Numerical modeling of loss mechanisms resulting from non-uniform illumination in multijunction concentrator solar cells

Alexander W. Haas  
*Purdue University, Birck Nanotechnology Center, awhaas@purdue.edu*

John R. Wilcox  
*Purdue University, Birck Nanotechnology Center, jrwilcox@purdue.edu*

Jeffery L. Gray  
*Purdue University, Birck Nanotechnology Center, grayj@purdue.edu*

Richard J. Schwartz  
*Purdue University, Birck Nanotechnology Center, schwartz@purdue.edu*

Follow this and additional works at: [http://docs.lib.purdue.edu/nanopub](http://docs.lib.purdue.edu/nanopub)

Part of the [Nanoscience and Nanotechnology Commons](http://docs.lib.purdue.edu/nanopub)

[http://dx.doi.org/10.1002/jnm.1922](http://dx.doi.org/10.1002/jnm.1922)

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.
Numerical modeling of loss mechanisms resulting from non-uniform illumination in multijunction concentrator solar cells

A. W. Haas*,†, J. R. Wilcox, J. L. Gray and R. J. Schwartz

School of Electrical and Computer Engineering & Birck Nanotechnology Center, Purdue University, West Lafayette, IN 47907, USA

ABSTRACT

Quasi-3D distributed emitter models utilizing a unit cell-based methodology have been successfully applied to the analysis of lateral current flow in solar cell emitter layers. However, the analysis of the specific loss mechanisms resulting from this flow has not been given adequate attention. In this work, a quasi-3D model for the simulation of effects related to the lateral flow of current in a solar cell emitter layer, particularly under non-uniform illumination, is developed. The model is applied to a specific case in which a GaInP/GaAs-like two-terminal solar cell is illuminated with a Lorentzian irradiance pattern, which is the expected pattern for parabolic trough concentrator. It is shown that bias-point loss, which results from the variation in the local operating condition of the cell, is significant under highly non-uniform illumination at low–moderate optical concentration. Understanding this loss mechanism is useful in considering trade-offs related to the design of the optical concentrator system, as well as, the design of a grid electrode pattern for maximum power output. Copyright © 2013 John Wiley & Sons, Ltd.

Received 22 August 2012; Revised 24 June 2013; Accepted 26 June 2013

KEY WORDS: solar cell; multijunction; concentrator; distributed diode

1. INTRODUCTION

Figure 1 shows a cross section of a simplified dual-junction concentrator solar cell layer structure with a bus bar and grid electrode; the assumed flow of light-generated current is shown qualitatively using arrows. Only one unit-section, or tile, is shown. The full device is composed of many tiles along its length.

It is well-known that the lateral flow of current through the emitter layers, particularly the top sub-cell emitter, which is typically thin and thus has relatively high-sheet resistance, causes significant power loss [1–5], results in a reduction in the open-circuit voltage, Voc [4, 6], and may affect the experimentally observed effective ideality factor of the cell [6, 7]. Non-uniform illumination compounds these issues.

This effect can be captured using a full 3D detailed numerical model, which simulates cell performance by numerically solving the semiconductor equations (Poisson's equation and hole and electron continuity). However, these simulations are computationally intensive and require a high degree of expertise and knowledge about the material parameters of the cell structure. More traditionally, this effect is modeled using a quasi-3D model [4, 5, 8–12].

The quasi-3D model discretizes the cell into small area elements. An interconnected grid of resistors represents the emitter layer and metal electrodes, and a sub-model is connected in the third-dimension to simulated carrier generation and recombination. Resistances may also be included in the model to represent contact resistance, tunnel junction resistance, and base resistance; however, losses in these
resistances are typically much smaller than those in the emitter and grid electrodes, and thus they will be considered negligible for the purposes of this study. The formulation of the model will be discussed in the next section.

Though the quasi-3D formulation has been applied extensively to the modeling of cell output under both uniform and non-uniform illumination, the contribution of the various loss mechanisms has been given little attention [5, 11, 12]. In particular, the so-called bias-point loss, which results from variations in the local operating condition across the cell area, has been given very little consideration. This loss mechanism plays a significant role in determining cell performance and may strongly influence grid electrode optimization. Understanding the role of these loss mechanisms, particularly under non-uniform illumination, provides a significant insight into the operation of the cell and is valuable in considering design trade-offs, particularly in the design of the optical concentrator system. Further, multijunction structures have been given only a small amount of attention [10].

In this work, a simple quasi-3D model for dual-junction solar cells, based upon the previous work by Haas et al. [5, 11, 12], is developed, and the extraction of loss mechanisms is discussed. The model is applied to a dual-junction GaInP/GaAs-like two-terminal device under nonuniform, Lorentzian illumination. It is shown that the bias-point loss strongly influences cell performance and grid electrode optimization, especially when the illumination is highly nonuniform and the optical concentration is low–moderate.

2. THE QUASI-3D MODEL

2.1. Dual-junction model formulation

The quasi-3D model employed in this work follows that of earlier work [5, 11, 12], which is very similar to the quasi-3D models employed by other research groups [4, 8–10] except that it has been extended to multiple junctions and focuses on the flow of current in the emitter layer. However, while the model is typically solved using SPICE, this formulation is implemented using MATLAB, which allows for greater flexibility and simplifies the manipulation and visualization of the resulting solution.

