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UNDERSTANDING THE IMPACT OF SHILL BIDDING IN ONLINE ENGLISH AUCTIONS

ABSTRACT

Increasing popularity of online auctions and the associated frauds have drawn the attention of many researchers. It is found that most of the auction sites prefer English auction to other auction mechanisms. The ease of adopting multiple fake identities over the Internet nourishes shill bidding by fraudulent sellers in English auction. In this paper we derive an equilibrium bidding strategy to counteract shill bidding in online English auction. Due to mere fear of cheating, the buyers may deviate from their normal behavior. Thus, there is a chance that an honest auctioneer may suffer from the loss of revenue because of lack of bidders' faith on him. Sometimes an honest bidder has to pay more due to unfair bidding practices. It is important to see which auction is most suitable from bidder's and auctioneer's point of view in cheating environment. We also make a comparison of honest bidder's expected gain and honest auctioneer's revenue loss for three important types of auctions: English auction, first price sealed-bid auction, and second price sealed-bid auction. The analysis of the results reveal that English auction should be the most preferred mechanism from both honest buyer's and honest seller's point of view. This fact can be used to explain the popularity of English auction over the Internet.

Keywords: Online English auction, Shill bidding, equilibrium bidding strategy, buyer's expected gain, seller's expected revenue loss.

INTRODUCTION

Online auctions account for a large volume of economic activities over the Internet. Lack of security has created a conducive environment for adopting unfair practices by bidders and auctioneers. Internet Fraud Complaint Center (IFCC) reports that Internet auction fraud comprises 64% and 46%of referred complaint in the years 2001 and 2002 respectively [1]. IFCC classifies auction frauds into six categories: Non-delivery of goods, miss representation of the items, triangulation, fee staking, selling of black-market goods, multiple bidding and shill bidding [2]. The last two categories of fraud are termed as cheating.

As noted by Chui and Zwick [3] most auctions that run over the Internet are English outcry type. Shill bidding is an age-old problem in traditional English auctions. Shill bidding is intentional fake bidding by the seller to drive up the price of his/her own item that is up for bid. This is accomplished by the seller himself or by someone colluded with the seller. We consider the problem of shill bidding in English auction from three different perspectives.

Firstly, when a bidder is aware of shill bidding he may show unanticipated behavior. For example, a bidder may bid a value much less than his personal valuation of the item to avoid the trap of possible shilling. Such low bid values may obstruct his winning process. Thus an important question is - what should be the equilibrium bidding strategy of an honest bidder to fight against shilling?

Secondly, when a buyer has the option for buying an item from different sites through different auction mechanisms he will be in a dilemma to select the appropriate mechanism which can help him to fetch the item in the lowest possible value. All the auction sites are prone to cheating. Thus, in spite of its popularity – Is cheating English auction is the best from bidder's perspective?

Thirdly, due to the lack of privacy and security measures in the online auction sites the bidders will always have the fear of cheating. Such imaginary fear of the bidders can lead to a revenue loss for an honest auctioneer. As noted by Porter and Shoham [4], even if the auctioneer does not cheat, the mere fear of cheating by the bidders result into a loss of revenue. An auctioneer also has the freedom to choose a site with appropriate auction mechanism to sell his item. In such an environment of insecurity and doubt – Should an honest auctioneer adopt English auction as the mechanism to sell his item?

This paper addresses the above questions in the following way:

(1) Developing an equilibrium bidding strategy for the bidders in the English auction when there is shill bidding.
(2) Comparing an honest bidder's expected gain in three important types of auction (English auction, First price sealed-bid auction, and Second price sealed-bid auction).
(3) Comparing an honest auctioneer's revenue loss (when compared to revenue from the optimal auction) for the above three types of auctions.

We refer the work of Porter and Shoham [4] for understanding bidders' behavior in cheating first price and second price sealed-bid auction. We use the same setting used by them for understanding shill bidding in English auction and make the three types of auctions comparable.

The rest of the paper is organized as follows. First various types of auctions found in the real world and the basis of these variations are discussed. Followed by a discussion on cheating in electronic auction which includes a taxonomy of cheating and types of cheating possible in three important auction mechanisms. Next an equilibrium bidding strategy in English auction is derived. Using this strategy as the winner's bidding strategy, in the subsequent two sections, three types of auctions are compared from bidders' and auctioneer's point of view. Before concluding the paper, a survey of the related works that has motivated this research is presented.

INTERNET AUCTIONS

McAfee and McMillan [5] define an auction as a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participant. An auction may consist of three basic activities: receiving bids, supplying intermediate information and clearing. Any auction at least has the first and the third activities. Wurman *et al* [6], classify auctions by a set of orthogonal features: ratio of buyer-seller, duration time, closing conditions, settlement price, and information revealed. Ratio of buyer-seller determines whether an auction allows multiple buyers or sellers. Three possible combinations are: one to many, many to one, and many to many. A restriction to "one" indicates that the auction is single-sided (i.e. existence of sole buyer or seller). An auction with many to many buyer-seller ratio is called double-sided auction. Duration time classifies auctions into single-round and multi-round auctions. At the end of each round some matching algorithm produces an allocation. Closing conditions determine when an auction ends. Auctions could close when a pre-specified time is reached, after a period of inactivity, or when a reserve price is reached. Settlement price is the price that a bid winner pays for the auction. The policies determining settlement price can be the Mth and (M+1)st or chronological matching policies [6]. The first and second price policies used by English and Second price sealed-bid (Vickrey) auction respectively are special cases of Mth and (M+1)st price policies. Information revealed determines what information is disclosed during and after the auction. The revealed information can be price quote, order book, transaction history, and so on [7]. Price quote informs participants of the hypothetical auction clear result. Order book refers to the current set of active bids. Transaction history is the selected publicized information about past transactions, including the prices, quantities or even the identities of the transaction agents. Auctions revealing no intermediate information are called "sealed-bid" auctions. Based on these features approximately 25 million types of auctions are possible. We concentrate on single side auctions.

