-Clock travel time by 2 minutes/vehicle-day (From 20 minutes to 18 minutes), and reduce Off-the-Clock travel time by 3 minutes/vehicle-day (From 25 minutes to 22 minutes). The average traffic volume is 10,000 per day. The On-the-Clock travel time value is about $34.50/h, the Off-the-Clock travel time value is about $17.55/h. Then On-the-Clock Benefit: \( 2/60 \times 10,000 \times 34.5 \times 365 = \$4.2M \), and Off-the-Clock Benefit: \( 3/60 \times 10,000 \times 17.55 \times 365 = \$3.2M \). So the total benefit is $7.4M.

A summary of these results, for all the candidate projects and all three performance measures, is provided in Table 2.3. It can be seen that the scaled value of the congestion project is highest and that of the safety project is least. So on the basis of monetary values alone, the safety project is the most attractive.

<table>
<thead>
<tr>
<th>Projects</th>
<th>Performance Measures</th>
<th>Performance Measure Changes</th>
<th>Monetized Benefit (million dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Pavement Project</td>
<td>IRI</td>
<td>Before Improvement: 120 inches/mile</td>
<td>After Improvement: 50 inches/mile</td>
</tr>
<tr>
<td>B: Safety Project</td>
<td>Crash Rate</td>
<td>15 fatal crashes per 100 million VMT</td>
<td>5 fatal crashes per 100 million VMT</td>
</tr>
<tr>
<td>C: Congestion Project</td>
<td>Travel time</td>
<td>On-the-Clock: 20 minutes Off-the-Clock: 25 minutes</td>
<td>On-the-Clock: 18 minutes Off-the-Clock: 22 minutes</td>
</tr>
</tbody>
</table>

As a scaling technique, monetization has serious drawbacks. First, there has not been enough research to quantify all transportation impacts in their monetary equivalents. Secondly, there can be ethical issues in the attempt to assign monetary values to safety impacts. Thirdly, the
use of monetary values yields a scale that is unbounded and this could cause some computation
problems.

2.2.4 “Distance from Specified Goal” Scaling Technique

This scaling method requires the decision-maker to establish target levels or goals that
ideally need to be achieved. For each alternative and performance measure, the scaled value of
the performance measure is the deviation from the specified target. On a Cartesian axis, this
simply is the distance from the target – the smaller the deviation, the more preferred the
alternative from the perspective of that performance measure. For one performance measure, this
simply is the vertical deviation (Figure 2.14 (a); for two performance measures, this is the
diagonal distance (Figure 2.14(b)); for three performance measures, this is the diagonal distance
in a three dimensional space (Figure 2.14(c)); for \( J \) performance measures, this overall distance is
given by Equation (2.5):

\[
Z = \left( \sum_{j=1}^{J} (A_j - M_j)^p \right)^{1/p}
\]  

(2.5)

Where \( Z \) represents the sum of deviations from the goal;

\( A_j \) represents the value of the \( j^{th} \) performance measure;

\( M_j \) is the target value of the \( j^{th} \) performance measure;

There are different norm metrics that can be used in the minimization of the goal
programming function. The parameter ‘\( p \)’ is varied to determine the type of distance metric being
measured. The three most commonly considered metric norms in goal programming are:

- If \( p = 1 \), “city block” distance
- If \( p = 2 \), “Euclidean” distance
- If \( p = \infty \), “Minmax” distance (or infinity norm)

At the subsequent stage of amalgamation, the distances of all the alternatives are
compared and the following example illustrates the application of the goal programming
technique.
The City of Megapolis is planning a long distance transit service connecting suburban areas to downtown. Four alternatives are being considered. The goal of the city is to have a maximum project cost of $3M, at least 6,000 people should be served, and the land lost should not exceed 150 acres. The extent to which each alternative achieves the performance measures are shown in the table below.

<table>
<thead>
<tr>
<th>Performance Measure (PM)</th>
<th>Alternative A</th>
<th>Alternative B</th>
<th>Alternative C</th>
<th>Alternative D</th>
<th>GOAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($M)</td>
<td>4.5</td>
<td>3.1</td>
<td>6.6</td>
<td>5.2</td>
<td>3</td>
</tr>
<tr>
<td>Pop served (1,000s)</td>
<td>2.1</td>
<td>1.9</td>
<td>5.5</td>
<td>4.1</td>
<td>6.0</td>
</tr>
<tr>
<td>Land Lost (acres in 100s)</td>
<td>1.7</td>
<td>2.3</td>
<td>2.9</td>
<td>2.7</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Determine the scaled values of the performance measures for each alternative.

**Solution:**

<table>
<thead>
<tr>
<th></th>
<th>Alternative A</th>
<th>Alternative B</th>
<th>Alternative C</th>
<th>Alternative D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($M)</td>
<td>4.5 – 3 = 1.5</td>
<td>3.1 – 3 = 0.1</td>
<td>6.6 – 3 = 3.6</td>
<td>5.2 – 3 = 2.2</td>
</tr>
<tr>
<td>Pop served (1,000s)</td>
<td>2.1 – 6 = -3.9</td>
<td>1.9 – 6 = -4.1</td>
<td>5.5 – 6 = -0.5</td>
<td>4.1 – 6 = -1.9</td>
</tr>
<tr>
<td>Land Lost (acres in 100s)</td>
<td>1.7 – 1.5 = 0.2</td>
<td>2.3 – 1.5 = 0.8</td>
<td>2.9 – 1.5 = 1.4</td>
<td>2.7 – 1.5 = 1.2</td>
</tr>
</tbody>
</table>

**Discussion**

It can be seen here that even though the units of the different performance measures are different ($, population, acres), they were expressed in similar scales. The assumption here is that they are all linear. This means that (a) for a given performance measure, the changes in desirability is constant from one level to the next, so for example, moving from a cost of $1M to $2M is equally undesirable as moving from $3M to $4M, (b) for any two performance measures, there is equal desirability for same levels of the performance measures. For example, $1M in cost is equally undesirable as 1 (in 100’s of acres) in land lost. In reality, these rather simple assumptions may not hold, and it will be necessary to develop utility functions to translate the levels of the performance measures to their corresponding utility before applying the distance-from-goal concepts.

Also, the example shown does not incorporate the weights of the performance measures. In other words, the same weights are assumed for the calculation. Weights, if known, could easily be added in the formulation to reflect the relative importance across the performance measures, and ultimately, the relative importance across the project alternatives.

**2.3 Preference-based scaling methods**

Preference-based scaling methods are those that involve a survey of asset management experts (and/or other stakeholders) so that their preferences regarding the various levels of a given performance measure can be expressed on a dimensionless scale showing the desirability of utilities of the different levels. For a given performance measure, such a scale can be established from 0-1, 0-10, or 1-100. If this is repeated for several performance measures that originally had different units, one obtains a normalized scale that can be used to compare or combine the different performance measures.
Of the preference-based scaling methods, the most popular and most widely-used measure of decision-makers desirability is utility theory (Keeney and Raiffa, 1976). In this study, the concept of utility is used as the measure of decision-makers desirability, and this will be done for all the different preference-based scaling methods herein discussed.

In utility theory, the basic element is value function or utility function, which reflects the preference structure of decision-makers. In the process of decision making, if there are \( n \) performance measures \((X_1, X_2, \ldots, X_n)\), let us assume \((x_{i1}, x_{i2}, \ldots, x_{in})\) and \((x_{j1}, x_{j2}, \ldots, x_{jn})\) are the performance measures values of any two alternatives. If one can find a scalar-valued function \( v() \) with the following property

\[
v(x_{i1}, x_{i2}, \ldots, x_{in}) \geq v(x_{j1}, x_{j2}, \ldots, x_{jn}) \iff (x_{i1}, x_{i2}, \ldots, x_{in}) \succeq (x_{j1}, x_{j2}, \ldots, x_{jn})
\]

where the symbol \( \succeq \) means “Preferred or indifferent to”, then one can call the function \( v() \) a value function or utility function (Keeney and Raiffa, 1976). The process of scaling, therefore, yields the value function or utility function for the performance measure in question.

What is the difference between a utility function and a value function?

The difference lies in the level of certainty of the project outcome in terms of the given performance measure. For instance, when we resurface a highway, the change in pavement performance (say, surface roughness in IRI units) is not known with certainty. Where there is more certainty than uncertainty regarding the project outcome, the resulting scaling function is referred to as a value function; in uncertainty condition, it is called a utility function. So, in a general sense, a value function is a special case of the utility function where uncertainty is zero.

We now discuss the various methods of developing a preference-based scaling function for a given performance measure. We present two categories of these methods: scaling methods under certainty scenario and scaling methods under risk scenario. These methods are shown in Figure 2.1.

2.3.1 Certainty Scenario

2.3.1.1 Direct Rating

The simplest scaling method, the Direct Rating technique (Keeney and Raiffa, 1976) asks the decision-maker to indicate directly the value or desirability he/she attaches to each level of the performance measure on a scale of say, 0 to 1. This method is most appropriate where the
performance measure has only a few levels and when these levels are discrete. Thus, it can be used for Present Serviceability Index (PSI) which ranges from 0 to 5; and congestion levels of service (LOS) which ranges from A to F, but is not appropriate for IRI (in/mile). The process of direct rating is described as follows:

Step 1: List all possible values of the performance measure: for performance measure X, its values are $x_1, x_2, \ldots, x_n$.

Step 2: Find out the least preferred value of X, denote it as $x^0$ and define its value function as $v(x^0) = 0$;

Step 3: Find out the most preferred value of X, denote it as $x^n$ and define its value function as $v(x^n) = 1$;

Step 4: Directly assign intermediate values $v(x^i)$ to the various values of the performance measure $x_i$'s between $x^0$ and $x^n$;

Step 5: List all the values of X and their corresponding scaling values.

The flow chart of this method is shown in Figure 2.15. An example showing how a scaling function is developed using this method, is provided in Appendix 1.1.

![Flow Chart](image)

**Figure 2.15: Steps for Developing a Scaling Function using the Direct Rating Method**

### 2.3.1.2 Midvalue Splitting Technique

The midvalue splitting method (Keeney and Raiffa, 1976) is based on the identification of the concept of midvalue point and differentially-value equivalent points. For two performance measures $X$ and $Y$, the pair $(x_1, x_2)$ ($x_1 < x_2$) is said to be *differentially value-equivalent* to the pair $(x_3, x_4)$ ($x_3 < x_4$) if the decision-maker is willing to forgo the same amount of $Y$ for the
increase of $X$ from $x_1$ to $x_2$ as for the increase from $x_3$ to $x_4$ at any point of $Y$. Thus, for any interval $[x_1, x_2]$ of $X$, its midvalue point $x_3$ is such that the pair $(x_1, x_3)$ and $(x_3, x_2)$ are differentially value-equivalent (Keeney and Raiffa, 1976). Based on the concept of midvalue splitting, the following steps can be used to develop a value function for performance measure $X$.

Steps:

Step 1: Determine the range of $X$, and define $u_X(x_0) = 0$ and $u_X(x_1) = 1$, where $x_0$ is the least preferred value and $x_1$ is the most preferred value.

Step 2: Determine the midvalue point of $[x_0, x_1]$, denote it as $x_{0.5}$, and let $u_X(x_{0.5}) = 0.5$;

Step 3: Determine the midvalue point of $[x_0, x_{0.5}]$, denote it as $x_{0.25}$, and let $u_X(x_{0.25}) = 0.25$;

Step 4: Determine the midvalue point of $[x_{0.5}, x_1]$, denote it as $x_{0.75}$, and let $u_X(x_{0.75}) = 0.75$;

Step 5: Consistency check. Determine whether the midvalue point of $[x_{0.25}, x_{0.75}]$ is $x_{0.5}$, if not, repeat steps 2 to 4;

Step 6: Plot points $((x_i, u_X(x_i)))$ and draw the curve using these points; the resulting curve is the value function of $X$ (Figure 2.16);

This easy-to-use method is applicable only in the certainty condition. Appendix 1.2 presents an example.

![Figure 2.16: Midvalue Splitting Method](image-url)
2.3.1.3 Statistical Regression to Enhance the Outcome of Scaling

In practice, decisions are always by a group of people and not a single decision-maker. So for each person in the decision group, the direct rating or midvalue splitting methods can be used to generate a number of observations for each level of the performance measure. Then statistical regression can be used to obtain the line of best fit through these points – this gives the value function that represents the preference structure of the entire decision group.

2.3.2 Risk Scenario

The risk scenario is used when the project outcome in terms of a given performance measure is not known with certainty, but a probability distribution can be developed for the levels of that performance measure. The distribution can be developed using historical data from similar projects. Under the risk scenario, scaling functions can be developed using the direct questioning approach and certainty equivalent approach. These are herein described:

2.3.2.1 Direct Questioning Approach (Keeney and Raiffa, 1976)

There are two variations to this approach, depending on whether the variable representing the performance measure is a discrete or continuous.

(i) where the performance measure is a discrete variable

In such cases, especially where the discrete levels of the performance measure are relatively few, the following direct assessment procedure can be used to develop the utility function.
Step 0: Determine all possible values of X, e.g. $x_1, x_2, \ldots, x_m$.

Step 1: Denote the least preferred value of X as $x_w$, the most preferred value of the performance measure as $x_h$; then define $u(x_w) = 0$ and $u(x_h) = 1$.

Step 2: For each $x_i$, determine the probability $p_i$ which render the following situations indifferent:

1. A guaranteed prospect of an outcome of $x_i$;
2. A risk prospect of obtaining an outcome of as $x_h$ with probability $p_i$ and an outcome of $x_w$ with probability $1 - p_i$.

Step 3: Calculate the utility of $x_i$

$$u(x_i) = p_i u(x_h) + (1 - p_i) u(x_w) = p_i ;$$

Step 4: Repeat step 2 and step 3 until the utilities of all the other levels of the performance measure have been determined.

Step 5: Check for consistency. Choose any three levels of the performance measure: $x_1, x_2$ and $x_3$. Then consider these two situations:

1. A guaranteed prospect of an outcome of $x_2$;
2. A risk prospect of obtaining an outcome of as $x_1$ with probability $p$ and an outcome of $x_3$ with probability $1 - p$ ;

If the decision-maker considers the above two situations as indifferent, then for consistency, $p$ should be equal to:

$$p = \frac{u(x_2) - u(x_3)}{u(x_1) - u(x_3)}$$

An example of this method is given in Appendix 1.3. Where there are many possible outcome levels of the performance measure, this method can be rather cumbersome and laborious. In such cases, it is recommended to utilize relative simple preferential techniques such as the direct rating method.
(2) where the performance measure is a continuous variable

If the performance measure is continuous, it is impossible to establish utilities for all the infinite possible levels it could take. In such cases, a number of discrete levels are taken from the continuum to adequately represent its spread, and the utilities of these discrete values are determined using a survey. The detailed steps are as follows:

Step 0: Determine the value range of X;

Step 1: Denote the least preferred value of the performance measure as $x_w$, the most preferred as $x_b$; then define $u(x_w) = 0$ and $u(x_b) = 1$;

Step 2: Compare the following situations:

1. A guaranteed prospect of an outcome of $X = 0.5(x_b - x_w)$
2. A risk prospect of obtaining an outcome of $x_b$ with probability $p$ and an outcome of $x_w$ with probability $(1-p)$

This is to determine the probability $p$ which renders the above situations indifferent. Then $p$ is $p_{0.5}$.

Step 3: repeat step 2 by setting the guaranteed prospect as $0.25(x_b - x_w)$ and $0.75(x_b - x_w)$, and get $p_{0.25}$ and $p_{0.75}$.

Step 4: Consistency check. Compare the following situations:

1. A guaranteed prospect of an outcome of $X = 0.5(x_b - x_w)$
2. A risk prospect of obtaining an outcome of $0.25(x_b - x_w)$ with probability $p$ and an outcome of $0.75(x_b - x_w)$ with probability $(1-p)$

This is to determine the probability $p$ that renders the above situations indifferent.

3. Check if $p$ equals to $\frac{P_{0.5} - P_{0.25}}{P_{0.75} - P_{0.25}}$. If yes, continue to step 5. If no, go back to step 2.

Step 5: Plot $(X_w, 0)$, $(0.25(x_b - x_w), p_{0.25})$, $(0.5(x_b - x_w), p_{0.5})$, $(0.75(x_b - x_w), p_{0.75})$, and $(X_b, 1)$, then use statistical regression to obtain the utility function.

For multiple survey respondents, further regression can be used to obtain the line of best fit for all observations, thus enhancing the scaling function further.
2.3.2 Certainty Equivalent Approach (Keeney and Raiffa, 1976)

From the literature, this technique appears to be the most popular approach for developing utility functions under the risk situation. To develop the utility function for a performance measure $X$, the following steps are used:

Step 1: Define the worst level of the performance measure $X$ as $X_w$, the best level of $X$ as $X_b$; then define $u(X_w) = 0$ and $u(X_b) = 1$;

Step 2: Compare the following two situations:

   (a) A guaranteed prospect of an outcome of $X_{0.5}$;
   
   (b) A risk prospect of obtaining an outcome of as $X_w$ with probability 50% and an outcome of $X_b$ with probability 50%;

   Determine $X_{0.5}$ that renders the above situations indifferent.

Step 3: Repeat step 2 by setting the guaranteed prospects $X_{0.25}$ and $X_{0.75}$, and get final $X_{0.25}$ and $X_{0.75}$.

Step 4: Consistency check. Compare the following situations:

   (a) A guaranteed prospect of an outcome of $X_{0.5}$;
   
   (b) A risk prospect of obtaining an outcome of as $X_{0.75}$ with probability 50% and an outcome of $X_{0.25}$ with probability 50%;

   If the decision-maker considers either situation (a) and (b) as superior to the other, then go back to step 2, until the decision-maker considers two situations as being indifferent.

Step 5: Plot $(X_w, 0)$, $(X_{0.25}, 0.25)$, $(X_{0.5}, 0.5)$, $(X_{0.75}, 0.75)$, and $(X_b, 1)$, chose the utility function form and calibrate the parameters in the function.

Appendix 1.4 shows how the certainty equivalent approach could be used to develop a scaling function.
2.3.3 Discussion

In some cases, previous research studies have established probability distributions of the outcome of a given performance measures. In this situation, it is needed only to calibrate the parameters in the distribution function to derive the scaling function. A relatively smaller expert survey may be necessary to generate data for carrying out the calibration. For example, if the utility function form is \( u(x) = 1 - e^{-(x-a)} \) and the range of \( X \) is \( (x_w, x_b = +\infty) \); and \( x_w \) is the least preferred value and \( x_b \) is the most preferred value, then:

\[
\begin{align*}
    u(x_w) &= 1 - e^{-(x_w-a)} = 0 \\
    u(x_b) &= 1 - e^{-(x_b-a)} = 1
\end{align*}
\]

Solving this equation would yield \( a = x_w \). So the utility function is \( u(x) = 1 - e^{-(x-x_w)} \).

If the utility function forms have more parameters than those shown above, additional surveys would be needed to calibrate the utility function. In that case, the direct question approach or the certainty equivalent method may be used.

For purposes of illustration, this report (in Appendix 2) presents some equations and graphs of utility functions that were developed in past research for several different performance measures. These are categorized by performance in terms of system preservation (pavement and bridge condition, remaining service life), user cost (in terms of average speed), mobility (in terms of average speed, detour length, and intersection delay), safety (in terms of skid resistance, sight distance, bridge structural and functional adequacies, etc.) environment (in terms of speed which is an air pollution surrogate), facility vulnerability to disaster (in terms of earthquake, scour, fatigue, and other vulnerability).
2.4 Shapes of Scaling Functions and Their Implications

2.4.1 Shapes of Scaling Functions

Irrespective of scaling method used, there generally are four major shapes that a scaling function can take: monotonically-increasing, monotonically-decreasing, concave, and convex.

(a) Monotonically-Increasing Scaling Functions

These functions, which may be linear or non-linear, represent the performance measure for which higher values are more desirable to the decision-maker. Examples include IRI Change (but not IRI) Bridge Health Index, Bridge Sufficiency Rating, Pavement Condition Index (PCI), Present Serviceability Rating (PSR), Pavement Quality Index, reductions in roughness, reductions in crash rates, etc. So, for example, a higher PCI translates into a good condition while a lower PCI translates into a poorer condition. Also, a higher IRI change is more desirable while a lower IRI change is less desirable.

(b) Monotonically-Decreasing Scaling Functions

These typically represent the performance measure for which higher values are less desirable to the decision-maker. The function shape may be linear or non-linear. Examples include IRI, Rutting, Bridge Corrosion Index, Crash Rate, Delay, reduction in speed, reduction in travel time, reduction in facility health/condition, etc. So, for example, a higher IRI translates into a poor condition and has a lower value or scale while a lower IRI translates into a superior condition and has a higher value or scale.

(c) Non-monotonic Scaling Functions

Scaling functions are not always monotonically increasing or decreasing. In some cases, the function is monotonically increasing up to a point and then monotonically decreasing thereafter. In other cases, it is monotonically decreasing up to a point and monotonically increasing thereafter. This happens where it is desired that the performance measure should not be too small or too large, or where it is desired that the performance measures is desirable only when it is lower than some threshold or when it exceeds some threshold. For instance, where speed is a performance measure, it is often desired that speed should not be too low or too high as either extreme is associated with higher fuel consumption. Non-monotonic scaling functions may be linear or non-linear.
Figure 2.19: Examples of Monotonically-Increasing Scaling Functions
Figure 2.20: Examples of Monotonically Decreasing Scaling Functions
2.4.2 Implication of the Shapes of Scaling Functions

A scaling function developed from the preferences of decision-makers can show revealing patterns of the risk taking attitude of the decision-makers. The risk-taking attitude is reflected in the concavity or convexity of the scaling function. It can be proven mathematically that a risk taking decision-maker has a strictly convex utility function, a risk averse decision-maker has a strictly concave scaling function, and a risk neutral decision-maker has a linear scaling function. Figure 2.21 presents the relation between the concavity and risk-taking tendency of a decision-maker.

![Figure 2.21: Relation between Risk Attitude and Scaling Function](image)

For scaling functions derived using preference-based methods, the final shape of the scaling functions is a reflection of the risk attitudes of the decision-maker [Keeney and Raiffa 1993]. A risk-averse decision-maker is one who behaves conservatively. In contrast a risk prone decision-maker is one who is willing to gamble with his/her resources to obtain a possibly superior consequence of his/her actions even though that may be less probable than the expected outcome. Most asset management decision-makers tend to be risk averse or risk neutral.
2.5 Chapter Summary

The performance measures typically encountered in asset management have different units or metrics and this makes it difficult for the Asset Manager to compare between projects on the basis of the different performance measures. This problem is addressed through scaling (or normalization). Scaling is carried out separately for each performance measure. In this chapter, we present and discuss the merits and demerits of a number of alternative techniques. The presented methodologies can be used by the Asset Manager to develop scaling functions that could be used to scale the performance measures so that (i) the different impacts of a given project can be expressed on the same scale or combined to yield an overall value, utility, or desirability, (ii) the impacts of different projects can be compared in a bid to select superior projects (iii) trade-off analysis can be carried out between the performance measures.

Scaling techniques may be categorized as “objective” methods and preference-based methods. The objective methods include linear scaling, cumulative distribution functions, and monetization. The preference-based methods are considered by some schools-of-thought as being subjective because they are developed on the basis of expert opinion: through surveys. Scaling functions developed using preference-based methods can be categorized into the value functions and utility functions. A utility function is considered a more general form of a value function: like value functions, utility functions incorporate the Asset Manager’s innate value of different levels of the performance measures; unlike value functions, utility functions incorporate the Asset Manager’s attitudes toward risk (i.e., whether the Asset Manager is risk prone, risk neutral, or risk averse).
CHAPTER 3 TECHNIQUES FOR AMALGAMATING THE OVERALL IMPACTS OF A
PROJECT ON THE BASIS OF THE MULTIPLE CRITERIA

3.1 Introduction

In Chapter 2, this report discussed various scaling methods which render performance measures with different units into a unit is commensurate across all the performance measures under consideration. Thus for any given candidate project, the Asset Manager can determine the dimensionless values of the impacts of the project separately for safety, congestion, preservation, etc. So the question that now arises is how best to combine them to get the overall impact for the project. It is needed to combine the different impacts because the candidate projects need to be ordered or optimized (for purposes of priority ranking or optimization), and also because it is sought to determine the trade-offs among the performance measures. The combination of the different impacts for each candidate project in the Asset Manager’s portfolio is known as amalgamation.

3.2 General Discussion of Amalgamation Methods

Where the decision problem involves the use of multi-attribute utility functions, the weighted sum or weighted product methods of amalgamation are widely considered most appropriate. These involve the use of the performance measure weights and scaling functions (which, in this case, are single attribute utility functions) into a multi-attribute utility function either in additive or multiplicative form. The expected values of the multi-attribute utility function are then used to rank the candidate projects and the project with maximum expected utility value is picked. Two assumptions are made for the multi-attribute utility functions: utility independence and preference independence. Utility independence means that the each attribute’s utility function does not depend on the levels of other attributes. Preference independence holds that if the trade-off one is willing to make between two attributes does not depend on the levels of other attributes. Of the two methods, the weighted sum method is the most widely used due to its simplicity and ease of use.
As seen in Chapter 2, the distances of a given project from an agency goal can be used as a measure of scaling. This distance could involve the unscaled or raw level of the performance measure, or better still, the utility of the raw level. In this context, the amalgamation of the different performance measures, for a given project, is simply the distance to the goal in 2, 3 or \( n \) dimensions depending on the number of performance measures. This is part of the overall decision process known as **goal programming**.