The cell (or tile, as defined in Figure 1) is discretized into \( n \times m \) area elements (or nodes) per sub-cell. An interconnected grid of resistors connected to each node represents the sheet resistance of the emitter layer seen by current flowing through the area element, which can be calculated to first order using the layer doping, \( N \), thickness, \( t \), and majority-carrier mobility, \( \mu \).

\[
R_{s}^{-1} = (q\mu Nt)^{-1}
\]  

(1)

Here, \( q \) is the elementary charge, and the sheet resistance is given in \( \Omega/\text{sq} \). Note that the sheet resistance of the top and bottom sub-cells is denoted by \( R_{s,B} \) and \( R_{s,T} \), respectively. In
the third-dimension, a sub-model is connected to each node to simulated carrier generation and recombination in each sub-cell. Each element area is represented by a unit-cell with the circuit visualization shown in Figure 2 for nodes \((i,j,k)\) \((k = 1\) for the top sub-cell and \(k = 2\) for the bottom sub-cell\) in the field of the device \(i.e.,\), away from the perimeter. Figures 1 and 2 show two junctions for the purposes of this work; however, this methodology could be easily extended to three or more junctions, though the solution time and required computer memory increases rapidly as junctions are added to the simulation.

Each node is composed of four resistors, \(R_r\), \(R_l\), \(R_t\), and \(R_b\), which represent the sheet resistance seen by current flowing in each of the four directions. For a node of length \(a\) and width \(b\) in the field of the device,

\[
R_r = R_l = \frac{(a/2b)R_S^2}{(2)}
\]

\[
R_t = R_b = \frac{(b/2a)R_S^2}{(2)}
\]

Nodes along the perimeter of the device are centered at the edge of the area element, and thus the resistor values given in Equations 2 and 3 may be up to twice the value shown, or may not exist at all, as current cannot flow out of the device except at the contacts.

Although defining the model in this way is convenient, the solution only incorporates the sum of each resistor with its nearest-neighbor node, as indicated in Figure 2 by \(R_{RR}\), \(R_{RL}\), \(R_{RT}\), and \(R_{RB}\). This formulation assumes that the back surface is metalized and thus forms an equipotential. More significantly, it is assumed that all lateral current flow is through the emitter layers (as implied in Figure 1), and thus the sheet resistance for each sub-cell is determined only by the lateral resistances of these layers. This is reasonable for III-V concentrator solar cells because the ratio of device thickness to grid electrode spacing is usually relatively small, as device thicknesses are typically less than 10–15 µm [13–18] with grid electrode spacing an order of magnitude (or more) larger [19]. Additionally, the probability of a carrier tunneling through the tunnel junction decreases exponentially with tunneling

Figure 2. Circuit visualization of cell discretization and node definitions. \(V_R\), \(V_L\), \(V_T\), and \(V_B\) are the voltages across the nearest-neighbor nodes, and \(V\) is the node voltage.
distance [20], and thus the carriers will tunnel normal to the interface. The role of the tunnel junction could be included in the model by adding a series resistance between sub-cells; however, this resistance should be insignificant for a high-quality tunnel junction at moderate concentration. Finally, it assumed that any resistive loss because of the metal/semiconductor interface and losses because of the vertical base resistance are negligible compared with those in the emitter layers and electrodes. Including these resistances could improve the accuracy of the model as compared with a particular measured solar cell; however, the purpose of this work is the analysis of particular loss mechanisms, and thus the role of these other resistances are assumed to be insignificant in this case.

The sub-model function used to simulate carrier generation and recombination in the cell, $I_D(V)$ is a function of the voltage across each junction or sub-cell and can take on many forms. An ideal diode or two-diode model is typically used [9, 10], though an analytic curve-function of the voltage across each junction or sub-cell and can take on many forms. An ideal diode or numerical device model have also been employed [5]. Note that $I_D(V)$ will vary spatially depending upon the size of the node, as well as, the intensity of the local illumination—in dark regions of the solar cell, such as under grid electrodes, the $I_D(V)$ model will include a recombination term but not a generation term.

At each node $i,j,k$, Kirchoff’s current law is used to define a relationship between nearest-neighbor node voltages with $V_B$ being the voltage of the node one row below ($V_B = V_{i+1,j,k}$), $V_T$ being the voltage of the node one row above ($V_T = V_{i-1,j,k}$), $V_L$ being the voltage at the node one column to the left ($V_L = V_{i,j-1,k}$), and $V_R$ being the voltage of the node one column to the right ($V_R = V_{i,j+1,k}$)—these definitions are also clarified by Figure 2. For $V_a = V_{i,j,1}$, $V_b = V_{i,j,2}$, and letting $I_{D,a}$ be the sub-model current in sub-cell $a$ (recall $k = 1$ for the top sub-cell and $k = 2$ for the bottom sub-cell) with $I_{D,a}$ being the sub-model current in the top sub-cell above the bottom sub-cell $i,j,2$ , the equations defined by Kirchoff’s laws at nodes in the field of the device at the same lateral position (given by $(i,j)$) in the top and bottom sub-cells are given respectively by

$$f_{i,j,1} = 0 = \left[ R_T R_B R_L I_D (V - V_b) - R_B R_T R_R (V - V_T) - R_L R_T R_R (V - V_B) \right]_{i,j,1} \tag{4}$$

$$f_{i,j,2} = 0 = \left[ R_T R_B R_L I_D (V - I_{D,a} (V_a - V)) - R_B R_T R_R (V - V_T) \right]_{i,j,2} \tag{5}$$

where the subscripts after the brackets have been incorporated to compact Equations 4 and 5, and imply that the values of the variables for the node given by the subscript should be used in the calculation. As an example, $V$ given in Equation 4 is in its most accurate form, $V_{i,j,1}$, as implied by the subscript after the bracket. Also note that the $I_D$ terms are functions of the node voltages in the top and bottom sub-cells, as discussed earlier, whereas the resistance terms ($R_L$, $R_T$, $R_B$, and $R_R$) are not voltage dependent.