There exist three broad categories of single sided action mechanisms: English auction, sealed-bid auction and Dutch auction. In English auction known as open cry auction, the price of the product increases with time as bidders compete with each other. At the end of the auction the highest bidder can take the item after paying the price he bids. This kind of auction has many disadvantages: (1) Time to conduct the auction is very high. (2) Many round of communication needs to take place and (3) Reveals maximum amount of information to both bidder and the auctioneer. This however associates economic advantages to both the auctioneer and the bidders. Auctioneer gets the highest possible value for the item. The bidder who is most interested to buy has a chance to out bid the competitors. Dutch auction on the other hand is decreasing price in nature. The auctioneer sets an expected price for the product and decreases the price at each time unit till some bidder bids. The first bidder takes the item. This auction type reveals the least amount of information and is most privacy preserving. This however is very time consuming and not economically efficient for the both auctioneer and the bidder.

In case of sealed-bid auction each party sends a sealed-bid for the item to an auctioneer who opens all the entire bids after a predefined time period. The highest bidder gets the item. There are two variations of sealed-bid auctions differentiated by the settlement price policy. In case of first price sealed-bid auctions the highest bidder pays exactly the amount he bids where as in case second price sealed-bid auction the highest bidder pays the amount of the second highest bid. The second price auction is otherwise known as Vickrey Auction based on the name of its designer William Vickrey [8]. The advantages are: (1) Time preserving as only two rounds of communication take place. (2) In case of second price auction the most interested party gets the item in most economic terms.

In the real world most of the important auctions are sealed-bid in nature. On the contrary most Internet auctions are of open out cry type. According to [3], about 88% Internet auctions are English auction and its variants (Straight and Yankee auctions). English auction and straight auction are similar. Yankee auctions bid on multiple items. The winners are determined by ranking bids in order of highest price, then by largest quantity, and then by earliest time. Dutch auction consists of 1% of Internet auction. Other forms of auctions, such as Vickery auction and double auction, account for the rest 11%. The reasons why English auction and its variants are popular with online auctions are: (1) English auction is well understood by all consumers, not just economists. (2) A typical online auction will last for several days to allow more bidders to participate. This rules out those auctions that need to be finished within a short time span, such as Dutch auction. (3) The possibility of cheating prevents sealed-auctions being widely accepted.

CHEATING IN ELECTRONIC AUCTION

Cheating is a common phenomenon in Internet auctions. According to [9], Internet auction fraud accounts for 87% of all online crime. We outline the reasons that cheating occurs frequently in Internet Auction.

- Cheap pseudonyms facilitate cheating in Internet auctions [15].
- Lack of personal contact prevents bidders or sellers from identifying the suspicious entities [16].
- The tolerance of bidders motivates the cheating. Harris survey (http://www.harrispollonline.com) reports that 21% buyers take no action when they have problems in Internet transactions.
- Openness of Internet auction increases the chance of successful cheating. For example, the chance that an honest bidder will overbid shill bids in Internet auction is larger than in traditional auction given the large number of potential bidders in Internet auction.

The type of cheating possible is dependent on the auction mechanism. In this section we first give a taxonomy of cheating in electronic auction. Followed by, their possibility in three major types of auctions: English, second price sealed-bid and first price sealed-bid.

Taxonomy of Cheating

We build taxonomy of cheating in electronic auctions based on the literatures. The cheating can be induced either by the bidder or the auctioneer. A cheating seller tries to sell the item in a price as high as possible in order to increase his expected revenue. Interest of the buyer is just the opposite. Figure 1 illustrates the taxonomy.

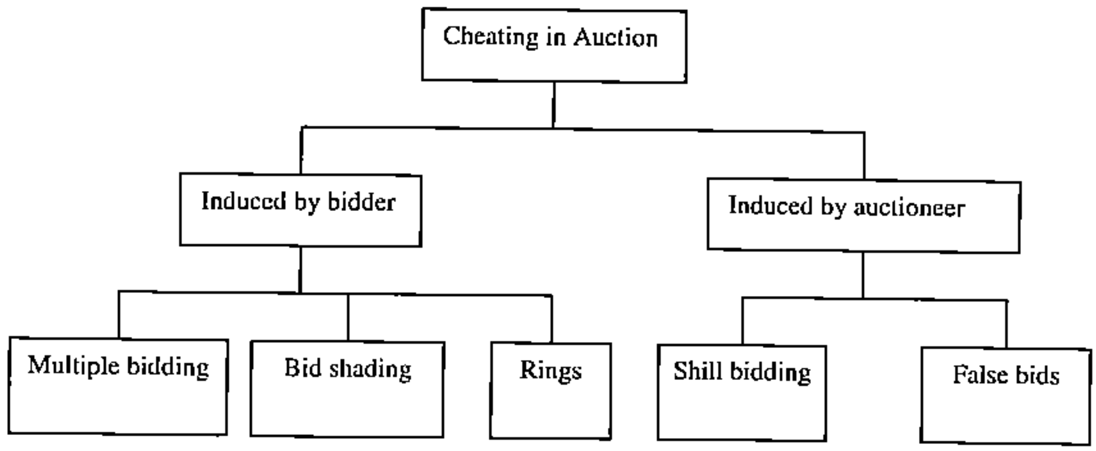

Figure 1: Cheating in electronic auction

Multiple bidding: A bidder can places multiple bids on the same item using different aliases [15].

Bid shading: If the bids are open a bidder tends to bid below his valuation of the product after examining the entries of other bidders. This is called bid shading [10].

Rings: Sometimes some bidders form a coalition called the ring. These ring members collude not to compete with each other and raise the price of the object [14].

Shill bidding: A corrupt auctioneer can appoint shills who place fake bids simply to increase the price of the item without the intention of buying it [22, 23].

False bids: An auctioneer can profitably cheat in a second price sealed-bid auction by looking at the bids before the auction clears and submitting an extra bid just below the price of the highest bid. Such extra bids are often called false bids [4].

