Plasmon mode characteristics of metallic nanowire in uniaxial anisotropic dielectric

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The plasmon mode characteristics of metallic nanowire embedded in a uniaxial anisotropic dielectric are investigated theoretically. The hybrid plasmon modes found in this structure are significantly different from the traditional plasmon modes of metallic nanowire in a homogeneous isotropic dielectric. In contrast to the transverse-magnetic-like wave for a traditional fundamental mode, the hybrid fundamental mode of metallic nanowire involves a nonzero longitudinal magnetic field component. The degenerate behaviors of adjacent order plasmon modes are demonstrated in the case of a strongly anisotropic dielectric. Moreover, the dependence of the degenerate characteristics on the radius of a metallic nanowire and the excited wavelength are clearly shown by dispersion relation. The results of the study provide a useful approach to modulate surface plasmon polaritons with anisotropic medium. © 2014 Optical Society of America

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Surface plasmon polaritons (SPPs) are the excitations of the electromagnetic waves coupled to collective oscillations of free electrons in a conductor. The excitation of SPPs can concentrate light into the subwavelength scale and produce high optical local-field enhancement. These unique advantages of SPPs open a significant opportunity for their application in a broad range of scientific and engineering fields, such as surface-enhanced Raman scattering [1,2], enhanced nonlinear optical effects [3,4], and highly sensitive biological and chemical sensors [5,6]. In addition, SPPs have been considered as one of the most promising candidates for constructing the next-generation ultracompact integrated photonic circuits [7]. The tunability of SPPs at a nano-optical device is very important for achieving functional SPPs circuits. Recently, the unique SPPs characteristics that exist in the plasmon waveguide with anisotropic medium have triggered an explosion of research interest due to many potential applications [8–11]. Different from general planar isotropic dielectric–metal waveguides, the hybrid SPPs with coexisting TE- and TM-polarized waves can be supported in an anisotropic dielectric–metal waveguide due to dependence of permittivity on the optical axis (OA) [12]. It is demonstrated that the propagation constants and mode distributions of SPPs can be greatly modulated by varying the relative angle between the propagation direction and the OA of the anisotropic dielectric [13].

The cross section of a metallic nanowire functions like a confining potential compared to a planar dielectric–metal waveguide in which the one transverse direction is uniform. Similar to confining potentials in quantum mechanics that lead to discrete states, metallic nanowires can support plasmon modes with different orders. Recently, metallic nanowire waveguides have received considerable attention due to the highly confined electromagnetic wave and relative low propagation loss [14,15]. Many functional devices based on metallic nanowires have been demonstrated, such as quarter-wave plates [16], Fabry–Perot resonators [17], plasmonic routers [18], and logic gates [19]. In these applications, the interference effects between plasmon modes with different orders play a critical role in determining the electromagnetic behaviors of the devices. The propagation constants and field localization behaviors of plasmon modes can be greatly modified with anisotropic dielectric surrounding the metallic nanowire, so the excitation behaviors and the corresponding interference effects between plasmon modes will be modified, which can realize the functional nano-optical devices with metallic nanowire. Therefore, understanding the mode characteristics of metallic nanowire in the anisotropic dielectric is essential to design the active plasmonic devices.

In this Letter, the dispersion equations of metallic nanowire in a uniaxial anisotropic dielectric are derived. The dispersion relations and field evolutions of the plasmon modes with different orders are analyzed by solving the dispersion equations. Owing to the coupling between field components, the plasmon modes in the metallic nanowire represent the hybrid behaviors. The adjacent order plasmon modes even demonstrate the degenerate characteristics in the strongly anisotropic dielectric.