Of course, Equations 4 and 5 only hold for nodes in the field of the device (i.e., regions of the device away from the edges of the cell) and must be modified for those nodes along the perimeter, because the available direction of current flow is restricted for these nodes (i.e., current does not flow out of the cell except at the contacts). Note that this restriction can be because of physical limitations of the cell (again, current cannot flow out of the cell except at the contacts) or imposed by how the cell is simulated, which results from the symmetry of the cell and illumination profile, as will be assumed in section 3 of this paper. Nodes that represent grid electrodes or the bus bar are formulated in the same way, except that the sheet resistance employed in Equations 2 and 3 are calculated from the resistivity of the electrode metallization, which effectively assumes negligible contact resistance.

One or more nodes in the top sub-cell must be used as a current sink. This is accomplished by using a voltage boundary condition. At these nodes,

$$f_{i,j,1} = 0 = [V - V_{out}]ig|_{i,j,1} \tag{6}$$

Here, $V_{out}$ is an applied bias that is used as the boundary condition.
Because $I_{pf}(V)$ is nonlinear, the system of equations formed by $f(F)$ in vector form, must be solved iteratively using a generalized version of Newton’s method. For the $k^{th}$ iteration,

$$[J]^k \Delta V^{k+1} = -F^k$$

where

$$\Delta V^{k+1} = V^{k+1} - V^k$$

The vectors $V$ and $F$, which are the node voltages ($V$) and $f$ at iteration $k$, are vectors of length $2nm$,

$$V^k = \begin{bmatrix} V^k_{(1,1)}, V^k_{(1,2)}, V^k_{(1,2)}, \cdots, V^k_{(1,m,1)}, V^k_{(1,m,2)}, V^k_{(2,1,1)}, V^k_{(2,1,2)} \\ \vdots, V^k_{(n,1)}, V^k_{(n,2)}, K, V^k_{(n,m,1)}, V^k_{(n,m,2)} \end{bmatrix}^T$$

$$F^k = \begin{bmatrix} f^k_{(1,1)}, f^k_{(1,2)}, f^k_{(1,2)}, \cdots, f^k_{(1,m,1)}, f^k_{(1,m,2)}, f^k_{(2,1,1)}, f^k_{(2,1,2)} \\ \vdots, f^k_{(n,1)}, f^k_{(n,2)}, K, f^k_{(n,m,1)}, f^k_{(n,m,2)} \end{bmatrix}^T$$

and $J$ is the Jacobian matrix of dimensions $2nm \times 2nm$, given by

$$J^k = \begin{bmatrix} \frac{\partial f_{1,1}}{\partial V_{1,1}}, \frac{\partial f_{1,1}}{\partial V_{1,2}}, \frac{\partial f_{1,1}}{\partial V_{2,1}}, \frac{\partial f_{1,1}}{\partial V_{2,2}}, \cdots, \frac{\partial f_{1,1}}{\partial V_{n,m,1}}, \frac{\partial f_{1,1}}{\partial V_{n,m,2}} \\ \frac{\partial f_{1,2}}{\partial V_{1,1}}, \frac{\partial f_{1,2}}{\partial V_{1,2}}, \frac{\partial f_{1,2}}{\partial V_{2,1}}, \frac{\partial f_{1,2}}{\partial V_{2,2}}, \cdots, \frac{\partial f_{1,2}}{\partial V_{n,m,1}}, \frac{\partial f_{1,2}}{\partial V_{n,m,2}} \\ \vdots \end{bmatrix}$$

The iteration process given by Equation 7 is repeated for a particular $V_{out}$ until $\max(\Delta V) < \varepsilon$, where $\varepsilon$ is some small voltage magnitude (on the order of $1 \times 10^{-8}$ V) that acts as a convergence criteria. The maximum power output of the tandem, $P_{\text{max}}$, is easily found by repeating this procedure in conjunction with a numerical method such as bisection, or the secant method until the $V_{out}$ producing maximum power is found, the global voltage and current of the cell at this condition is given by $V_{\text{max}}$ and $I_{\text{max}}$, respectively.

2.2. Extraction of loss mechanisms

Once $P_{\text{max}}$ has been determined, the various power loss mechanisms are extracted using the solved node voltages, $V$. The equations that will be given in this section apply to nodes in the field of the
device and must be modified for those around the perimeter. Additionally, all of the loss mechanisms except for joule losses in the grid electrodes and bus bar must be computed for both sub-cells.