Cheating in Second price Sealed-Bid Auction

As proposed by Vickrey [8] the second price sealed-bid auctions have many advantages for both buyers and sellers. In spite of those advantages, the Vickrey auctions are rare outside the financial market. Rothkopf, Teisberg and

Khan [10] offer two explanations for its rarity. One of the explanations is the fear of auctioneers cheating. An auctioneer can profitably cheat in second price auction by looking at the bids before the auction clears and submitting an extra bid [4]. The value of such a *false bid* is almost same as that of the winners bid. For example if the highest bid is $1000 and the second highest bid is $ 800, then a cheating seller can introduce a false bid of value $999. So the winner has to pay $999 (Almost same as his bid) instead of $800, and thus decreasing the expected gain of the winner. Rothkopf and Harstad [10] have shown that if the cheating of a seller is found the buyers start *shading* their bids and in the long run the second price auction becomes less profitable than any other auction. Shoham and Potter note that even when the seler is not cheating the mere fear of cheating makes the buyers *shade* their bids.

Cheating in First Price Sealed-Bid Auction

Unlike second price auction a seller can not cheat in a first price. But a bidder can cheat in a first price auction by bidding a value below his valuation of the product (*bid shading*) in order to have a positive utility if he wins [4]. In the internet environment due to lack of security measures enables the bidders to see others' bids before the auction closes. So the cheating bidders who have access to such information can go on revising their bids with *multiple* identities in order to bid the minimum amount necessary to win the bid. Thus a scenario equivalent to English auction will emerge in which all the cheating agents keep on revising their bid until all but one cheater wants the good at the current winning price.

Cheating In English Auction

Cheating in English auction can take place either in the form of shill bidding or multiple bidding [22,23]. In case of shill bidding the auctioneer cheats whereas in case of multiple bidding a bidder cheats.

A shill tries to escalate the price without the intention of buying it. In this process occasionally the shill wins the auction if no other higher bid comes from the other bidders. So the item to be sold remains with the auctioneer. Such items are re-auctioned at a latter time. If the item is auctioned in a site which does not charge any entry fee then the auctioneer neither loses nor gains in the process of *shill bidding*. But if there is some entry fee then the auctioneer has to bear the loss. This scenario of seller's cheating in English auction is completely different from that of the second price auction. In case of second price auction the seller can increase his profit up to the declared bid price of the winner. This declared bid price may be less than or equal to the maximum valuation of the product by the winner. On the contrary, in case of English auction the seller can drive the bidders to go up to their maximum valuation.

In case of *multiple bidding* a cheating agent submits many bids adopting multiple identities. Some of these bids are higher than that of their personal valuation of the product. They drive the bid to such an extent that no other bidder dares to bid and withdraw themselves from the auction. At this point the cheater also withdraws all his bids except the one lowest value. So he acquires the product in a much cheaper price increasing his own gain. This kind of cheating is possible in the sites that allow bid withdrawal.

THE EQUILIBRIUM BIDDING STRATEGY FOR ONLINE ENGLISH AUCTION

As mentioned earlier, cheating take place in English auction in the form of multiple bidding and/or shill bidding. In the current model we assume that the auction site does not allow bid withdrawal. Thus we remove the possibility of cheating through multiple bidding. We also assume that the probability of a shill winning the auction is zero. With these assumptions, in this section, we develop equilibrium bidding strategy for English auction when there is shilling. We adopt the same formulation used in [4] and adopt the same variables and symbols to make both the works comparable.

Problem Formulation

We consider the auction for a single indivisible object. The auction consists of *N*-1 bidders and an auctioneer. Each bidder associates two values with the product – the reservation value and the bid. The reservation value is the maximum price a bidder is willing to pay for the product based on his personal valuation. This information is private to each bidder. A bid on the other hand is the publicly declared price that a bidder is willing to pay for the product. Each bidder has a reservation value $\theta_i (i = 1,2,...N)$ for the object. Without the loss of generality we assume $\theta_i \in [0,1]$. Each agent's reservation value is independently drawn from a cumulative distribution function (*cdf*) *F* over [0, 1], where $F(0) = 0$ and $F(1) = 1$. We assume $F(.)$ is strictly increasing and differentiable in the interval [0,1]. The derivative of *cdf*, $f(\theta)$ is then the probability density function (*pdf*). Each bidder knows his reservation value and the distribution *F* of other agents. A bidding strategy $b_i : [0,1] \rightarrow [0,1]$ maps a bidder's reservation value to its bid. As we mention earlier $\theta = (\theta_1, \theta_2, ...\theta_n)$ is the vector of reservation values of all the agents and $b(\theta) = (b_1(\theta_1), b_2(\theta_2), ..b_n(\theta_n))$ is the vector of bids.

An honest bidder *i* can bid up to his reservation value, i.e. $\theta_i \geq b_i(\theta_i)$. On the other hand a dishonest bidder (shill) *j* can bid well above his reservation value in order to escalate the bid values of the honest bidders, i.e. $\theta_j \leq b_j(\theta_j)$. According to assumption that the shill never wins the auction, so his bid value has to be less than that of the reservation value of the winner *i*, i.e. $b_j(\theta_j) \leq \theta_i$.

Bidder's Expected Gain (Utility)

The expected utility (gain) of a winner is the difference between his reservation value and his expected payment.

