Unstructured Peer-to-Peer Networks for Sharing Processor Cycles

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FOR SHARING PROCESSOR CYCLES

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Abstract
Motivated by the needs and success of projects such as SETI@home and genome@home, we propose an architecture for a sustainable large-scale peer-to-peer environment for distributed cycle sharing among Internet hosts. Such networks are characterized by highly dynamic state due to high arrival and departure rates. This makes it difficult to build and maintain structured networks and to use state-based resource allocation techniques. We build our system to work in an environment similar to current file-sharing networks such as Gnutella and Freenet. In doing so, we are able to leverage vast network resources while providing resilience to random failures, low network overhead, and an open architecture for resource brokering.

This paper describes the underlying analytical and algorithmic substrates based on randomization for job distribution, replication, monitoring, validation, and aggregation. It also describes a rendezvous service that allows oblivious resource sharing and communication between participating hosts. We support our claims of robustness and scalability analytically with high probabilistic guarantees. Our algorithms do not introduce any state dependencies, and hence are resilient to dynamic node arrivals, departures, and failures. We support all analytical claims with a detailed simulation-based evaluation of our distributed framework.

1 Introduction
The use of a large number of unreliable hosts over a wide area network to solve compute-intensive problems has been pioneered by projects such as SETI@home [25], genome@home [9], and distributed.net [7] among others. While key advantages such as increased performance, reliability, and scalability motivate the use of decentralized peer-to-peer (P2P) systems instead of traditionally used client-server models, we consider the broader goal of an open environment for cycle sharing as a major driver for a move towards a P2P solution for harnessing idle CPU cycles of Internet hosts. Such an environment would allow any participant to submit tasks, in contrast to the inflexible nature of a client-server model. Furthermore, an open P2P system provides an incentive for peers to contribute their resources, expecting cycles in return, as opposed to the altruistic basis for current systems.

In this paper we propose and evaluate an unstructured P2P architecture for distributed cycle sharing among Internet hosts. The dynamic nature of P2P networks resulting from high arrival and departure rates motivates our choice of an unstructured (stateless) model. A majority of Internet-deployed examples of successful, resilient, large-scale, and massively-distributed systems rely on such unstructured topologies (e.g., Gnutella [11], Freenet, etc. for file sharing). Our design decision trades off the overheads of building and maintaining a structured overlay (with associated guarantees on network path lengths) for a less expensive model (with probabilistic guarantees on network delays). The use of efficient randomized algorithms in our network affords simplicity and scalability. Our system capitalizes on the large number of participating nodes to achieve robustness through redundancy. The underlying approach is novel and carries its share of complexities, solutions to which form the key contributions of this paper. The design goals of our system are:

1. Low job makespans by ensuring load balance.
2. Resilience to node failures and frequent node arrivals and departures.
3. Validation of computed output by redundant distributed computations.
5. An accounting framework for the resources contributed by various nodes.
In the context of our system, uniform sampling allows us to design randomized algorithms for load balancing, applying redundancy to support fault tolerance, and building a probabilistic rendezvous service for monitoring task progress and contributions of participating nodes.

**Uniform Sampling in Unstructured Networks**

Uniform sampling in a network requires randomly selecting a node, such that every node in the network has the same probability of being selected. A trivial approach to this problem would be to collect the entire set of node identifiers at each node and index randomly into this table of identifiers. This simple approach, however, does not work for our target applications because the overhead of frequently updating system state at each node (if at all possible) would be extremely high. An alternate approach to this problem relies on the notion of a random walk. Starting from an initial node, a random walk (of predetermined length) transitions through a sequence of intermediate nodes with probabilities defined for each link and ends at a destination node. The likelihood of terminating a random walk at any node determines whether the walk is a uniform sampling random walk or not. Formally, we define a uniform sampling random walk as follows:

**Definition 1.1 (Uniform sampling using random walk)**

A random walk of a given length samples uniformly at random from a set of nodes of a connected network if and only if the walk terminates at any node belonging to the network, with probability $1/N$, where $N$ is the number of nodes in the network.

The key parameters of interest in sampling via random walk are: (i) it should provide a uniform sample irrespective of the topology of the network, and (ii) the length of the walk required to reach stationarity (mixing time of the walk) should be small. A number of researchers, over the years, have studied properties of random walks. Lovász [18] provides an excellent survey of these techniques. The simplest random walk algorithm selects an outgoing edge at every node with equal probability. E.g., if node $i$ has degree four, each of the edges is traversed with a probability $0.25$. It can be shown that the probability distribution associated with target nodes becomes stationary after a finite length random walk (also known as the mixing time for the corresponding Markov chain). This length can be shown to approach $O(\log n)$.