2.2.1. Joule losses. Joule losses, or $I^2R$ loss, are calculated for the emitter layers ($P_{L,EJ}$), bus bar ($P_{L,BJ}$), and grid electrodes ($P_{L,GJ}$) simply by summing the contribution of each node. For nodes in the field of the tandem, $P_{L,EJ}$ for each sub-cell is calculated as

$$P_{L,EJ} = \sum \text{Emitter} \left( \frac{V - VR}{RR} \right)^2 + \text{Bus Bar} \left( \frac{V - VL}{RL} \right)^2 + \text{Grid Electrodes} \left[ \left( \frac{V - VT}{RT} \right) + \text{Bus Bar} \left( \frac{V - VB}{RB} \right) \right]$$

Recall from the Section 2 and Figure 2, that the small subscript resistors ($R_r$, $R_p$, $R_n$, and $R_b$) are related to the sheet resistance of each tile, whereas the large subscript resistors ($R_R$, $R_L$, $R_T$, and $R_B$) are the sum of each tile’s resistance and that of the neighboring tiles. Similarly, $V$ and $V_R$, $V_L$, $V_T$, and $V_B$ are the voltage of each node and its nearest neighbors, respectively.

$P_{L,BJ}$ and $P_{L,GJ}$ are calculated similarly using the nodes representing the bus bar and grid electrodes, respectively.

2.2.2. Bias-point loss. As shown in Figure 1, current flows laterally across the emitter and into the grid electrodes and bus bar. Because the emitter and electrodes have finite resistance, a potential gradient is developed across the cell area. In addition to an overall maximum power point for the tandem, a local maximum power condition exists, which depends upon the details of the sub-model and the local irradiance. Only some regions of the tandem can operate at the local maximum power point because of the potential gradient: much of the cell does not produce as much as it could under ideal conditions and thus constitutes a power loss. This bias-point loss, $P_{L,BP}$, is easily calculated by finding the difference between the power produced locally by each node and the power that could be produced if it were operating at the local maximum power condition. $P_{L,BP}$ is only calculated for illuminated emitter nodes that are illuminated. For illuminated emitter nodes in the field of the device,

$$P_{L,BP} = \sum_{\text{Illuminated Emitter Nodes}} \left[ V_{mp, local} I_{mp, local} - V_{solved, D, local} I_{D, local} \right]$$

Here, $V_{mp, local}$ and $I_{mp, local}$ (note that $I_{mp, local} = I_D, local (V_{mp, local})$) are the local maximum power voltage and current at each node respectively calculated (in isolation) using the sub-model for each sub-cell. Likewise, $V_{solved, local}$ is the voltage at the node with the cell as a whole operating at its global maximum power point, and $I_D, local (V_{solved, local})$ is the current produced by the node at this voltage—$V_{solved, local}$ for each node was found during the iteration process described in Section 2.1 (at the global $V_{mp}$), and $I_D, local$ for each node is easily calculated using $V_{solved, local}$. These values vary spatially with the illumination profile. $P_{L,BP}$ is typically a minor loss under uniform illumination and/or low-sheet resistance, but, as will be shown, may be substantial under non-uniform illumination because of the rapid change in the local maximum power voltage across the cell area.

2.2.3. Grid electrode shadowing. Grid electrode shadowing loss, $P_{L,S}$, is due to the electrode metallization reflecting away all of the incident spectral energy in these locations. $P_{L,S}$ is easily calculated by summing the power that could potentially be produced by the regions of the cell shadowed by grid electrodes and the bus bar. For illuminated grid electrode nodes in the field of the device,

$$P_{L,S} = \sum_{\text{Illuminated Grid Electrode Nodes}} V_{mp, local} I_{mp, local}$$

Again, $V_{mp, local}$ and $I_{mp, local}$ are the local maximum power voltage and current respectively and are calculated using the sub-model for each sub-cell.
2.2.4. Dark-diode loss. Non-illuminated areas of the solar cell act as a simple diode with an exponential dependence on voltage. These areas of the cell, which may be under the electrodes or in other non-illuminated regions, are termed dark-diodes. They cause dark-diode loss, \( P_{LDD} \), which results from the recombination current in these regions (which flows in a direction opposite to the light generated current). \( P_{LDD} \) is calculated by summing the power loss in these non-illuminated regions. Here, \( V_{solved_{local}} \) is the voltage at the node, and \( I_D(V_{solved_{local}}) \) the current produced each node with the solar cell operating at its global maximum power condition.

\[
P_{LDD} = \sum_{\text{Non-Illuminated Emitter Nodes}} V_{solved_{local}} I_D(V_{solved_{local}})
\]  

(15)

2.2.5. Total loss. The total power loss, \( P_{LT} \), is found by summing the contribution of each of the six mechanisms discussed here (\( P_{LEJ}, P_{LBJ}, P_{LGI}, P_{LBP}, P_{L,S}, \) and \( P_{LDD} \)). As already mentioned, additional loss mechanisms, such as contact resistance and current flow in the tunnel junction, cause additional loss. However, the focus of this work is on understanding the primary losses resulting from the lateral flow of current in the sub-cell emitter layers, and thus these additional loss mechanisms have been omitted.

3. LOSS MECHANISMS IN A GaINP/GAAS CONCENTRATOR CELL RESULTING FROM LORENTZIAN ILLUMINATION

In order to understand the role of the various loss mechanisms described in Section 2, it is necessary to apply the model to a specific dual-junction case. This requires choosing both a particular dual-junction tandem and illumination pattern.