The expected gain of a buyer is defined by Riley and Samuelson [11] as follows:

Expected Buyers Gain = Probability of Winning *(Reservation Value – Bid)

= Probability of Winning *($\theta_i - b_i(\theta_i)$) (1)

As per the model an honest bidder can win this auction if his final bid is higher than that of the reservation values of all other honest bidders and bids of all the dishonest bidders (shills).This fact can be formalized in the similar way as that of first price auction [4] as follows: Let the seller has a reservation value θ_s , which is a constant for a specific auction. The shill's bid has to be greater than that of the seller's reservation value. It is also less than that of the reservation value of the winner (honest bidder) so that the shill's probability of winning to zero. The probability that an honest bidder i beats a shill j is then $\Pr ob(\theta_s \le b_j(\theta_j) \le \theta_i) = F(\theta_i) - F(\theta_s)$. Since it is not profitable for a seller to accept any bid below his reservation value [11], it makes $F(\theta_s) = 0$ .Thus the probability that bidder i has a higher bid than a cheater can be represented by $F(\theta_i)$. Each honest bidder's reservation value has to be less than that of the bid value of the winner. So the probability that an honest bidder's bid is it is higher than that of another honest bidder is $F(b_i(\theta_i))$. So the probability that an honest agent bid is higher than that of any other agent is the weighted average these two probabilities, the weights being the probability of cheating (P^b) and non-cheating (1 - P^b) respectively. Probability that he wins the auction is therefore this probability raised to the power N-1 (His bid is higher than other N-1 agents). This can be represented as :

$$[P^b.F(\theta_i) + (1-P^b).F(b_i(\theta_i))]^{N-1} \tag{2}$$

Thus, we can write bidder i's expected utility as:

$$E_{\theta_{-i}} u_i(b(\theta), \mu^b, \theta_i) = (\theta_i - b_i(\theta_i)).[P^b.F(\theta_i) + (1-P^b).F(b_i(\theta_i))]^{N-1} \tag{3}$$

θ_{-i} is the vector of reservation values of all the agents except the agent i.

Equilibrium

Our aim is to find the equilibrium bidding strategy of an honest agent in English auction in the presence of shills. It is assumed that the agents (bidders) are rational and maximizes their utility. All the dishonest agents bid a higher then their reservation value, all the honest agents bid according to a symmetric bidding strategy. To find the equilibrium bidding strategy, we will maximize the expected utility function (Equation 3) by taking its derivatives with respect to $b_i(\theta_i)$ and setting it to zero. The equilibrium $b_i(\theta_i)$, derived from this equation is presented in theorem 1. This theorem is similar to that of theorem 3 of [4] and can be proved accordingly.

Theorem 1: In an English auction in which each bidder cheats with the probability P^b , it is a Bayes Nash equilibrium for each non-cheating bidder i to bid according to the strategy that is a fixed point in the following equation:

$$b_i(\theta_i)=\theta_i-\frac{\int_0^{\theta_i}(P^b.F(x)+(1-P^b).F(b_i(x)))^{N-1}dx}{(P^b.F(\theta_i).+(1-P^b).F(b_i(\theta_i)))^{N-1}} \tag{4}$$

Proof:

We define $\phi_i:[0,b_i(\theta_i)]\rightarrow[0,1]$ as the inverse function of $b_i(\theta_i)$. That is, it takes the bid of the agent i as the input and returns the reservation value θ_i that induces this bid. Thus, we can rewrite bidder i's expected utility as:

$$E_{\theta_{-i}}u_i(b(\theta),\mu^b,\theta_i)=(\theta_i-b_i(\theta_i)).[P^b.F(\phi_i(b_i(\theta_i)))+(1-P^b).F(b_i(\theta_i))]^{N-1} \tag{5}$$

Finding the equilibrium

The agent i's reservation value is private and is constant for him in the expected utility function. To find the equilibrium $b_i(\theta_i)$, we take the derivative and set it to zero.

$$\begin{aligned}0=[&(\theta_i-b_i(\theta_i)).(N-1).(P^b.F(\phi_i(b_i(\theta_i)))+(1-P^b).F(b_i(\theta_i)))^{N-2}.\\&(P^b.f(\phi_i(b_i(\theta_i))).\phi_i'(b_i(\theta_i))+(1-P^b).f(b_i(\theta_i)))]-\\&(P^b.F(\phi_i(b_i(\theta_i)))+(1-P^b).F(b_i(\theta_i)))^{N-1}\end{aligned}$$

To further simplify we use the formula $f'(x)=\frac{1}{g'(f(x))}$ where $g(x)$ is the inverse function of $f(x)$. Plugging in function from our setting gives us: $\phi_i'(b_i(\theta_i))=\frac{1}{b_i'(\theta_i)}$ Applying both this equation and $\phi_i(b_i(\theta_i))=\theta_i$ gives us:

$$\begin{aligned}0=[&(\theta_i-b_i(\theta_i)).(N-1).(P^b.f(\theta_i).\frac{1}{b_i'(\theta_i)}+(1-P^b).f(b_i(\theta_i)))]-\\&(P^b.F(\theta_i)+(1-P^b).F(b_i(\theta_i)))\end{aligned}$$

Rearranging the terms yields:

$$b_i(\theta_i)=\theta_i-\frac{(P^b.F(\theta_i)+(1-P^b).F(b_i(\theta_i)))b_i'(\theta_i)}{(N-1).(P^b.f(\theta_i).+(1-P^b).f(b_i(\theta_i))b_i'(\theta_i))} \tag{6}$$

In order to verify Equation 4, we first take its derivative:

$$b_i'(\theta_i)=1-[1-\frac{(N-1).(P^b.F(\theta_i).+(1-P^b).F(b_i(\theta_i)))^{N-2}.(P^b.f(\theta_i).+(1-P^b).f(b_i(\theta_i)).b_i'(\theta_i)).\int_0^{\theta_i}(P^b.F(x)+(1-P^b).F(b_i(x))^{N-1}dx}{(P^b.F(\theta_i).+(1-P^b).F(b_i(\theta_i)))^{2(N-1)}}]$$

This equation simplifies to:

$$b_i'(\theta_i)=\frac{(N-1).(P^b.f(\theta_i).+(1-P^b).f(b_i(\theta_i))b_i'(\theta_i)).\int_0^{\theta_i}(P^b.F(x)+(1-P^b).F(b_i(x))^{N-1}dx}{(P^b.F(\theta_i).+(1-P^b).F(b_i(\theta_i)))^N}$$

Plugging this equation in the numerator of Equation 6 yields Equation 4. □

Uniform distribution as a special case

Here we consider uniform distribution as a special case This result, like that of the first price auction [4], is particularly robust for uniform distribution .