The rest of this paper is organized as follows. In Section 2, we summarize related results. In Section 3, we present an overview of our randomization-based P2P computing architecture. In Section 4, we show how uniform sampling can be achieved via random walks. We also present an algorithm that allows efficient (short length) random walks to obtain uniform sampling. In Section 5, we empirically evaluate the performance of our architecture. We show that our architecture yields high efficiencies for distributed computations. We also evaluate strategies for job replication. We derive conclusion from our work in Section 6.
2 Related Work

SETI@home [25], genome@home [9], and distributed.net [7] are among the early examples of distributed computing projects that set the bar for systems that support a large number of Internet hosts. However, these systems are custom made for executing tasks originating at a single source. In contrast our system allows sharing CPU cycles between peers and running jobs from multiple users in the network.

The Condor [6] project aims at utilizing distributed computing resources in a network to provide high throughput. A mechanism called ClassAd is used to advertise attributes of available resources and jobs. Condor acts as a broker between the resources and the jobs, using the attributes provided to it. Similar to our system, this provides an open environment in which multiple users can submit jobs. However, task management in Condor is centralized, which makes the system fragile in the face of high node departures and failures. Our main emphasis is to architect allocation and communication mechanisms that yield high efficiency and are robust in the face of high node departure and failures. More importantly, they are simple to implement and incur virtually no overhead in topology maintenance. Consequently, many real-world large-scale peer-to-peer networks are unstructured. However, the lack of a well-defined topology in the overlay and completely decentralized control. Each node in the network maintains a list of hosts it can connect to. The rules for connecting to peers and to maintain this list are usually heuristic-based and do not enforce a structured topology in the resulting network. Some nodes in the network are designated as (or choose to be) super-nodes. Super-nodes have large degrees and connect to low degree leaves and other super-nodes. The emergence of self-organized network graphs has a highly skewed degree distribution with few nodes having high degrees while most nodes have only a few neighbors. Furthermore, our system can provide a distributed CPU sharing system on top of an unstructured P2P network, motivated by the success of massive unstructured networks for file sharing and many are connected to few.

Our work can also be considered complementary to much of the work on grid computing including, Globus [10, 8], Legion 16, Avaki 11, Purdue University Network Computing Hub (PUNCH) [14] and Sun's Grid Engine [27]. Each of these systems implement a centralized or distributed control. Each node in the network maintains a list of resources and the jobs, using the attributes provided to it. In contrast we build our system on top of an unstructured P2P network, and describe a simple randomized job allocation scheme that achieves good load balance. We also motivate the need for redundancy, in the context of target applications and show how our protocol caters to replicating tasks in the network. A key aspect of our P2P cycle sharing environments is a decentralized rendezvous service for monitoring job progress, supporting loosely coupled inter-task communication, and aggregating completed tasks. We describe the distributed construction and the probabilistic guarantees on the performance of this service. We also show how our architecture can be leveraged to manage reputation of participating hosts in the network as a means to counter "free-riders" in the system.

3 Architectural Overview

In this section we provide a brief overview of unstructured P2P networks and describe a simple randomized job allocation scheme that achieves good load balance. We also motivate the need for redundancy, in the context of target applications and show how our protocol caters to replicating tasks in the network.

3.1 Unstructured Peer-to-Peer Networks

Unstructured P2P networks are characterized by lack of well-defined topology in the overlay and completely decentralized control. Each node in the network maintains a list of hosts it can connect to. The rules for connecting to peers and to maintain this list are usually heuristic-based and do not enforce a structured topology in the resulting network. Some nodes in the network are designated as (or choose to be) super-nodes. Super-nodes have large degrees and connect to low degree leaves and other super-nodes. Mutual, self-organized network graphs have a highly skewed degree distribution with few nodes having high degrees while most nodes have only a few neighbors and are often connected to the super-nodes. Peers are connected to many, and many are connected to few. Such networks are often referred to as "small-world networks." These networks have some highly desirable features such as high diameter and resilience to random failures and frequent node arrival and departures. More importantly, they are simple to implement and incur virtually no overhead in topology maintenance. Consequently, many real-world large-scale peer-to-peer networks are unstructured. However, the lack of a well-defined topology makes it difficult to locate resources and there are no guarantees of finding an object that might exist in the network. In such networks, the naive method for locating resources is by flooding the query to a predetermined number of hops. This approach generally has high overhead in terms of network messages.

In this paper we build our system on top of an unstructured network, and present novel algorithms that have low overhead, and at the same time provide performance guarantees with high probability (w.h.p).

\[ \text{w.h.p} \quad \text{implies that the probability of failure is } \frac{1}{2^N}. \]
3.2 Job Allocation with Redundancy

We first present a simple job allocation strategy that achieves appropriate redundancy and good load balance. Conventional unstructured P2P networks comprise of tens, even hundreds of thousands of nodes. Consequently, computational resources exist for building sufficient redundancy into the system. There are two main motivating factors for redundancy:

1. Resilience. In an open Internet environment failures (and departures) of nodes must be expected. In such an environment replication of the same task to multiple hosts is needed to account for failure of nodes.