A GaInP/GaAs-like device is a good choice for this analysis, as these devices are a very common part of a solar concentrator system, as a standalone device [13–15, 17, 21], as part of a three or more junction tandem [16, 22], or as an element in a spectrally split system [23–25].

Similarly, a specific illumination pattern must be employed. A useful illumination pattern for this study is one that is given by a Lorentzian distribution along the cell width and is constant along the cell length. As will be shown, this is the expected pattern for a very simple parabolic concentrator and is convenient because the length symmetry can be taken advantage of to improve the accuracy of the simulation and reduce the time required to achieve a solution.

3.1. Lorentzian illumination

As noted, the Lorenzian illumination pattern is the expected pattern resulting from a very simple parabolic trough concentrator. Applying a basic ray-tracing technique to a simple parabolic mirror using the law of reflection in vector form shows that this is the case. For an incident ray, \( r \), reflected ray, \( r' \), and a normal vector, \( \mathbf{n} \), at the point of reflection,

\[
r' = r - 2(n \cdot r)n
\]

(16)

A set of light rays, assumed in this simple case to be normal to the minima of the parabola, is incident on the parabolic mirror. Using Equation 16, the reflected ray is calculated and traced, and its intersection with a receiving plane, which is the location of the solar cell, is easily determined. The incident ray density along the length of the receiving plane gives the relative irradiance at that point. An illustration of the ray-tracing technique is shown for 10 incident rays in Figure 3 [12]. Here, the parabolic mirror is defined by \( z = ay^2 \), with \( a = 0.025 \text{ mm}^{-1} \).

An example of the relative spectra incident on the receiver plane (using \( 10^5 \) rays) is shown in Figure 4 for receiver planes close to and far from the focus of the parabolic mirror [12]. Note that the shadowing of incident rays because of the receiver plane has been ignored for this simple case.

As illustrated in Figure 4, a Lorentzian distribution fits the results of the ray trace simulation almost exactly. The normalized form of this distribution is given by Equation 17.
\[ I(y) = \frac{A}{\pi} \left[ \frac{\Gamma/2}{y^2 + (\Gamma/2)^2} \right] \]

Figure 3. Example of ray trace procedure for a simple parabolic trough concentrator with \( a = 0.025 \text{ mm} \) and 10 incident rays. Incident rays are shown with red lines, normal vectors with green lines, and the reflected rays with blue lines. Shadowing of incident rays because of the receiver plane (shown in magenta) has been ignored for this simple case ([12], 2011).

Figure 4. Irradiance profile for parabolic trough with \( a = 0.025 \text{ mm} \) for a receiver plane close (a) to and far and (b) from the focus. Shadowing of the incident rays because of the receiver plane has been ignored ([12], 2011).

The distribution half-maximums are located at \( y = \pm \Gamma/2 \), and \( A \) is a scaling parameter that accounts for factors such as the average concentration and incident spectral power. Here, it has been assumed...
that the profile is centered along the cell width (the y-axis, also along the length of the grid electrodes shown in Figure 1).

As already noted, it is assumed that the illumination profile does not vary along the cell length (along the length of the bus bar, as shown in Figure 1)—this symmetry allows the simulation to focus on a single tile, as defined in Figure 1. Further, for all \( \Gamma \), \( A \) will be adjusted so that the total spectral energy incident on the cell surface is the same as the uniform illumination case. Though somewhat unrealistic, this is carried out so that a comparison of the losses incurred by each value of \( \Gamma \) are not obscured by the total incident irradiance in each case.

3.2. Sub-model parameters

As noted in Section 2.1, the quasi-3D model requires choosing an analytic sub-model that accounts for light-induced carrier generation and carrier recombination within each sub-cell. For the purposes of this analysis, an ideal diode sub-model and with the form given in Equation 18, is sufficient.

\[
J(V) = J_{SC} - J_0 (\exp[V/nV_{th}] - 1)
\]

Here, \( J_{SC} \) is the short-circuit current density, \( J_0 \) is the dark recombination current density, \( n \) is the ideality factor, and \( V_{th} \) is the thermal voltage (\( =kT/q\approx0.0259 \) V at 300 K). High-quality sub-cells may have \( n \approx 1 \) and are designed so that they are nearly \( J_{SC} \)-matched. The dark recombination current density, \( J_0 \), for a sub-cell with a bandgap \( E_g \) in this case is given by the ‘state of the art’ approximation proposed by Gray et al. [26], which gives a realistic approximation for world class, but realistic, solar cells.

\[
J_0 = 1.14 \times 10^9 \exp[-40.5E_g] \text{[mA/cm}^2]\]

In this case, the top sub-cell is assumed to be GaInP with a bandgap of 1.9 eV, and the bottom sub-cell is assumed to be GaAs with a bandgap of 1.42 eV. Further, \( J_{SC} = 14 \text{ mA/cm}^2 \) (at one sun average optical concentration) is assumed for both sub-cells. This is a reasonable assumption for a high-quality tandem under the terrestrial standard spectrum.