Corollary 1. In the English auction where the shills cheat with probability P^h, and $F(\theta_i)=\theta_i$, it is a Bayes-Nash equilibrium for each non cheating agent to bid according to the strategy $b_i(\theta_i)=\frac{N-1}{N}\theta_i$.

The proof of this can be found in the appendix.

Proof: Putting $F(\theta_i)=\theta_i$ in equation 4 yields:

$$b_i(\theta_i)=\theta_i-\frac{\int_0^{\theta_i}(P^h.x+(1-P^h).b_i(x))^{N-1}dx}{(P^h.\theta_i+(1-P^h).b_i(\theta_i))^{N-1}}$$

Now plugging the strategy $b_i(\theta_i)=\frac{N-1}{N}\theta_i$, into this equation in order to verify this as a fixed point we get:

$$b_i(\theta_i)=\theta_i-\frac{\int_0^{\theta_i}(P^h.x+(1-P^h).\frac{N-1}{N}x)^{N-1}dx}{(P^h.\theta_i+(1-P^h).\frac{N-1}{N}\theta_i)^{N-1}}=\theta_i-\frac{\int_0^{\theta_i}x^{N-1}dx}{\theta_i^{N-1}}=\theta_i-\frac{(1/N)\theta_i^N}{\theta_i^{N-1}}=\frac{N-1}{N}\theta_i$$

COMPARISON OF CHEATING AUCTIONS

Cheating brings short term benefits to the cheater. The ease of adopting fake and multiple identities boosts the cheaters' confidence of not loosing reputation when they are caught. With state-of-the-art technologies, it is difficult to prevent cheating in Internet auctions. Law enforcement is also difficult over the Internet. In this situation it is worth seeing which type of auction is robust in the online marketplace. In this section we compare two types of sealed-bid auction with the English auction from a non-cheater's point of view. A non-cheater can be an auctioneer or a bidder. We analyze three types of auction in the specific case where the reservation values are drawn from uniform distribution.

Comparison from Honest Bidder's Perspective

Suppose an honest bidder has the option for choosing the type of auction in a cheating environment. He should have the fair idea whether his expected gains are same or different in each type of auctions. So that he can choose a site accordingly. We consider the definition of expected gain of a buyer by Riley and Samuelson [11] represented in Equation 1.

The probability of cheating and the equilibrium bidding strategy for an honest bidder in English auction are given in Equation 2 and Equation 4 respectively. We refer the work of Porter and Shoham [4] for understanding cheating in sealed-bid auctions and adopt the probabilities of cheating and equilibrium strategies as derived by them for both first price and second price auction.

The expected gain of the winner is considered as the expected revenue loss for the seller [4]. As specified earlier the expected gain is the difference between the winner's reservation value and the final bid. Thus, if we assume the expected revenue loss as defied by [4], in turn we assume that when there is no cheating the bidder would have placed the final bid equal to that of his reservation value - which seldom is the case. So in the following paragraph we present a different approach for evaluating the expected revenue loss of a seller.

Comparison from Honest Auctioneer's Perspective

We define expected revenue loss to be the difference between the maximum possible expected revenue in the non-cheating environment and expected revenue in cheating environment. The expected revenue for a seller is defined Riley and Samuelson [11] to be N times the expectations on the expected payment of a typical buyer. Assuming that the seller knows the distribution $F(\theta_i)$ of the reservation values his expected revenue is:

$$R = N\int_{0}^{\theta_i} P(\theta)F'(\theta)d\theta \tag{7}$$

Where, $P(\theta)$ is the expected payment by a single buyer and $P(\theta_i)$=$Prob\{\, b(\theta_i)$ is high bid$\}$ $b(\theta_i)$.

We use the fact that the optimal auctions maximize a seller's expected revenue [12]. According to revenue equivalence theorem the expected revenues from all the optimal auctions are same [12]. We refer Riley and Samuelson [11] for finding the expected revenue in a non-cheating environment for any optimal auction using the following formula:

$$R = N\int_{0}^{\theta_i} [\theta\ F'(\theta)+F(\theta)]F^{N-1}(\theta)d\theta \tag{8}$$

The expected revenue in case of three types of cheating auctions can be derived from Equation 7.

Uniform Distribution as a special case

Now we can derive the expected gain of the winner and revenue loss for an honest bidder, assuming that all the bidders draw their bids from a uniform distribution and the winner bids according to the equilibrium bidding strategy. Following the work of Porter and Shoham [4] we say, the equilibrium bidding strategy $b_i(\theta_i)$ in case of uniform distribution for second and first price auction are $\frac{N-1}{N-1+P^c}\theta_i$ (P^c is the probability of cheating in the second price auction) and $\frac{N-1}{N}\theta_i$ respectively. Corollary 1 gives the equilibrium bidding strategy in the English auction. It can be easily verified that when $F(\theta)=\theta$, the probability of winning in second price, first price and English auction are θ_i^{N-1}, $[\frac{N-P^a}{N}\theta_i]^{N-1}$ and $[\frac{N-1+P^b}{N}\theta_i]^{N-1}$ respectively.

Theorem 2: In the second price auction where an auctioneer cheats with probability P^c, the winner bids according to the equilibrium bidding strategy, and $F(\theta)=\theta$ then the expected gain of the winner is $\frac{P^c}{N-1+P^c}\theta_i^N$

Theorem 3: In the first price auction where the bidders cheat with probability P^a, the winner bids according to the equilibrium bidding strategy, and $F(\theta)=\theta$ then the expected gain of the winner is $\frac{(N-P^a)^{N-1}}{N^N}\theta_i^N$

Theorem 4: In the English auction where the shills cheat with probability P^b, the winner bids according to the equilibrium bidding strategy, and $F(\theta)=\theta$ then the expected gain of the winner is $\frac{(N-1+P^b)^{N-1}}{N^N}\theta_i^N$

These results can be easily verified by appropriately replacing the values of probability of winning and $b_i(\theta_i)$ in equation 1 for each type of auction forms. Some important observations can be made from the above theorems. If the probability of cheating is 1 (certain) second price auction and English auction generate the same expected gain which is lower than the expected gain in first price auction. In case of first price and second price auction the seller cheats, thus the expected gain of the buyer decreases. Whereas, in first price auction, the buyers' shade there bids in order to increase their expected gain. When the probability of cheating is 0.5, first price auction and English auction generate same expected gain for the buyer which is higher than the expected gain from second price auction.