2. Validation. We can expect that some of the nodes would return wrong results, either because of malicious or other reasons. Results from several nodes can be cross-checked to detect faults, and possibly select a correct output (e.g., simply using majority) from the available set of reported results. Indeed, several current systems such as SETI@home use similar methods.

We assume that a job, J, can be broken down into n independent subtasks. We denote, using N, the number of Internet hosts that are engaged in computing the job J. The subtasks of J can be clustered into batches (b1, b2, . . . , bl, each with K subtasks). We discuss various considerations while choosing K later. While submitting a job each batch is replicated by a factor r ≥ 1. For example, r = 2 implies that two nodes will be assigned the same batch to compute.

A simple randomized job submission algorithm that allows replication can be constructed as follows:

1. A host, A that wants to submit subtasks of its job sets the batch size K and a replication factor r.

2. For each batch, host A selects a node uniformly at random by performing a random walk and submits a batch to it. The replication factor is r is sent with the batch.

3. Each node that receives a batch decrements r by one and if r > 0 sends a copy of the batch to another node chosen uniformly at random. The updated value of r is sent with the batch. Thus, each batch is replicated at a total of r nodes in the network.

Note that the number of messages sent by A remains n/K since redundant application is taken care of by nodes downstream. Similarly, note that replication of each batch occurs in parallel. However, the total number of messages in the network is given by nr/K. In real-world situations this translates to reducing the cost of job submissions for master nodes. Any node failures during this process are handled by re-submitting jobs until r replicas exist.

Several jobs can be initiated on the network concurrently. Since our job allocation protocol is based on randomization, a node may be assigned more than one batch, either of the same job or of different jobs. A node processes batches one at a time on a first come first served basis. We would like to emphasize that this randomized protocol is extremely simple, does not maintain any state information, and has a minimal overhead arising from random walks for uniform sampling.

3.2.1 Uniform Sampling and Load Balancing

Assume that a total of m batches need to be assigned to the N processors. If we assign batches uniformly at random to the N processors, we can provide bounds on the quality of load balance achieved. Given m batches, we answer the following questions:

1. What is the expected fraction of nodes that will have a batch assigned to them?

2. What is the probability that a node gets a given number of jobs to perform?

3. What is the maximum load on a node, with high probability?

The arguments presented here suggest that using m = N log N provides good utilization of the network w.h.p., and at the same time yields a low probability of high load imbalance.

Lemma 3.1 Given m batches the expected fraction of processors that will have a batch assigned to them is 1 − e−m/N.

Proof: We consider the question, how many processors do not have tasks assigned to them? The probability that a given processor did not get a task, when m tasks are distributed is given by:

\[ P(X = 0) = \frac{N!}{m!(N - m)!} \approx e^{-m/N} \]

assuming that N is large. We define random variable X, which is 1 if the ith host did not get any batch, and 0 otherwise. Then by linearity of expectation:

\[ E[X] = E\left(\sum_{i=1}^{N} X_i\right) = \sum_{i=1}^{N} E[X_i] = N(1 - \frac{1}{N})^m \approx N e^{-m/N} \]

Thus, the fraction of nodes that will get at least one batch to process is \( N - Ne^{-m/N} \)/N = 1 − e−m/N. □
This lemma simply implies that if $m \leq N$ at most, approximately, 65% of the nodes will be used. Similarly, if $m = N \log N$, with high probability all nodes will be used.

Now we find the probability that a given node gets $M$ batches to process. Given that $m$ and $N$ are large compared to $M$, this probability is given by:

$$P(m \leq M) = \frac{1}{M} e^{-m/M}$$

Similarly, the probability that a node gets at least $M$ batches is given by:

$$P(m \geq M) = 1 - e^{-m/M}$$

**Lemma 3.2** If $m = \Omega(N \log N)$, the probability that the maximum load on any node is more than $\Omega(\log N)$, is low.

**Proof:** For simplicity, we set $m = N \log N$ and maximum load to be $M = e \log N$. The probability that any node has at least $e \log N$ is bounded as follows:

$$P(m \geq M) = \frac{N}{N \log N} \left(1 - e^{-M/N \log N}ight) \leq \frac{1}{N}$$

**Theorem 3.1** When $m = \Omega(N \log N)$ the load imbalance on a node diminishes.

**Proof:** A perfect deterministic algorithm allocates $O(\log N)$ batches to each machine, when $m = \Omega(N \log N)$. From the previous lemma we know that the probability that the maximum load on a machine is $\Omega(\log N)$, is low. Therefore, a randomized algorithm that uses uniform sampling to distribute tasks approaches a deterministic algorithm, and yields good load balance.