Typical GaInP and GaAs sub-cell emitter layers have thicknesses between approximately 50 and 200 nm [13–15, 21] and doping densities between \( 10^{18} \text{ cm}^{-3} \) and \( 3 \times 10^{18} \text{ cm}^{-3} \) [13, 15]. An n-type GaInP emitter has a majority carrier mobility of approximately 400 to 600 V/cm\( \cdot \)s [27], whereas an n-type GaAs emitter has a majority carrier mobility of approximately 2000 to 4000 V/cm\( \cdot \)s [28]. Using Equation 1, reasonable top and bottom sub-cell emitter sheet resistances, \( R_{s,T} \) and \( R_{s,B} \), are calculated to be \( \approx 500 \Omega/\text{sq.} \) and \( \approx 10 \Omega/\text{sq.} \), respectively.

The bus bar is assumed to be composed of 100 \text{um} wide (\( w_b \)) and 5 \text{um} thick (\( t_b \)) silver and extend the length of the device. Similarly, the grid electrodes are composed of 10 \text{um} wide (\( w_g \)) and 5 \text{um} thick (\( t_g \)) silver, and extend the width of the device, which in this case is assumed to be 2 \text{mm}. Table I summarizes the assumed cell and sub-model parameters. Note that because of the symmetry of the cell layout and illumination pattern and the assumption of an electrical contact along the entire length of the bus bar, the results will be presented as per unit cell length, and thus this length is not given in Table I.

Table I. Sub-model and cell layout parameters. It is assumed that current is collected along the entire bus bar length.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Top Sub-Cell</th>
<th>Bottom Sub-Cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_{SC} ) (mA/cm(^2))</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>( J_0 ) (mA/cm(^2))</td>
<td>4.345( \times 10^{-25} )</td>
<td>1.204( \times 10^{-16} )</td>
</tr>
<tr>
<td>( n )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( V_{th} ) (V)</td>
<td>( \approx 0.0259 )</td>
<td>( \approx 0.0259 )</td>
</tr>
<tr>
<td>( R_s ) (( \Omega/\text{sq.} ))</td>
<td>500</td>
<td>100</td>
</tr>
<tr>
<td>( w_b ) (\text{um})</td>
<td>100</td>
<td>—</td>
</tr>
<tr>
<td>( w_g ) (\text{um})</td>
<td>10</td>
<td>—</td>
</tr>
<tr>
<td>( t_b ) (\text{um})</td>
<td>5</td>
<td>—</td>
</tr>
<tr>
<td>( t_g ) (\text{um})</td>
<td>2</td>
<td>—</td>
</tr>
<tr>
<td>( W ) (\text{mm})</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
3.3. Variation in power loss mechanisms with N/L

The grid electrode density, $N/L$ (where $N$ is the number of grid electrodes, and $L$ is the device length parallel to the bus bar), plays a significant role in determining the performance of the tandem. As such, it is useful to consider how these losses vary with $N/L$ and how this influences the optimal grid electrode density, $N_{\text{opt}}/L$, particularly because $N_{\text{opt}}/L$ is often calculated by considering only the trade-off between grid electrode shadowing and emitter joule loss.

A GaInP/GaAs-like two-terminal tandem, with the sub-model parameters given in Table I, and a device width, $W$, of 2 mm was simulated using the quasi-3D model for a wide range of grid electrode density. Note that, as illustrated in Figure 1, the grid electrodes are assumed to extend the entire width of the tandem and the bus bar the entire length of the tandem. Again, because of the symmetry of the device layout and illumination pattern, only one tile was simulated; thus, the power loss mechanisms are expressed as per unit device length.

As an example, the results of this simulation for an average optical concentration (or geometric concentration, i.e., the ratio of the system input area to the cell area), $X$, of 100 suns with $\Gamma = 0.05$ mm (highly non-uniform illumination, similar to Figure 4(a)) and $\Gamma = 1000$ mm (uniform illumination) is shown in Figure 5. Again, the total incident irradiance has been adjusted so that it is the same as the uniform illumination case, independent of $\Gamma$. Note that the node areas were $\approx 30 \text{ um}^2$ in all cases, and the loss shown here is the sum of the loss in each sub-cell. Additionally, the resulting joule losses in the grid electrodes were found to be relatively insignificant at this optical concentration and cell dimensions and have thus been omitted.

As $N/L$ increases, more of the cell is shadowed and thus $P_{L,S}$ increases. On the other hand, $P_{L,EJ}$ decreases because of a reduction in the distance that current must flow through the resistive emitter to get into the grid electrode or bus bar. $P_{L,S}$ changes only slowly with $\Gamma$, whereas $P_{L,EJ}$ is a strong function of $\Gamma$. This results from a significant increase in the density of current that flows across the length of the tile to the grid electrode in the regions of peak irradiance ($y = 0$) and the proportionality of joule losses to $I^2$. The linear density of current flow into the grid electrode for both values of $\Gamma$ with $N/L = 5 \text{ mm}^{-1}$ is shown in Figure 6 to illustrate this point. Note that the decrease in current flow into the grid electrode near $y = 1$ mm in the uniform illumination, $\Gamma = 1000$ mm, case is because of the current flowing directly into the bus bar rather than into the grid electrode.