Another approach of amalgamation is **compromise programming**, a variation of goal programming (Zeleny 1973) which identifies solutions closest to the ideal solution as determined by some measure of distance. The solutions identified are called compromise solutions and constitute the compromise set. If the compromise set is small enough to allow the decision-maker to choose a satisfactory solution, then the process is terminated. Otherwise, the ideal solution is redefined and the whole process is repeated.

**Outranking methods**, a class of multi-criteria decision making techniques that provide an ordinal ranking (sometimes only a partial ordering) of the alternatives, are exemplified by the Elimination and Choice Translating Algorithm (ELECTRE) method (Benayoun et al. 1966; Roy and Bertier 1971). ELECTRE establishes a set of outranking relationships among alternatives. A candidate project is deemed to outrank another only if (i) the sum of normalized weights (i.e., the concordance index) for the candidate exceeds a predetermined threshold value, and (ii) the number of performance measures for which the latter candidate is superior by an amount greater than a tolerable threshold value (i.e., discordance index), is zero. An extension of the ELECTRE method by incorporating uncertainty was discussed by Mahmassani (1981).

The **Step Method** (STEM) (Benayoun and Tergny 1969) is the first interactive method introduced to solve linear and nonlinear problems. The method assumes that the best compromising solution has the minimum combined deviation from the ideal point, and the decision-maker has a pessimistic view of the worst component of all deviations (of individual candidate projects) from the ideal point. The technique essentially consists of two steps: (i) a non-dominated solution in the minimax sense to the ideal point for each objective function is sought, and a payoff table is constructed to obtain the ideal criterion vector (ii) the decision-maker then compares the solution vector with the ideal vector of a payoff table by modifying the constraint set and the relative weights of objective functions. The process terminates when the decision-maker is satisfied with the current solution.
In the sections below, this chapter presents details of selected amalgamation methods that are recommended for combining the different impacts of any given candidate project that exists in INDOT Asset Manager’s portfolio.

### 3.3 Amalgamation Methods

#### 3.3.1 Weighted Sum Method (WSM)

The weighted sum method is commonly used by many decision-makers. It uses additive function form to obtain the final value of a candidate project (or alternative). The final value of alternative $A_i$ can be calculated as (Fishburn, 1967; Triantaphyllou, 2000):

$$ U_{A_i} = \sum_{j=1}^{n} w_j a_{ij} \quad i = 1, 2, ..., m $$

(3.1)

Where $w_j$ is the weight of performance measure $j$;

$a_{ij}$ is the scaled value of performance measure $j$ for alternative $i$;

$n$ is the number of performance measures; $m$ is the number of alternatives.

The alternative with the highest $U_{A_i}$ is the best choice.

When the WSM is used, the value of performance measures must be dimensionless or have the same units (e.g., scaled value). If the scaled values are from preference-based scaling methods, the multiple performance measures must be *utility independent* and *preference independent*. Utility independence means that each criterion’s utility function does not depend on the levels of other performance measures. Preference independence assumes the trade-offs between two performance measures do not depend on the levels of other performance measures. In addition, in the risk condition, the expected values of performance measures are used in equation 3.1.

**Example:**

Consider five alternative highway projects: $A$, $B$, $C$, $D$, and $E$. Four performance measures ($P_1$, $P_2$, $P_3$, and $P_4$) are used to evaluate these alternatives. The weights of these performance measures, $w_i$, are given below. Also, for each project alternative, the scaled impacts of the project, for each performance measure, are provided in the table below. For each project, determine the overall or amalgamated impact.
Alternatives  \( P_1 \)  \( P_2 \)  \( P_3 \)  \( P_4 \)
\[ \begin{array}{cccc}
A & 0.59 & 0.95 & 0.06 & 0.60 \\
B & 0.07 & 0.18 & 0.81 & 0.85 \\
C & 0.80 & 0.26 & 0.06 & 0.90 \\
D & 0.58 & 0.36 & 0.13 & 0.97 \\
E & 0.86 & 0.09 & 0.15 & 0.35 \\
\end{array} \]

Solution:

Using the weighted sum method, the amalgamated values of performance measures for each candidate project or alternative are found as follows:

<table>
<thead>
<tr>
<th>Alternative, ( i )</th>
<th>( C_1 ) ((w_1=0.2))</th>
<th>( C_2 ) ((w_2=0.1))</th>
<th>( C_3 ) ((w_3=0.4))</th>
<th>( C_4 ) ((w_4=0.3))</th>
<th>Amalgamated Impact of the Alternative ( i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.59</td>
<td>0.95</td>
<td>0.06</td>
<td>0.60</td>
<td>0.59<em>0.2+0.95</em>0.1+0.06<em>0.4+0.60</em>0.3=0.42</td>
</tr>
<tr>
<td>B</td>
<td>0.07</td>
<td>0.18</td>
<td>0.81</td>
<td>0.85</td>
<td>0.07<em>0.2+0.18</em>0.1+0.81<em>0.4+0.85</em>0.3=0.61</td>
</tr>
<tr>
<td>C</td>
<td>0.80</td>
<td>0.26</td>
<td>0.06</td>
<td>0.90</td>
<td>0.80<em>0.2+0.26</em>0.1+0.06<em>0.4+0.90</em>0.3=0.48</td>
</tr>
<tr>
<td>D</td>
<td>0.58</td>
<td>0.36</td>
<td>0.13</td>
<td>0.97</td>
<td>0.58<em>0.2+0.36</em>0.1+0.13<em>0.4+0.97</em>0.3=0.50</td>
</tr>
<tr>
<td>E</td>
<td>0.86</td>
<td>0.09</td>
<td>0.15</td>
<td>0.35</td>
<td>0.86<em>0.2+0.09</em>0.1+0.15<em>0.4+0.35</em>0.3=0.35</td>
</tr>
</tbody>
</table>

3.3.2 The Multiplicative Utility Function

The multiplicative utility function of alternative \( A_i \) is defined as follows (Keeney and Raiffa, 1976):

\[
U_i = \frac{1}{k} \left( \prod_{j=1}^{n} \left[ 1 + k w_j u(x_{ij}) \right] - 1 \right) \tag{3.2}
\]

Where: \( u(x_{ij}) \) is the utility of alternative \( i \) on the \( j \)th performance measure;

\( w_j \) is the relative weight of performance measure \( j \);

\( k \) is a scaling constant that is determined from the equation \( 1 + k = \prod_{j=1}^{n} (1 + k w_j) \).

The premise of using multiplicative utility function is that all the criteria must be mutually utility independent. If \( X_1, X_2, \ldots, X_n \) are the \( n \) criteria, we say criteria \( X_i \) is utility independent if \( X_i \)'s utility function does not depend on the levels of other criteria. Also \( X_1, X_2, \ldots, X_n \) are mutually utility independent if every subset of \( \{ X_1, X_2, \ldots, X_n \} \) is utility independent of its complement (Keeney and Raiffa, 1976). The project alternative with higher final utility is superior to that with lower final utility.
**Example:** Consider the following five alternative projects A to E. These projects are being evaluated and ranked on the basis of three performance measures: C₁, C₂, and C₃ with relative weights \( w_1 = 0.4, w_2 = 0.3, w_3 = 0.3 \), respectively.

<table>
<thead>
<tr>
<th>Project</th>
<th>Project Impacts in terms of the Respective Performance Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C₁</td>
</tr>
<tr>
<td>A</td>
<td>0.59</td>
</tr>
<tr>
<td>B</td>
<td>0.07</td>
</tr>
<tr>
<td>C</td>
<td>0.8</td>
</tr>
<tr>
<td>D</td>
<td>0.58</td>
</tr>
<tr>
<td>E</td>
<td>0.86</td>
</tr>
</tbody>
</table>

**Solution**

First, the value of the parameter \( k \) is obtained by solving the following equation

\[
1 + k = \prod_{j=1}^{n} (1 + k w_j)
\]

Plugging in the \( w_j \) values in the equation yields \( k = 0 \) or \( k = -9.1667 \).

But \( k \) cannot be 0, so \( k = -9.1667 \). Thus, the multiplicative equation for amalgamating the performance measures is:

\[
u_i = \frac{1}{-9.1667 \left( \prod_{j=1}^{n} [1 - 9.1667 w_j u(x_{ij})] - 1 \right)}
\]

Using this equation, the amalgamated value of each project can now be calculated:

<table>
<thead>
<tr>
<th>Project</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>Amalgamated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.59</td>
<td>0.95</td>
<td>0.2</td>
<td>0.017002</td>
</tr>
<tr>
<td>B</td>
<td>0.07</td>
<td>0.18</td>
<td>0.81</td>
<td>0.159358</td>
</tr>
<tr>
<td>C</td>
<td>0.8</td>
<td>0.26</td>
<td>0.06</td>
<td>0.159281</td>
</tr>
<tr>
<td>D</td>
<td>0.58</td>
<td>0.36</td>
<td>0.13</td>
<td>0.10988</td>
</tr>
<tr>
<td>E</td>
<td>0.86</td>
<td>0.09</td>
<td>0.15</td>
<td>0.212942</td>
</tr>
</tbody>
</table>

From the results, it can be concluded that alternative E is the best choice, followed by B, C, D and lastly, A.
3.3.3 The Weighted Product Model (WPM) Method

The WPM method compares two candidate projects at a time, on the basis of the multiple performance measures, to determine the superior project. First, WPM takes the ratio of the values of the levels of performance of two projects; and then uses the product model to obtain the final result upon which the Asset Manager could make a decision regarding which project is most superior or could draw up a project list ordered by superiority. The formula is as follows: (Miller and Starr, 1969; Bridgman, 1992; Triantaphyllou, 2000):

$$ r_{SL}(A_S / A_L) = \prod_{j=1}^{n} \left( \frac{x_{Sj}}{x_{Lj}} \right)^{w_j} $$

(3.3)

Where $x_{Sj}$ is level of performance measure $j$ for Project $S$

$x_{Lj}$ is level of performance measure $j$ for Project $L$

$r_{SL}$ = ratio between the performance impacts of $S$ and $L$

If $r_{SL} \geq 1$, Project $S$ is more desirable than Project $L$;

If $r_{SL} = 1$, Project $S$ is indifferent to Project $L$;

If $r_{SL} < 1$, Project $L$ is less desirable than Project $S$;

$w_j$ is the weight of performance measure $j$.

This procedure can be repeated for all projects in the Asset Manager’s portfolio until all the alternatives are ranked in order of superiority. The WPM amalgamation process, therefore, yields a set of ratios for each project to determine how well it performs, overall, compared to the other candidate projects. This method is simple and easy to use. The biggest advantage of this is that it can use the original raw value and units of the performance measures thus obviating the need for scaling. The limitation is that the value of any performance measure cannot be zero. A second limitation is that the pairwise comparison can be onerous when the number of projects alternatives is large.

**Example:** For the problem posed in the weighted sum method, use the weighted product method to amalgamate the overall impacts of Project C and Project D and compare the impacts of the two projects.
Solution:

\[ r_{CD} (A_C / A_D) = (0.8^{0.2} + 0.26^{0.1}) + (0.96^{0.4} + 0.97^{0.3}) = 0.81 < 1 \]

The results suggest that Project D is superior to Project C. However, it may be noted that in reaching the conclusion, the solution actually goes beyond amalgamation and carries out project selection which is an MCDM phase that is subsequent to amalgamation.

3.3.4 Analytic Hierarchy Process (AHP) Method

The AHP method, which was first introduced by Saaty in 1980, is one of the most popular MCDM methods. In AHP, there are two parts: a pairwise comparison part and an eigenvector part. In scaling, only the eigenvector part is used.

Assume the decision matrix is \( X \) as shown.

\[
X = \begin{bmatrix}
    x_{11} & x_{12} & \cdots & x_{1n} \\
    x_{21} & x_{22} & \cdots & x_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{m1} & x_{m2} & \cdots & x_{mn}
\end{bmatrix}
\]  

(3.4)

Where \( x_{ij} \) can represent the scaled value or the raw value of the performance measure \( j \) of alternative project \( i \).

The matrix is then transformed as follows:

\[
\begin{bmatrix}
    x_{11} / \sum_{i=1}^{m} x_{i1} & x_{12} / \sum_{i=1}^{m} x_{i2} & \cdots & x_{1n} / \sum_{i=1}^{m} x_{in} \\
    x_{21} / \sum_{i=1}^{m} x_{i1} & x_{22} / \sum_{i=1}^{m} x_{i2} & \cdots & x_{2n} / \sum_{i=1}^{m} x_{in} \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{m1} / \sum_{i=1}^{m} x_{i1} & x_{m2} / \sum_{i=1}^{m} x_{i2} & \cdots & x_{mn} / \sum_{i=1}^{m} x_{in}
\end{bmatrix}
\]  

(3.5)

So the overall desirability of project alternative \( i \) can be calculated as

\[
S_i = \sum_{k=1}^{n} w_k (x_{ik} / \sum_{j=1}^{m} x_{jk})
\]  

(3.6)
Then the alternative with highest value of $S_i$ is the best alternative. It can also be used to compare two alternatives and do trade-off analysis between them. The alternative with higher $S_i$ is better than one with a lower $S_i$ value.

**Critique:** The AHP method is widely used by decision-makers in various disciplines. In this method, it is not necessary to scale the performance measure into a dimensionless unit, and thus can be relatively less demanding in its application. However, it becomes inaccurate when there are some missing values or zero values in the decision matrix.

**Example.** For the problem posed in the weighted sum method, use AHP to determine the amalgamated impacts of the different projects.

Solution: The calculation process and the final amalgamated value are shown in the following table.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>$C_1$ ($w_1=0.2$)</th>
<th>$C_2$ ($w_2=0.1$)</th>
<th>$C_3$ ($w_3=0.4$)</th>
<th>$C_4$ ($w_4=0.3$)</th>
<th>Amalgamated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.59</td>
<td>0.95</td>
<td>0.06</td>
<td>0.60</td>
<td>$0.59/2.9<em>0.2+0.95/1.84</em>0.1+0.06/1.21<em>0.4+0.60/3.67</em>0.3=0.16$</td>
</tr>
<tr>
<td>B</td>
<td>0.07</td>
<td>0.18</td>
<td>0.81</td>
<td>0.85</td>
<td>$0.07/2.9<em>0.2+0.18/1.84</em>0.1+0.81/1.21<em>0.4+0.85/3.67</em>0.3=0.35$</td>
</tr>
<tr>
<td>C</td>
<td>0.80</td>
<td>0.26</td>
<td>0.06</td>
<td>0.90</td>
<td>$0.80/2.9<em>0.2+0.26/1.84</em>0.1+0.06/1.21<em>0.4+0.90/3.67</em>0.3=0.16$</td>
</tr>
<tr>
<td>D</td>
<td>0.58</td>
<td>0.36</td>
<td>0.13</td>
<td>0.97</td>
<td>$0.58/2.9<em>0.2+0.36/1.84</em>0.1+0.13/1.21<em>0.4+0.97/3.67</em>0.3=0.18$</td>
</tr>
<tr>
<td>E</td>
<td>0.86</td>
<td>0.09</td>
<td>0.15</td>
<td>0.35</td>
<td>$0.86/2.9<em>0.2+0.09/1.84</em>0.1+0.15/1.21<em>0.4+0.35/3.67</em>0.3=0.14$</td>
</tr>
<tr>
<td>SUM</td>
<td>2.9</td>
<td>1.84</td>
<td>1.21</td>
<td>3.67</td>
<td>--</td>
</tr>
</tbody>
</table>

### 3.3.5 The ELECTRE Method

The ELECTRE (Elimination and Choice Translating Algorithm) Method was first introduced in 1966 by Benayoun, et al. The basic idea of the ELECTRE method is to address “outranking relations” by using pairwise comparisons among alternatives to establish a set of outranking relationships. The steps of this method are as follows (Triantaphyllou, 2000):

**Step1: Normalizing the Decision Matrix**

Use the following method to transform the value of each criterion to yield dimensionless entries:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{m} x_{ij}^2}}$$  \hspace{1cm} (3.7)
Step 2: Weighting the Normalized Decision Matrix

\[ Y = XW = \begin{bmatrix} w_1 x_{11} & w_2 x_{12} & \cdots & w_n x_{1n} \\ w_1 x_{21} & w_2 x_{22} & \cdots & w_n x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_1 x_{m1} & w_2 x_{m2} & \cdots & w_n x_{mn} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{bmatrix} \]  

(3.8)

Step 3: Determine the concordance and discordance sets

**Concordance Set.** The concordance set of two alternatives \( A_S \) and \( A_L \), denoted as \( C_{SL} \), is defined as the set of all the criteria for which \( A_S \) is preferred to \( A_L \). That is:

\[ C_{SL} = \{ \text{criterion } j, y_{sj} \geq y_{lj} \} \text{ for } j = 1,2,\ldots,n \]  

(3.9)

The complementary subset is called the discordance set, denoted as \( D_{SL} \) (Triantaphyllou, 2000),

\[ D_{SL} = \{ \text{criterion } j, y_{sj} < y_{lj} \} \text{ for } j = 1,2,\ldots,n \]  

(3.10)

Step 4: Construct the concordance and discordance matrices

The following formulae are used to calculate the entries in concordance and discordance matrices:

\[ c_{sl} = \sum_{j \in C_{sl}} w_j, \text{ for } j = 1,2,\ldots,n \]  

(3.11)

When \( S = L \), \( c_{SL} \) is not defined.

\[ d_{sl} = \frac{\max_{j \in D_{sl}} |y_{sj} - y_{lj}|}{\max_{j} |y_{sj} - y_{lj}|} \]  

(3.12)

When \( S = L \), \( d_{SL} \) is not defined.

Step 5: Determine the concordance and discordance dominance matrices

\[ c = \frac{1}{m(m-1)} \sum_{s=1}^{m} \sum_{l=1}^{m} c_{sl} \]  

(3.13)

Then calculate the concordance dominance matrix \( F \), in which the entries are defined as:

\[ f_{sl} = 1, \text{ if } c_{sl} \geq c, \]
\[ f_{sl} = 0, \text{ if } c_{sl} < c \]  

(3.14)

Then

\[ d = \frac{1}{m(m-1)} \sum_{k=1}^{m} \sum_{l=1}^{m} d_{kl} \]  

(3.15)
Calculate the concordance dominance matrix $G$, in which the entries are defined as

$$
g_{ij} = 1, \text{if} \quad d_{ij} \geq d_j \quad \text{and} \quad d_{ij} \neq d_j
$$

$$
g_{ij} = 0, \text{if} \quad d_{ij} < d_j
$$

(3.16)

Step 6: Calculate the Aggregate Dominance Matrix $Q$

$$
q_{ij} = f_{ij} \times g_{ij}
$$

(3.17)

In the matrix $Q$, if $q_{ij} = 1$, then the alternative $A_i$ dominates (or is superior to) alternative $A_j$.

**Example:** For the problem posed in the weighted sum method, use the ELECTRE method to determine the superior project(s) based on their amalgamated impacts.

**Solution:** First, use the formula is (3.17) to normalize the matrix. Then multiply each element using the appropriate weight. The results are presented in the tables below.

(a) Scaled Performance Measure Matrix

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.59</td>
<td>0.95</td>
<td>0.06</td>
<td>0.6</td>
</tr>
<tr>
<td>B</td>
<td>0.07</td>
<td>0.18</td>
<td>0.81</td>
<td>0.85</td>
</tr>
<tr>
<td>C</td>
<td>0.8</td>
<td>0.26</td>
<td>0.06</td>
<td>0.9</td>
</tr>
<tr>
<td>D</td>
<td>0.58</td>
<td>0.36</td>
<td>0.13</td>
<td>0.97</td>
</tr>
<tr>
<td>E</td>
<td>0.86</td>
<td>0.09</td>
<td>0.15</td>
<td>0.35</td>
</tr>
</tbody>
</table>

(b) Normalized Performance Measure Matrix

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.0820</td>
<td>0.0890</td>
<td>0.0286</td>
<td>0.1047</td>
</tr>
<tr>
<td>B</td>
<td>0.0097</td>
<td>0.0169</td>
<td>0.3865</td>
<td>0.1483</td>
</tr>
<tr>
<td>C</td>
<td>0.1112</td>
<td>0.0243</td>
<td>0.0286</td>
<td>0.1570</td>
</tr>
<tr>
<td>D</td>
<td>0.0806</td>
<td>0.0337</td>
<td>0.0620</td>
<td>0.1693</td>
</tr>
<tr>
<td>E</td>
<td>0.1196</td>
<td>0.0084</td>
<td>0.0716</td>
<td>0.0611</td>
</tr>
</tbody>
</table>

(c) Weighted Matrix

Then the concordance and discordance matrices are

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>--</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>B</td>
<td>0.7</td>
<td>--</td>
<td>0.4</td>
<td>0.4</td>
<td>0.8</td>
</tr>
<tr>
<td>C</td>
<td>0.5</td>
<td>0.6</td>
<td>--</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>D</td>
<td>0.7</td>
<td>0.6</td>
<td>0.2</td>
<td>--</td>
<td>0.4</td>
</tr>
<tr>
<td>E</td>
<td>0.6</td>
<td>0.6</td>
<td>0.4</td>
<td>0.4</td>
<td>--</td>
</tr>
</tbody>
</table>

Concordance Matrix

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>--</td>
<td>0.20</td>
<td>1.00</td>
<td>0.52</td>
<td>1.05</td>
</tr>
<tr>
<td>B</td>
<td>1.00</td>
<td>--</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>C</td>
<td>0.81</td>
<td>0.28</td>
<td>--</td>
<td>0.92</td>
<td>1.00</td>
</tr>
<tr>
<td>D</td>
<td>1.00</td>
<td>0.22</td>
<td>1.00</td>
<td>--</td>
<td>1.00</td>
</tr>
<tr>
<td>E</td>
<td>0.53</td>
<td>0.35</td>
<td>0.45</td>
<td>0.36</td>
<td>--</td>
</tr>
</tbody>
</table>

Discordance Matrix
Using the formula in 3.13 and 3.15, the threshold value $c$ for concordance matrix is obtained as 0.45, and the threshold value $d$ for concordance matrix is obtained as 0.7348. Thus, the concordance dominance matrix and the discordance dominance matrix become:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>--</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>--</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>1</td>
<td>--</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>--</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>--</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>--</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>--</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0</td>
<td>--</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>--</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>--</td>
</tr>
</tbody>
</table>

And thus, the aggregate dominance matrix is:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>--</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>--</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0</td>
<td>--</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>--</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>--</td>
</tr>
</tbody>
</table>

From the aggregate dominance matrix it can be seen that alternative A is dominated by B, C, and D; the alternative E is dominated by alternative B. So B, C, and D are the three best choices.

3.3.6 The Goal Programming Method of Amalgamation

Figure 3.1 presents a 3-D example of how the amalgamated impacts of a project can be found on the basis of the project impact in terms of three performance measures, using goal programming.

![Figure 3.1: Amalgamation of Distances from Goal (for 3 Performance Measures)]
Example
Consider the example in Chapter 2.2.4 where it was considered that the City of Megapolis is planning a long distance transit service connecting suburban areas to downtown. Four candidate projects (alternatives) are being evaluated. The city’s goal is to have a maximum project cost of $3M, at least 6,000 people should be served, and the land lost should not exceed 150 acres. The extent to which each alternative achieves the performance measures are shown in the table below. The scaled values of the performance measures for each alternative are shown in each cell of the 2nd, 3rd, and 4th rows. The amalgamated values of these scaled values are shown in the 5th row (the solution to the example in Chapter 2.2.4 may be referred to ascertain the relevant assumptions in deriving the scaled values).

<table>
<thead>
<tr>
<th></th>
<th>Alternative A</th>
<th>Alternative B</th>
<th>Alternative C</th>
<th>Alternative D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($M)</td>
<td>4.5 – 3 = 1.5</td>
<td>3.1 – 3 = 0.1</td>
<td>6.6 – 3 = 3.6</td>
<td>5.2 – 3 = 2.2</td>
</tr>
<tr>
<td>Pop served (1,000s)</td>
<td>2.1 – 6 = -3.9</td>
<td>1.9 – 6 = -4.1</td>
<td>5.5 – 6 = -0.5</td>
<td>4.1 – 6 = -1.9</td>
</tr>
<tr>
<td>Land Lost (acres in 100s)</td>
<td>1.7 – 1.5 = 0.2</td>
<td>2.3 – 1.5 = 0.8</td>
<td>2.9 – 1.5 = 1.4</td>
<td>2.7 – 1.5 = 1.2</td>
</tr>
<tr>
<td>Distance from Goal</td>
<td>√1.5² + (-3.9)² + 0.2² = 4.18</td>
<td>√0.1² + (-4.1)² + 0.8² = 4.178</td>
<td>√3.6² + (-0.5)² + 1.4² = 3.89</td>
<td>√2.2² + (-1.9)² + 1.2² = 3.14</td>
</tr>
</tbody>
</table>

3.3.7 The TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) Method
This method was developed by Yoon and Hwang in 1980. The basic idea of the TOPSIS method is that the best alternative should have the shortest distance from the ideal solution and have the farthest distance from the worst ideal solution. This method assumes that the preference structure for each criteria is monotonically decreasing or increasing, which means “the more the better” or “the fewer the better”. This method follows these steps (Triantaphyllou, 2000):

Step1: Normalize Decision Matrix
In the decision matrix formula 3.4, each entry is transformed into a normalized value

\[ r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{m} x_{kj}^2}} \] (3.18)

This step has the same transformation as the ELECTRE method.