### 3.2.2 Choosing Batch Sizes

Selecting an appropriate batch size in a distributed environment is a challenging task and depends on several variables. Krukel and Weiss [15] show that when the running time of the subtasks are independent and identically distributed (i.i.d.) random variables with mean $\mu$ and variance $\sigma^2$, then estimated completion time is given by:

$$E(T) = \frac{\mu}{p} + \frac{\mu \sigma^2}{pK} + \sigma \sqrt{2K \log p}$$

It is assumed that $K/\log K$ is large, and for smaller values the error is not substantial. This expression is quiet general and holds for processing time distributions including exponential, gamma, Weibull, uniform, deterministic, and truncated normal distributions. The variance in the time required to complete the processing of a task, $\sigma$, depends on the following parameters:

1. The number of concurrent jobs, scheduled on the processors. This corresponds to the number of subtasks allocated to each processor, and depends on the load balance achieved by the job submission algorithm.
2. The processing capability of participating hosts.
3. The variation and non-deterministic nature of processing requirement of each subtask.

Each term in Equation 3.1 has important implications. The first term corresponds to the time it would take to aggregate all jobs and perform the required computation. The second term captures the benefit from aggregating jobs in terms of reducing the communication cost. The final term represents the overhead due to uneven finishing times of the individual tasks. The most important implication of this result is that the tradeoff between communication, which decreases as $1/K$, and the variance in processing times which increases as $\sqrt{K}$.

In the above discussion there is no mention of the failure (and departure) of the nodes in the network. There is an important relationship between the size of the batch and the lifetime of a node. As a simple illustration consider a batch of size $n$ and $m = \Omega(n \log n)$. The load imbalance on such nodes implies that it would take much longer to complete the jobs, which further reduces the set of nodes that have suitable lifetimes. Furthermore, being able to use the long lived nodes would require a job allocation scheme that maintains state information. This can be prohibitively expensive in large dynamic networks. Our use of a randomized strategy avoids these overheads, but at the same time is susceptible to failure under the conditions discussed. Thus, a batch size should be small enough, so that the required time for processing the batch is comparable to the lifetime of the node. Lifetime of a node is also important in the context of developing a replication strategy as discussed in Section 3.2.3.

In summary, the size of a batch, $K$, should be:

1. Large enough so that the network overhead is reduced.
2. Small enough so that total job completion is minimally affected by variation in processing times taken by hosts, $\sigma$, and
3. Small enough so that computation time required for a given host is compatible to the host's lifetime.

Furthermore, the results from Section 3.2.1 suggest that if the total number of tasks to be computed in the network (from all jobs) is $n$, then the number of batches, $n' \geq N$. 


3.2.3 Multi-Step Replication

We revisit the replication strategy keeping in view the fact that increased replication at submission time implies increased time to completion of the jobs. Increased makespan of a job implies that more nodes would leave the network during that time, conditioned on the distribution of lifetime of the nodes in the network.

Let, the number of nodes leaving the network, over unit time be \( y \). We denote the time for completion of \( n \) tasks by \( T(n) \). As defined earlier, \( r \) denotes the number of replicas of a given task in the network. Recall that the protocol given earlier in this section performs replication at job submission time. Then, \( n r = T(n) \), gives the number of nodes leaving the network during the makespan of the job. The level of replication to deterministically counter the failures would require \( r = \frac{n}{y} \). This in turn assures, the stability condition: \( y = \frac{n}{T(n)} \leq \frac{1}{T(n)} \), i.e., at most one node failure over the job makespan. This stability requirement is degenerate.

Thus, a job submission protocol should use a multi-step replication strategy instead of replication-at-initiation. This algorithm is well suited for a high node failure rate environment. The protocol works as follows:

1. A host, \( A \), that wishes to submit subtasks of its job, sets the batch size, \( K \). The replication factor \( r \) is set to one.
2. For each batch, host \( A \) selects a node uniformly at random, by performing a random walk, and submits a batch to it.
3. Host \( A \) also calculates \( T \), which is the time it would take to complete an ideal parallel ensemble to complete the job, as \( T = \frac{n}{y} \). It then waits for a time \( T \), where \( T \) is the expected efficiency of our distributed system without failures. This can be estimated to be roughly 40% using Kruskal's equation for realistic Internet environments.

4. After waiting for this time period, it collects the jobs that have been computed using the algorithm presented in Section 3.3.2.
5. Host \( A \) determines the jobs that did not run to completion and resubmits them with \( r = 2 \). If \( n_f \) jobs were missing \( T \) is calculated again as \( \max(r - n_f/N, 1) \mu \). The waiting and re-submission, with \( r \) incremented at each step, continues until results for all tasks are retrieved.

In Section 5, we show using simulation that this multi-step protocol in fact executes significantly faster than the replication-at-initiation strategy when this strategy uses more than 3 replicas. However, the key advantage is that this algorithm achieves 100% job completion, which is an important characteristic of any feasible service.