The amount of current generated in each tile, and thus that flows across it, is governed by $N/L$. Consequentially, when $N/L$ is small a larger voltage drops across the emitter because of the sizeable amount of current flowing across it. This results in high $P_{L,BP}$ at low $N/L$ because most of the tile
operates far from the local maximum power condition. Thus, as $N/L$ increases, the voltage drop across the tile decreases, driving down $P_{LPBP}$. This is illustrated in Figure 7, which shows the potential along the back of the tile ($y = -1$ mm, opposite the bus bar) for $N/L = 3$ mm$^{-1}$ and 8 mm$^{-1}$ with $\Gamma = 1000$ mm.

The smaller voltage drop at high $N/L$, which is due to less current generation and subsequent flow across the emitter in each tile, allows for more of the tile to operate near the local maximum power condition (though at very high $N/L$, $P_{LPBP}$ begins to increase because the grid electrode pulls the potential of the entire tile down to reduce dark-diode losses).

As illustrated in Figure 5, when the illumination is highly uniform, the optimal grid electrode density, $N_{opt}/L$, is largely determined by grid electrode shadowing and emitter joule loss, as these are clearly the dominant loss mechanisms. On the other hand, when the illumination is highly non-uniform (small $\Gamma$), bias-point loss plays a significant role and aids in driving up the optimal $N/L$ considerably. As an example, consider the potential gradient across the tile for $N/L = 5$ mm$^{-1}$, as shown in Figure 8.

When the illumination is highly non-uniform, the potential gradient cannot adequately follow the illumination profile, and thus the bias-point loss is much higher than the uniform illumination case. Consequently, in this case, $P_{LPBP}$ plays a significant role in setting the optimal grid electrode density, which is higher than would be expected if only the shadowing and emitter joule losses were considered. As will be shown in Section 3.4, at very high concentration, the significance of bias-point loss is reduced, and thus its role in setting the optimal grid electrode density is most important for moderate concentrations.

Figure 6. Linear density of current flow into grid electrode for $N/L = 5$ mm$^{-1}$. Note that the bus bar at $y = 1$ mm.

Figure 7. Potential along the back of the tile ($y = -1$ mm, opposite the bus bar) for $N/L = 3$ mm$^{-1}$ and 8 mm$^{-1}$ with $\Gamma = 1000$ mm.
3.4. Variation in power loss mechanisms with $\Gamma$

Having discussed how the power losses vary with the grid electrode density, $N/L$, it is useful to consider how they vary with $\Gamma$ for fixed $W$. The comparison may be skewed if the same grid electrode density is assumed in all cases, as the power output will not be optimal for most values of $\Gamma$. Thus, the variation in power loss is compared under fully grid electrode optimized conditions for each concentration and $\Gamma$.

Figure 9 shows the variation in power loss mechanisms with $\Gamma$ for $X = 1$, 100, and 500 suns. In all cases, the grid electrode density has been set to the optimal value for each pair of $\Gamma$ and $X$.

The variation in the emitter joule loss and shadowing loss follow the expected trends with concentration: as the concentration increases, the joule loss in the emitter increases because of their proportionality to $I^2$ (and thus roughly to $X^2$), which also increases the optimal grid electrode density, $N_{\text{opt}}/L$, and thus the shadowing loss. The emitter joule loss (and also $N_{\text{opt}}/L$) decrease with increasing $\Gamma$. 

Figure 8. Potential profile across tile for highly nonuniform ($\Gamma = 0.05$ mm, top) and uniform illumination ($\Gamma = 1000$ mm, bottom) for $N/L = 5$ mm$^{-1}$. Note that the grid electrode is located at $x = 0$ mm and the bus bar at $y = 1$ mm. The mesh plot indicates the local maximum power voltage.
because the density of current flow through the emitter becomes uniform across its width rather than being concentrated near $W=0$ (as shown in Figure 6).

It is also clear (as expected from Figure 8) that the bias-point loss is more significant when the illumination pattern is highly nonuniform, as the potential gradient across the emitter cannot adequately follow the local maximum power voltage. This loss decreases rapidly with $\Gamma$ and to a lesser degree, with $X$. In all cases, the dark-diode loss is relatively low.

Figure 8 shows the variation in absolute power loss with $\Gamma$; however, it is useful to consider how significant the role of each loss mechanism is relative to the total. Consequentially, Figure 10 shows how the power loss mechanisms vary with $\Gamma$ as a fraction of the total loss in each $\Gamma$ and $X$.

When both $X$ and $\Gamma$ are small, bias-point loss is the dominant mechanism, constituting approximately 70% of the total loss at 1 sun with $\Gamma=0.05$ mm. As $X$ increases, the significance of

---

Figure 9. Variation in power loss mechanisms with $\Gamma$ under full grid electrode optimized conditions expressed as loss per unit sun concentration. Joule losses in the bus bar and grid electrodes constituted less than 5% of the total loss in all cases and have thus been omitted.

Figure 10. Variation in power loss mechanisms with $\Gamma$ under full grid electrode optimized conditions expressed as a fraction of the total power loss. Joule losses in the bus bar and grid electrodes have again been omitted.
the bias-point loss decreases because of a relative increase in shadowing and emitter joule losses, but is still significant at approximately 10–20% of the total loss. Even when the illumination is uniform, bias-point loss accounts for approximately 5–10% of the total loss, which is not negligible. This result is important, as it causes a reduction in the potential advantage that may be achieved by going to higher concentration (to reduce cell area and thus cost) or a higher degree of non-uniformity (to possibly reduce the necessary solar tracker accuracy).