Step2: Weigh Normalized Decision Matrix
In this step, the normalized entries in formula 3.18 are multiplied by the relative weights of each criterion. So the normalized decision matrix becomes:
Step 3: Find the ideal and the worst ideal alternative

Assume there are 2 alternatives \( A^b \) and \( A^w \), and the entries in their decision matrix are defined as

\[
A^b = \{a_{b1}, a_{b2}, \ldots, a_{wn}\} \quad (3.20)
\]

Where \( a_{mi} = \text{the most preferred value among } u_{i1}, u_{i2}, \ldots, u_{mi} \).

\[
A^w = \{a_{w1}, a_{w2}, \ldots, a_{wn}\} \quad (3.21)
\]

Where \( a_{wi} = \text{the least preferred value among } u_{i1}, u_{i2}, \ldots, u_{mi} \).

Step 4: Calculate the distance from the ideal alternative and the worst ideal alternative

The distance from \( i^{th} \) alternative to the ideal alternative is defined as

\[
D_{i^+} = \sqrt{\sum_{k=1}^{n} (u_{ik} - a_{bk})^2} \quad (3.22)
\]

The distance from \( i^{th} \) alternative to the worst alternative is defined as

\[
D_{i^-} = \sqrt{\sum_{k=1}^{n} (u_{ik} - a_{wk})^2} \quad (3.23)
\]

Step 5: Calculate the relative closeness to the ideal alternative

The relative closeness of the \( i^{th} \) alternative to the ideal alternative is defined as

\[
C_i = \frac{D_{i^-}}{D_{i^+} + D_{i^-}} \quad (3.24)
\]

So the alternative with the highest \( C_i \) is considered the best.

Example Using the data from the “Weighted Sum Method” example, find the amalgamated value of the different performance measures for each project, using TOPSIS. First, use the formula in (3.18) to normalize the matrix. And then multiply each element using the weight. The results are in the following tables. Note that A, B, C, D are the alternative projects. C1 to C4 are the performance measures.
\[ r_{ij} = \sqrt{\sum_{k=1}^{m} x_{ij}^2} \]

(a) Scaled Performance Measure Matrix

\[
\begin{array}{cccc}
\text{C1} & \text{C2} & \text{C3} & \text{C4} \\
\text{A} & 0.59 & 0.95 & 0.66 & 0.6 \\
\text{B} & 0.07 & 0.18 & 0.81 & 0.85 \\
\text{C} & 0.8 & 0.26 & 0.06 & 0.9 \\
\text{D} & 0.58 & 0.36 & 0.13 & 0.97 \\
\text{E} & 0.86 & 0.09 & 0.15 & 0.35 \\
\end{array}
\]

(b) Normalized Performance Measure Matrix

\[
\begin{array}{cccc}
\text{C1} & \text{C2} & \text{C3} & \text{C4} \\
\text{A} & 0.41 & 0.89 & 0.07 & 0.35 \\
\text{B} & 0.05 & 0.17 & 0.97 & 0.49 \\
\text{C} & 0.56 & 0.24 & 0.07 & 0.52 \\
\text{D} & 0.40 & 0.34 & 0.16 & 0.56 \\
\text{E} & 0.60 & 0.08 & 0.18 & 0.20 \\
\end{array}
\]

(c) Weighted Matrix

Based on matrix (c), the ideal alternative is determined as: \( A^*(0.1196, 0.0890, 0.3865, 0.1693) \), the worst ideal alternative is \( A^w(0.0097, 0.0084, 0.0286, 0.0611) \). The distances of each alternative from the most ideal and the worst ideal alternatives are shown in the following table.

\[
\begin{array}{cccc}
\text{Distance from the} & \text{Distance from the worst} & \text{Relative closeness to the} \\
\text{ideal alternative} & \text{ideal alternative} & \text{ideal alternative} \\
\text{A} & 0.0179 & 0.0002 & 0.0103 \\
\text{B} & 0.0003 & 0.0184 & 0.9833 \\
\text{C} & 0.0175 & 0.0004 & 0.0218 \\
\text{D} & 0.0121 & 0.0003 & 0.0275 \\
\text{E} & 0.0138 & 0.0002 & 0.0139 \\
\end{array}
\]

From the results, it can be seen that alternative B is the best choice.

3.4 Chapter Summary

Amalgamation combines the scaled, weighted, or weighted-and-scaled performance measures to yield a single value that reflects the desirability of a project alternative. This chapter presented seven amalgamation techniques that could be used by the Asset Manager. For each method, the chapter provides a numerical example to demonstrate its application.
CHAPTER 4 INDOT ASSET PROGRAM DEVELOPMENT

After scaling and amalgamation, the Asset Manager now knows the utility (the overall combined desirability) of each candidate project in his/her asset program. This desirability is in terms of the performance measures for each project in each program area as a function of not only the principal driving purpose (e.g., bridge asset improvement) but also secondary effects on other, non-driving performance goals (e.g., traffic safety and mobility). It may be recalled from Chapter 1 that these candidate projects are derived from the individual program areas (pavement, bridges, safety, and congestion). For each project, the desirability of the project implementation can be calculated. The benefit can be the utility of the project, or the benefit changes between “with implementation” or “without implementation” of the project in the lifecycle, or the change in consumer surplus. A common objective of the Asset Manager is to choose that set of candidate projects that he/she (i) can afford given the limited budget, (ii) would, as much as possible, maximize the network level benefits yet minimize the network level costs as much as possible. This choice process is known as program development, portfolio development, or program selection and is consistent with populating the asset program only those projects that are deemed relatively high-performing.

This chapter presents and discusses various useful optimization methodologies that could be used by INDOT’s program manager to develop the asset program for any given year or other appropriate programming period. The methodologies that are presented are those, which, from our literature review, have been tried and tested in the literature for managing assets, pavements, and bridges. Thus, the chapter presents what can be considered the best and easily implementable optimization methodologies for the problem at hand.

4.1 Some General Optimization Approaches used in Past Management Systems

The past few decades has seen considerable research in program development not only in highway management but also in other sectors and disciplines. This is been done largely in response to research requests by agencies seeking to maximize returns for asset investments within a limited budget. The present section discusses details of past studies.
Gruver et al. (1976) developed a methodology to select highway projects including bridge preservation with the objective to maximizing user benefits (vehicle operating costs, travel times and accidents). A similar study for North Carolina in 1988 by the Texas Transportation Institute (TTI) developed a methodology for prioritizing safety programs (McFarland et al. 1983), which was subsequently expanded to include other highway assets (bridges and pavements). In the methodology, scaling of the different performance measures was carried out (albeit implicitly) through monetization (expressing their impacts in dollar equivalents). The selection of projects was carried out using optimization tools such as dynamic and integer programming (Subramanian 1983; McFarland 1983). Hudson et al. (1987) also developed a methodology for project selection on the basis of multiple performance criteria.

In the early eighties, research in the area of pavement management such as the development of Arizona’s PMS (Golabi et al. 1982) yielded advances that still have a large influence on asset management system methodologies that are being developed today. In the Arizona PMS, the network optimization system consists of two interrelated models: a short- and long-term model. In the short-term model, the network performance is expressed in terms of proportions. The objective of the network optimization is to identify the least cost actions that would maintain a pre-established proportion of road sections in condition states desired by policy makers. A linear program was used to find the solution to the short-term model, which is a steady-state optimal policy. Such goals reflected the desire of the Arizona DOT decision-makers to be able to influence the time taken for the network to reach the optimal steady state and to be able to impose different short and long-term performance standards. The long-term model establishes a policy that minimizes the long-term expected costs subject to a variety of constraints including performance regarding acceptable and unacceptable states. These aspects of the Arizona PMS study influenced the subsequent FHWA Bridge Management Systems Demonstration Program (O’Connor and Hyman 1989) which was geared towards the selection of bridge deck preservation projects. In 1995, a similar research project sought to identify bridge projects on the basis of minimizing maintenance costs and maintaining acceptable structural reliability (Tao et al., 1995). Also, on the premise that bridge management at the network level is concerned with the twin goals of ensuring an adequate level of safety at the lowest possible life-cycle cost, Frangopol et al. (2000) investigated the optimization of network level bridge maintenance planning on the basis of minimum expected cost. The researchers offered a framework for optimal network-level bridge maintenance planning that minimizes the expected maintenance cost of a bridge stock and maintains the lifetime reliability of each bridge above an
acceptable (target) level. The framework supports the optimal allocation of resources to manage a stock of gradually deteriorating bridges.

In the mid to late eighties, Zimmermann (1987) surveyed different approaches to multi-criteria decision-making and capital budgeting under uncertainty using fuzzy sets. A generalized and simplified version of the stated problem was formulated to simultaneously satisfy both the objective function and constraints – each of which were expressed as membership functions in the form of fuzzy sets. In the case of multi-attribute decision making, the author discussed how to express “fuzzy utilities” under uncertainty. Also, Crum and Derkinderen (1981) discussed a number of issues associated with capital budgeting under conditions of uncertainty.

From the above discussion, some valuable lessons can be learned for purposes of asset management at INDOT. In most of the methodologies reviewed, performance measures being optimized largely comprised some form of facility condition, either expressed as a structural index, roughness index, sufficiency rating, or some index to indicate the overall or elemental health of the facility. Also, constraints used in the methodologies were typically single – an annual budgetary constraint. Also, it was seen that most studies carried out this optimization at the network level. While some studies sought to optimize the percentage of facilities in a certain desirable condition, others sought to determine the best set of actions to carry out, at which facility, and in which year. The use of the incremental benefit cost procedure (initially developed for safety management applications), has been embraced by most other facility optimization methodologies. Also, a few researchers have explored the use of mathematical programming techniques with some success. The subsequent section provides a more detailed discussion dedicated to past work on mathematical programming.

### 4.2 Formulation of Asset Management Optimization Problem

INDOT’s selection of projects from different program areas (management systems) to constitute an asset program under budgetary limitations, is identical to the classic Knapsack Problem (KP), a well-known integer programming problem that has had a vast range of applications in several fields. The knapsack problem is \textit{NP-hard}. The multi-dimensional 0-1 knapsack problem (MDKP) is a special case of general 0-1 linear programs. Historically, one of the first example applications was by Lorie and Savage (1955) in their bid to solve a capital budgeting problem.

The knapsack problem can be explained using a simple anecdotal situation as follows: A shopper with a shopping cart wishes to purchase a number of items. For each possible item on the
shelf, there is an associated utility or reward value to the shopper. Each item also has a certain weight or cost. The shopper seeks to fill the shopping cart with as many items as possible to maximize her overall satisfaction (objective). However, the cart is constrained by a certain total of weight, size or the shopper has a budget constraint. Then the knapsack problem is to determine which items the consumer should select. This is the simplest Knapsack problem. It has many variations some of which are discussed below.

4.2.1 Multi-Choice Knapsack Problem (MCKP)

In a more generalized form of the Knapsack problem, the consumer has a set of $n$ classes, where each class contains a number of items. The consumer faces a “multi-choice” problem because he/she has a set of choices for each class. Assume that the consumer needs to pick exactly one item from each class. So, for example the consumer needs to select one item from items of class 1, one item items of class 2, and so on, to maximize the reward gained with the constraint that the cart cannot hold more than a certain volume. In the asset management problem at hand, each class represents a project in the larger pool of candidate INDOT projects. The choice for each project is binary: do or do not do. The reward is measured in terms of multiple-criteria such as cost, condition, etc. The “size constraint” of the knapsack corresponds to the budget constraint for the program period. Thus, the context of INDOT’s Asset Manager’s problem, as defined in Chapter 1, is not consistent with a MCKP.

4.2.2 Multi-Dimensional Knapsack Problem (MDKP)

Another variation of the Knapsack problem is that the consumer seeks to select from a set of distinct items subject to more than one “size” constraint, and each item has a known weight, volume and width. For example, the shopping cart cannot hold more than a certain weight, more than a certain volume and more than a certain length or width, the shopper cannot spend money beyond some limit, etc. This gives the multi-dimensionality aspect to the problem. In the context of asset management, a scenario with multiple “size” constraints could be one having a budget constraint, network-wide condition constraint (minimum condition target), network-wide safety constraint etc. Mathematically, the multi-dimensional Knapsack problem (MDKP) can be stated as:
\[
\max z = \sum_{k=1}^{n} \sum_{j \in L_k} r_{jk} x_{jk} \\
\text{s.t. } \sum_{k=1}^{n} \sum_{j \in L_k} a_{ijk} x_{jk} \leq b_i, \ i = 1,2,\ldots,m \\
\sum_{j \in L_k} x_{jk} = 1, \ k = 1,2,\ldots,n \\
x_{jk} \in \{0,1\} \ \ k = 1,2,\ldots,n, \ j \in L_k
\] (4.1)

Where:

- \( n \) is the number of classes,
- \( L_k \) represents the set of items for class \( k \) and,
- \( m \) is the number of knapsack constraints (size constraints) with capacities \( b_i \).

Each item \( j \in L_k \) is associated with \( r_{jk} \) units of profit and \( a_{ijk} \) units of weight.

The goal is to choose one item from each class such that the profit is maximized without exceeding the capacities of the knapsack. If the number of size constraints is one and there is only one item in each class, then the problem reduces to a simple 0-1 knapsack problem (KP).

### 4.2.3 Multi-Choice Multi-Dimensional Knapsack Problem (MCMDKP)

This is a further generalization of the Knapsack problem which contains both the multi-choice (more than one item or activity in each class) and the multi-dimensional (more than one size constraint) aspects as explained above.

### 4.2.4 Different Formulation of Asset Management Optimization for INDOT

In any typical year or programming period, the different management systems and special program managers provide the Asset Manager (AM) with a list of their top priority projects. These projects constitute the pool of “candidate” projects. Due to budgetary constraints, the AM can only carry out a selected subset of these candidate projects. The selected subset or the optimal solution is one that yields maximum returns to the AM. The returns are the overall benefits of implementing the selected projects, and how to express these “returns” in related to what the AM’s objectives are, as discussed below.
1. Formulation of the Asset Manager’s Objective Function

As the Asset Manager proceeds to select the optimal set of projects and to carry out the subsequent trade-off analysis, he/she may have any of several objectives in mind. It is important to establish the specific objective that is sought and to select the appropriate mathematical formulation to represent that objective. For a given set of assets, performance measures and constraints, the optimal solution may differ depending on the way the objective function is formulated. This section herein presents four different ways of formulating the objective function. Note that in each of the first three formulations, $u$ represents the benefit for each project.

(1) **Maximize the ratio of the total benefits to total project costs**

\[
\text{Max } Z = \frac{\sum_{i=1}^{m} x_i u_i}{\sum_{i=1}^{m} x_i c_i}
\]  
\[
(4.2)
\]

Where: $Z$ is the total benefit/cost ratio;
$m$ is the number of projects in the candidate pool;
$x_i$ is a binary variable that represents the selection of a project ($x = 1$) or otherwise ($x = 0$);
$c_i$ is the agency cost of implementing project $i$;
$u_i$ is the amalgamated benefit or utility of project $i$.

(2) **Maximize the total of the ratio of individual project benefits to project cost**

\[
\text{Max } Z = \sum_{i=1}^{m} x_i \frac{u_i}{c_i}
\]  
\[
(4.3)
\]

Where: $Z$ is the sum of benefit/cost ratio;
$m$ is the number of projects in the candidate pool;
$x_i$ is a binary variable that represents the selection of a project ($x = 1$) or otherwise ($x = 0$);
$c_i$ is the agency cost of implementing project $i$;
$u_i$ is the amalgamated benefit of implementing project $i$.
(3) Maximize the net total benefit

\[ \text{Max } Z = \sum_{i=1}^{m} x_i u_i \]  

Where: 
- \( Z \) is the total amalgamated value;  
- \( m \) is the number of projects in the candidate pool;  
- \( x_i \) is a binary variable that represents the selection of a project \((x = 1)\) or otherwise \((x = 0)\);  
- \( u_i \) is the amalgamated benefit or utility of project \(i\) in terms of the multiple performance measures

(4) Minimize the distance from an established goal

\[ \text{Max } Z = \sum_{i=1}^{m} x_i \left[ \sum_{j=1}^{l} (A_{ij} - M_{ij})^p \right]^{\frac{1}{p}} \]

Where: 
- \( Z \) represents the sum of deviations from the goal;  
- \( m \) is the number of projects in the candidate pool;  
- \( x_i \) is a binary variable that represents the selection of a project \((x = 1)\) or otherwise \((x = 0)\);  
- \( A_{ij} \) represents the value of the jth performance measure;  
- \( M_{ij} \) is the target value of the jth performance measure;  
- The parameter ‘\( p \)’ is varied to determine the type of distance metric being measured. The three most commonly considered metric norms in goal programming are:  
  - If \( p = 1 \), “city block” distance  
  - If \( p = 2 \), “Euclidean” distance  
  - If \( p = \infty \), “Minmax” distance (or infinity norm)

Discussion

In the above objective functions, formulation (2), (3), and (4) can be solved rather easily. Formulation (1) may have some conceptual inconsistencies, can be very difficult to solve, and is rarely used in practice. As such, this study focuses on the last three formulations only.
2. Formulation of the Asset Manager’s Constraints

Due to his/her desire to duly incorporate limited agency funding, political considerations, or the strategic mission and goals of the agency, the Asset Manager typically faces a number of constraints on his/her performance measures. The next section first discusses the various possibilities of the Asset Manager's budgetary constraints, followed by those of his/her performance constraints.

A. Budgetary Constraints

Clearly, the overall total cost of the finally selected projects in the Asset Manager’s knapsack should not exceed the overall budget of the agency. There could also exist constraints on the overall costs of the individual program areas. For each program area, upper-bound constraints, or ceilings, ensure that spending in a given program area should not exceed some threshold; lower-bound constraints, or “floors” ensure that a given program area gets a minimal guaranteed level of funding and therefore, a minimum guaranteed number of projects. In any given problem setting, there may be budgetary ceilings for all or some program areas, budgetary floors for all or some program areas, or both ceilings and floors in some or all program areas. There are several variations to the above problem, as itemized in Table 4.1. Pursuant to this table, the mathematical formulations are discussed below.

<table>
<thead>
<tr>
<th>Overall Asset Program (Budgetary Ceiling)</th>
<th>Individual Program Areas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Budgetary Ceilings for at least 1 Program Area</td>
</tr>
<tr>
<td>Formulation 1</td>
<td>Yes</td>
</tr>
<tr>
<td>Formulation 2</td>
<td>Yes</td>
</tr>
<tr>
<td>Formulation 3</td>
<td>Yes</td>
</tr>
<tr>
<td>Formulation 4</td>
<td>Yes</td>
</tr>
<tr>
<td>Formulation 5</td>
<td>None</td>
</tr>
<tr>
<td>Formulation 6</td>
<td>None</td>
</tr>
<tr>
<td>Formulation 7</td>
<td>None</td>
</tr>
</tbody>
</table>
(1) A budgetary ceiling for overall asset program; but no budgetary ceiling or floors for each program area

This is the simplest formulation of the problem. There is no budgetary restriction on the individual program areas. Instead, there is a budgetary ceiling on the overall asset program.

\[ \sum_{i=1}^{m} x_i c_i \leq B \]

Where: 
- \( m \) is the number of projects (or alternatives) in the overall candidate pool;
- \( c_i \) is the agency cost of implementing project \( i \);
- \( x_i \) is a binary variable that represents the selection of a project (\( x = 1 \)) or otherwise (\( x = 0 \));
- \( B \) is the total budget for the asset program.

For this type of constraint, if the objective function is consistent with equations (2), (3) or (4), then it is a kind of classical Knapsack Problem.

(2) A budgetary ceiling for overall asset program; and budgetary ceilings for one or more program areas

Here, there is a budgetary ceiling on the overall asset program. In addition, a number (ranging from one to all) of the program areas have their individual budgetary ceilings.

\[ \sum_{i=1}^{m} x_i c_i \leq B \]
\[ \sum_{i=1}^{m} x_i y_i^1 c_i \leq b_1 \]
\[ \sum_{i=1}^{m} x_i y_i^2 c_i \leq b_2 \]
\[ \ldots \]
\[ \sum_{i=1}^{m} x_i y_i^s c_i \leq b_s \]

\( y_i^j = 1 \) (project \( i \) belongs to program area \( j \)) or 0 otherwise;
- \( s \) is the number of program areas constituting the overall asset program;
- \( x_i \) is a binary variable that represents the selection of a project (\( x = 1 \)) or otherwise (\( x = 0 \));
- \( c_i \) is the agency cost of implementing project \( i \);
- \( b_j \) is the budget of program area \( j \); \( B \) is the total budget for the asset program.
\( m \) is the number of projects (or alternatives) in the overall candidate pool.

Where a program area has no budgetary ceiling (upper spending limit), the \( b_i \) constraint is excluded from the mathematical formulation.

(3) **A budgetary ceiling for overall asset program; and budgetary floors for one or more program areas**

Here, there exists a budgetary ceiling on the overall asset program. In addition, a number (ranging from one to all) of the constituent program areas have a budgetary floor.

\[
\sum_{i=1}^{m} x_i c_i \leq B \\
\sum_{i=1}^{m} x_i y_i^j c_i \geq f_j^1 \\
\sum_{i=1}^{m} x_i y_i^j c_i \geq f_j^2 \\
\vdots \\
\sum_{i=1}^{m} x_i y_i^j c_i \geq f_j^s
\]

\( y_i^j = 1 \) (project \( i \) belongs to program area \( j \)) or 0 otherwise;

\( s \) is the number of program areas constituting the overall asset program;

\( x_i \) is a binary variable that represents the selection of a project (\( x = 1 \)) or otherwise (\( x = 0 \));

\( c_i \) is the agency cost of implementing project \( i \);

\( f_j \) is the budgetary floor (or minimum spending limit) for program area \( j \);

\( B \) is the total budget for the asset program.

\( m \) is the number of projects (or alternatives) in the overall candidate pool.

Where a program area has no minimum spending limit, \( f_j \) is set to 0.

(4) **Budgetary ceiling for overall asset program; and budgetary ceilings and/or floors for one or more program areas**

This is the most restrictive of all the budgetary constraint formulations. Here, there is a budgetary ceiling on the overall asset program. In addition, a number of program areas have either a budgetary ceiling or a budgetary floor or both.
\[
\sum_{i=1}^{m} x_i c_i \leq B \\
\sum_{i=1}^{m} x_i y_i^1 c_i \leq b_i \\
\quad \ldots \\
\sum_{i=1}^{m} x_i y_i^s c_i \leq b_s \\
\sum_{i=1}^{m} x_i y_i^1 c_i \geq f_i \\
\quad \ldots \\
\sum_{i=1}^{m} x_i y_i^s c_i \geq f_s 
\]

\( y_i^j = 1 \) (project \( i \) belongs to program area \( j \)) or 0 otherwise;

\( s \) is the number of program areas constituting the overall asset program;
\( x_i \) is a binary variable that represents the selection of a project \((x = 1)\) or otherwise \((x = 0)\);
\( c_i \) is the agency cost of implementing project \( i \);
\( b_j \) is the budgetary ceiling for program area \( j \);
\( f_j \) is the budgetary floor (or minimum spending limit) for program area \( j \);
\( B \) is the total budget for the asset program.

Where a program area has no minimum spending limit, \( f_j \) is set to 0.
Where a program area has no budgetary ceiling (upper spending limit), the \( b_i \) constraint is not included in the mathematical formulation.

(5) No budgetary ceiling for overall asset program; but has budgetary ceilings for one or more program areas

Here, there is no budgetary ceiling on the overall asset program. Instead, a number (ranging from one to all) of the program areas have a budgetary ceiling.
\[
\sum_{i=1}^{m} x_i y_i c_i \leq b_i
\]
\[
\sum_{i=1}^{m} x_i y_i^2 c_i \leq b_2
\]
\[
\ldots
\]
\[
\sum_{i=1}^{m} x_i y_i^s c_i \leq b_s
\]

\(y_i^j = 1\) (project \(i\) belongs to program area \(j\)) or 0 otherwise;

\(s\) is the number of program areas constituting the overall asset program;

\(x_i\) is a binary variable that represents the selection of a project \((x = 1)\) or otherwise \((x = 0)\);

\(c_i\) is the agency cost of implementing project \(i\); \(b_j\) is the budget of program area \(j\);

\(m\) is the number of projects (or alternatives) in the overall candidate pool.

Where a program area has no budgetary ceiling (upper spending limit), the \(b_i\) constraint is not included in the mathematical formulation.

(6) **No budgetary ceiling for overall asset program; but has budgetary floors for one or more program areas**

Here, there is no budgetary ceiling on the overall asset program. Instead, a number (ranging from one to all) of the program areas have a budgetary floor.

\[
\sum_{i=1}^{m} x_i y_i c_i \geq f_1
\]
\[
\sum_{i=1}^{m} x_i y_i^2 c_i \geq f_2
\]
\[
\ldots
\]
\[
\sum_{i=1}^{m} x_i y_i^s c_i \geq f_s
\]

\(y_i^j = 1\) (project \(i\) belongs to program area \(j\)) or 0 otherwise; \(s\) is the nr. of program areas;

\(x_i\) is a binary variable that represents the selection of a project \((x = 1)\) or otherwise \((x = 0)\);

\(c_i\) is the agency cost of implementing project \(i\);

\(f_j\) is the budgetary floor (or minimum spending limit) for program area \(j\);

\(m\) is the number of projects (or alternatives) in the overall candidate pool.
No budgetary ceiling for overall asset program; Has budgetary ceilings and/or floors for one or more program areas

Here, there is a budgetary ceiling on the overall asset program. In addition, a number (ranging from 0 to all) the program areas have either a budgetary ceiling or a floor or both.

\[ \sum_{i=1}^{m} x_{ij} y_{ij} c_i \leq b_j \]

\[ \cdots \]

\[ \sum_{i=1}^{m} x_{ij} y_{ij} c_i \leq b_s \]

\[ \sum_{i=1}^{m} x_{ij} y_{ij} c_i \geq f_j \]

\[ \cdots \]

\[ \sum_{i=1}^{m} x_{ij} y_{ij} c_i \geq f_s \]

\[ y_{ij} = 1 \text{ (project } i \text{ belongs to program area } j) \text{ or } 0 \text{ otherwise; } \]

\[ m \text{ is the number of projects (or alternatives) in the overall candidate pool.} \]

B. Performance Constraints

From the perspective of performance, there could be a constraint on the overall scaled and amalgamated performance but this would not make sense to the layman who thinks in terms of the raw values of the individual performance measures. Instead, therefore, the Asset Manager could establish constraints on these raw values. For example, she/he could specify for example, a minimum average pavement condition (IRI) for the entire network, a maximum average crash rate, maximum average delay, etc. Details of such constraints are herein discussed.
(1) Performance Constraint Possibility 1
For all assets, the overall impact, in terms of the average final value of a given performance measure, should be greater (or less) than some established threshold.

\[
\sum_{i=1}^{m} \left( \frac{a_{ij} + \Delta a_{ij} x_i}{m} \right) \geq (\leq) L_j
\]

\(a_{ij}\) is the original value of project \(i\) for performance measure \(j\);
\(x_i = 1\) or \(0\) (alternative project \(i\) is selected or otherwise);
\(\Delta a_{ij}\) is the \(\Delta\) value of performance measure \(j\) due to project \(i\) if project \(i\) is selected;
\(L_j\) is the threshold of performance measure \(j\).