3.3 Rendezvous Service

The rendezvous service provides a communication fabric between nodes that are oblivious to each other. The key used for communication is a resource identifier or a resource query string, rather than a node address. This is analogous to the directory service in a client-server architecture. However, unlike its client-server model based counterpart, rendezvous service does not have a centralized repository and the peers need not know or register with any predefined server node. The required information is maintained in a completely distributed fashion among the peers.

3.3.1 Construction

Nodes become a part of the rendezvous service by creating a "rendezvous service set" (RS-Set). The RS-set of each node contains pointers to \( V \log N \) peers selected uniformly at random. The creation of the RS-set at each node occurs in a distributed fashion asynchronously. Each node is responsible for maintaining \( V \log N \) live peers in its RS-set, when there are node failures. The provider of a resource publishes its resource identifier to its RS-set. Similarly, the node looking for this resource sends a query to its own RS-set. If there is an intersection between RS-sets of the producer and the consumer then, the consumer can access the resource.

**Theorem 3.2** Any two RS-sets of size \( V \log N \) nodes intersect w.h.p.

**Proof:** Since the RS-sets contain nodes selected uniformly at random, the probability that a given node of one RS-set is not in the other RS-set is given by \( 1 - \frac{V \log N}{N} \). Thus, the probability that none of \( V \log N \) nodes of one RS-set are in the other RS-set is \( \left( 1 - \frac{V \log N}{N} \right)^{V \log N} \approx \frac{1}{N} \). The probability that at least one node in the two sets intersect is \( 1 - \frac{1}{N} \).

6
Note that this service has very low overhead. The overhead of creating the RS-set is amortized over the life of the node. Similarly, only V(N log N) messages are required for each query. We can compare this with flooding, where the number of network messages increases exponentially with the number of hops. In the rest of this section, we show how this service is used in our architecture.

3.3.2 Monitoring and Merging Jobs
When a node completes the execution of a batch, it informs its RS-set using the job identifier. With high probability, one of the nodes in the RS-set of this node also belongs to the RS-set of the owner of the job. Such an intersecting node receives the results. Each of the nodes in the RS-set of job-owners maintains independent bit vectors, where the jobs received by them are marked. The owner requests these vectors from its RS-set, once in a given interval. It may also download the results for the completed tasks. Alternatively, the owner may ask its RS-set nodes to submit completed jobs to it once a certain number of new results are available. If replication factor r > 1 is used in submitting jobs the owner would get multiple copies of the same job. This information can be used to verify that the results match, and thus provide a validation mechanism.

3.3.3 Reputation Monitor
The RS-set of the master receives results computed by all participating nodes. Some free-riders may not compute the tasks given to them, or similarly, some malicious users might return wrong results. The node’s RS-set maintains information about the results submitted by its owner. Similarly, the owner of the job publishes the information (ID) about the nodes that reported inaccurate results. If node x wants to query the reputation of node y, it simply inquires its RS-set. The overlap between the RS-sets reveals the required information. Using this information, nodes may reject jobs coming from free-riders and malicious users, hence discouraging such activities. This system is robust to collaborative malicious activity since a large number of nodes (2 RS-sets) keep this information. Such systems have been proposed in the context of conventional file-sharing P2P networks and have been shown to handle free-riders in a scalable manner.

3.3.4 Decoupled Communication
A node might need to search for information without knowing which node has the required information. For example, a node processing a task may need to know the result of some preceding task. Such cases may arise if the subtasks of a job are not completely independent. Similarly, several other examples arise for various scenarios, a node needs to send a query about the requested resource to its RS-set and with high probability it will intersect with the RS-set of the provider of the resource.

4 Uniform Sampling With Random Walks
In this section we introduce random walks and show that how they can be used to perform random sampling. If the underlying network does not have a regular degree distribution, i.e., if few nodes are connected to many, and many nodes are connected to a few nodes, then a random walk, with transitions from a node to its randomly chosen neighbor, does not yield a uniform sample. We revisit Kruskal’s equation and argue that a skewed sampling results in a bad load balance and a long job make-up. We show how the transition of the walk from one neighbor to another must be modified to achieve a uniform sampling, and give an algorithm that computes the required transition probabilities for random walks.

4.1 Sampling With Random Walks
Random walks can be abstracted as Markov chains defined over a state space and a given state transition matrix. The network nodes form the state space and the probability of moving from a node to its neighbor form the transitions. Using a Markov chain model, we show in this section, that (1) a random walk of a given minimum length on a connected aperiodic graph (which represents the network) reaches a stationary node sampling distribution, and (2) a simple random walk cannot achieve uniform sampling unless each node in the network has an identical number of connections. We also discuss various parameters that determine the length of the random walk required to achieve a stationary sample distribution.