We now find the expected revenue loss for an honest seller. When the values are drawn from a uniform distribution, following Equation 5, the expected revenue in non-cheating environment can be found to be

$$R = N\int_0^{\theta_i}[2\theta-1]\theta^{N-1}d\theta \tag{9}$$

Theorem 5: In the second price auction where an auctioneer cheats with probability P^c, the winner bids according to the equilibrium bidding strategy, and $F(\theta)=\theta$ then the seller's expected revenue loss is $[\frac{N(N-1+2P^c)}{(N+1)(N-1+P^c)}\theta_i - 1]\theta_i^N$

Theorem 6: In the first price auction where the bidders cheat with probability P^a, the winner bids according to the equilibrium bidding strategy, and $F(\theta)=\theta$ then the seller's expected revenue loss is $[\frac{2N^N(N+1)-(N-P^a)^{N-1}(N-1)}{N^{N-1}(N+1)}\theta_i - 1]\theta_i^N$

Theorem 7: In the English auction where the shills cheat with probability P^b, the winner bids according to the equilibrium bidding strategy, and $F(\theta)=\theta$ then the seller's expected revenue loss is $[\frac{2N^N(N+1)-(N-1+P^a)^{N-1}(N-1)}{N^{N-1}(N+1)}\theta_i - 1]\theta_i^N$

These results can be verified by replacing the variables appropriately in equation (7) and subtracting the result from equation (9). It can be found that when the probability of cheating is 1 the seller's revenue loss is negative ($0<\theta_i \le 1$). In other words the seller does not suffer a loss of revenue in case of second price auction when there is cheating. This is true because seller cheats in the second price auction by placing a fake bid increases his revenue but he never wins the auction. This is not the case for other two types of auctions. The seller makes a loss in these auctions if the left hand side of the bracketed expression becomes greater than 1. In the following section we experimentally show the revenue loss curves to visualize this result.

EXPERIMENTAL RESULTS

In the last section we used uniform distribution to compare three auction types. Such analytical comparison becomes difficult with other distributions. Moreover the results can be visualized and comprehended if shown graphically. So, in this section we present the experimental results of comparison of three types of cheating auction. We consider three distributions like that of [4] for θ: uniform distribution ($F(\theta_i)=\theta_i$), normalized exponential distribution ($F(\theta_i)=\frac{e^{\theta_i}-1}{e-1}$)and an arbitrary polynomial distribution ($F(\theta_i)=-\frac{1}{2}\theta_i^2+\frac{3}{2}\theta_i$). Throughout this section we assume the transactions are taking place in a highly cheating environment. We assume probability of cheating to be 0.9 for each type of auction.

Comparison from Honest Bidder's Perspective

Figure 2- (a), (b) and (c) shows the comparative plots for expected buyer's gain vs. the reservation values for uniform, exponential and polynomial distribution respectively when the number of buyer's is equal to 25. The nature of the curves is same irrespective of the distribution. So we can safely assume that this result holds for other distribution of the reservation value (θ). The observations are:

(1) The buyer's expected gain is same for second price and English auction when cheating takes place. The seller cheats in both these cases - In second price auction by placing a fake bid and in English auction by shilling. The awareness of cheating by the seller makes the bidders *shade* their bids, hence the gain.

(2) Expected gain from the cheating first price auction is less than other two auctions. This result is quite peculiar because in first price auction the buyers cheat the seller by shading their bid where as in other two auctions the buyers act defensively against seller's cheating. So it turns out that the buyers can get more expected revenue when the seller cheats than when they cheat the seller. Or in other words they shade their bid more when they are aware of cheating than cheating them selves.

(3) Irrespective of the auction mechanism a buyer can have some gain only when his personal valuation of the product (reservation value, θ) is very high. The buyers, who do not value the product very much if wins the auction, are not going to have much expected gain gain.

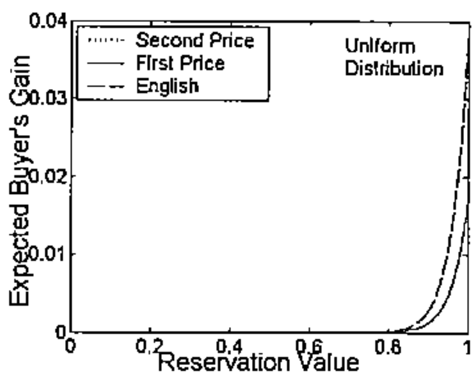

Figure 2(a)

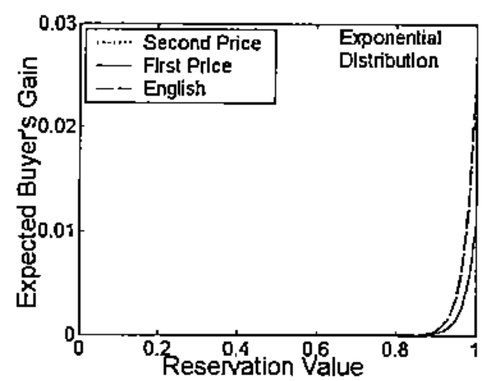

Figure 2(b)

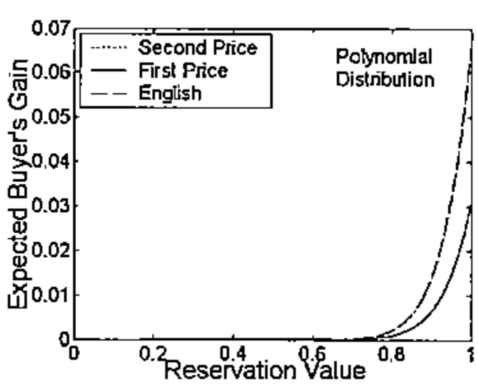

Figure 2(c)