(2) Performance Constraint Possibility 2
Often, the Asset Manager seeks the impact of projects on the overall network under consideration. It should be noted that this includes all assets in the network (whether the asset is slated for some project or not). In fact, it makes little practical sense to find an average performance only for assets that received some project. Thus, a constraint could be that the overall impact of the selected project(s) on the average network value of a given performance measure, should exceed some established standard. Mathematically, this is written as:

\[
\sum_{i=1}^{m} \left( \frac{a_{ij} + \Delta a_{ij} x_i}{m} \right) + \sum_{i=1}^{q} a_{ij} \geq (\leq) L_j
\]

\(\sum_{i=1}^{q} a_{ij}\) is the total value of performance measure \(j\) for all projects in the network;
\(a_{ij}\) is the original value of performance measure \(j\) for project \(i\);
\(x_i = 1\) or \(0\) (alternative project \(i\) is selected or not);
\(\Delta a_{ij}\) is the \(\Delta\) value of performance measure \(j\) due to project \(i\) if that project is selected;
\(L_j\) is the threshold of performance measure \(j\);
\(m\) is the number of projects (or number of assets which received projects in the overall candidate pool);
\(q\) is the number of assets in the network which received no project in the overall candidate pool.

(3) Performance Constraint Possibility 3
In this case, for each project, the project impact in terms of the value a given performance measure \(j\), should exceed some specified minimum threshold.
If $a_g < L_f$, then $x_f = 1$

In optimization formulation, if the Asset Manager chooses any type of the above objective functions (except formulation (1)) and any type of constraints regarding budget and performance, then the problem is a Knapsack Problem. If there is only one constraint, then the problem is a classical Knapsack Problem. If there is more than one constraint, then the problem is a Multi-Dimensional Knapsack Problem.

MDKP problems are considered as NP-hard in the sense that no known deterministic polynomial algorithm exists for their solution. The time requirement for the optimal solution grows exponentially with the size of the problem. There are two classes of methods that exist to solve this problem: exact methods (or algorithms) and heuristics. Exact methods are guaranteed to arrive at the optimal solution but are typically associated with lower computational speeds. On the other hand, heuristic methods strive to achieve “good” approximate (near optimal) solutions quickly and provide error bounds for the solution. In this research study, both methods were explored for use. The literature was examined for the best available algorithms and heuristics for the simpler problems of 0/1 KP, and MDKP, and then the research investigated how to improve and tailor these methods to suit the specific problem at hand.

For purposes of demonstrating the problem formulation, this research study used only a small subset of projects and thus the issues of computational speed were not so apparent. MS Excel Solver was used as the platform to carry out the optimization and project selection. However, as discussed in the section below, there are certain issues associated with the use of MS Solver to solve problems of this nature.

4.3 Discussion of MS Solver Limitations and Computational Experiments

In order to examine the suitability of the widely available Solver tool in MS excel for solving the optimization problem faced by INDOT’s Asset Manager, this research study carried out a computational experiments. These involved gradually increasing the problem size and dimensionality and examining the impact of the computational time. In the sections below, this report discusses MS Solver’s limitation on the number of decision variables, the limitation on the number of constraints, and the relationship between the computational time and the number of decision variables.
4.3.1 Limitation on the Number of Decision Variables

The standard Microsoft Excel Solver (e.g., solver in Microsoft Excel 2007) places upper limits on the number of decision variables (see “cells to change” in Solver’s dialog box). The maximum number of decision variables is 200 (Microsoft Excel 2007 and www.solver.com). As INDOT’s Asset Manager would need to establish a decision variable for each candidate project under consideration (1 for implement the project; 0 for do not implement the project) in those Excel cells, it is clear that the standard solver cannot solve the optimization problem if there are over 200 projects under consideration.

If INDOT’s Asset Manager faces an optimization problem that has over 200 projects, he/she may purchase an updated version of the standard MS Excel Solver tool, or other MS Excel add-ons such as Premium Solver, Premium Solver Platform, or Solver engines. Premium Solver has a limit of 2,000 decision variables for linear problems, and 500 variables for nonlinear problems. Premium Solver Platform handles linear and quadratic problems of up to 8,000 variables; Solver engines for the Premium Solver Platform can handle problems of virtually unlimited size (www.solver.com). In addition, Premium Solver Platform, and Solver engines are much faster than the standard solver. The dollar prices of these upgrading resources can be found on the internet at www.solver.com.

4.3.2 Limitation on the Number of Constraints

The Asset Manager typically faces several constraints on budget (overall asset, and/or individual program areas) and/or performance (network average or individual minimums). In MS Solver, the AM inputs these constraints in the “Constraints” dialog box. For the standard solver tool, if the optimization problem is linear, there is no limit on the number of constraints. However, if it is nonlinear, then the maximum number of constraints is 100 (www.solver.com). In Premium Solver, Premium Solver Platform, or Solver engines, a greater number of constraints are allowed than are allowed in the standard solver tool.

4.3.3 The Relationship between the Runtime and the Number of Decision Variables

The Asset Manager often seeks to carry out optimization in a reasonable amount of time, so that the impacts of different funding scenarios and other trade-offs can be investigated quickly and fed to the top management. The time for MS Solver to reach an optimal solution increases rapidly as
the number of decision variables and constraints increase. *Premium Solver* can usually solve problems much faster than the standard Solver -- up to 100 times faster in some cases (www.solver.com).

In a bid to investigate the efficacy of using the standard version of MS solver to solve INDOT’s asset management optimization problem, this research carried out computational experiments that revealed the relationship between the run time and the number of decision variable, using a very liberal constraint scenario (i.e., only one constraint).

![Figure 4.1 Computational Experiments using MS Excel Solver Tool - Standard Version](image)

*Note: The runtime depends on several factors such as the technical features of the computer used and the option settings in the solver. The data in this chart are from a computer with the following features: RAM = 1.0GB, CPU = Genuine Intel 1.73GHz. Number of decision variables can be represented as the number of candidate projects.*

INDOT’s project selection tasks faced by the Asset Manager will likely involve a far greater number of candidate projects and performance constraints than in the spreadsheets use in this study. It is therefore clear that in actual practice, this would preclude the reaching of an optimal solution using MS Excel’s standard Solver tool even after several days or months of running the algorithm. Thus, there will be a need to purchase commercially-available optimization add-ons such as the advanced Solver tools identified above, the General Algebraic Modeling System (GAMS) (see www.gams.com), CPLEX (www.ilog.com) or other appropriate packages. Alternatively, INDOT could develop an appropriate heuristics to find near-optimal solutions in reasonably good time, as was done by Patidar et al. (2007) in their bridge management optimization research for the National Cooperative Highway Research Program. In any case, the sections below present a discussion on various approximate and exact algorithms
that could be used, for large problems of this nature, to reduce the dimensionality of the problem to render it more tractable and solvable in reasonably good computational times.

4.4 Literature Review of Solution Methods for the Formulated Optimization Problem

There is a vast body of knowledge on solution methods for the resource allocation problem. A recent valuable addition to the body of literature on the subject is a text by Cohon (2003). This author described non-inferior set estimation method to determine and evaluate the extreme points and the properties of the line segments between them. In the sections below, this report discusses solution methods, exact algorithms and heuristics for solving the problem that faces INDOT Asset Manager as described in Chapter 1 of this report.

4.4.1 Solution Methods involving Mathematical Programming (MP)

Patidar et al. (2007) reviewed existing literature on solution methods involving MP and found that a variety of techniques had been recommended or used by past researchers to allocate resources optimally to achieve a certain objective. These included variations of linear programming, integer programming, dynamic programming, goal programming, etc.

A classical reference for the optimal control problem is the text by Intriligator (1971) which formulated solution methods to allocate scarce resources among competing ends over a period of time. Three traditional, interrelated approaches have been identified for addressing optimal control problems: calculus of variations, maximum principle, and dynamic programming. Dynamic programming is capable of handling decision variables that are discrete (such as those in the present research study). However, unlike the problem context in the present study, the use of DP is consistent with temporal relationships between the alternatives. Dynamic programming is derived from Bellman’s equations and the principal of optimality – an optimal policy has the property that whatever the initial state and decision are, the remaining decisions must also be an optimal policy with respect to the state resulting from the first decision. Therefore asset management that takes into account time trends in deterioration stand to benefit from DP application. In the present research, it is assumed that all life-cycle and deterioration issues are taken care of in the individual program areas, and thus these issues are already addressed by the time the project is sent to the Asset Manager for consideration as a candidate project. DP is considered a general case of the calculus of variations and implies the maximum principle’s conditions. The equations associated with calculus of variations and maximum principle are continuous or piecewise continuous functions.
In the late seventies, Sinha et al. (1981) used goal programming techniques to achieve optimal allocation of federal and state funds for highway system improvement and maintenance. Their work involved a multi-objective framework (four system objectives were used), and six alternative activities were considered for each facility. Jiang and Sinha (1989) developed solution methods for optimizing bridge investments in Indiana BMS, based on dynamic and integer linear programming. The solution method involved selection of projects while maximizing the effectiveness or benefit to the system subject to the constraints of available budget over a given program period. Markov chain transition probabilities of bridge conditions were used to predict or update bridge conditions at each stage of the dynamic programming. The solution method selected projects by maximizing yearly system effectiveness subject to different budget spending. The effectiveness was measured in terms of the coefficient of safety condition; the coefficient of community impact of the project (such as detour length and bridge ADT in the case of bridge management); and the change in the area under performance curves achieved by the activity. In terms of dynamic programming, each year of the program period was considered a “stage”. Each activity for a facility is a 0-1 decision variable of the dynamic programming as well as the integer linear programming. Jiang and Sinha (1990) utilized ranking and optimization techniques to select bridge projects. A similar optimization model was used as in the 1989 study, the only difference being measure of project effectiveness – as the change or reduction in disutility of a bridge after performing the activity. Vitale et al. (1996) showed how the Indiana BMS could be used to conduct trade-off analyses by varying the parameters (such as funding levels) to analyze the effect of various spending policies on bridge condition and other performance measures.

In another application to bridge assets and other assets, Harper et al. (1990), Ravirala et al. (1996) and Guignier et al. (1999) applied Markovian techniques. Harper et al. utilized linear programming techniques to optimize decisions, duly recognizing that optimization parameters are stochastic in nature. The module consisted of a long-term (steady state) goal-setting model; a multiyear (short term) planning model (both of which were based on Markovian decision models using linear programming techniques), and a financial model. Bridges were stratified according to bridge type, climate and functional class, and a separate linear program was solved for each stratum. The long-term model first establishes the steady state performance goals that provide targets for the multi-year and financial models; the steady-state model takes inputs as desirable and undesirable condition states, proportions in these states, maximum and minimum allowable proportions and Markovian transition probabilities. The model optimized for proportion of segments in a given condition state receiving a given action and the average cost for each segment. The multi-year model determined the optimal maintenance policy for each year in the
planning horizon. The financial model imposed a network wide budget constraint across all strata. Using a model that yielded a multi-year program minimizing the weighted sum of treatment costs and deviations from goals, Ravirala et al. (1996) developed an optimal solution for a capital improvement program for bridges in New York State. The goals defined in that study included annual budget goals and an annual average system conditions. The model was solved as a linear program. Guignier et al. (1999) carried out budget allocation using a Markov decision model that jointly optimized maintenance and improvement activities. The infinite horizon model was used to study steady-state policies while relaxing the assumption of age homogeneous condition state transition probabilities. The model also allowed for carry-over of annual budget which could be spent more efficiently in later years. Facility-specific representation was used in the model because the improvements were selected for individual facilities, whereas maintenance could be optimized at the network level. Because joint optimization is considered a large problem due to a large number of decision variables and constraints, issues of computational complexity arose, but the authors downplayed these consequences by stating that the problem was to be applied only at the planning level and thus computational time was not a critical issue.

Li and Sinha (2004) utilized the Lagrangian relaxation technique for solving the multi-choice multi-dimensional knapsack problem for selecting projects across different program areas: bridges, pavements, congestion, and safety, under scenarios of risk and certainty.

4.4.2 Solution Approaches that utilized Meta-Heuristic Techniques

In the last 15 years, the use of non-traditional solution techniques for optimal control has blossomed - these included fuzzy logic, neural networks, and genetic algorithms.

Using artificial neural networks (ANNs), Mohamed et al. (1995) optimized available resources that for facility improvements that minimize the loss of network benefits. The problem was perceived to have a facility-specific dimension and a network dimension. A dynamic programming model was used to handle the facility-specific dimension and a two-layer ANN was developed for the network dimension. Each neuron receives a number of inputs – these are converted to a single output by using activation and output functions. The number of neurons in the second layer of the ANN was equal to the product of the number of facilities and the number of activities for each facility. The network is supplied with the loss and initial cost associated with each alternative activity and the available budget. Once the network reached a steady state, the output of neuron (0 or 1) indicated which activities were to be carried out.
The asset management research described in this report stands to benefit from the several decades of similar research carried out in the pavement management arena. In a study that analyzed an evolutionary neural network model for the selection of pavement strategy, Taha and Hanna (1995) used genetic algorithms to design the “best” neural network model for optimal maintenance of flexible pavements. The researchers described an evolutionary-learning system using gradient descent learning and a genetic algorithm to determine the network connection weights. The input vector consists of factors that affect the selection of a specific flexible pavement maintenance strategy. The output vector consists of different pavement maintenance strategies available. That research demonstrated that genetic algorithms and neural networks can be combined to handle multi-objective optimization problems. Pilson et al. (1999) used genetic algorithms to solve the multi-objective optimization of pavement scheduling problems at both network level and facility level, arguing that the context of the pavement management problem makes it suited for directed random search heuristics such as genetic algorithms. Showing how to solve the general network optimization problem using efficient surfaces for the investments, the authors contend that using efficient surfaces to break down the network problem into project sub-problems was a viable solution technique to such kinds of optimal control problems. Fwa et. al. (2000) used a genetic algorithm procedure to solve multi-objective network level pavement maintenance resource allocation problems. Their work sought a Pareto optimal solution set and a rank-based fitness evaluation. Demonstrating their research with a numerical example, the authors concluded that the robust search characteristics and multiple-solution handling capability of genetic algorithms were well suited for optimization analysis. Genetic algorithm optimization techniques also were used by Chan et al. (2003) to allocate pavement maintenance funds across various hierarchies of government in a region. The used a 2-stage genetic algorithm and showed that the optimal allocation yielded superior network performance. Hegazy et al. (2004) also used genetic algorithms to optimize repair actions over facility life-cycle. Their solution method optimized resources at both project and network levels under yearly budget limits, minimum allowable condition state for individual bridges and for the network. The solution representation took the form of chromosomes: a string of (N by T) elements was constructed, where N is the number of bridges and T is the planning horizon. The methodology was applied to a small network where the algorithm reached near-optimal solution.

Other non-traditional solution techniques for resource allocation problems similar to the one at hand include fuzzy logic approaches. These were used in Indiana in 1988 as part of the IBMS decision-making procedure (Tee et al. 1988).
The information search showed that the use of non-traditional techniques, such as neural networks, fuzzy logic, and genetic algorithms, for integrated asset management optimization shows considerable promise, given its apparent success with bridge management and in pavement management. This offers encouragement to look beyond traditional methods to solve particularly hard problems such as the one in the present study.

4.4.3 Exact Algorithms for Solving the Decision Problem

In the literature, exact algorithms typically utilize a variety of solution methods including branch-and-bound, dynamic programming, enumeration, and reduction techniques. Morin and Marsten (1976) demonstrated the use of branch-and-bound methods to reduce computational requirements in discrete dynamic programs. They used relaxations and fathoming criteria for identifying and eliminating irrelevant states whose sub-policies were inconsistent with optimal policies during the dynamic programming computation. Marsten and Morin (1977; 1978) combined dynamic programming and branch-and-bound approaches for solving the MDKP problem. Also, Morin and Esogbue (1974) presented a solution method that reduces dimensionality in finite dynamic programs. Using appropriate mathematical properties of the functional equation associated with dynamic programming, they reduced the $M$-dimensional state space to a 1-dimensional search over an imbedded state space. An algorithm was thus developed for non-linear knapsack problem which recursively generates the complete family of undominated feasible solutions (Morin and Marsten 1976). Shih (1979) designed a linear programming based branch-and-bound method for MDKP. The estimation of an upper bound and the branching rule at any node are based on the information provided by the solutions of the LP relaxations associated with each of the $m$ single-constraint knapsack problems.

An exact algorithm for large 0-1 knapsack problems discussed by Balas and Zemel (1980) was based on three concepts: a core problem whose size is usually a small fraction of the full problem size and does not seem to increase with the latter (a satisfactory solution approximation can be found by solving the linear relaxation of the problem, or the LKP); a binary-search type solution method for solving the LKP without sorting the variables (the computational complexity of this procedure is $O(n)$); the use of a simple heuristic which involve 0-1 assignments with a probability that increases exponentially with the problem size. Gavish and Pirkul (1985) proved the existence of theoretical relationships between various MDKP relaxations. They reduced the dimensionality of the MDKP problem size using Lagrangian,
Surrogate and Composite relaxations and developed an algorithm for computing surrogate multipliers, rules for reducing problem size and an efficient branch-and-bound procedure.

A number of researchers established upper bounds for hard 0-1 knapsack problems by adding valid inequalities on the cardinality of an optimal solution and then relaxing it in a Lagrangian fashion (Martello and Toth, 1997). The authors developed a polynomial-time branch-and-bound algorithm which incorporated an iterative technique determine the optimal Lagrangian multipliers for the linear relaxation of the problem. In subsequent research, this approach was combined with the concepts of surrogate relaxation and core problem to develop an efficient algorithm for 0-1 knapsack problem (Martello et al. 1999). The core was enumerated through dynamic programming. An overview of solution methods for solving hard KPs was presented by Martello et al. (2000) where the roles played by cardinality constraints and dynamic programming in reaching optimal solutions quickly, were stressed.

4.4.4 Heuristics

Unlike exact algorithms, heuristics reach near optimal solutions, and they do so in relatively quicker computation times (Patidar et al., 2007). The early heuristic approaches for solving problems such as the asset management problem at hand were based on greedy algorithms. Fast and simple to implement, these use profit-to-weight ratios to solve the single constraint knapsack problem. For example, for the MDKP, Senju and Toyoda (1968) developed a dual heuristic that starts with the all-ones as the solution and gradually sets the variables to zero one-at-a-time in order of increasing ratios until all feasibility requirements are satisfied. In subsequent research, Marsten and Morin (1977) found the optimal solutions to the problem and showed that the heuristic was very effective. Toyoda (1975) developed a primal method which started from the origin, setting variables to one according to decreasing ratios until no more variables can be added without violating the constraints. The concept of dual multipliers has been used to develop effective heuristics in the form of competitive greedy algorithms. Magazine and Oguz (1984) combined Senju and Toyoda’s dual algorithm with a Lagrangian relaxation approach, yielding a heuristic that provided upper bounds with approximate solutions at no additional cost. This heuristic was improved by Volgenant and Zoon (1990) who calculated the Lagrangian multipliers simultaneously and sharpened the upper bounds. Magazine and Oguz’s research was furthered by Moser et al. (1997) who generalized the heuristic for a multi-choice MDKP. To solve MDKPs, Pirkul (1987) developed a greedy heuristic based on a descent procedure to determine the surrogate constraints (a linear relaxation of the surrogate problem is considered to help
computational efficiency). In developing a heuristic for MDKP, Lee and Guignard (1988) utilized three concepts that influence the trade-off between solution quality and computation time: the use of a modified Toyoda’s procedure, reduction of the problem size using the LP relaxation, and improvement of the solution by complementing certain set of variables. The pivot and complement heuristic (Balas and Martin 1980) find approximate solutions to large binary programming problems. This heuristic has performed remarkably well in past research, in terms of solution efficiency and computational time. An LP-based heuristic for solving bi-objective binary knapsack problems which was developed by Zhang and Ong (2003) also performed well.

Like other combinatorial optimization problems, knapsack problems have also been investigated using the meta-heuristics. Chu and Beasley (1998) presented a heuristic based on genetic algorithms for the MDKP. The GA is restricted to search only the feasible region of the solution space by using a heuristic operator to covert an infeasible solution to a feasible one. This operator is based on a greedy-like heuristic which uses the profit-to-weight ratios. To convert the MDKP to a single-constraint KP, the surrogate relaxation of the problem is considered. The surrogate duality approach of Pirkul (1987) is then used to determine the surrogate multipliers by solving the linear relaxation of the original MDKP. The heuristic was shown to provide good solutions with a modest computational effort.

Hanafi and Freville (1998) developed a heuristic based on tabu search for the MDKP. Strategic oscillation and surrogate constraint information is used to balance the intensification and diversification strategies. Vasquez and Vimont (2004) used geometric constraint and cutting planes combined with tabu search method to solve the MDKP.

4.5 Chapter Summary

A possible aim of the Asset Manager is to compare different candidate projects from different program areas and finally choose some projects which can maximize the total benefit under limited total budget. The total benefit can be expressed as a utility value. The Knapsack Problem is the basic approach to address this optimization problem. In optimization, two important parts are objective function and constraints. Some formulations of objective function, budget constraints and performance measure constraints are provided in this chapter. In addition, this chapter also discusses past research studies on a number of techniques, exact algorithms and heuristics that could be used to reduce the size of the Knapsack optimization problem at hand.
CHAPTER 5: UNCERTAINTY CONSIDERATIONS IN ASSET MANAGEMENT DECISION-MAKING AND TRADE-OFFS

5.1 Introduction

In Chapters 2 and 3, this report discussed various scaling and amalgamation methods, respectively. In Chapter 4, the report discussed how to use optimization to help the Asset Manager conduct network level optimization. An important issue in asset management is that project outcomes (in terms of each performance measure) are not always known with certainty: thus any decision analysis can be carried out on for either the deterministic (certainty) scenario where the impacts of each project are known, or the probabilistic (uncertainty) scenarios where the impacts of each project are not known with certainty and thus are considered variable over a given range. In classical literature, and indeed in real life such as INDOT practice, there are two subcases for the uncertainty scenario: the risk case, where the project outcomes have a known probability distribution; and the pure uncertainty case, where the probability distributions of project outcomes are unknown. It is useful for INDOT’s Asset Manager to possess the capability for carrying out the analysis under all these cases and subcases.

The issue of uncertainty is all too real. Fluid changes in the funding environment can render uncertainty and variabilities to budgets; unpredictable weather and traffic patterns can lead to deviations in deterioration rates from those predicted; differences in contractor quality can lead to different performance changes after a project, changes in the surrounding economy or gas price fluctuations may cause traffic volumes and speeds to be different from what was predicted subsequent to a congestion mitigation project. Thus, recognizing that the outcome of performance measures after project implementation may be not known with certainty, this chapter discusses the methods to deal with the uncertainty of project outcomes in terms of the relevant performance measures.

The case of certainty implies that the possible outcomes of performance measures occur deterministically. Risk is defined as the situation where the set of all possible outcomes of a performance measure is known and the probability distribution of the outcomes is also known. The term uncertainty is defined for situation where only part of all possible outcomes of an action is known, but the probability distribution of such outcomes is not fully definable for a lack of
reliable information (Young, 2001). In the previous chapters, the examples presented for scaling and amalgamation methods are based on certainty case, and incorporation of uncertainty has been only implicit at best. In the present chapter, the focus is explicitly on the risk and uncertainty.

5.2 Risk Considerations in Asset Management

In the risk scenario, the outcomes of performance measures are not known with certitude, but a probability distribution is known. With the probability distribution, the probabilistic risk assessment can be performed to ultimately establish mathematical expectations (or expected values) for the performance measures after project implementation. The expected value of the performance measure can then be used to conduct scaling, amalgamation and overall utility or combined impact of the project. The following sections discuss the basic concept of probabilistic risk assessments of performance measures under the risk scenario.

5.2.1 Bayesian Updating for Probability Distributions

The probability risk analysis may produce a family of probability distributions (or risk curves) for each individual performance measure – one distribution for each confidence level. The average of these distributions yields the mean probability distribution. As seen in Figure 5.1, the corresponding risk curves associated with a specific performance measure as the complementary of cumulative distributions (CCD) can be derived. The expected value of the performance measure (corresponding to CCD value of 0.5) based on the mean risk curve or a risk curve at higher confidence level, may be used for estimating the project benefits (Paté-Cornell, 2002; Winkler, 2003). Bayesian updating can help improve the confidence level of a distribution, thus improving the confidence level of the risk curve.