Let G(V,E) be a simple connected undirected graph representing a distributed system with [V] = N nodes and [E] = e links. The degree, or number of links, of a node i, 1 ≤ i ≤ N, is given by d_i. The set of neighbors of a node i is given by F(i), where edge (i,j) ∈ E, Vj ∈ F(i). The N × N adjacency matrix of G is given by A = [aij], where 1 ≤ i,j ≤ N, a_i,j = 1 if the edge (i,j) ∈ E, and 0 otherwise. The corresponding N × N transition probability matrix, given by P = [pij], is the probability of moving from node i to a node j in one hop. P is a row-stochastic matrix, i.e., Σ_j pij = 1.

For a simple random walk the transition from node i to its neighbor is governed by the transition probability matrix P, where ∀ j ∈ F(i), pij = 1/|i| and 0 otherwise. The
sequence of nodes can be denoted as \( X_t = \{X_t, X_{t+1}, \ldots \} \), where \( X_t = i \) implies that at step \( t \) the walk is at node \( i \).

If we consider nodes in \( G \) as states in a finite state space, then the random walk represents a discrete-time stochastic process, \( \{X_t\}_{t \geq 0} \). For this stochastic process we have:

\[
Pr(X_{t+1} = j | X_0 = i, \ldots, X_{t-1} = k, X_t = l) = \begin{cases} 
Pr(X_{t+1} = j | X_0 = l) = p_{lj} 
\end{cases}
\]

Equation (2) simply implies that a random walk is memoryless, i.e., during a random walk the probability of transition from node \( i \) to node \( j \) in one step depends only on node \( i \). Thus, a random walk can be conveniently modeled as a Markov chain, more specifically a homogeneous Markov chain, since the right hand side of Equation (2) is independent of \( t \).

It is well known that an irreducible and aperiodic Markov process, \( \{X_t\} \), shows that the component sequence of nodes can be denoted as a Markov chain, more specifically a homogeneous Markov chain has the following properties:

1. It is irreducible if the graph \( G \) is connected and is aperiodic. A graph \( G \) is aperiodic if the greatest common divisor of the length of all cycles in the graph is 1. In particular, an undirected aperiodic graph cannot be bipartite, which is a reasonable assumption for real networks in which connections are established randomly.

4.1.1 Convergence to Random Sampling

It is well known that an irreducible and aperiodic Markov chain has a stationary distribution \( \pi = \pi P \), where \( P \) follows where \( P \) implies \( t \)-step transition. It is easy to show (21, page 132) that \( \pi_i \), the component corresponding to \( i \), is equal to \( \pi_j \), the component corresponding to \( j \), since the right hand side of Equation (2) is independent of \( t \). Thus, a Markov chain has the following properties:

1. It is irreducible if the graph \( G \) is connected and is aperiodic. A graph \( G \) is aperiodic if the greatest common divisor of the length of all cycles in the graph is 1. In particular, an undirected aperiodic graph cannot be bipartite, which is a reasonable assumption for real networks in which connections are established randomly.

4.1.2 Length of Walk for Random Sampling

The sample distribution at step 1 of the walk depends on \( P \), which in turn depends on the eigenstructure of \( P \). From the Perron-Frobenius theorem, we have

\[
P = \lambda_1 v_1 v_1^T + O(t^{\alpha-1})|v_1|^2.
\]

where \( v_1 \) is the right eigenvector corresponding to eigenvalue \( \lambda_1 \) and \( p_{11} \) is the algebraic multiplicity of \( \lambda_1 \). Rewriting the above equation, we have

\[
P = \lambda_1^t + O(t^{\alpha-1})|v_1|^2.
\]

These results imply that

\[
P = \lambda_1^t + O(t^{\alpha-1})|v_1|^2.
\]

As \( |v_1|^2 < 1 \), when \( t \) is large, \( \lambda_1^t \) is the second largest eigenvalue modulus (SLEM), the larger the second largest eigenvalue modulus (SLEM), the faster the convergence to the stationary distribution. As a result, a walk of smaller length is required for random sampling. The length of the required walk, or the mixing time, is often approximated as \( O(\log N) \) [18], however the exact factors involved depend on the construction (and thus the SLEM) of the transition probability matrix.

4.2 Uniform Sampling in Nonuniform Networks

As mentioned in Section 4.1.1, a random walk of a given minimum length converges to a stationary distribution \( \pi \). If the stationary distribution \( \pi \) is uniform, i.e., \( \pi_i = \frac{1}{|G|} \) for all \( i \), then the Kruskal's equation would be written as:

\[
E(T) = \frac{\alpha}{p} + \frac{ab}{pK} + \sqrt{\frac{\alpha}{p}} + t^2/2(\log p).
\]

This directly impacts the expected running times of jobs. Note that variance of the degree of the nodes may be very high if the underlying graph follows power-law degree distribution. In Section 5.2.1, we experimentally demonstrate that using simple random walks for sampling in nonuniform networks yield a poor load balance, and consequently a large job turnaround time.