The studied structure is composed of a cylindrical metallic nanowire embedded in homogeneous uniaxial anisotropic medium, and the coordinates systems are illustrated in Fig. 1. The OA of the anisotropic dielectric is set in the x–y plane. In general, we assume that the OA orientation of the anisotropic dielectric is parallel to the x axis in view of the rotational symmetry of the nanowire. The relative dielectric tensor \( \epsilon \) in the Cartesian coordinate \((x, y, z)\) is described as follows:

\[
\epsilon = \begin{pmatrix}
  n_x^2 & 0 & 0 \\
  0 & n_y^2 & 0 \\
  0 & 0 & n_z^2
\end{pmatrix}.
\]
Because the geometrical structure of the nanowire is independent of \(z\) and the plasmon modes are confined near the surface of the metallic nanowire and propagate along the \(z\) axis (Fig. 1), a term \(\exp(i\beta z)\) can be factored out from the Maxwell’s equations, where \(\beta\) is the propagation constant of the plasmon mode. The electromagnetic wave exists as an evanescent wave both inside and outside the nanowire. Considering the anisotropic dielectric environment, it is convenient to describe the field components of the eigenmode outside the metallic nanowire in the Cartesian coordinate system. After some algebraic operations, the coupled-wave equations can be derived as [20, 21]

\[
\frac{\partial^2 E_y^0}{\partial x^2} + \frac{n_e^2}{n_o^2} \frac{\partial^2 E_y^0}{\partial y^2} + \frac{n_e^2}{n_o^2} (k_0^2 n_e^2 - \beta^2) E_y^0 = 0, \tag{2}
\]

\[
\frac{\partial^2 E_y^0}{\partial x^2} + \frac{n_e^2}{n_o^2} \frac{\partial^2 E_y^0}{\partial y^2} + (k_0^2 n_e^2 - \beta^2) E_y^0 = \frac{n_e^2}{n_o^2} \frac{\partial^2 E_z^0}{\partial x \partial y}. \tag{3}
\]

In order to solve Eq. (2), a new variable \(y' = n_e/y_o\) is introduced as \(y' = n_e/n_o\). Then, Eq. (2) can be expressed as

\[
\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y'^2} + \frac{n_e^2}{n_o^2} (k_0^2 n_e^2 - \beta^2) E_y = 0. \tag{4}
\]

Then \(E_y\) can be analytically solved in the new cylindrical coordinate \((\rho', \theta', z)\) from Eq. (4), where \(\rho' = \sqrt{\rho^2 + y'^2}, \quad \theta' = \tan^{-1}(y'/x)\). The solutions of \(E_y\) can also be obtained according to the form of the \(E_y\). At the interface between the nanowire and anisotropic dielectric \((\rho = R)\), the field components in the new coordinate \((\rho', \theta', z)\) system can be written as a Fourier sine or cosine series in the coordinate \((\rho, \theta, z)\) system. Then, applying the boundary conditions of continuity for tangential electromagnetic field components at the metal–dielectric interface, the dispersion equations of the structure can be obtained [22].

The mode characteristics and dispersion relations of Ag nanowire in the uniaxial anisotropic dielectric are theoretically studied by solving the dispersion equations. To describe the anisotropy of the dielectric medium, the refractive index anisotropy of the dielectric is defined as

\[
d_{eo} = (n_e - n_o)/n_{rm}.
\]

where \(n_{rm} = \max\left((n_e, n_o)\right)\). The permittivity of Ag is chosen to be \(\varepsilon = -18.2 + i0.5\) at wavelength 632.8 nm [23]. The refractive index \(n_o = 1.8\).

Due to the anisotropy of the dielectric surrounding the Ag nanowire, the coupling between \(E_x\) and \(E_y\) components leads to a new set of elementary modes, denoted as \(H_0, H_{1s}, H_{1o}\), and so on, where the numerical subscripts denote the order of the elementary mode, \(c\) and \(s\) denote the mode with cosine and sine angular distributions, respectively. The propagation constants of plasmon modes with different orders as a function of \(d_{eo}\) are shown in Fig. 2. The real and imaginary parts of the effective refractive index \((\beta/k_0)\) for the positive uniaxial medium \((d_{eo} > 0)\) are shown in Figs. 2(a) and 2(b), respectively. The dotted line in Fig. 2(a) represents the light line along the OA in the anisotropic dielectric. Only the modes with effective refractive index higher than that of the anisotropic dielectric can be bounded and propagate effectively. As the \(d_{eo}\) increases, the effective refractive indices of these plasmon modes are increased. Moreover, the adjacent order plasmon modes (e.g., \(H_0\) and \(H_{1c}\) modes, \(H_{1s}\) and \(H_{2s}\) modes) demonstrate almost the same propagation constants as the \(d_{eo}\) is larger than 0.15. For the negative uniaxial medium \((d_{eo} < 0)\), the real and imaginary parts of the effective refractive index are shown in Figs. 2(c) and 2(d), respectively. In this case, the effective refractive indices of the \(H_0\) and \(H_{1c}\) modes \((H_{1s}\) and \(H_{2s}\) modes) are decreased and approach each other with decreasing of \(d_{eo}\).