Figure 5.1: Characterization of Risks Associated with the Values of a Performance Measure

Note: X_M, X_L, X_H- the expected value of a performance measure according to a distribution at mean, low or high confidence
Without loss of generality, the procedure for Bayesian updating for a continuous performance measure variable $X$ representing the project impact is:

$$f(X=F|\varepsilon) = \frac{f(X=F|\varepsilon) f(\varepsilon|X=F)}{f(\varepsilon)}$$

where

$\varepsilon$ = Newly available information for assessing $X$

$f(X=F)$ = Prior distributions for $X$; $f(X=F|\varepsilon)$ = Posterior distributions for $X$

$f(\varepsilon|X=F)$ = Likelihood function of having $X$ equal to $F$, and

$f(\varepsilon)$ = Prior predictive distribution of new information, $f(\varepsilon) = \int_{-\infty}^{\infty} f(\varepsilon|X)f(X)dX$

**Example**

Assuming that travel speed is the performance measure for a given congestion mitigation project. Also assume that this follows a normal distribution. The normal parameters for the prior distribution of speed have been determined as $m_1=60$ mph and $\sigma_1=10$ mph. If additional information from 100 simulation runs implies a mean and standard deviation of $m=45$ mph and $\sigma=5$ mph, the normal parameters for the posterior distribution become:

$$m_2 = \frac{1}{\sigma_1^2}m_1 + \frac{n}{\sigma^2} \cdot \frac{n}{\sigma^2} = \frac{1}{1/10^2} \cdot 60 + \frac{100/5^2}{1/10^2} \cdot 45 = 45.04 \text{ mph and}$$

$$\sigma_2 = \sqrt{\frac{\sigma_1^2 \sigma^2}{n \sigma_1^2 + \sigma^2}} = \sqrt{\frac{10^2 \times 5^2}{100 \times 10^2 + 5^2}} = 0.5 \text{ mph.}$$

As such, the expected speed is herein “updated” from 60 mph to 45.04 mph, while the standard deviation reduces from 10 mph to 0.5 mph. This provides a firmer and more confident distribution for travel speed as a performance measure for the computation.
5.2.2 Selection of Input Probability Distributions

Strictly speaking, the possible outcomes of performance measures such as physical asset conditions, agency and user costs, travel speed, crash rates, etc. are bounded by non-negative minimum and maximum values. In addition, the distributions of the possible outcomes could be either symmetric or skewed. Such characteristics can be modeled by the beta distribution that is continuous over a finite range and allows for virtually any degree of skewness and kurtosis. The general beta distribution has four parameters: lower range (L), upper range (H), and two shape parameters referred to as $\alpha$ and $\beta$. The beta density function is given by:

$$f(x|\alpha, \beta, L, H) = \frac{\Gamma(\alpha + \beta) - (x - L)^{\alpha - 1} - (H - x)^{\beta - 1}}{\Gamma(\alpha) \cdot \Gamma(\beta) \cdot (H - L)^{\alpha + \beta - 1}} \quad (L \leq x \leq H)$$

where

the $\Gamma$ -function factors serve to normalize the distribution so that the area under the density function from L to H is exactly one.

The mean and variance for the beta distribution are given as

$$\mu = \frac{\alpha}{\alpha + \beta} \quad \text{and} \quad \sigma^2 = \frac{\alpha \beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)}.$$

It is seen that the distribution mean is a weighted average of L and H such that when $0 < \alpha < \beta$ the mean is closer to L and the distribution is skewed to the right; whereas for $\alpha > \beta > 0$ the mean is closer to H and the distribution is skewed to the left. When $\alpha = \beta$ the distribution is symmetric. Also, it can be noted that for a given $\alpha/\beta$ ratio, the mean is constant and the variance varies inversely with the absolute magnitude of $\alpha + \beta$. Thus, by increasing $\alpha$ and $\beta$ by proportionate amounts, the variance may decrease while the mean is constant; and conversely, by decreasing $\alpha$ and $\beta$ by proportionate amounts, the variance may increase while the mean remains unchanged. In practice, the skewness and variance (kurtosis) can be categorized as high, medium or low based on the magnitude of $\alpha$ and $\beta$. Table 5.1 presents the combinations of skewness and variance (kurtosis) for beta distributions that best approximate the risk factor.
Table 5.1: Approximate Values of Shape Parameters for Beta Distributions

<table>
<thead>
<tr>
<th>Combination Type</th>
<th>Skewness</th>
<th>Variance (Kurtosis)</th>
<th>α</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Skewed to the left</td>
<td>High</td>
<td>1.50</td>
<td>0.50</td>
</tr>
<tr>
<td>2</td>
<td>Symmetric</td>
<td>High</td>
<td>1.35</td>
<td>1.35</td>
</tr>
<tr>
<td>3</td>
<td>Skewed to the right</td>
<td>High</td>
<td>0.50</td>
<td>1.50</td>
</tr>
<tr>
<td>4</td>
<td>Skewed to the left</td>
<td>Medium</td>
<td>3.00</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>Symmetric</td>
<td>Medium</td>
<td>2.75</td>
<td>2.75</td>
</tr>
<tr>
<td>6</td>
<td>Skewed to the right</td>
<td>Medium</td>
<td>1.00</td>
<td>3.00</td>
</tr>
<tr>
<td>7</td>
<td>Skewed to the left</td>
<td>Low</td>
<td>4.50</td>
<td>1.50</td>
</tr>
<tr>
<td>8</td>
<td>Symmetric</td>
<td>Low</td>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>9</td>
<td>Skewed to the right</td>
<td>Low</td>
<td>1.50</td>
<td>4.50</td>
</tr>
</tbody>
</table>

5.2.3 Determination of Distribution Controlling Parameters

For state-maintained highway networks, historical data on projects outcomes in terms of relevant performance measures are generally available. Such data can be processed to obtain the values of performance measures for the risk-based estimation of overall project impacts or benefits.

5.2.4 Sensitivity of Overall Project Impact to Changes in Individual Performance Outputs

Simulation is essentially a rigorous extension of sensitivity analysis that uses randomly sampled values from the input probability distributions to calculate separate discrete results. Two types of sampling techniques are commonly used. The first type is Monte Carlo sampling that uses random numbers to select values from probability distributions. The second type is Latin Hypercube sampling where the probability scale of the cumulative distribution curve is divided into an equal number of probability ranges. The number of ranges used is equal to the number of iterations performed in the simulation. Because of the stratified sampling nature of Latin Hypercube simulation, it is possible to achieve convergence in fewer numbers of iterations as compared to Monte Carlo simulation (FHWA, 1998; Reigle, 2000).
5.3 Uncertainty Considerations in Asset Management

As a practical matter, the probability distribution for the possible values or even the full range of possible values of a certain performance measure for computing individual project impacts may not be known. In such cases, the mathematical expectation of the performance measure cannot be determined and the expected utility gain as the overall project impacts cannot be estimated correspondingly. This section introduces an approach extended from Shackle’s model to explicitly address cases where those performance measures are under uncertainty with no definable probability distributions.

Shackle’s model overcomes the limitation of inability to compute the mathematical expectation for each performance measure for computing project impacts according to the following procedure. First, it uses degree of surprise as a measure of uncertainty associated with the performance measure for computing project impacts in place of probability distribution. Then, it introduces a priority weight by jointly evaluating each known outcome of a performance measure for computing project impacts and its degree of surprise pair. Finally, it identifies and standardizes the focus gain and focus loss values relative to an expected outcome from maximum priority weights (Ford and Ghose, 1998; Shackle, 1949; Young, 2001).

(a) Degree of Surprise Function

The degree of surprise reflects the decision-maker’s reaction to a certain degree of uncertainty regarding possible outcomes of a performance measure for computing specific impacts resulting from a candidate project, with gains and losses from the expected outcome being considered separately (Figure 5.2). A degree of surprise function for a performance measure for computing a specific item of project impacts can be established using the following steps:

- Assume a range of \( s \) possible outcomes of a performance measure \( X \) for computing a specific item of project benefits from an investment option (\( X = F_1, F_2, \ldots, F_s \) ranging from \( F_{\text{min}} \) to \( F_{\text{max}} \))
- Denote \( F(E) \) as the expected outcome for the performance measure for computing a specific item of project impacts or benefits
- Let the deviation of an outcome of the performance measure \( X \) relative to the expected outcome \( F(E) \) to be \( x, x = X - F(E) \)
- Assign a value to represent the degree of surprise \( y \) ranging from 0 (no surprise) to 10 (very surprised), to reflect the decision-maker’s degree of belief for a given outcome \( X \) as captured by the deviation \( x \),

- Establish a degree of surprise function \( y = f(x) \).
respect to the deviation of the performance measure from the expected outcome and degree of surprise.

- Priority function $\phi$ is a saddle shaped curve that maintains a maximum priority weight on the gain side from expected outcome and a maximum priority weight on the loss side from expected outcome. The deviations of the two outcomes corresponding to the two maximum priority weights are called focus gain ($x_{FG}$) and focus loss ($x_{FL}$) values.

![Figure 5.3: Illustration of a Typical Priority Function](image)

Note: $F_{min}, F_{max}$ are the minimum and maximum values of a performance measure for computing a specific item of project impacts or benefits; $F_L, F_U$ are the lower and upper extreme values of the performance measure with no degree of surprise; $F_E$ is the expected outcome of the performance measure; $x$ is the deviation of a possible outcome $X$ from $F_E$; $x = X - F_E$; $x_{FG}, x_{FL}$ are the focus gain and focus loss.

(c) Standardized Focus Gain and Loss Values

The focus gain and loss values involve uncertainty because they have non-zero degrees of surprise. It is therefore necessary to filter out such uncertainty to establish the standardized focus gain and loss values with zero degree of surprise. The standardization process can be accomplished by using the priority indifference curves at both the gain and loss side from the expected outcome that retain the maximum priority weights consistent with those of the focus gain and focus loss values.
Figure 5.4: Illustration of the Standardized Focus Gain and Loss Values

Notations:

\[ x = \text{Deviation of a possible outcome of a performance measure } X \text{ from the expected outcome } F(E) \]

\[ y(x) = \text{Degree of surprise function, set } y(x) = c \cdot x^2 \]

\[ \phi_1(x, y) = \text{Priority indifference curve, set } \phi_1(x, y) = \alpha_1 x^{0.5} - \beta_1 y^2 = k (k \geq 0) \]

\[ \phi_2(x, y) = \text{Maximum priority indifference curve on the gain side, } \phi_2(x, y) = \alpha_2 x^{0.5} - \beta_2 y^2 = \phi_{\text{max}(G)} \]

\[ x_{SG} = \text{Standardized gain value on indifference curve } \phi_1(x, y) \text{ with no surprise} \]

\[ x_{FG} = \text{Focus gain value on maximum priority indifference curve } \phi_2(x, y) \]

\[ x_{SFG} = \text{Standardized focus gain value on maximum priority indifference curve } \phi_2(x, y) \text{ with no surprise} \]

\[ A, B, C \text{ are points on } \phi_1(x, y), \text{ and } O \text{ is a point on } \phi_2(x, y). \]

The purpose is to find the standardized focus gain \( x_{SFG} \) from the underlying focus gain \( x_{FG} \) on the maximum priority indifference curve \( \phi_2(x, y) \). As \( \phi_2(x, y) \) only intersects with the degree of surprise function \( y(x) \) at point \( O \), it would be impractical to further progress the
standardization process. This is because it is impossible to simultaneously calibrate two
parameters \( \alpha_2 \) and \( \beta_2 \) for \( \phi_2(x, y) \) based solely on one point on the curve. To overcome this
restriction, the indifference curve \( \phi_1(x, y) \) closest to \( \phi_2(x, y) \) that intersects with the degree of
surprise function \( y(x) \) twice at points \( A \) and \( B \) can be utilized. As shown in Figure 5.4, when the
priority indifference curve \( \phi_1(x, y) \) approaching \( \phi_2(x, y) \) (i.e., \( \phi_1(x, y) = k \rightarrow \phi_{\text{max}(G)} \)), the
standardized gain value \( x_{SG} \) for \( \phi_1(x, y) \) will overlap with the standardized focus gain \( x_{SFG} \). Hence,
the process reduces to establishing a mathematical expression for the standardized gain value \( x_{SG} \).

For points \( A \) and \( B \) on priority indifference curve \( \phi_1(x, y) \),

\[
\alpha_1 x_A^{0.5} - \beta_1 y_A^2 = k \tag{5.1}
\]

\[
\alpha_1 x_B^{0.5} - \beta_1 y_B^2 = k \tag{5.2}
\]

Substituting \( y_A = c \cdot x_A^2 \) and \( y_B = c \cdot x_B^2 \) into Equations (5-1) and (5-2) yields

\[
\alpha_1 = \frac{k(x_B^4 - x_A^4)}{(x_B^4 - x_A^4)^{0.5} - x_A^{0.5} \cdot x_A^{0.5}} \tag{5.3}
\]

For point \( C(x_{SG}, 0) \) on \( \phi_1(x, y) \), we get \( \phi_1(x, y) = \alpha_1 x_{SG}^{0.5} - \beta_1 0^2 = \alpha_1 x_{SG}^{0.5} \)

Thus,

\[
x_{SG} = \left[ \frac{\phi_{\text{max}(G)}}{\alpha_1} \right]^{\frac{1}{2}}
\]

and \( x_{SFG} \approx \left[ \frac{\phi_{\text{max}(G)} \cdot (x_B^4 - x_A^4) - x_B^{0.5} \cdot x_A^{0.5}}{k(x_B^4 - x_A^4)^{0.5}} \right]^2 \)

Following this procedure, the standardized focus gain and loss values for a performance
measure for computing a specific project benefit item, \( x_{SFG} \) and \( x_{SFL} \), corresponding to the
maximum priority indices, \( \phi_{\text{max}(G)} \) and \( \phi_{\text{max}(L)} \), on the gain side and loss side from the expected
outcome can be determined (Li and Sinha, 2004).

(d) Extension of Shackle’s Model for Project Benefit Estimation under Uncertainty

Shackle’s model first assigns degrees of surprise to possible outcomes of a performance measure
for computing a specific item of project benefits (in utility value) that deviate from the expected
outcome. It then designates a priority weight for the deviation of each outcome from the expected
outcome and its degree of surprise pair. The deviations of two outcomes (which separately
maintaining the highest priority weights on the gain side and loss side from the expected
outcome) are identified and denoted as focus gain and focus loss values. Finally, the focus gain and loss values are standardized to remove associated uncertainty.

The process of identifying focus gain and loss values and further standardizing those values facilitates complete filtration of uncertainty associated with a performance measure for computing a specific item of project benefits. In the original Shackle’s model, the ratio of standardized focus gain over focus loss is utilized to assess the project merits. The theory behind this is that a project is more preferred if it preserves a higher focus gain-over-loss ratio. For highway project evaluation that compares various projects using utility value benefits, it is desirable to simultaneously consider the expected outcome with the focus gain and focus loss values regarding the performance measure. With this in mind, an extension of Shackle’s model can be introduced using the following notations:

\[ F_{(E)} = \text{Expected outcome of a performance measure for computing a specific item of project outcome} \]

\[ x_{SFG} = \text{Standardized focus gain from the expected outcome} \]

\[ x_{SFL} = \text{Standardized focus loss from the expected outcome} \]

\[ F_{SFG} = \text{Outcome of a performance measure with standardized focus gain, } F_{SFG} = F_{(E)} + x_{SFG} \]

\[ F_{SFL} = \text{Outcome of a performance measure with standardized focus loss, } F_{SFL} = F_{(E)} - x_{SFL} \]

\[ F = \text{A single value determined according to a decision rule for a performance measure under uncertainty} \]

Given a triple \(< F_{SFL}, F_{(E)}, F_{SFG} >\) concerning a performance measure for computing a specific item of project impacts or benefits, a decision rule can be set in order to determine a single value that will be eventually used for project benefit computation. Assuming that the decision-maker only tolerates loss of the value from the expected outcome for a performance measure for computing a specific item of project impacts or benefits by \( \Delta X \), the decision is set below:

\[
F = \begin{cases} 
 F_{(E)}, & \text{if } |F_{SFL} - F_{(E)}| \leq \Delta X \\
 F_{SFL} \left[ 1 - \Delta X / F_{(E)} \right], & \text{otherwise} 
\end{cases}
\]

(5.4)

In some cases, lower values for a performance measure such as the IRI or crash rate, are preferred. In such cases, the decision rule becomes:
\[ F = \begin{cases} 
F_{(E)}, & \text{if } |F_{\text{SFL}} - F_{(E)}| \leq \Delta X \\
F_{\text{SFL}} \frac{1}{1 + \Delta X / F_{(E)}}, & \text{otherwise} 
\end{cases} \] (5.5)

If the deviation of focus loss \( F_{\text{SFL}} \) from the expected outcome \( F_{(E)} \) does not exceed \( \Delta X \), the expected outcome will be assigned. This will yield identical decision outcome between uncertainty-based analysis and risk-based analysis thus maintaining methodological consistency. Different tolerance levels \( \Delta X \) may be used for different performance measures under uncertainty.

**Numerical Example for the Uncertainty Scenario**

For a certain congestion mitigation project, it has been established that while there are some impacts in terms of average travel speed. The exact value of this average travel speed after the project implementation is not known. Also, there are no historical data to calibrate the distribution of past similar projects.

Shackle’s model can be used to deal with this uncertainty situation. Through a survey of experts, the surprise and priority functions can be calibrated as illustrated in the Table 5.2. Then, based on these functions, the standard focus gain and loss values can be calculated as shown in the table.

<table>
<thead>
<tr>
<th>Range of Deviation</th>
<th>Data Set</th>
<th>Surprise Functions ( y(x) = c\cdot(x-55)^2 )</th>
<th>Priority Functions ( \varphi(x) = a\cdot(x-55)^{0.5} + b\cdot y^2 )</th>
<th>Standardized Focus Gain and Loss Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Coefficient ( c )</td>
<td>Coefficient ( a )</td>
<td>Coefficient ( b )</td>
</tr>
<tr>
<td>13%</td>
<td>All Loss</td>
<td>0.2059</td>
<td>0.7363</td>
<td>-0.0112</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2301</td>
<td>0.9519</td>
<td>-0.0167</td>
</tr>
<tr>
<td>25%</td>
<td>All Loss</td>
<td>0.0515</td>
<td>0.5207</td>
<td>-0.0112</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0575</td>
<td>0.6731</td>
<td>-0.0167</td>
</tr>
<tr>
<td>38%</td>
<td>All Loss</td>
<td>0.0229</td>
<td>0.4251</td>
<td>-0.0112</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0256</td>
<td>0.5496</td>
<td>-0.0167</td>
</tr>
<tr>
<td>50%</td>
<td>Gain Loss</td>
<td>0.0114</td>
<td>0.4332</td>
<td>-0.0266</td>
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<tr>
<td></td>
<td></td>
<td>0.0144</td>
<td>0.4759</td>
<td>-0.0167</td>
</tr>
<tr>
<td>63%</td>
<td>Gain Loss</td>
<td>0.0073</td>
<td>0.3875</td>
<td>-0.0266</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0092</td>
<td>0.4257</td>
<td>-0.0167</td>
</tr>
<tr>
<td>75%</td>
<td>Gain Loss</td>
<td>0.0050</td>
<td>0.3537</td>
<td>-0.0266</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0064</td>
<td>0.3886</td>
<td>-0.0167</td>
</tr>
<tr>
<td>88%</td>
<td>Gain Loss</td>
<td>0.0037</td>
<td>0.3275</td>
<td>-0.0266</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0047</td>
<td>0.3598</td>
<td>-0.0167</td>
</tr>
<tr>
<td>100%</td>
<td>Gain All</td>
<td>0.0028</td>
<td>0.3063</td>
<td>-0.0266</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0032</td>
<td>0.2603</td>
<td>-0.0112</td>
</tr>
<tr>
<td>113%</td>
<td>Gain All</td>
<td>0.0022</td>
<td>0.2888</td>
<td>-0.0266</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0027</td>
<td>0.2379</td>
<td>-0.0097</td>
</tr>
<tr>
<td>125%</td>
<td>Gain All</td>
<td>0.0018</td>
<td>0.2740</td>
<td>-0.0266</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0023</td>
<td>0.2194</td>
<td>-0.0084</td>
</tr>
</tbody>
</table>
For instance, if the decision-maker expects that the travel speed after the project implementation is 40 miles/hour, and the tolerance of deviation is $\Delta X$, then the value of performance measure is

$$F = \begin{cases} 
40 & |X_{SFL}| \leq \Delta X \\
F_{SFL} \over (1 - \Delta X / 30) & \text{otherwise}
\end{cases}$$

Where $F_{SFL} = 40 - X_{SFL}$ and $X_{SFL}$ is the value of the Standardized Focus Loss value.

The range of deviation can be the difference between the expected travel speed and the maximum/minimum possible of the outcome of travel speed. If the range of deviation is less than 12.5% and the $\Delta X=5$. Thus, it can be seen that $X_{SFL}$ at 12.5% is 3.11, which is less than $\Delta X=5$.

Thus, the value of F is determined as $F = 40$.

If $\Delta X=2$, then $\Delta X = 2 < X_{SFL}$. Thus:

$$F = \frac{40 - 3.11}{(1 - 2/40)} = 38.83$$

Therefore, 38.83 miles/hour is the raw value of the speed performance measure, and this can be used in subsequent steps of the MCDM process – scaling and amalgamation just as was done for the certainty scenario.

### 5.4 Chapter Summary

In asset management, the overall outcome of project in terms of the performance measures can be under the certainty, risk, or uncertainty scenario. In certainty case, the value of performance measure can be directly used to calculate the impact of project implementation. Under the risk scenario, a probability distribution can be used and this can be further refined using Bayesian updating as more and more data becomes available. From the probability distribution for performance measures, the mathematical expectation of the outcomes can be measured in terms of the performance measure. The mathematical expectation could then be used to compute the expected impact. In the uncertainty scenario, a single value can be determined using a pre-specified decision rule as the extension of Shackle’s model. This value can either be equivalent to the expected outcome or the outcome corresponding to the standardized focus loss with penalty. This single value could be adopted to compute the overall project impact.
CHAPTER 6 TRADE-OFF ANALYSIS

6.1 Introduction

A trade-off can be generally defined as “a barter situation that involves losing a quality or aspect of something in return for gaining a quality or aspect of another”. Trade-off analysis is useful in the process of decision-making in many fields. In field of transportation asset management, relatively little research has been conducted in the area of trade-off analysis.

Amekudzi et al. (2001) addressed the analysis of investment trade-offs for competing infrastructure in the context of uncertainty using Shortfall Analysis to determine minimum levels of investments for heterogeneous facilities and Markowitz Theory to analyze the marginal utilities of investments in competing facilities in the context of data uncertainty. The researchers used the National Bridge Investment Analysis System (NBIAS) and Highway Economic Requirements System (HERS) to illustrate the application of the method.

In its Project 08-36 (2004), the National Cooperative Highway Research Program (NCHRP) developed a multimodal trade-offs methodology for use in statewide transportation planning. That report lists a five-step evaluation process for trade-off analysis: (1) establish structure for inter-program analysis; (2) establish structure for intra-program analysis; (3) identify program areas of interest; (4) apply analysis procedures; (5) present trade-off information. Also, the NCHRP study examined two applications of trade-off analysis: trade-off between investing in ferry service versus road improvements; and trade-off among several alternatives for improving transportation in a corridor.

Li and Sinha (2004) used the utility theory to establish the foundation of trade-off for certainty and risk situation. The researchers used Shackle’s Model to address the uncertainty situation. Based on these methods, they developed a highway asset management framework and a software package to conduct project selection across different program areas for the Indiana Department of Transportation.

Cambridge Systematics (2005) developed two software packages AssetManager NT and AssetManager PT for NCHRP. AssetManager NT analyzes highway investment versus performance across infrastructure categories over 10-to 20-year timeframe. AssetManager PT assesses the impacts of investment choices on a short-term program of projects. Using these packages, it is possible to carry out trade-off analysis to some extent. Of the few past studies that carried out highway asset management trade-off analysis, most focused on only one or two types
of trade-offs. In the present study, a wider number of possible trade-offs types at INDOT are identified and examples are provided. These include trade-offs that involve performance measures, performance measure thresholds, and budgets.

6.2 Types of Trade-off Analysis for Asset Management at INDOT

The different types of trade-off analysis considered in this study are listed in Table 6.1.

Table 6.1: Trade-off Analysis Types

<table>
<thead>
<tr>
<th>Trade-off Analysis Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Change total budget levels of the overall asset program and determine the influence on the AVERAGE (or CHANGE) values of performance measures</td>
</tr>
<tr>
<td>2</td>
<td>Set the threshold of each performance measure and determine the minimum required funding</td>
</tr>
<tr>
<td>3</td>
<td>Change the threshold on one performance measure and determine the influence on other performance measures (total budget is fixed)</td>
</tr>
<tr>
<td>4</td>
<td>Change the threshold of one performance measure and determine the influence on program area budget (total budget is fixed)</td>
</tr>
<tr>
<td>5</td>
<td>Shift budget between program areas and determine the influence on the AVERAGE (or CHANGE) in performance measure (total budget is fixed)</td>
</tr>
</tbody>
</table>

Trade-off Analysis Type 1: Change total budget levels and find out the influence on the AVERAGE (or CHANGE) values of performance measures

The Asset Manager at INDOT may seek the trade-offs between the total asset budget and levels of the performance measures. That is, if the asset budget is increased or decreased: (a) by how much will each performance change? (b) what will be the resulting final levels of each performance measure? To answer this question, the following procedure (Figure 6.1) is used.