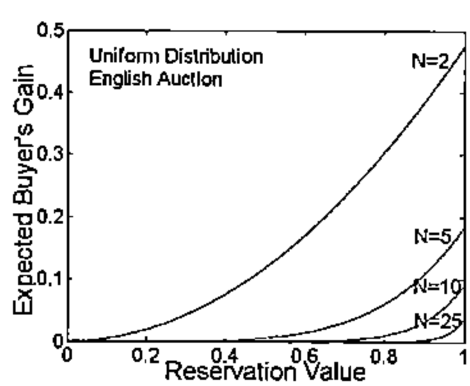

Figure 2(d)

Figure 2 Comparative buyer's expected gain plots for various distributions when N=25

Figure 2 (d) shows the buyer's expected gain for uniform distribution, English auction and different number of competing buyers. It turns out that the expected gain decreases when the number of buyers increase in the system. The seller can exploit this opportunity particularly in English auction to introduce more number of shills to a give the buyer an idea of intense competition. We got similar results for other type of auctions and other distributions. Thus irrespective of the distribution of the reservation value and underlying auction mechanism, the seller can deceive the buyers by falsely showing a high number of competing bidders and can induce increased the bid values.

Comparison from Honest Auctioneer's Perspective

In this section we compare the expected revenue loss for an honest seller in cheating environment compared to the expected revenue from any optimal auction in non-cheating environment. It should be kept in mind that the expected revenue is N times expected payment by all the buyers assuming that all the buyers have equal chance of winning. The figures 3 – (a), (b) and (c) shows the comparative seller's expected revenue loss plots for uniform distributions and different numbers of competing bidders. Following are the observations from these figures:

(1) In first price auction the seller never cheats and the buyers shade their bids to increase their expected gain. Thus this bid shading by each individual buyer will contribute to make revenue loss for the seller. Figure 3 (a) confirms this fact and shows that when the number of buyers in the system is higher than 5, the expected revenue loss becomes non-negative. Even for lower number of buyers in the system there is a loss when the bids are drawn from a higher θ value. Or in other words when the competition becomes intense either in terms of the number of competing buyers and/or each buyer have a high valuation of the product, the buyers shade their bids more and more.

(2) The expected revenue loss in case of second price auction is never positive (Figure 3 (b)). Which means a seller never makes a loss if adopts second price auction as the mechanism in an environment where the buyers are aware of cheating than what he would have got in a non-cheating optimal auction. Thus, when the number of buyer's increases in the system the buyers shade their bid more and the revenue loss tends to be zero. A seller can cheat in a second price auction by introducing a fake bid. If he chooses to do so then he can increase his revenue but there is no chance of seller winning the auction.

(3) It is found from Figure 3 (c) that the expected revenue loss in English auction for the higher values of θ is positive. But for the higher values of θ the nature of the curves are similar to the second price auction. This fact can be interpreted in the following way. The nature of seller's cheating in English auction is different from that of the second price auction. During the process of shilling there is a chance that the shill may win which contributes to

increase the expected revenue loss in English auction. When the bid values are drove up very high by the shill the chance of shill winning the auction also becomes very high leading to a positive expected revenue loss towards the higher values of θ .

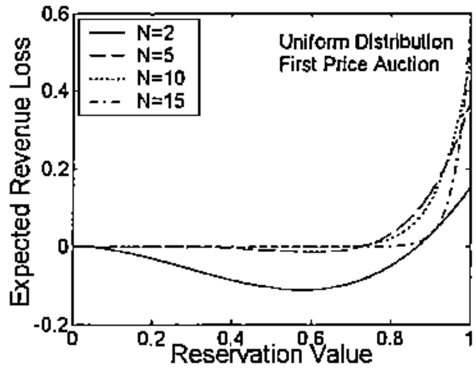

Figure 3(a)

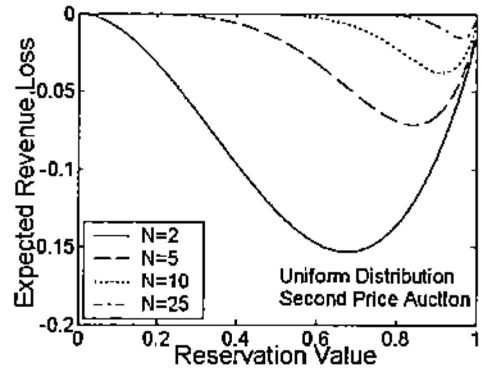

Figure 3(b)

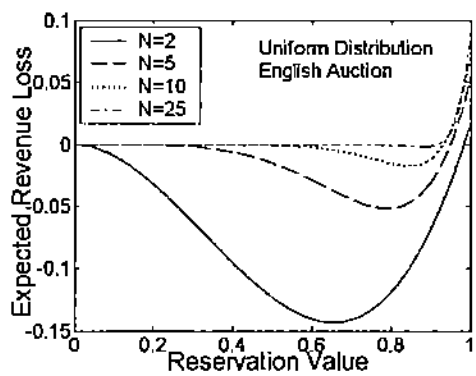

Figure 3(c)

Figure 3 Comparative seller's expected revenue loss plots for uniform distribution and different number of buyers

The figures 4 – (a), (b) and (c) shows the comparative sellers expected revenue loss plots for three distributions and 25 numbers of competing bidders show similar results. Irrespective of the distribution, for lower reservation values the expected revenue loss for the seller is zero. The second price auction always gives a negative loss for higher values of θ . Maximum revenue loss for a seller occurs in case of first price auction independent of the distribution of θ . Revenue loss in case of English auction for higher θ values is less than that of the first price auction.