To understand the underlying physical mechanism, the cases for \(d_{eo} > 0\) are particularly studied. First, the field characteristics of the fundamental \(H_0\) mode are analyzed. The electric field and magnetic field distributions of the fundamental \(H_0\) mode are calculated at different \(d_{eo}\), which are shown in Figs. 3(a) and 3(b), respectively. As the dielectric surrounding the Ag nanowire is isotropy \((d_{eo} = 0)\), it is known that the mode is TM polarized, which means that the mode has no magnetic field parallel to the nanowire. Moreover, the mode distribution is axially symmetric, and the direction of electric field is...
radially polarized, as noted in Fig. 3(a). But for the anisotropic dielectric, the mode characteristics of the H₀ mode will be changed. Owing to the coupling between field components in the anisotropic dielectric, the plasmon modes of the metallic nanowire will be the combinations of different order solutions. For example, the angular momentum of the zeroth-order mode will be modulated due to the periodic change of dielectric function on the cylindrical surface of the nanowire, which contributes to the coupling among the zeroth-order, the second-order, the fourth-order, and other higher-order modes. Therefore, the H₀ mode becomes the hybrid mode. The longitudinal magnetic field component (H₀) of the H₀ mode appears in the anisotropic structure, and demonstrates four nodes of field on the nanowire surface, as shown in Fig. 3(b). It means that this field component comes from that of the second-order and other higher-order modes. As the dₑₒ increases, the energy of the mode is gradually localized along the direction of the OA and becomes nonaxially symmetric. The directions of electric field vectors are also gradually changed from radial polarization to linear polarization.

For the higher-order H₁ and H₂ modes, the original degenerate modes with cosine and sine angular distributions will eliminate the degeneration, and they will have different propagation constants with increasing of dₑₒ, as shown in Fig. 2(a). The effective refractive indices of modes with cosine angular distribution (e.g., H₁c mode) are higher than that of sine angular distribution (e.g., H₁s mode) for the odd-order modes, while the case is opposite for that of even-order modes, such as H₂c and H₂s modes. This is attributed to the fact that the odd-order and even-order modes have the different field localizations on the nanowire surface. Furthermore, the adjacent-order plasmon modes in the Ag nanowire even exhibit the new degenerate behaviors in the strongly anisotropic dielectric environment, as noted in Figs. 2(a) and 2(b). For example, as the dₑₒ increases, the effective refractive index of the H₁c mode gradually approaches that of the H₀ mode; finally, the two modes demonstrate almost the same propagation constants as the dₑₒ is larger than 0.15. It is the same case for that of the H₁s and H₂s modes. To get insight into the physical origin, the field distributions for high-order modes at different dₑₒ are calculated and shown in Fig. 4. It is well known that 2m nodes of the field can be found on the nanowire surface for the mth-order mode in the isotropic dielectric. But the case will be different for the metallic nanowire embedded in the strongly anisotropic dielectric. For the H₁c mode, the two nodes of the field are observed along the direction of the OA at dₑₒ = 0. As dₑₒ increases, the localization of the field is enhanced and the most energy of the mode is localized in the vicinity of the OA, as noted in Fig. 4(a). In comparison with the field distribution of the H₀ mode at dₑₒ = 0.16, as shown in Fig. 3(a), the H₀ and H₁c modes exhibit almost the same energy localization patterns on the nanowire surface. Owing to different angular distributions for the two modes, it is noted that the fields of the H₀ and H₁c modes demonstrate the symmetric and antisymmetric distributions perpendicular to the direction of OA, respectively.