![Figure 6.1: Flowchart for Trade-off Analysis Type 1](image-url)
From the flowchart in Figure 6.1, it can be seen that this type of trade-off analysis is relatively straightforward. The AM first sets the total budget constraint, conducts optimization, and then calculates the average (change) value of performance measures based on the optimization result. Then the budget is increased or decreased and the process is repeated.

Based on the data (see the spreadsheets in the CD that accompany this report), an example has been developed to demonstrate the procedure for this type of trade-off analysis. In Table 6.2, the average performance measure values yielded by the different total budget levels are listed. Figure 6.2 illustrates the results of this trade-off analysis.

Table 6.2: Total Budget Levels and Average Performance Measure Values

<table>
<thead>
<tr>
<th>Budget Levels ($M)</th>
<th>Average IRI (Inch/mile)</th>
<th>Average Speed (Mile/hour)</th>
<th>Average CR (Crashes/100 million VMT)</th>
<th>Average SR (Rating)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>151.8</td>
<td>39.9</td>
<td>13.1</td>
<td>2.8</td>
</tr>
<tr>
<td>60</td>
<td>92.1</td>
<td>43.2</td>
<td>12.1</td>
<td>5.8</td>
</tr>
<tr>
<td>100</td>
<td>67.8</td>
<td>45.6</td>
<td>10.3</td>
<td>5.8</td>
</tr>
<tr>
<td>120</td>
<td>67.8</td>
<td>45.7</td>
<td>9.6</td>
<td>7.2</td>
</tr>
<tr>
<td>140</td>
<td>67.8</td>
<td>47.1</td>
<td>8.9</td>
<td>7.2</td>
</tr>
<tr>
<td>190</td>
<td>67.8</td>
<td>48.4</td>
<td>7.4</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Note: IRI—International Roughness Index, SR—Sufficiency Rating, CR—Crash Rate

Figure 6.2: Trade-off Analysis between Total Budget and Final Values of Performance Measures
From the above table and figure, it can be seen that when the total budget increases, all the performance measures improve considerably. For example, average IRI becomes lower, average speed increases, crash rate decreases, and bridge sufficiency rating increases.

This example utilizes the average final (or post-implementation) values of performance measures. However, a high average “final” value could be only because highway network already has very good performance before project implementation. As such, it may be more meaningful, from a practical standpoint, to measure the change in the performance measures and not their final values. Table 6.3 and Figure 6.3 present the trade-offs in terms of changes in values of performance measures.

Table 6.3: Total Budget Levels and Corresponding Changes in Performance

<table>
<thead>
<tr>
<th>Budget Levels ($M)</th>
<th>Average ΔIRI (Inch/mile)</th>
<th>Average ΔSpeed (Mile/hour)</th>
<th>Average ΔCR (Crashes/100 million VMT)</th>
<th>Average ΔSR (Rating)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>60</td>
<td>59.7</td>
<td>4.3</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>100</td>
<td>84.0</td>
<td>6.6</td>
<td>2.8</td>
<td>3.0</td>
</tr>
<tr>
<td>120</td>
<td>84.0</td>
<td>6.8</td>
<td>3.5</td>
<td>4.4</td>
</tr>
<tr>
<td>140</td>
<td>84.0</td>
<td>8.2</td>
<td>4.2</td>
<td>4.4</td>
</tr>
<tr>
<td>190</td>
<td>84.0</td>
<td>9.5</td>
<td>5.7</td>
<td>5.2</td>
</tr>
</tbody>
</table>

Note: IRI—International Roughness Index, SR—Sufficiency Rating, CR—Crash Rate

Figure 6.3: Trade-off Analysis: Asset Budget vs. Change in Performance
From Figure 6.3, it can be seen that at lower budgetary levels, incremental changes in budget yields significant changes in IRI, but after $100M, the change in IRI is very small – this is suggestive of the scale economies: beyond some funding limit, there is little marginal change in the performance measure and may not be worth spending beyond that amount.

**Trade-off Analysis 2:** Trade-off between threshold of each performance measure and the minimum required funding to achieve that threshold

In the Asset Manager’s decision setting, there often exist minimum requirements for performance or levels of service often to accommodate the perspectives of stakeholders within or outside INDOT. For instance, it could be desired that the network crash rate should be less than a certain specified value or the average travel speed in the network should be higher than a certain value. In these types of trade-off, the Asset Manager seeks an answer to the following question: “How much money does INDOT need to achieve a certain minimum standard network performance?” The following steps can be used to investigate this type of trade-off.

![Flowchart for Trade-off Analysis Type 2](image)

Using the data in the spreadsheet, type 2 trade-off analysis was carried out. Table 6.4 and Figure 6.5 present the results. It can be seen that as higher standards of performance are specified (that is, the more aggressive the thresholds), the minimum funding required to achieve these
thresholds, increases. While this is consistent with intuition, what is not known (and what this analysis provides) is the exact pattern of the mathematical relationship that represents this trade-off. This relationship can be used by INDOT to predict the funding consequences of tightening or loosening of the performance thresholds.

Table 6.4: Minimum Level of Total Budget to Achieve Specified Performance Thresholds

<table>
<thead>
<tr>
<th></th>
<th>Maximum Average IRI</th>
<th>Minimum Average Speed</th>
<th>Maximum Average CR</th>
<th>Minimum Average SR</th>
<th>Minimum Required Funding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thresholds</td>
<td>130.00</td>
<td>42.00</td>
<td>13.00</td>
<td>4.00</td>
<td>19.50</td>
</tr>
<tr>
<td>Scenario 1</td>
<td>120.00</td>
<td>45.00</td>
<td>12.00</td>
<td>4.50</td>
<td>64.06</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>100.00</td>
<td>45.00</td>
<td>11.50</td>
<td>4.00</td>
<td>65.91</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>80.00</td>
<td>45.00</td>
<td>9.00</td>
<td>5.00</td>
<td>90.54</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>80.00</td>
<td>40.00</td>
<td>8.60</td>
<td>6.00</td>
<td>105.10</td>
</tr>
<tr>
<td>Scenario 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.5: Trade-off Analysis between Total Budget and Performance Thresholds
Trade-off Analysis 3: *Change the threshold (or standard of performance) assigned to a “base” performance measure and investigate the influence on other performance measures (the total budget remains fixed)*

In some cases, the total budget is limited, and yet the Asset Manager seeks to increase the performance in a certain aspect (e.g., reduce the network crash rate). Recognizing that this may have adverse consequences on the other measures of performance, the Asset Manager will seek to investigate the impact of this situation on the other performance measures such as average travel speed and IRI. Figure 6.6 presents the flowchart for analyzing such trade-offs.

![Flowchart for Trade-off Analysis Type 3](image)

Figure 6.6: Flowchart for Trade-off Analysis Type 3

Figure 6.7 presents an example of the results for this type of trade-off, using data in the spreadsheet provided in the project CD. The figure suggests that the network pavement performance (average IRI) exhibits the most sensitive trade-off relationship with network safety performance (average crash rate): when the AM’s threshold for crash rate is more aggressive, the pavement performance decreases (average IRI increases) considerably. In other words, as the safety standards are tightened by the AM, the pavement condition suffers, obviously because the optimization program diverts more funds to the safety program area in order to satisfy the aggressive safety requirement. It is seen that the change in average speed and bridge condition, however, is fairly constant across the different safety performance thresholds.
The above case analyzed the changes on the other performance measures. In practice, the AM may require trade-offs between only two performance measures at a time. For example, how much speed reduction can be traded off for a certain decrease on the crash rate? Figure 6.8 shows an example of trade-off results for travel speed and crash rate, based on data in the spreadsheet.
In this figure, the gradient of the regression line is about 0.7. This **marginal rate of substitution** suggests that 10-mph speed reduction can “buy” a decrease of 7 crashes/100MVMT. In this example, the regression line is almost linear, but in fact, it could take any shape. If it is non-linear, it will be possible to obtain different marginal rates of substitution for different levels of a given performance measure by carrying out point differentiation at that point.

**Trade-off Analysis 4:** *Change the threshold of the “base” performance measure and find out the influence on program area budget (total budget is fixed)*

This type of trade-off is somewhat the converse of that described for Trade-off type 3. Here, the AM seeks the distribution of the asset program budget in each program area in order to achieve some specified threshold level of a specified “base” performance measure.

![Flowchart for Trade-off Analysis Type 4](image)

Figure 6.9: Flowchart for Trade-off Analysis Type 4

Based on the data in the CD, an example is herein developed to show how this trade-off analysis works. Table 6.5 and Figure 6.10 show the changes the minimum amounts (budgets) needed for each program area corresponding to different safety thresholds in terms of crash rate.
Table 6.5: Program Area Budget Distribution ($M) vs. Threshold of Base PM

<table>
<thead>
<tr>
<th>Average CR Level (Crashes/100 million VMT)</th>
<th>Pavement Budget</th>
<th>Congestion Budget</th>
<th>Bridge Budget</th>
<th>Safety Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.78</td>
<td>32.40</td>
<td>1.99</td>
<td>29.30</td>
<td>25.68</td>
</tr>
<tr>
<td>9.91</td>
<td>21.00</td>
<td>18.08</td>
<td>17.70</td>
<td>32.44</td>
</tr>
<tr>
<td>10.27</td>
<td>32.40</td>
<td>11.09</td>
<td>29.30</td>
<td>16.55</td>
</tr>
<tr>
<td>10.91</td>
<td>44.80</td>
<td>8.98</td>
<td>17.70</td>
<td>17.75</td>
</tr>
<tr>
<td>11.41</td>
<td>44.80</td>
<td>18.08</td>
<td>17.70</td>
<td>8.63</td>
</tr>
<tr>
<td>Sum</td>
<td>89.365</td>
<td>89.2175</td>
<td>89.34</td>
<td>89.23</td>
</tr>
</tbody>
</table>

Figure 6.10: Trade-off Analysis between Threshold of a “Base” Performance Measure and Program Area Budgets

From the figure, it can be seen that when the crash rate threshold becomes more relaxed (a loosened standard), the safety budget decreases. Also (not surprisingly), the pavement budget increases. An interesting observation too is that when the threshold of crash rate increases, other program area budgets do not increase monotonically. This is because the projects in each program area have influences on other performance measures besides their respective primary
performance measures. For example, a pavement projects not only have impacts of pavement preservation but also could have safety impact such as reduced crash rate and congestion impacts such as increased travel speed. The consideration of multiple performance impacts for each program area therefore adds some interesting complexity to the trade-off analysis, and this issue could be investigated further when real data become available.

**Trade-off Analysis Type 5:** *Shifting funds across budgets of different program areas and investigating the result on CHANGE (or FINAL values) of performance (total budget is fixed)*

One of the most important kinds of trade-off analysis is “budget shifting” analysis. The Asset Manager’s candidate projects are typically generated and “housed” or sponsored in select program areas, such as pavement program, bridge program, etc. As always, there can be severe competition between different program areas for the limited funding. Changes in agency policy and mission or the desire to address public concerns in a particular program area, sudden disaster, and other circumstances can lead to shifts of substantial funds from one program area to another. To address trade-off problems of this nature, the procedure illustrated in the flowchart (Figure 6.11) can be used.

Figure 6.11: Flowchart for Trade-off Analysis Type 5

An example based on the data in the CD is shown in Table 6.6 and Figure 6.12.
In this example, the sum of safety budget and congestion budget is 70 million dollars. The budget can shift between safety budget and congestion budget, but the sum is fixed (70 million dollars). In Table 6.6 and Figure 6.12, it can be seen that values of the performance measures change in response to the different budget allocations in the different program areas. In Figure 6.12, a function could be derived to describe the marginal rate of substitution. In this example, the changes in the values of performance measures are used.

In certain cases, the Asset Manager may be more interested in the final (or post-implementation) values of performance instead of the changes therein. These values can be easily
calculated by adding the changes to the original values of performance measures. In this example, the results are in Table 6.7 and Figure 6.13.

Table 6.7: Trade-off Type 5 – Program Area Budget Shifting Analysis II

<table>
<thead>
<tr>
<th>Safety Budget ($M)</th>
<th>25</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Congestion Budget ($M)</td>
<td>45</td>
<td>35</td>
<td>30</td>
<td>25</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average speed (mile/hour)</th>
<th>49.2869</th>
<th>47.8599</th>
<th>48.1141</th>
<th>46.6871</th>
<th>45.0119</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average crash rate (Crashes/100 million VMT)</td>
<td>9.7797</td>
<td>8.8458</td>
<td>8.9418</td>
<td>7.9538</td>
<td>7.6461</td>
</tr>
</tbody>
</table>

\[ y = 0.0882x^2 - 7.7999x + 179.95 \]

Figure 6.13: Trade-off Type 5 – Program Area Budgets Shifting Analysis II
**Trade-off Analysis 6: Trade-off Analysis between Expected Performance Level (or Impact) and Variability in Performance (Risk)**

In some cases, the AM seeks to investigate the trade-offs between the expected value of the performance impact and the risk (variability). Ideally, the AM seeks to maximize the beneficial impacts (of benefit performance measures) but minimize the variability of such benefits. Often, however, higher performance comes at a price: increased variability or uncertainty. Using Monte Carlo simulation of the Excel Spreadsheets for various projects, it is possible to construct tables and graphs that show the trade-off relationship between performance impact and impact variability.

The case of variability has interesting applications also in project selection, and we herein present an example problem in this area. This example, which utilizes the Markowitz model, also sheds light on how a parameter could be derived from the project selection analysis, to analyze trade-offs.

**The Markowitz Model**

In practice, some projects (such as two project alternatives that are adjacent to each other in the same corridor) are not completely independent and may have some interrelationships. When one uses the project impact to conduct optimization instead of just using the amalgamated value, there may be some bias because of the correlation between the projects. For example, in calculating the benefits of a project implementation, the Average Daily Traffic (ADT) is usually a common parameter. However, ADT is not always known with certainty and follows some probability distributions. Thus, the impact of a project implementation also follows some probability distribution and has some risk associated with its expected benefit. Typically, projects that have high impact also have high risk. So there is a trade-off analysis between the project impacts (or “benefit”) and the uncertainty associated with the impact (or “risk”). In this section, the term “benefit” is used to represent project impacts or desirability. The following Markowitz Mean-Variance Model (Markowitz, 1987) can be used to address this problem.

\[
\text{Maximize } (1 - w) \sum_{i=1}^{n} x_i E(r_i) - w \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \text{cov}(r_i, r_j)
\]

Subject to \( \sum_{i=1}^{n} x_i \leq 100\% \), \( x_i \geq 0 \), \( i = 1, 2, \cdots, n \)

(1) \( n \) represents the number of candidate projects;
(2) $r_i$ denotes the benefit rate of project $i$. It is a random variable, and its expected value is $E(r_i)$. For the project $i$, if the benefit is $b_i$, and the cost is $c_i$, then the benefit rate $r_i$ can be calculated as $(b_i - c_i)/c_i$;

(3) $x_i$ denotes the proportion of the total budget that is invested in project $i$. For a project, the total cost is $c_i$ and the total budget $TB$ are always known, then $x_i$ can be easily determined by $x_i = c_i/TB$;

(4) $E(r_i)$ is the expected value of benefit of candidate project $i$;

(5) $\text{cov}(r_i, r_j)$ is the covariance of $r_i$ and $r_j$. In particular, when $i = j$, $\text{cov}(r_i, r_j) = \text{Var}(i)$

(6) $\sum_{i=1}^{n} x_i \leq 100\%$ is the budget constraint;

(7) $w$ is the weight of risk that lies between 0 and 1. A larger weight $w$ implies that the decision-makers are more concerned about the risk; while a smaller weight $w$ implies that the decision-maker is more concerned about the expected benefit.

If $w = 0$, then the decision-maker is not concerned about the risk and only pursues alternative (candidate project) with the largest benefit.

If $w = 1$, then the decision-maker is very concerned about the risk, and wants only to choose the alternatives with the lowest risk.

In this model, therefore, it is clear that the weight $w$ is the key factor that drives the conclusion of the trade-off analysis because it directly reflects the trade-off between risk and benefit.

Example

A highway between city A and city B has two bridges which “divide” the highway into three segments. Thus there are five assets on this highway (two bridges and three pavement sections) that are candidates for rehabilitation. Assume the total budget is 3 million dollars and each alternative project has a cost of 1 million dollars. Obviously, not all the 5 projects can be implemented because the budget is limited. Thus a decision has to be made on which project to undertake.

\[
\begin{array}{ccccccc}
A & 1 & 2 & 3 & 4 & 5 & B \\
\end{array}
\]

In order to simplify the benefit function, assume the benefit functions of the five alternatives only contain one common variable $X$, the annual average daily traffic volume on this highway. The benefit rate function of each alternative is
\[ B_i = \frac{U_i + a_i X}{C_i} \quad i = 1,2,3,4,5 \]

Where \( c_i = 1 \) million dollars represents the cost of each alternative \( i \)

\( X \) represents annual average daily traffic volume (thousands vehicle per day) on this highway during the service life and is a random variable.

Assume \( X \) has a homogeneous distribution.

\[
f(X) = \begin{cases} 
1/4 & 8 \leq X \leq 10 \\
0 & X \leq 8 \text{ or } X \geq 12 
\end{cases}
\]

\( U_i \) and \( a_i \) are constants whose values are presented in the table below.

<table>
<thead>
<tr>
<th>Project</th>
<th>( U_i )</th>
<th>( a_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.02</td>
<td>0.018</td>
</tr>
<tr>
<td>2</td>
<td>0.018</td>
<td>0.012</td>
</tr>
<tr>
<td>3</td>
<td>0.024</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>0.015</td>
<td>0.008</td>
</tr>
<tr>
<td>5</td>
<td>0.026</td>
<td>0.02</td>
</tr>
</tbody>
</table>

From the data provided above, it can be seen that \( B_i \) is a variable because \( X \) is a random variable.

The following can be determined:

\[
E(B_1) = 0.2 \quad E(B_2) = 0.138 \quad E(B_3) = 0.224 \quad E(B_4) = 0.095 \quad E(B_5) = 0.226
\]

And the resulting variance matrix is:

\[
\delta^2 = \begin{pmatrix}
0.000432 & 0.000288 & 0.000480 & 0.000192 & 0.000480 \\
0.000192 & 0.000320 & 0.000128 & 0.000320 & 0.000533 \\
0.000533 & 0.000213 & 0.000533 & 0.000213 & 0.000533 \\
0.000085 & 0.0000213 & 0.000085 & 0.0000213 & 0.0000533 \\
0.0000533 & 0.0000213 & 0.0000533 & 0.0000213 & 0.000085
\end{pmatrix}
\]

Overall, the five alternative projects can have \( 5 \times 5 - 1 = 24 \) selection sets. However, from the budget and cost of the projects, it is known that there can be three alternatives for implementation. So only the selection sets with three elements are considered. There are 10 selection sets. The expected total benefit rate and the variance of the total benefit rate can be calculated. These values are listed in the following table.
Expected benefit and risk for each project bundle

<table>
<thead>
<tr>
<th>Selection set (Project bundle)</th>
<th>Selected projects</th>
<th>E(B)</th>
<th>Var(B)</th>
<th>$\delta(B) = \sqrt{\text{Var}(B)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Project 1,2,3</td>
<td>0.1873</td>
<td>0.000499</td>
<td>0.0223</td>
</tr>
<tr>
<td>2</td>
<td>Project 1,2,4</td>
<td>0.1443</td>
<td>0.000293</td>
<td>0.0171</td>
</tr>
<tr>
<td>3</td>
<td>Project 1,2,5</td>
<td>0.1880</td>
<td>0.000499</td>
<td>0.0223</td>
</tr>
<tr>
<td>4</td>
<td>Project 1,3,4</td>
<td>0.1730</td>
<td>0.000430</td>
<td>0.0207</td>
</tr>
<tr>
<td>5</td>
<td>Project 1,3,5</td>
<td>0.2167</td>
<td>0.000665</td>
<td>0.0258</td>
</tr>
<tr>
<td>6</td>
<td>Project 1,4,5</td>
<td>0.1737</td>
<td>0.000430</td>
<td>0.0207</td>
</tr>
<tr>
<td>7</td>
<td>Project 2,3,4</td>
<td>0.1523</td>
<td>0.000327</td>
<td>0.0181</td>
</tr>
<tr>
<td>8</td>
<td>Project 2,3,5</td>
<td>0.1960</td>
<td>0.000540</td>
<td>0.0232</td>
</tr>
<tr>
<td>9</td>
<td>Project 2,4,5</td>
<td>0.1530</td>
<td>0.000327</td>
<td>0.0181</td>
</tr>
<tr>
<td>10</td>
<td>Project 3,4,5</td>
<td>0.1817</td>
<td>0.000469</td>
<td>0.0217</td>
</tr>
</tbody>
</table>

Assuming that the decision-maker is concerned about benefit and risk equally, that means $w = 0.5$, then the objective function is:

$$\text{Max} \quad 0.5\left(\sum_{i=1}^{n} x_i E(r_i) - \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \text{cov}(r_i, r_j)\right)$$

From the above table, the selection set 5 (Project 1, 3, 5), on the basis of the objective function, is found to be the best.

In practice, trade-off analysis can be carried out by specifying different $w$ values and examining the impact on the final selection.

6.3 Chapter Summary

The basic idea of trade-off involves losing one quality or aspect of something in return for gaining another quality or aspect. There are many types of trade-off that can be encountered in asset management practice, for instance, trade-off between performance measures, between program area budgets, and between the levels of performance and their uncertainty. In this chapter, at least five types of trade-off are presented. For each type, a methodology and numerical example based on the data in the project spreadsheet are presented in this chapter. The results show that the trade-off analyses can offer interesting and useful insights for the Asset Manager.
CHAPTER 7 SUMMARY AND CONCLUSION

7.1 Research Products

The need for this research project arose from current and ongoing trends in the transportation environment. The current environment is characterized by funding limitations and uncertainties, increased stakeholder participation, and the need of increased accountability and transparency. As such, INDOT seeks to further enhance its existing evaluation processes for decision-making in the highway sector. Consistent with such evaluation processes is the incorporation of multiple performance criteria from different program or functional areas, optimization of decisions under constrained budgets, and investigation of performance and budgetary trade-offs.

At the current time, INDOT lacks explicit tools to help decision-makers evaluate decisions on the basis of multiple performance measures across asset classes and also to conduct trade-off analysis in asset management. In order to address these issues, INDOT requires a portfolio of methodologies that can help its Asset Manager(s) compare, through scaling and amalgamation, projects from different program areas that have different sets of performance impacts and to conduct various types of tradeoff analysis.

This report describes a number of scaling methods that could be used in asset management decision analysis in a multiple-criteria context. Of these methods, linear scaling is the easiest and can be used when there is little or no information about the performance measure but may not reflect the true distribution of the decision-makers’ preferences towards the various levels of the performance measure. Scaling based on the probability distribution or cumulative probability function can be somewhat effective when historical are available to calibrate the distribution of performance measures, but this method may suffer from inherent relativity bias. Utility/value function scaling, which captures the preference structure of experts and decision-makers, was found to be widely used and is considered the best theoretical method to conduct scaling of different performance measures. Its limitation of subjectivity can be mitigated using a number of techniques such as the Delphi process and regression analysis. Also, unlike most other scaling methods, utility/value functions can be used to scale performance measures of a qualitative nature.

Amalgamation combines the scaled performance measures often to yield a single number that represents the desirability of each candidate project. This report discussed several
amalgamation methods that could be used in asset management decision-making involving multiple performance criteria. Of these methods, the weighted sum method and multiplicative function are widely used, but these methods require that the performance measures must be preference independent or mutually utility independent. Goal programming, deemed the most practical amalgamation method, is consistent with practical situations where the Asset Manager has a pre-specified target for each performance measure and seeks to work towards those goals.

Optimization is the process to choose the best group of projects that can maximize the total performance utility or benefit under given constraints such as budget ceilings or floors, or performance standards. In other words, this process involves the build-up of an asset program which is an optimal “knapsack” of projects from a larger pool of candidate projects provided by the individual program areas. Optimization often serves as a prelude to (and/or component of) trade-off analysis. The asset management optimization problem can be viewed in terms of different classes of Knapsack problems according to different considerations. This study developed mathematical formulations for different conceptual setups for the objective function, budgetary constraints, performance measure constraints, and uncertainty constraints that INDOT’s Asset Manager is likely to encounter in practice.

There is inevitable uncertainty associated with project impacts in terms of the performance outcomes. As such, the final list of selected projects or trade-off patterns could be different from those derived under the certainty scenario. This study provided methodologies that could be used to carry out project selection and trade-offs under circumstances of risk and uncertainty. For the risk scenario where the probability distribution of performance outcomes is known, mathematical expectation can be used to derive performance outputs that subsequently could be scaled, amalgamated, and optimized. For the pure uncertainty scenario, Shackle’s model is recommended for use.

Also, the report describes techniques for carrying out several types of trade-off analysis. These trade-offs involve performance levels, performance thresholds (standards), asset budgets, program area budgets, and uncertainty. After a discussion of each concept in the text, numerical examples are presented to facilitate comprehension and application of the concepts, and future replication of the analyses.

Finally, a set of spreadsheets were developed and submitted in a CD as an addendum to this report to demonstrate how the Asset Manager could carry out the processes of scaling, amalgamation, optimization for project selection, and trade-off analysis in an automated manner. There are four spreadsheets: (1) the certainty scenario and multiple performance measures per
program area, (2) the uncertainty scenario and multiple performance measures per program area, (3) the certainty scenario and only 1 “benefit” performance measure per program area, (4) the uncertainty (risk) scenario and only 1 “benefit” performance measure per program area.