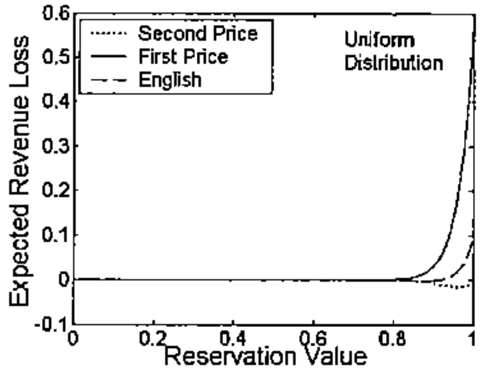

Figure 4(a)

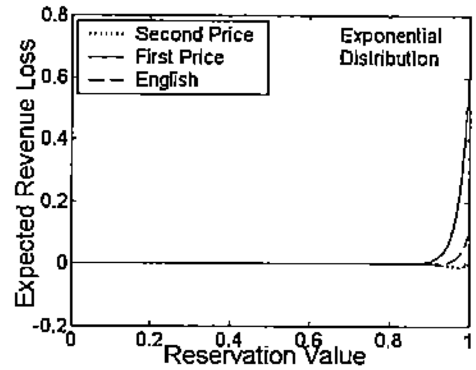

Figure 4(b)

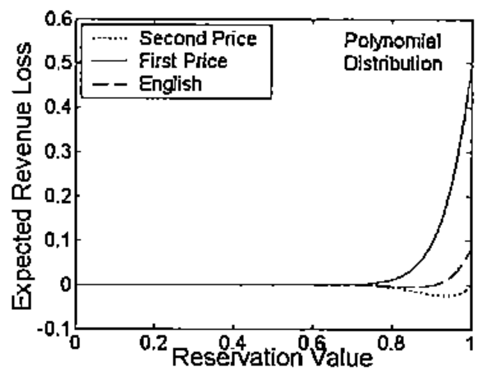

Figure 4(c)

Figure 3 Comparative seller's expected revenue loss plots for various distributions when N=25

After consolidating all the results some of the interesting observations are:

(1)The bidders shade their bid more while counteracting cheating by the seller in second price and English auction than the cheating by the fellow buyers in first price auction.

(2) Cheating has very little effect on bidder's expected gain and seller's revenue loss for the lower bid values and very high number of bidders in the system.

(3) Both buyers and sellers are not likely to prefer first price auction compared to the other two types.

(4) Surprisingly the second price auction in spite of its rarity is found to be most preferable type of auction from both buyers' and seller's point of view. We refer to the work of Rothkopf and Harstad [13] to explain this rather strange result. They have shown using a dynamic model that if a seller adopts cheating in second price auction his revenue will continue to decrease over the time and reach a stage where it will be less profitable than a first price auction. Since our model is static in nature, we can assume that that as the time passes the second price auction will be no more profitable for the seller.

(5) Based on the above observations and particularly the assumption of decreasing profit in second price auction in the last observation English auction emerges as the most preferable mechanism for both honest buyers and the honest seller. It provides the highest expected gain for the winning buyer and very little revenue loss for the seller in

the cheating environment. This result can be used to explain the popularity of English auction over the Internet where there is fear of cheating.

RELATED WORK

The study of bidder's behavior is an interesting field of research in the auction literature. Riley and Samuelson [11], for example, present a framework for generating equilibrium bidding strategy for a large class of auctions. They also provide a methodology for comparing all types of auctions in terms of bidder's and auctioneer's gains. Literature admits cheating is a common phenomenon in the auction process. Graham and Marshal [14]model the collusive bidder behavior in second-price and English auction. In this type of cheating the bidders form a coalition called ring. Ring members never compete seriously against each other. When one of the ring members wins, the gain is divided equally among ring members. The authors extend the revenue equivalence of the second-price and English auction to accommodate such cooperative behavior. They conclude that both the auctioneer's reserve price and the expected payoff to a ring member is an increasing function of the size of the coalition. Rothkopf and Harstad present two models, namely static game-theoretic model and dynamic bid-taker reputation model, to exhibit that Vickery auctions will be driven out by sealed-bid auctions when there exists possibility of cheating [13].In Vickrey auctions, a bid-taker (seller) may introduce a fictitious bid just below the highest submitted bid after observing the submitted bids such that the payment of the winner will be increased. The game-theoretic model shows if cheating exists only the most dishonest type prefers Vickery auction to first-price auctions. The dynamic reputation model indicates that in the setting of repeated Vickery auction, a dishonest bid-taker have no reason to conduct this kind of auction because he will be caught eventually and loose his reputation afterwards. In a combinatorial auction, multiple goods are sold simultaneously. Bidders bid on any combination of goods. A dishonest bidder may submit false bids under fake identities, which is called false-name-bid. A protocol is false-name-proof if truth-telling without using false-name bids is a dominant strategy for bidders. Yokoo *et. al.* [15] prove that Vickrey-Clarke-Groves(VCG) mechanism is not false-name-proof. They conclude that when surplus function is concave over bidders the VCG mechanism is false-name-proof.

Internet based auctions are more prone to cheating due to the lack of security. Porter and Shoham [4] derive the equilibrium bidding strategies for an honest bidder who is aware of cheating in sealed-bid auctions. They consider two forms of cheating. In case of second price auction a seller inserts a fake bid to increase the payment of the winner. In case of first price auction a bidder examines the competing bids and submits a bid to win the auction with minimum payment. They also find the expected revenue loss for an honest seller due to the possibility of cheating. Ba *et. al.* [16] propose to reduce online fraud through the use of reputation mechanism maintained by a trusted third party. According to their proposition a rusted third party not only issues a certificate but also maintains reputations associated with the certificate holder. They define a trusted third party system stage game to formalize the online transaction process with aid of trusted third parties and propose a symmetric sequential equilibrium strategy. Kauffman and Wood [22] distinguish between two different types of shilling that exhibit different motivation and certain stage. While purpose of shill bidding is to increase the seller's revenue, the multiple bidding is for decreasing the revenue. It will be interesting to observe the effect when both type of cheating together takes place. Our model assumes that the shill never wins the auction. But in real-life sometimes the shills win. So this model can be extended to accommodate this situation

The auction process over the internet involves three parties- the buyer (bidder), the seller (auctioneer) and the site that hosts the auctions. Much work has been done to see the reputation effect of sellers. Some work can be done to see reputation effect of the site that hosts the auction.

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