For the H₁s mode, the two nodes of the field originally localized along the y axis are gradually shifted toward the direction of the OA with increasing of the dₑₒ. Finally, the field energy localized along the y axis becomes very weak, and the four nodes of the field are observed on the nanowire surface, as shown in Fig. 4(b). Figure 4(c) demonstrates the field distributions of the H₂s mode at different dₑₒ. As the dₑₒ increases, the four nodes of the field are also shifted toward the direction of the OA due to the increasing of dielectric function. Finally, the same energy localization patterns are found on the metallic nanowire surface for the different order H₁s and H₂s modes at dₑₒ = 0.16, which contribute to the same propagation constants for the two modes as noted in Figs. 2(a) and 2(b). The same case is for that of the H₀ and H₁c modes. For the negative uniaxial medium (nₑ < nₒ), the energy of the modes will be gradually localized along the direction of the y axis with decreasing of dₑₒ. Thus, the H₀ and H₁c modes (H₁c and H₂s modes) will exhibit the same energy localization patterns on the nanowire surface in the strongly anisotropic dielectric, which results in almost the same propagation patterns for these modes as noted in Figs. 2(c) and 2(d). Therefore, a conclusion can be made that the adjacent-order plasmon modes will exhibit the degenerate behaviors when the metallic nanowire is embedded in the strongly anisotropic dielectric.

The geometry-dependent dispersion relations of the Ag nanowire embedded in the strongly anisotropic dielectric are calculated and displayed in Fig. 5. As illustrated in Fig. 5(a), the degenerated H₀ and H₁c modes are gradually split as the radius of the Ag nanowire decreases, the
Effective refractive index of the $H_0$ mode grows sharply, but that of the $H_{1c}$ mode decreases. Hence, as the radius of the Ag nanowire decreases, the $H_0$ mode becomes more confined, which originates from more field energy penetrating into the metal region, while the field distribution of the $H_{1c}$ mode spreads quickly. The different field localizations are accompanied with growing and decreasing loss for the $H_0$ and $H_{1c}$ modes, respectively, with decreasing of the nanowire radius, as noted in Fig. 5(b). As the effective refractive index of the $H_{1c}$ mode closes to the light line along the OA, the mode becomes poorly confined and is effectively cut off. For the $H_{1c}$ and $H_{2c}$ modes, the effective refractive indices of the modes decrease and split, then cut off with decreasing of the radius. Figure 6 demonstrates the dispersion relations of the plasmon modes in the Ag nanowire at $d_{co} = 0.16$. When the radius of the nanowire is 100 nm, only the $H_0$, $H_{1c}$, and $H_{1s}$ modes are supported in the Ag nanowire at a wavelength of 632.8 nm. As the wavelength decreases, other higher-order modes appear, and the energy of the field penetrating into the metal will increase, which enhances the field localization and increases the effective refractive indices of plasmon modes. Meanwhile, the high localization of the field improves the sensitivity of the effective refractive indices of plasmon modes to the dielectric environment. Therefore, as the medium surrounding the Ag nanowire has the anisotropic dielectric behavior, the coupling effect between different order plasmon modes is enhanced due to the appearance of higher-order modes and the high localization of the mode field at the short wavelength. Then, the effective refractive indices of adjacent-order plasmon modes approach each other with decreasing of the wavelength. Finally, these adjacent-order plasmon modes demonstrate the degenerate behaviors at the short wavelength, as shown in Fig. 6.

In conclusion, we have shown the plasmon mode characteristics of metallic nanowire embedded in a uniaxial anisotropic dielectric, which are quite different from the traditional plasmon modes of metallic nanowire in a homogeneous isotropic dielectric. The fundamental mode of the metallic nanowire in the anisotropic dielectric is a hybrid mode and the direction of the electric field is no longer radially polarized. The adjacent-order plasmon modes even exhibit the degenerate behaviors in the strongly anisotropic dielectric. Moreover, the radius of the nanowire and the wavelength can also affect the degenerate behaviors of plasmon modes. The studied results have great potential applications in the active plasmonic devices and the design of novel functional devices.

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References and Notes

22. The detailed description of the solution process will be published elsewhere.

Fig. 5. (a) Real and (b) imaginary parts of the propagation constants of plasmon modes as a function of the radius of nanowire at $d_{co} = 0.16$. The wavelength is 632.8 nm.

Fig. 6. Dispersion diagram of plasmon modes in the Ag nanowire with radius $R = 100$ nm. The refractive index anisotropy $d_{co}$ is set to 0.16.