7.2 Future Work and Final Comments

Future research or practice work in asset management at INDOT could involve collection of data on expected or project outcomes and full scale testing of the concepts presented in this report. This would bring into sharper focus the concepts discussed herein and would yield insights that could serve as basis for agency policy formulation and/or monitoring.

Data for applying the asset management processes described in this report relate to the projects and their outcomes. These data come from the individual program areas or management systems. Thus, future researchers in this area could make a clear demarcation between asset management where the concepts (even though they mean well) are developed using data from historical practice (for which there often is a preponderance of data in program-area archives); and asset management where the concepts are developed on the basis of optimal-practice data (which are often derived from simulation rather than real life). There is a school of thought who contend that there is very little to be learned from analysis based on historical data because decisions in the past were likely based on more on funding availability rather than engineering considerations and need. Proponents of the use of historical data do not discount the position of their opponents that optimal practice in the program areas would yield “superior” data and consequently “superior” results from the various asset management processes. However, the proponents argue that optimal practice in the program areas may not be a practical reality in many years to come because there will always be funding restrictions that would preclude such practice. These philosophical issues could be examined in future research and the relationships between these two positions in asset management practice could be explored.

Furthermore, future work could investigate trade-off concepts at (1) the network level using project-level data (as was done in this research study), (2) the network level using network-level data (see Appendix 4), or (3) the project level using project-level data (as is done in most program areas or management systems). Results between the three categories could be compared and contrasted to shed more light on any differences in trade-off results at the various levels.

Pursuant to the second category listed above, Appendix 4 presents implicit, interesting trade-offs between state-level highway investments and highway system performance for all
states in the USA (Noureldin, 2008). This table shows how the state of Indiana is performing relative to other states and to the national average. In the future, research on such statewide trends could investigate these and other network-level trade-offs by duly incorporating other variables such as traffic levels, climatic severity, surficial geology, etc., in furtherance to the research recently carried out by Anastasopoulos et al. (2009).

Finally, the practice of asset management, like the management of its constituent program areas, involves the use of processes and concepts that are intended for purposes of decision support. These concepts are not meant to serve as a religious panacea for all asset investment questions. While it is true that the concepts herein may be needed for balanced and rational decision-making, they alone may not always be sufficient in the evaluation of project consequences, final selection of projects, or analysis of trade-offs. Often, arriving at the final solution will also require sound engineering judgment, candid consideration of project constraints (especially those that are not readily quantifiable), due consideration of a project’s local environment, administrative practices, and culture at the area of its jurisdiction, and a healthy dose of flexibility.
LIST OF REFERENCES AND RESOURCES


Austroads (1997), Strategy for Improving Asset Practice. Sydney, Australia.


APPENDIX 1: Illustrations of the Preference-Based Scaling Processes

Appendix 1.1: Certainty Scenario—Direct Rating Example

*Developing utility function for Bridge Superstructure Condition Rating (NBI Item-59)*

Superstructure condition rating describes the physical condition of all structural members. It uses the scores ranging from 0 (failed) to 9 (excellent condition) in Table A.1 to reflect the superstructure condition.

**Step 1:** Assume that after a project, the Bridge Superstructure Condition Rating (BSCR) is \( X \) with certainty; Define the value function of \( X=0 \) as \( v(X=0)=0 \), and define the value function of \( X=9 \) as \( v(X=9) = 1 \).

**Step 2:** Ask the decision maker the following questions in the table to get the utility function.

<table>
<thead>
<tr>
<th>No.</th>
<th>Question</th>
<th>Answer(^1)</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>If we assign ( v=1 ) to the perfect condition when ( X=9 ), and assign ( v=0 ) to the failed condition when ( X=0 ), to reflect the degree of your satisfaction, what is the number you will assign to the “Imminent” Failure Condition when ( X=1 )?</td>
<td>0.05</td>
<td>( v(X=1)=0.05 )</td>
</tr>
<tr>
<td>2</td>
<td>So what number will you assign to the Critical Condition (when ( X=2 )) to reflect the degree of your satisfaction?</td>
<td>0.1</td>
<td>( v(X=2) = 0.1 )</td>
</tr>
<tr>
<td>3</td>
<td>And what number will you assign to the Serious Condition (when ( X=3 )) to reflect the degree of your satisfaction?</td>
<td>0.3</td>
<td>( v(X=3) = 0.3 )</td>
</tr>
<tr>
<td>4</td>
<td>And what number will you assign to the Poor Condition (when ( X=4 )) to reflect the degree of your satisfaction?</td>
<td>0.4</td>
<td>( v(X=4) = 0.4 )</td>
</tr>
<tr>
<td>5</td>
<td>And what number will you assign to the Fair Condition (when ( X=5 )) to reflect the degree of your satisfaction?</td>
<td>0.65</td>
<td>( v(X=5) = 0.65 )</td>
</tr>
<tr>
<td>6</td>
<td>And what number will you assign to the Satisfactory Condition (when ( X=6 )) to reflect the degree of your satisfaction?</td>
<td>0.8</td>
<td>( v(X=6) = 0.8 )</td>
</tr>
<tr>
<td>7</td>
<td>And what number will you assign to the Good Condition when (( X=7 )) to reflect the degree of your satisfaction?</td>
<td>0.9</td>
<td>( v(X=7) = 0.9 )</td>
</tr>
<tr>
<td>8</td>
<td>And what number will you assign to the Very Good Condition (when ( X=8 )) to reflect the degree of your satisfaction?</td>
<td>0.95</td>
<td>( v(X=8) = 0.95 )</td>
</tr>
</tbody>
</table>

\(^1\) Answer is assumed for purposes of illustration.

Step 3: From the above questions and the answers we can derive the value function for the Bridge Superstructure Condition Rating (BSCR) as follows:
\[ v(X) = \begin{cases} 
0 & \text{when } X = 0; \\
0.05 & \text{when } X = 1; \\
0.1 & \text{when } X = 2; \\
0.3 & \text{when } X = 3; \\
0.4 & \text{when } X = 4; \\
0.65 & \text{when } X = 5; \\
0.8 & \text{when } X = 6; \\
0.9 & \text{when } X = 7; \\
0.95 & \text{when } X = 8; \\
1 & \text{when } X = 9. 
\end{cases} \]

Table A.1: Superstructure Condition Descriptions and Scores

<table>
<thead>
<tr>
<th>Score</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Not Applicable</td>
</tr>
<tr>
<td>9</td>
<td>Excellent Condition</td>
</tr>
<tr>
<td>8</td>
<td>Very Good Condition - no problems noted.</td>
</tr>
<tr>
<td>7</td>
<td>Good Condition - some minor problems.</td>
</tr>
<tr>
<td>6</td>
<td>Satisfactory Condition - structural elements show some minor deterioration.</td>
</tr>
<tr>
<td>5</td>
<td>Fair Condition - all primary structural elements are sound but may have minor section loss, cracking, spalling or scour.</td>
</tr>
<tr>
<td>4</td>
<td>Poor Condition - advanced section loss, deterioration, spalling or scour.</td>
</tr>
<tr>
<td>3</td>
<td>Serious Condition - loss of section, deterioration, spalling or scour have seriously affected primary structural components. Local failures are possible. Fatigue cracks in steel or shear cracks in concrete may be present.</td>
</tr>
<tr>
<td>2</td>
<td>Critical Condition - advanced deterioration of primary structural elements. Fatigue cracks in steel or shear cracks in concrete may be present or scour may have removed substructure support. Unless closely monitored it may be necessary to close the bridge until corrective action is taken.</td>
</tr>
<tr>
<td>1</td>
<td>“Imminent” Failure Condition - major deterioration or section loss present in critical structural components or obvious vertical or horizontal movement affecting structure stability. Bridge is closed to traffic but corrective action may put back in light service.</td>
</tr>
<tr>
<td>0</td>
<td>Failed Condition - out of service and beyond corrective action.</td>
</tr>
</tbody>
</table>
Appendix 1.2: Certainty Scenario—Midvalue Splitting Technique Example

Developing utility function for Average Speed

This example develops a value function for the average speed (X) on a freeway whose speed limit is 80 mph.

Step 1: Define the value function of X=0 as v(X=0)=0, and define the value function of X=80mph as v(X=80)=1.

Step 2: Ask the decision-maker the following questions in the table to derive the utility function.

<table>
<thead>
<tr>
<th>No.</th>
<th>Question</th>
<th>Answer</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>What number will you assign to X so that you would be equally delighted with:</td>
<td>25</td>
<td>v(X=25)=0.5</td>
</tr>
<tr>
<td></td>
<td>(1) An improvement of average speed from X=0 to X;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) An improvement of average speed from X to X=80.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>What number will you assign to X so that you would be equally delighted with:</td>
<td>10</td>
<td>v(X=10)=0.25</td>
</tr>
<tr>
<td></td>
<td>(1) An improvement of average speed from X=0 to X;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) An improvement of average speed from X to X=25.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>What number will you assign to X so that you would be equally delighted with:</td>
<td>50</td>
<td>v(X=50)=0.75</td>
</tr>
<tr>
<td></td>
<td>(1) An improvement of average speed from X=25 to X;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) An improvement of average speed from X to X=80.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>What number will you assign to X so that you would be equally delighted with:</td>
<td>About 25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1) An improvement of average speed from X=10 to X;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) An improvement of average speed from X to X=50.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Answer is assumed for purposes of illustration.

Step 3: Plot the points (x=0, v=0), (x=10, v=0.25), (x=25, v=0.5), (x=50, v=0.75) and (x=80, v=1), and use statistical regression to fit the value function. This yields the value function

\[ V = -0.0001X^2 + 0.0222X + 0.0172 \] as plotted below.
Example 1.3: Risk Scenario—Direct Question Approach for Discrete Performance Measure
Developing utility function for Bridge Superstructure Condition Rating (NBI Item-59)

Superstructure condition rating (NBI Item-59) describes the physical condition of all structural members. It uses the scores (0 to 9) in Table A.2 (NBI Rating scales).

**Step 1:** Assume that after improvement, the Bridge Superstructure Condition Rating (BSCR) is X.

Then theoretically, X has ten possible values (from 0 to 9); Define the utility of X=0 as u(X=0)=0, and define the utility of X=9 as u(X=9)=1.

Step 2: Ask the decision maker the following questions in the table to get the utility function.

<table>
<thead>
<tr>
<th>No.</th>
<th>Question</th>
<th>Answer</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Consider the following 2 situations:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1) The BSCR will be 1 for certain</td>
<td>p₁=0.05</td>
<td>u(X=1)=0.05</td>
</tr>
<tr>
<td></td>
<td>(2) The BSCR will be 9 with a probability of p₁, and will be 0 with a probability of (1- p₁); What number will you assign to p₁ so that the above two situations are indifferent?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Consider the following 2 situations:</td>
<td>p₂=0.15</td>
<td>u(X=2)=0.15</td>
</tr>
<tr>
<td></td>
<td>(1) The BSCR will be 2 for certain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) The BSCR will be 9 with a probability of p₂, and will be 0 with a probability of (1- p₂); What number will you assign to p₂ so that the above two situations are indifferent?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Consider the following 2 situations:</td>
<td>p₃=0.25</td>
<td>u(X=3)=0.25</td>
</tr>
<tr>
<td></td>
<td>(1) The BSCR will be 3 for certain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) The BSCR will be 9 with a probability of p₃, and will be 0 with a probability of (1- p₃); What number will you assign to p₃ so that the above two situations are indifferent?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Consider the following 2 situations:</td>
<td>p₄=0.3</td>
<td>u(X=4)=0.3</td>
</tr>
<tr>
<td></td>
<td>(1) The BSCR will be 4 for certain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) The BSCR will be 9 with a probability of p₄, and will be 0 with a probability of (1- p₄); What number will you assign to p₄ so that the above two situations are indifferent?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Consider the following 2 situations:</td>
<td>p₅=0.7</td>
<td>u(X=5)=0.7</td>
</tr>
<tr>
<td></td>
<td>(1) The BSCR will be 5 for certain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) The BSCR will be 9 with a probability of p₅, and will be 0 with a probability of (1- p₅); What number will you assign to p₅ so that the above two situations are indifferent?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Consider the following 2 situations:</td>
<td>p₆=0.8</td>
<td>u(X=6)=0.8</td>
</tr>
<tr>
<td></td>
<td>(1) The BSCR will be 6 for certain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) The BSCR will be 9 with a probability of p₆, and will be 0 with a probability of (1- p₆); What number will you assign to p₆ so that the above two situations are indifferent?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Consider the following 2 situations:</td>
<td>p₇=0.9</td>
<td>u(X=7)=0.9</td>
</tr>
<tr>
<td></td>
<td>(1) The BSCR will be 7 for certain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) The BSCR will be 9 with a probability of p₇, and will be 0 with a probability of (1- p₇); What number will you assign to p₇ so that the above two situations are indifferent?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Consider the following 2 situations:</td>
<td>p₈=0.95</td>
<td>u(X=8)=0.95</td>
</tr>
<tr>
<td></td>
<td>(1) The BSCR will be 9 for certain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) The BSCR will be 9 with a probability of p₈, and will be 0 with a probability of (1- p₈); What number will you assign to p₈ so that the above two situations are indifferent?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Consider the following 2 situations:</td>
<td>About 0.7</td>
<td>It seems consistent</td>
</tr>
<tr>
<td></td>
<td>(1) The BSCR will be 5 for certain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) The BSCR will be 8 with a probability of p, and will be 2 with a probability of (1- p); What number will you assign to p so that the above two situations are indifferent?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Consider the following 2 situations:</td>
<td>About 0.82</td>
<td>It seems consistent</td>
</tr>
<tr>
<td></td>
<td>(1) The BSCR will be 6 for certain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2) The BSCR will be 6 with a probability of p, and will be 3 with a probability of (1- p); What number will you assign to p so that the above two situations are indifferent?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Answer is assumed for purposes of illustration.
Step 3: From the above questions and the assumed answers, the utility function for the Bridge Superstructure Condition Rating (BSCR) can be derived as follows:

\[
  u(X) = \begin{cases} 
    0 & \text{when } X = 0; \\
    0.05 & \text{when } X = 1; \\
    0.15 & \text{when } X = 2; \\
    0.25 & \text{when } X = 3; \\
    0.3 & \text{when } X = 4; \\
    0.7 & \text{when } X = 5; \\
    0.8 & \text{when } X = 6; \\
    0.9 & \text{when } X = 7; \\
    0.95 & \text{when } X = 8; \\
    1 & \text{when } X = 9. 
  \end{cases}
\]

Table A.2 NBI Rating Scale

<table>
<thead>
<tr>
<th>Score</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Not Applicable</td>
</tr>
<tr>
<td>9</td>
<td>Excellent Condition</td>
</tr>
<tr>
<td>8</td>
<td>Very Good Condition - no problems noted.</td>
</tr>
<tr>
<td>7</td>
<td>Good Condition - some minor problems.</td>
</tr>
<tr>
<td>6</td>
<td>Satisfactory Condition - structural elements show some minor deterioration.</td>
</tr>
<tr>
<td>5</td>
<td>Fair Condition - all primary structural elements are sound but may have minor section loss, cracking, spalling or scour.</td>
</tr>
<tr>
<td>4</td>
<td>Poor Condition - advanced section loss, deterioration, spalling or scour.</td>
</tr>
<tr>
<td>3</td>
<td>Serious Condition - loss of section, deterioration, spalling or scour have seriously affected primary structural components. Local failures are possible. Fatigue cracks in steel or shear cracks in concrete may be present.</td>
</tr>
<tr>
<td>2</td>
<td>Critical Condition - advanced deterioration of primary structural elements. Fatigue cracks in steel or shear cracks in concrete may be present or scour may have removed substructure support. Unless closely monitored it may be necessary to close the bridge until corrective action is taken.</td>
</tr>
<tr>
<td>1</td>
<td>“Imminent” Failure Condition - major deterioration or section loss present in critical structural components or obvious vertical or horizontal movement affecting structure stability. Bridge is closed to traffic but corrective action may put back in light service.</td>
</tr>
<tr>
<td>0</td>
<td>Failed Condition - out of service and beyond corrective action.</td>
</tr>
</tbody>
</table>
Appendix 1.4: Risk Scenario—Certainty Equivalent Approach Example
Developing utility function for Pavement Surface Condition (IRI)

IRI is a measure of ride quality obtained by road meters installed on vehicles or trailers, typically expressed in inches per mile. Higher values indicate a lower ride quality. New pavements typically can have IRI of approximately 75 in/mile to 100 in/mile. Theoretically, the range of IRI is from 0 to infinity, but in practice, IRI with 75 in/mile is often viewed as excellent condition, and IRI with more than 500 in/mile is viewed as failure condition. Assume that the IRI of a pavement section will be X after the resurfacing action. Use the certainty equivalent approach to develop utility function for IRI.

Step 1: Define \( u(\text{IRI}=75)=1; \ u(\text{IRI}=500)=0; \)

Step 2: Ask the decision makers the questions listed in the following table (the assumed answers are also given in the table).

<table>
<thead>
<tr>
<th>No.</th>
<th>Question</th>
<th>Answer</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>What IRI will you assign to ( X_{0.5} ) so that the following situations are indifferent to you? (1) IRI will be ( X_{0.5} ) for certain (2) IRI will be 75 in/mile with the probability 0.5 and will be 500 in/mile with the probability 0.5;</td>
<td>( X_{0.5} = 140 )</td>
<td>( u(\text{IRI}=140)=0.5 )</td>
</tr>
<tr>
<td>2</td>
<td>What IRI will you assign to ( X_{0.25} ) so that the following two situations are indifferent to you? (1) IRI will be ( X_{0.25} ) for certain (2) IRI will be 140 in/mile with the probability 0.5 and will be 500 in/mile with the probability 0.5;</td>
<td>( X_{0.25} = 200 )</td>
<td>( u(\text{IRI}=200)=0.25 )</td>
</tr>
<tr>
<td>3</td>
<td>What IRI will you assign to ( X_{0.75} ) so that the following situations are indifferent to you? (1) IRI will be ( X_{0.75} ) for certain (2) IRI will be 140 in/mile with the probability 0.5 and will be 75 in/mile with the probability 0.5;</td>
<td>( X_{0.75} = 100 )</td>
<td>( u(\text{IRI}=100)=0.75 )</td>
</tr>
<tr>
<td>4</td>
<td>What IRI will you assign to X so that the following situations are indifferent to you? (1) IRI will be X for certain (2) IRI will be 100 in/mile with the probability 0.5 and will be 200 in/mile with the probability 0.5;</td>
<td>About 140</td>
<td>Consistency check: it seems consistency</td>
</tr>
<tr>
<td>5</td>
<td>What IRI will you assign to ( X_{0.125} ) so that the following situations are indifferent to you? (1) IRI will be ( X_{0.125} ) for certain (2) IRI will be 200 in/mile with the probability 0.5 and will be 500 in/mile with the probability 0.5;</td>
<td>( X_{0.125} = 300 )</td>
<td>( u(\text{IRI}=300)=0.125 )</td>
</tr>
<tr>
<td>6</td>
<td>What IRI will you assign to ( X_{0.375} ) so that the following situations are indifferent to you? (1) IRI will be ( X_{0.375} ) for certain (2) IRI will be 140 in/mile with the probability 0.5 and will be 200 in/mile with the probability 0.5;</td>
<td>( X_{0.375} = 160 )</td>
<td>( u(\text{IRI}=160)=0.375 )</td>
</tr>
<tr>
<td>7</td>
<td>What IRI will you assign to ( X_{0.625} ) so that the following situations are indifferent to you? (1) IRI will be ( X_{0.625} ) for certain (2) IRI will be 140 in/mile with the probability 0.5 and will be 100 in/mile with the probability 0.5;</td>
<td>( X_{0.625} = 125 )</td>
<td>( u(\text{IRI}=125)=0.625 )</td>
</tr>
<tr>
<td>8</td>
<td>What IRI will you assign to ( X_{0.875} ) so that the following situations are indifferent to you? (1) IRI will be ( X_{0.875} ) for certain (2) IRI will be 100 in/mile with the probability 0.5 and will be 75 in/mile with the probability 0.5;</td>
<td>( X_{0.875} = 90 )</td>
<td>( u(\text{IRI}=90)=0.875 )</td>
</tr>
</tbody>
</table>

1. Answer is assumed for purposes of illustration.
Step 3: Plot the following points (IRI, \( u(\text{IRI}) \)):

<table>
<thead>
<tr>
<th>IRI</th>
<th>( u(\text{IRI}) )</th>
<th>IRI</th>
<th>( u(\text{IRI}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>1</td>
<td>160</td>
<td>0.375</td>
</tr>
<tr>
<td>90</td>
<td>0.875</td>
<td>200</td>
<td>0.25</td>
</tr>
<tr>
<td>100</td>
<td>0.75</td>
<td>300</td>
<td>0.125</td>
</tr>
<tr>
<td>125</td>
<td>0.625</td>
<td>500</td>
<td>0.1</td>
</tr>
<tr>
<td>140</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 4: Choose an appropriate functional form such as \( ae^{-bx} \) for the utility function, and use the above data to calibrate the parameters \( a \) and \( b \) as follows: \( a=1.938 \), \( b=0.009 \).