4.2.1 Modifying Transition Probabilities

To achieve a uniform stationary distribution in an irregular graph, we need to modify its probability transition matrix.
Let $P$ be a probability transition matrix of a Markov chain, then $\pi_{\text{uniform}} = \pi_0 p$, which is the same as $\pi = N^{-1}P$. This means that the sum of each column vector of $P$ is 1, i.e., $P$ is column stochastic. A probability transition matrix which is column stochastic in addition to being row stochastic is called doubly stochastic. Note that symmetric transition probability matrix are doubly stochastic. Thus, if we create a matrix with $P_{ij} = p_{ji}$ we will achieve a uniform stationary distribution, and hence a random walk using these transition probabilities will yield a uniform sample.

Two well-known algorithms, maximum-degree algorithm (MD) [2] and Metropolis-Hastings algorithm [20, 13], yield a symmetric transition probability matrix. However, these algorithms need a long walk to reach stationarity, if the graph has a highly skewed degree distribution. In our previous work [2], we present a detailed discussion and experimental evaluation of these algorithms, and suggested a new algorithm for building a transition matrix. Here, we reproduce the algorithm, however, the details and evaluation are omitted.

### 4.2.2 Random Weight Distribution Algorithm

In this section, we present our distributed algorithm, referred to as the Random Weight Distribution (RWD) algorithm. RWD is a completely decentralized algorithm that sets up transition probabilities in a connected network to enable efficient uniform sampling via random walks.

The algorithm proceeds as follows. In the initialization phase each node, locally, sets transitions probability as:

1. **Initialization**
   1. $N := r(i)$
   2. $\alpha := \text{Quantum}$
   3. $P_{ij} := \frac{1}{\max(d_i, \alpha)}$
   4. foreach $j \in r(i)$ repeat
      5. $P_{ij} := \frac{1}{\alpha}$
   6. end foreach

   Here, $\alpha$ is a static system parameter with the constraint that it should be greater than $\max d_i$. This parameter is static because we can sufficiently overestimate $\max d_i$ knowing system properties (e.g., popular P2P clients have a maximum connection limit [17]). Note that this phase results in a high self-transition probability for low degree node. Also note that the resulting transition probability matrix is symmetric.

   After the initialization is complete, each node attempts to distribute its self-transition probability randomly and symmetrically to its neighbors. The term weight of a node refers to the self-transition probability of the node at any given time during the execution of the algorithm. At a node $i$, the algorithm terminates when either the weight of the node becomes zero or the weight of all nodes $j \in r(i)$ becomes zero. Intuitively, a low self-transition probability implies that the walk mixes faster and converges to a stationary uniform distribution with a fewer number of steps. The pseudo code for the complete RWD algorithm is shown in Figure 1.

![Figure 1. The Random Weight Distribution algorithm.](image)
son job allocation using simple random walk based sampling yields a highly skewed load balance. A comparison of the efficiency achieved in this case is plotted in Figure 2. As predicted the system has the highest efficiency when $K = \log N$ and the number of batches is $\log N$. The efficiency achieved in this case is 43%, which is excellent for such loosely coupled dynamic environments. We expect that in the real world there would usually be enough job submission requests to meet the optimum value of number of tasks in the system.

### 5.2 Efficiency and Computational Throughput

We study the efficiency of our system in comparison to an ideal $N$ processor parallel ensemble, which computes $n$ tasks in $\mu N$ time. As stated in the discussion in Section 3.2, if $N \log N$ batches are submitted then, w.h.p, each processor has a task to perform. Similarly, if possible we would like to have batch sizes approaching $K = \log N$ tasks (which translates to an optimal $n = N \log N$). In the cases where $n/K < N$ we use $K = \max(n/N, 1)$. To evaluate the efficiency of our system for different job sizes and batch sizes we use the parameters described in Table 1. Note that although we would like to have $N \log N$ batches each time, due to substantial network overhead we prioritize larger batch size over the number of batches that can be submitted. As a result when there are only 10,000 batches, $\approx 30\%$ of the machines in the network had no tasks to process. This is consistent with the predictions from Section 3.2.1. The resulting efficiency of the system is plotted in Figure 2.
To compare the load balance of the two schemes we use the optimal parameters $n = \frac{N \log^3 N}{N}$ and $K = \log^2 N$. The resulting percentage of the total number of tasks assigned to each machine are plotted in Figure 3. The $x$-axis of the plot represents nodes sorted in ascending order by degree. On the other hand the number of tasks assigned to a machine by sampling using SRW is biased to the degree of the node. Thus, some nodes end up receiving almost 5 times higher load than other nodes.

A load imbalance implies that the job would take longer time to finish. We compare the system efficiency when batches are allocated with sampling using the two techniques. The number of jobs used are $n = \frac{N \log^3 N}{N}$, $\frac{N \log^2 N}{N}$, and the cluster sizes used in both cases is $\log^2 N$. The plot in Figure 4 shows this comparison. The system using RWD performs much better irrespective of the loads assigned to it and the performance advantage increases as the load increases.

5.3 Performance Under Node Failures

An important aspect of our architecture is its performance under node failures. The key parameters of interest are: percentage of tasks that are successfully retrieved by the owner of the job, increase in completion time due to replication, and performance under varying failure rates. For these experiments we use $K = \log^2 N$, $n = \frac{N \log^3 N}{N}$ and $N = 10,000$ nodes.

5.3.1 Node Failure Model

Node failure is modeled using a parameter $\alpha$, which represents the fraction of nodes that fail in the time it would have taken for the job to complete in an environment without failure. This definition is useful for modeling node lifetime in comparison to the lifetime of the job. Node lifetimes are modeled as Zipf random variables which are correlated to the degree of the node. The parameter $\alpha$ is used to normalize these lifetimes in relation to the job makespan time. The number of nodes in the network is kept roughly constant by matching the arrival and failure rates.

5.3.2 Evaluation

We compare the performance of the two schemes described in the paper, namely replication-at-initiation and multi-step replication. For the replication-at-initiation method we use a replication factor of two (i.e., two copies of the replicated processes are submitted to the system). The effect of higher replication levels is discussed later. We let the owner query the system (through the rendezvous service) for its running tasks, often enough so that it knows almost instantaneously if no more of its tasks are running. This is unrealistic in a real-world environment, but our results here are meant to provide a bound on how well the system can perform. For the multi-step replication method the system assumes an ef-
The plot in Figure 5 shows the percentage of unique tasks successfully retrieved by the owner, for the replication-at-initiation scheme. As the fraction of failing nodes increase, the success ratio decreases rapidly. In comparison the multi-step protocol achieves a 100% completion rate.

The plot on the bottom, in Figure 6 shows the average number of replicas found. The trends seen are interesting, because the average number of replicas found by the replication-at-initiation scheme decrease because of increased failures. On the other hand for the multi-step scheme, each iteration of replication submission increases the replication rate.

The plot on the top, in Figure 6 shows the average number of replicas submitted is higher and thus the average number of replicas received is higher. Quantitatively, the multi-step replication scheme has fewer redundant replicas and hence use resources more efficiently. However, as discussed earlier the replicas received can be useful for validating results.

Nevertheless, the multi-step scheme can be easily modified to increase the redundancy in a controlled fashion, simply by submitting replicated tasks even after one result for that task has been successfully received.
To evaluate the benefit of using different levels of replication in the replication-at-initiation scheme, we repeat the experiment with $\alpha = 0.5$, while varying the replication level. The number of replicas submitted is increased from 1 (i.e., no redundancy) to 9, i.e., $10^9$. The plots in Figure 7 summarize the results. The plot on the top shows that increasing the replication level results in the number of tasks completed to asymptotically approach 100%. However, a perfect result is not achieved even with very high replication levels. The plot in the middle shows that average number of results retrieved from replicas approaches 50% of the number of replicas submitted. This is consistent with the failure fraction $\alpha = 0.5$. When more replicas are submitted, there is a higher chance of reaching a long lived node. Due to the zipf distribution, such long lived nodes might have a much higher lifetime compared to the average. Finally, the plot at the bottom shows that the time for completion increases as the number of replicas increase. An important contrast can be drawn here to the performance of the multi-step technique. For $\alpha = 0.5$, the time taken by level replication, which achieves 99.2% completion, is almost 2.4 times more than the multi-step technique, which always achieves 100% replication.

6 Conclusion

In this paper we presented a distributed architecture for sharing processor cycles in unstructured P2P networks. The use of unstructured P2P networks is motivated by the success of massive real-world networks for file sharing. We present randomized algorithms for allocating tasks in the network, which achieve a good load balance and low job makespan. We analytically show that random job allocation using uniform sampling achieves good load balance. We present two protocols that incorporate redundancy for resilience against frequent node departures, and validation of the execution output. The parameters that affect job throughput are discussed in the context of our allocation scheme. Our architecture includes a rendezvous service that allows job progress monitoring, aggregation of tasks, node reputation management, and context-based communication between oblivious hosts. We show that the rendezvous service provides probabilistic guarantees for locating resources.

Our algorithms are built on the premise of uniform sampling in an unstructured network. We show that random walks are ideal for random sampling, however, the resulting samples are affected by the topology of the network. We present an algorithm that allows uniform sampling via random walks irrespective of the underlying network topology. This is done by building a transition matrix for the walk in a distributed fashion. The resulting transition probability matrix also reduces the length of the random walk required to converge to uniform stationarity. The efficiency of the resulting cycle sharing system is evaluated using comprehensive simulation. The system is also evaluated with varying rates of node failures. The simulation results reflect the efficiency and robustness of our randomization based protocols.
References