Step 5: Thus, the utility function for IRI is \( u(x) = 1.938e^{-0.009x} \).
<table>
<thead>
<tr>
<th>Category</th>
<th>Performance Measure(x)</th>
<th>Utility Function Form</th>
<th>Coefficient (a)</th>
<th>Factor (K)</th>
<th>Unit</th>
<th>Xmin</th>
<th>Xmax</th>
</tr>
</thead>
<tbody>
<tr>
<td>System Preservation</td>
<td>Pavement Surface Condition(IRI)</td>
<td>K<em>Exp(a</em>x²)</td>
<td>-0.000044</td>
<td>1.0729</td>
<td>inch/mile</td>
<td>0</td>
<td>infinity</td>
</tr>
<tr>
<td></td>
<td>Remaining Service Life</td>
<td>K*[1-Exp(a*x²)]</td>
<td>-0.0195</td>
<td>1.1659</td>
<td>years</td>
<td>0</td>
<td>infinity</td>
</tr>
<tr>
<td></td>
<td>Bridge Structural Condition</td>
<td>K*[1-Exp(a*x²)]</td>
<td>-0.0249</td>
<td>1.1535</td>
<td>rating scale</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Bridge Wearing Surface Condition</td>
<td>K*[1-Exp(a*x²)]</td>
<td>-0.025</td>
<td>1.1521</td>
<td>rating scale</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Historical Bridge Age</td>
<td>K*[1-Exp(a*(x-80))]</td>
<td>-0.0144</td>
<td>1.216</td>
<td>years</td>
<td>80</td>
<td>infinity</td>
</tr>
<tr>
<td></td>
<td>Historical Bridge Length</td>
<td>K*[1-Exp(a*(x-40))]</td>
<td>-0.0112</td>
<td>1.1999</td>
<td>ft</td>
<td>0</td>
<td>infinity</td>
</tr>
<tr>
<td></td>
<td>Deck Condition Rating</td>
<td>K*[1-Exp(a*x)]</td>
<td>-0.19</td>
<td>122.75</td>
<td>rating scale</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Superstructure Condition Rating</td>
<td>K*[1-Exp(a*x)]</td>
<td>-0.203</td>
<td>119.13</td>
<td>rating scale</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Substructure Condition Rating</td>
<td>K*[1-Exp(a*x)]</td>
<td>-0.202</td>
<td>119.49</td>
<td>rating scale</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Culvert Condition Rating</td>
<td>K*[1-Exp(a*x)]</td>
<td>-0.14</td>
<td>140.51</td>
<td>rating scale</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Health Index</td>
<td>K*[1397.9/(1+Exp(a*(85-x)))-1]</td>
<td>0.092</td>
<td>0.0852</td>
<td>rating scale</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Sufficiency Rating</td>
<td>K*[5.54/(1+Exp(a*(70-x))-1)]</td>
<td>37.96</td>
<td>0.0216</td>
<td>rating scale</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>User Cost</td>
<td>Average speed (x=&lt;55)</td>
<td>K*[1-Exp(a*(x-15))]</td>
<td>-0.0486</td>
<td>1.167</td>
<td>mph</td>
<td>0</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>Average speed (x&gt;55)</td>
<td>K*[1-Exp(a*(75-x))]</td>
<td>-0.0778</td>
<td>1.1668</td>
<td>mph</td>
<td>55</td>
<td>infinity</td>
</tr>
<tr>
<td>Mobility</td>
<td>Average speed</td>
<td>K*[1-Exp(a*x²)]</td>
<td>-0.0005</td>
<td>1.0425</td>
<td>mph</td>
<td>0</td>
<td>infinity</td>
</tr>
<tr>
<td></td>
<td>Detour length</td>
<td>K<em>Exp(a</em>x)</td>
<td>-0.2145</td>
<td>1</td>
<td>mile</td>
<td>0</td>
<td>infinity</td>
</tr>
<tr>
<td></td>
<td>Intersection delay time 1</td>
<td>K<em>Exp(a</em>x)</td>
<td>-0.0982</td>
<td>1</td>
<td>min/vehicle</td>
<td>0</td>
<td>infinity</td>
</tr>
<tr>
<td></td>
<td>Intersection delay time 2</td>
<td>K<em>Exp(b</em>x)</td>
<td>-0.1772</td>
<td>1</td>
<td>min/vehicle</td>
<td>0</td>
<td>infinity</td>
</tr>
<tr>
<td>Safety</td>
<td>Bridge load inventory rating</td>
<td>K*[1-Exp(a*x²)]</td>
<td>-0.0404</td>
<td>1</td>
<td>metric tones</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Bridge deck width (x=&lt;1.0)</td>
<td>K*[1-Exp(a*(x-0.8))]</td>
<td>-8.5113</td>
<td>1.2229</td>
<td>ratio</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Bridge deck width (x&gt;1.0)</td>
<td>K*[1-Exp(a*(1.1-x))]</td>
<td>-59.2952</td>
<td>1</td>
<td>ratio</td>
<td>1</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>Bridge vertical clearance (o)</td>
<td>K*[1-Exp(a*(x-0.8))]</td>
<td>-8.2612</td>
<td>1.237</td>
<td>ratio</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Bridge vertical clearance (u)</td>
<td>K*[1-Exp(a*(x-0.8))]</td>
<td>-8.2672</td>
<td>1.2367</td>
<td>ratio</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Bridge horizontal clearance</td>
<td>K*[1-Exp(a*(x-0.8))]</td>
<td>-8.2278</td>
<td>1.239</td>
<td>ratio</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Average speed</td>
<td>K<em>Exp(a</em>x²)</td>
<td>-0.0004</td>
<td>1</td>
<td>mph</td>
<td>0</td>
<td>infinity</td>
</tr>
<tr>
<td></td>
<td>Skid resistance</td>
<td>K*[1-Exp(a*(x-10))]</td>
<td>-0.0437</td>
<td>1.2108</td>
<td>number</td>
<td>0</td>
<td>infinity</td>
</tr>
<tr>
<td></td>
<td>Lane width (x=&lt;1.0)</td>
<td>K*[1-Exp(a*(x-0.8))]</td>
<td>-8.2827</td>
<td>1.2358</td>
<td>ratio</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td>Metric</td>
<td>Expression</td>
<td>Coefficient</td>
<td>Exponent</td>
<td>Ratio</td>
<td>Number</td>
<td></td>
<td></td>
</tr>
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<td>---------------------------------------------</td>
<td>-----------------------</td>
<td>-------------</td>
<td>----------</td>
<td>-------</td>
<td>--------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lane width (x&gt;1.0)</td>
<td>$K[1-\exp(a*(1.1-x))]$</td>
<td>-59.6314</td>
<td>1</td>
<td>ratio</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shoulder width</td>
<td>$K[1-\exp(a*x/2)]$</td>
<td>-2.4343</td>
<td>1.0961</td>
<td>ratio</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Railroad Crossing Adequacy</td>
<td>$K[1-\exp(a*x/2)]$</td>
<td>-0.6963</td>
<td>1.1413</td>
<td>number</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sight distance</td>
<td>$K[1-\exp(a*(x-0.6))]$</td>
<td>-4.5181</td>
<td>1.1963</td>
<td>ratio</td>
<td>0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Luminance</td>
<td>$K[1-\exp(a*(x-0.6))]$</td>
<td>-4.6399</td>
<td>1.1853</td>
<td>ratio</td>
<td>0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometric Rating</td>
<td>$K[1-\exp(a*x)]$</td>
<td>0.6963</td>
<td>1.1413</td>
<td>number</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inventory Rating</td>
<td>$K[1-\exp(a*x)]$</td>
<td>0.02</td>
<td>115.33</td>
<td>metric</td>
<td>infinity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operating Rating</td>
<td>$K[1-\exp(a*x)]$</td>
<td>0.014</td>
<td>134.13</td>
<td>metric</td>
<td>infinity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Environment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed for CO2, TSP, SO2 (X=&lt;55)</td>
<td>$K[1-\exp(a*(x-15))]$</td>
<td>-0.0478</td>
<td>1.1734</td>
<td>miles/hour</td>
<td>0 55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed for CO2, TSP, SO2 (X&gt;55)</td>
<td>$K[1-\exp(a*(75-x))]$</td>
<td>-0.0816</td>
<td>1.1495</td>
<td>miles/hour</td>
<td>55 infinity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed for NMHC</td>
<td>$K[1-\exp(a*x)]$</td>
<td>-0.0338</td>
<td>1.0717</td>
<td>miles/hour</td>
<td>0 infinity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed for CO (X=&lt;35)</td>
<td>$K[1-\exp(a*x)]$</td>
<td>-0.0618</td>
<td>1.1299</td>
<td>miles/hour</td>
<td>0 35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed for CO (X&gt;35)</td>
<td>$K[1-\exp(a*(65-x))]$</td>
<td>-0.0609</td>
<td>1.069</td>
<td>miles/hour</td>
<td>35 infinity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed for Nox (X=&lt;15)</td>
<td>$K[1-\exp(a*x)]$</td>
<td>-0.0949</td>
<td>1.3173</td>
<td>miles/hour</td>
<td>0 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed for Nox (X&gt;15)</td>
<td>$K[1-\exp(a*(65-x))]$</td>
<td>-0.0366</td>
<td>1.1021</td>
<td>miles/hour</td>
<td>15 infinity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Protection from Extreme Events</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scour vulnerability Rating</td>
<td>$K[1-\exp(a*(x-1))]$</td>
<td>-0.43</td>
<td>121.76</td>
<td>rating scale</td>
<td>1 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fatigue(concrete) vulnerability Rating</td>
<td>$K[1-\exp(a*(x-1))]$</td>
<td>-0.33</td>
<td>137.03</td>
<td>rating scale</td>
<td>1 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fatigue(steel) vulnerability Rating</td>
<td>$K[1-\exp(a*(x-1))]$</td>
<td>-0.4</td>
<td>125.35</td>
<td>rating scale</td>
<td>1 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earthquake vulnerability Rating</td>
<td>$K[1-\exp(a*(x-1))]$</td>
<td>-0.36</td>
<td>130.57</td>
<td>rating scale</td>
<td>1 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other disaster vulnerability Rating</td>
<td>$K[1-\exp(a*(x-1))]$</td>
<td>-0.37</td>
<td>129.5</td>
<td>rating scale</td>
<td>1 6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(From Li, 2004 and Patidar, 2007)

1. System Preservation

![Utility vs Pavement Surface Condition](chart1.png)

![Utility vs Remaining Service Life](chart2.png)
\[ U = 1.0729 e^{-0.00044x^2} \]

\[ U = 1.1659(1 - e^{-0.0195x^2}) \]

\[ U = 1.1535(1 - e^{-0.0249x^2}) \]

\[ U = 1.1521(1 - e^{-0.025x^2}) \]

\[ U = 1.2160(1 - e^{-0.0144(x-80)}) \]

\[ U = 1.1999(1 - e^{-0.0112(x-40)}) \]

\[ U = 1.2275(1 - e^{-0.19x}) \]

\[ U = 1.1913(1 - e^{-0.203x}) \]
\[ U = 1.1949(1 - e^{-0.202x}) \]

\[ U = 1.4051(1 - e^{-0.14x}) \]

\[ U = 0.00092 \times \left( \frac{1397.9}{1 + e^{0.0852(85-x)}} - 1 \right) \]

\[ U = 0.3796 \times \left( \frac{5.54}{1 + e^{0.0216(70-x)}} - 1 \right) \]

2. User Cost

\[ U = 1.167(1 - e^{-0.0486(x-15)}) \quad x \leq 55 \]

\[ U = 1.1668(1 - e^{-0.0778(75-x)}) \quad x > 55 \]
3. Mobility

\[ U = 1.0425(1 - e^{-0.0005x^2}) \]

\[ U = e^{-0.2045x} \]

\[ U = e^{-0.0982x} \]

\[ U = e^{-0.1772x} \]

4. Safety

\[ U = 1 - e^{-0.04041x} \]

\[ U = 1.2229(1 - e^{-8.0-8.5113(x - 0.8)}) \]

\[ x \leq 1.0 \]

\[ U = 1 - e^{-59.2952(1.1 - x)} \]

\[ x > 1.0 \]
\[ U = 1.2370(1 - e^{8.2612(x-0.8)}) \]
\[ U = 1.2367(1 - e^{8.2672(x-0.8)}) \]
\[ U = 1.2390(1 - e^{8.2278(x-0.8)}) \]
\[ U = e^{-0.0004x^2} \]
\[ U = 1.2108(1 - e^{-0.0437(x-10)}) \]
\[ U = 1.2358(1 - e^{-8.2827(x-0.8)}) \quad x \leq 1.0 \]
\[ U = 1 - e^{-59.6314(1.1-x)} \quad x > 1.0 \]
$U = 1.0961(1 - e^{-2.4343x^2})$

$U = 1.1413(1 - e^{-0.6963x})$

$U = 1.1963(1 - e^{-4.5181(x-0.6)})$

$U = 1.1853(1 - e^{-4.6399(x-0.6)})$

$U = 3.3215(1 - e^{-0.04x})$

$U = 1.1533(1 - e^{-0.02x})$
5. Environment

\[ U = 1.3413(1 - e^{-0.014x}) \]

\[ U = 1.1734(1 - e^{-0.0478(x-15)}) \quad x \leq 55 \]

\[ U = 1.0717(1 - e^{-0.0338x}) \]

\[ U = 1.1299(1 - e^{-0.0618x}) \quad x \leq 35 \]
6. Protection from Extreme Events

\[ U = 1.0690(1 - e^{-0.0609(65-x)^2}) \quad x > 35 \]

\[ U = 1.3703(1 - e^{-0.33(x-1)}) \]

\[ U = 1.1021(1 - e^{-0.0366(65-x)^2}) \quad x > 15 \]
\[ U = 1.2535(1 - e^{-0.40(x-1)}) \]

\[ U = 1.3057(1 - e^{-0.36(x-1)}) \]

\[ U = 1.295(1 - e^{-0.37(x-1)}) \]
APPENDIX 3

Weighting Methods

In multi-criteria decision-making, the weight of each performance measure represents the relative importance of that performance measure relative to other performance measures in the domain of the decision-making problem. Different weighting methods or even minor changes in the weight distributions can drastically alter the final decision. Thus, weighting is a critical aspect of multi-criteria decision-making. Common weighting methods are listed below.

Common Weighting Methods that can be used for Asset Management Decision-Making

<table>
<thead>
<tr>
<th>Index</th>
<th>Name</th>
<th>Index</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal Weighting</td>
<td>7</td>
<td>Simple Multi-attribute Rating Technique</td>
</tr>
<tr>
<td>2</td>
<td>Ranking</td>
<td>8</td>
<td>Multiple Regression</td>
</tr>
<tr>
<td>3</td>
<td>Point Allocation</td>
<td>9</td>
<td>Gamble/Lottery</td>
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<td>4</td>
<td>Direct Rating</td>
<td>10</td>
<td>Tradeoff Method</td>
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<tr>
<td>5</td>
<td>Swing Weighting Method</td>
<td>11</td>
<td>Pricing out</td>
</tr>
<tr>
<td>6</td>
<td>Analytical Hierarchy Process</td>
<td>12</td>
<td>Delphi Method¹</td>
</tr>
</tbody>
</table>

¹ Delphi is a process to enhance the outcomes of the other weighting methods.

1. **Equal Weighting** (Dawes and Corrigan, 1974)

   In the equal weighting method, all the performance measures are assigned the same weights, and the sum of weights should equal to 1. For example, if there are $n$ performance measures, their weights are assigned as $w_1, w_2, ..., w_n$. Thus,

   $$w_i = 1/n$$  \quad \text{and}  \quad \sum_{i=1}^{n} w_i = 1$$

   Equal weighting is an effortless method of weighting and obviously does not need any survey. However, this method does not reflect the different importance among the different performance measures. It can be used in the situation where there is no information about the weights of the performance measures.

2. **Ranking**

   The ranking method first ranks the performance measures according to their importance, then transform their ranking order to weights. There are many methods to do this:
(1) Rank-sum weights (Einhorn and McCoach, 1977; Stillwell et al, 1981)

For \( n \) performance measures \( c_1, c_2, \ldots, c_3 \), their rank positions are \( r_1, r_2, \ldots, r_3 \). Thus, their weights are calculated as follows:

\[
 w_i = \frac{n - r_i + 1}{\sum_{j=1}^{n} (n - r_j + 1)} \quad \text{and} \quad \sum_{i=1}^{n} w_i = 1
\]

(2) Rank reciprocal weights (Stillwell et al, 1981)

The rank reciprocal method derives weights from the normalized reciprocals of the performance measure ranks. For \( n \) performance measures \( c_1, c_2, \ldots, c_3 \), their rank positions are \( r_1, r_2, \ldots, r_3 \), then their weights are calculated as follows:

\[
 w_i = \frac{1/r_i}{\sum_{j=1}^{n} 1/r_j} \quad \text{and} \quad \sum_{i=1}^{n} w_i = 1
\]

(3) Rank exponent weights (Stillwell et al, 1981)

For \( n \) performance measures \( c_1, c_2, \ldots, c_3 \), their rank positions are \( r_1, r_2, \ldots, r_3 \), then their weights are calculated by

\[
 w_i = \left( \frac{n - r_i + 1}{\sum_{j=1}^{n} (n - r_j + 1)} \right)^x \quad \text{and} \quad \sum_{i=1}^{n} w_i = 1
\]

Respondents are asked to assign a weight to the most important performance measure. The weight should be between 0 and 1. Then use this weight to calibrate \( x \) and calculate the weight of other performance measures. When \( x = 0 \), it is equal weighting; when \( x = 1 \), it is rank-sum weighting.

(4) Rank-order centroid weights (Baron and Barrett, 1996)

This method computes the weights from the vertices of the simplex set up of the performance measures. Then the coordinates of the centroid are used as the weights (Baron, 1992).

For \( n \) ranked criteria \( c_1, c_2, \ldots, c_3 \), their rank positions are \( r_1, r_2, \ldots, r_3 \) and \( r_1 \leq r_2 \leq \cdots \leq r_n \), then if their weights are \( w_1, w_2, \ldots, w_n \), we have \( w_1 \geq w_2 \geq \cdots \geq w_n \). So the weights are calculated by

\[
 w_i = \frac{1}{n} \sum_{j=1}^{i} \frac{1}{j} \quad \text{and} \quad \sum_{i=1}^{n} w_i = 1
\]
3. **Point Allocation (PA)**

This method (Cook and Stewart, 1975) allocates 100 points to be shared among the different performance measures. The points allocated to a performance measure represent its relative importance; the sum of all the points should equal to 100. For example, there are $n$ performance measures, and their allocated points are $p_1, p_2, \ldots p_n$, then their weights are:

$$w_i = \frac{p_i}{100} \quad \text{and} \quad \sum_{i=1}^{n} w_i = 1$$

4. **Direct Rating**

The direct rating method rates each performance measure on a certain point scale, such as a 5-point scale, 10-point scale, or 100-point scale. The rating points reflect the importance of the performance measure. There is no restriction on the sum of the ratings. Then the ratings are transformed into the weights. For example, for $n$ performance measures, and a 100-point scale, the ratings are $r_1, r_2, \ldots, r_n$, and thus their weights are given by:

$$w_i = \frac{r_i}{\sum_{j=1}^{n} r_j} \quad \text{and} \quad \sum_{i=1}^{n} w_i = 1$$

5. **Swing Weighting Method** (Goicoechea et al, 1986)

Swing weighting method is similar to direct weighting. In this method, it is first assumed that all performance measures are in the worst condition; then choose one performance measure which can yield the largest improvement to the alternative if it moves from its worst condition to its best condition and rate it on a defined rating scale (e.g., a 100-point scale). Then for the remaining performance measures, determine that which can yield the largest improvement to the alternative if it moves from its worst condition to its best condition, and assign an appropriate rating accordingly.

Continue to do so until there are no performance measures left. For $n$ performance measures $c_1, c_2, \ldots, c_3$, whose ratings are $r_1, r_2, \ldots, r_n$, their weights can be calculated by

$$w_i = \frac{r_i}{\sum_{j=1}^{n} r_j} \quad \text{and} \quad \sum_{i=1}^{n} w_i = 1$$
6. **Analytical Hierarchy Process (AHP)** (Saaty, 1977)

AHP method is a pairwise comparison method. First, the survey respondents are asked to compare the criteria (or performance measures) with each other, and use the ratio in the following table to show the relative importance of each pair.

<table>
<thead>
<tr>
<th>Comparison</th>
<th>X/Y Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criterion $X$ is extremely more important than criterion $Y$</td>
<td>9</td>
</tr>
<tr>
<td>Criterion $X$ is strongly more important than criterion $Y$</td>
<td>8</td>
</tr>
<tr>
<td>Criterion $X$ is moderately more important than criterion $Y$</td>
<td>7</td>
</tr>
<tr>
<td>Criterion $X$ is slightly more important than criterion $Y$</td>
<td>6</td>
</tr>
<tr>
<td>Criterion $X$ is equally important than criterion $Y$</td>
<td>5</td>
</tr>
<tr>
<td>Criterion $X$ is slightly less important than criterion $Y$</td>
<td>4</td>
</tr>
<tr>
<td>Criterion $X$ is moderately less important than criterion $Y$</td>
<td>3</td>
</tr>
<tr>
<td>Criterion $X$ is strongly less important than criterion $Y$</td>
<td>2</td>
</tr>
<tr>
<td>Criterion $X$ is extremely less important than criterion $Y$</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: From Sinha and Labi, 2007

For example, if criterion $X$ is strongly more important than criterion $Y$, then the ratio 7 is used. After all the comparisons, the resulting ratios are used to fill the following matrix.

\[
\begin{bmatrix}
1 & z_{12} & \cdots & z_{1n} \\
1/z_{12} & 1 & \cdots & z_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
1/z_{1n} & 1/z_{2n} & \cdots & 1
\end{bmatrix}
\]

So for $n$ criteria, AHP needs $n(n+1)/2$ comparisons. Then weights of all criteria can be obtained by calculating the eigenvector of the matrix.

7. **Simple Multi-attribute Rating Technique (SMART)** (Edward 1977)

The SMART method is a kind of combination of ranking and direct rating method. This method first ranks all the performance measures according to their relative importance, and then assigns an arbitrary number 10 to the least important performance measure. The decision-maker judges how much more important each of the remaining performance measures is in relation to the least important and assigns numbers in multiples of 10 to each performance measure to reflect its relative. Then transform these numbers to weights. For $n$ criteria $c_1, c_2, \ldots, c_3$, their assigned numbers are $r_1, r_2, \ldots, r_n$, then their weights can be calculated as:
\[ w_i = \frac{r_i}{\sum_{j=1}^{n} r_j} \quad \text{and} \quad \sum_{i=1}^{n} w_i = 1 \]

This method is fairly similar to direct weighting. However, it ranks the attributes first and then rates them.

8. **Multiple Regression (MR)** (Hammond, Stewart and Steinmann 1975)

The multiple regression method does not involve a direct request for weights from the survey respondent. Rather, this method asks the respondent to provide an overall rating of each project alternative on a certain scale (e.g. 0-100) on the basis of the knowledge that the outcome of the project is in terms of some specific performance measures. So the respondent assigns the desirability of the project, but indirect assigns weights to the performance measures that the project is expected to elicit. Regression techniques are then used to derive the weight of each performance measure. In this method, it is helpful to know the scaled value of each performance outcome for each alternative.

9. **Gamble/Lottery** (Keeney & Raiffa, 1976)

The gamble method chooses a weight for one performance measure at a time by asking the decision-maker to compare a “sure thing” and a “gamble”. The first step is to determine which performance measure is the most important to move from its worst to its best possible level. Then, two situations are considered: first, the most important performance measure is at its best level and other performance measures are at their least desirable levels. Second, the chance of all performance measures being at their most desirable levels is set to \( p \), and the chance of all performance measures being at their worst values is \( 1-p \). If the two situations are equally desirable, the weight for the most important goal will be precisely \( p \). The same approach is repeated to derive the weights for remaining performance measure with decreasing relative importance.

10. **Tradeoff Method** (Keeney and Raiffa 1976)

The tradeoff method is similar to AHP method in its use of a pairwise comparison. In this method, subjects are asked how much change is required in one criterion to compensate a unit change in another criterion. This yields a tradeoff coefficient for these two criteria. Using the same method, the tradeoff coefficient between any other two criteria can be obtained. Based on these coefficients, the relative weights can be calculated.
11. Pricing out (Keeney and Raiffa 1976, 95)

The basic idea of the pricing-out method is similar to trade-off method. In the former, however, use is made of an explicit perception of trade-off between the performance measure and money. Respondents are asked the monetary worth of a unit change in a performance measures. Based on these monetary values, the weights can be derived.

For n criteria \( c_1, c_2, \ldots, c_n \), a unit change of these criteria is worth \( r_1, r_2, \ldots, r_n \), respectively. Thus, their weights can be calculated as:

\[
    w_j = \frac{r_i}{\sum_{j=1}^{n} r_j} \quad \text{and} \quad \sum_{i=1}^{n} w_i = 1
\]

12. Delphi Method (Dalkey and Helmer, 1963)

The Delphi method is a strategy to further refine the weights obtained in the other weighting methods. Respondents are shown the results of their surveys and given a chance to review their responses. This is continued until there are no significant differences between two successive surveys. The most important contribution of Delphi is that it can reduce the variance of the weights assigned by the respondents. When the weights of criteria are viewed as a range or distribution instead of fixed numbers, the use the Delphi approach is very beneficial.

Summary

In the literature, it is seen that several studies in different fields have been conducted to compare the different weighting methods and to ascertain the best method but no consensus has been reached on the matter. This may be due to the fact that the outcome of the weighting procedure depends on several factors such as the respondents, the performance measures, and the manner in which the survey is designed and performed. For purposes of asset management decision-making at INDOT, it is recommended to use several methods simultaneously, and then use the average of the derived weights as the final weights.

References


## APPENDIX 4

**Highway Relative Performance (System and Financial) of Indiana compared to Other States**

<table>
<thead>
<tr>
<th>Categories</th>
<th>Performance Indicator</th>
<th>Indiana</th>
<th>Highest</th>
<th>Lowest</th>
<th>Nation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Overall Performance</strong></td>
<td>Overall Performance</td>
<td>0.75</td>
<td>4.11(NJ)</td>
<td>0.45(ND)</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>System Performance</strong></td>
<td>Overall System Performance</td>
<td>0.51</td>
<td>4.16(AK)</td>
<td>0.36(AZ)</td>
<td>1.00</td>
</tr>
<tr>
<td>Safety</td>
<td>Fatality per 100 MVMT</td>
<td>1.26</td>
<td>2.364(MT)</td>
<td>0.785(MA)</td>
<td>1.421</td>
</tr>
<tr>
<td>Bridge</td>
<td>Percent Deficient (Structurally and Functionally)</td>
<td>21.65%</td>
<td>58.43%(RI)</td>
<td>3.92%(MA)</td>
<td>24.13%</td>
</tr>
<tr>
<td>Safety</td>
<td>Mobility and Congestion Percent Narrow Lanes (&lt;12 ft)</td>
<td>6%</td>
<td>41.1%(WI)</td>
<td>0%(AZ)</td>
<td>10.6%</td>
</tr>
<tr>
<td>Pavement</td>
<td>% Miles of V/C &gt; 0.7</td>
<td>30.77%</td>
<td>83.29%(CA)</td>
<td>0%(ND, MT, WY)</td>
<td>50.72%</td>
</tr>
<tr>
<td>Pavement</td>
<td>% Poor Rural Interstates (IRI&gt;170)</td>
<td>0%</td>
<td>23%(NH)</td>
<td>0%(IN)</td>
<td>2%</td>
</tr>
<tr>
<td>Pavement</td>
<td>% Poor Urban Interstate (IRI&gt;170)</td>
<td>1.9%</td>
<td>26.53%(HI)</td>
<td>0%(GA)</td>
<td>5.15%</td>
</tr>
<tr>
<td>Pavement</td>
<td>% Poor Rural &amp; Other Principal Arterials (IRI&gt;220)</td>
<td>0.17%</td>
<td>16.59%(AK)</td>
<td>0%(AZ)</td>
<td>0.76%</td>
</tr>
<tr>
<td><strong>Financial Performance</strong></td>
<td>Overall Financial Performance</td>
<td>1.08</td>
<td>7.31(NJ)</td>
<td>0.32(SC)</td>
<td>1.00</td>
</tr>
<tr>
<td>Financial Performance</td>
<td>Normalized Total Receipts Per CL Mile of Responsibility</td>
<td>1.013</td>
<td>17.189(NJ)</td>
<td>0.247(SC)</td>
<td>1.0</td>
</tr>
<tr>
<td>Financial Performance</td>
<td>Capital &amp; Bridge Disbursements Per CL Mile of Responsibility</td>
<td>1.085</td>
<td>8.785(NJ)</td>
<td>0.25(VA)</td>
<td>1.0</td>
</tr>
<tr>
<td>Financial Performance</td>
<td>Normalized Preservation, Maintenance &amp; HW Services Disbursements Per CL Mile of Responsibility</td>
<td>1.785</td>
<td>6.929(NJ)</td>
<td>0.222(ND)</td>
<td>1.0</td>
</tr>
<tr>
<td>Financial Performance</td>
<td>Preservation, Maintenance &amp; HW Services Disbursements as % of Total Budget</td>
<td>24.2%</td>
<td>37.5%(VA)</td>
<td>6.0%(AZ)</td>
<td>17.1%</td>
</tr>
<tr>
<td>Financial Performance</td>
<td>Normalized Administrative Disbursements Per CL Mile of responsibility</td>
<td>0.639</td>
<td>8.329(NJ)</td>
<td>0.131(KY)</td>
<td>1.0</td>
</tr>
<tr>
<td>Financial Performance</td>
<td>Administrative Disbursements as % of Total Budget</td>
<td>3.6%</td>
<td>19.4%(HI)</td>
<td>2.3%(KY)</td>
<td>7.0%</td>
</tr>
<tr>
<td>Financial Performance</td>
<td>Normalized Total Disbursements Per CL Mile of Responsibility</td>
<td>1.266</td>
<td>15.04(NJ)</td>
<td>0.268(WV)</td>
<td>1.0</td>
</tr>
</tbody>
</table>