It should be noted that the results presented here, particularly the fractional power loss shown in Figure 10 are sensitive to the assumed cell layout parameters. For example, if 5 um wide and 10 um thick grid electrodes were employed, the conductivity, and thus power loss for a particular N/L would remain the same, but the shadowing loss would be cut in half. However, the layout parameters employed in this work are reasonable for a typically concentrator solar cell.

3.5. Variation in power loss mechanisms with the y-location of peak illumination

This paper has focused on power losses for a non-uniform profile, with a Lorentzian distribution peaked along the y-axis at y=0. However, it is also interesting to consider, at least in brief, what may be expected to happen as the peak moves away from y=0, which could happen, for example, in a tracking concentrator system with nonzero-tracking error.

Of course, when off-normal light is incident on the parabolic concentrator, the profile will no longer have a Lorentzian shape; further, concentrators with broad illumination profiles (with large Γ) will rapidly fall off the cell edge for the movement along the y-axis, thus significantly decreasing the overall efficiency of the cell. For this discussion, it is assumed that the Lorentzian profile remains constant, and that Γ is sufficiently small that negligible light falls off the edge of the cell until the peak intensity is nearly at the cell edge. Additionally, only movement of the profile in the y-direction is considered.

Under these assumptions, one would expect the shadowing loss to remain nearly constant until the profile is almost at the cell edge, at which point shadowing loss would increase rapidly. Similar behavior would be anticipated for the emitter joule loss and dark-diode losses. On the other hand, joule losses in the grid electrodes would be expected to drop, significantly at high concentration, because of the decreased path length through the grid electrodes that carriers must flow before getting into the bus bar. The bias-point loss is somewhat more difficult to predict, but under these assumptions one would anticipate the bias-point losses would decrease because of the reduced potential drop across the grid electrodes, which, on average, would allow the illuminated regions to operate closer to their local maximum power point. As noted, this analysis is much more complex for large Γ as the decrease in total spectral power on the active area of the cell must also be considered.

4. CONCLUSIONS

In this work, a unit cell-based quasi-3D model was developed for use in simulating the effect of non-uniform illumination on the performance of multijunction concentrator solar cells. The model presented here focused on dual-junction devices, though it could be extended to three or more junctions easily. This formulation focused on the effect of current flow in the emitter layers and assumed that the effects of tunnel junction loss, current crowding, and contact resistance are negligible. These effects could also be included in the model.

The quasi-3D model was applied to the analysis of loss mechanisms that result from the lateral current flow in the emitter layers under Lorentzian illumination, which was shown to be the expected irradiance profile for a parabolic trough concentrator. It was shown that at moderate average optical concentration and bias-point loss, which results from the potential gradient across the cell emitter forcing most of it to operate away from the local maximum power condition, plays an important role in setting the optimal grid electrode density, particularly when the illumination is highly nonuniform.

Common grid electrode optimization procedures often ignore the role of this loss mechanism.

The variation in power loss mechanisms with the degree of nonuniformity was considered for 1, 100, and 500 suns. It was shown that when the illumination is highly nonuniform (and Lorentzian in profile), the bias-point loss is the dominant mechanism at low concentration, but also plays a significant
role at higher concentrations, though emitter joule loss and shadowing loss dominate in this case. Under uniform illumination the power loss is dominated by emitter joule loss and shadowing loss for all concentrations.

The results presented here, influence both the optimization of a grid electrode pattern and inform designers in considering tradeoffs in the optical concentrator design.

ACKNOWLEDGEMENTS

This work was supported by DuPont through the Very High Efficiency Solar Cell (VHESC) program funded by the Defense Advanced Research Projects Agency (DARPA), Contract Number HR0011-07-9-0005. The views, opinions, and/or findings contained in this article are those of the author and should not be interpreted as representing the official views or policies, either expressed or implied, of the Defense Advanced Research Projects Agency or the Department of Defense.

REFERENCES


AUTHOR’S BIOGRAPHIES

Alexander Haas was born in Hutchinson, Minnesota in 1983. He graduated from Dordt College in Sioux Center, Iowa in 2006 with a Bachelor of Science degree in Engineering with an electrical emphasis. In 2011, he earned a PhD in Electrical and Computer Engineering from Purdue University in West Lafayette, Indiana. He is currently a Principle Engineer at Emcore Corporation in Albuquerque, New Mexico.

John Wilcox received the BS degree in Electrical and Computer Engineering from Brigham Young University, Provo, UT, USA. He also received the MSECE degree in Electrical and Computer Engineering from Purdue University, West Lafayette, IN, USA, in 2009, where he is currently nearing completion of the PhD degree in Electrical and Computer Engineering. His research interests include semiconductor device physics, temperature dependencies of solar cells, and detailed numerical modeling. He has published numerous research papers in journals and conferences. He is a member of Institute of Electrical and Electronics Engineers (IEEE) and has been the President of the Purdue IEEE Electron Device Society.

Jeffery L. Gray was born in Forest City, Iowa, USA, in September 1954. He received his BS degree in Physics and Mathematics from the University of Wisconsin-River Fall in 1976. He earned his MSEE and PhD from Purdue University in 1978 and 1982, respectively. He is a senior member of the IEEE and is currently an Associate Professor of Electrical and Computer Engineering at Purdue University, West Lafayette, Indiana, USA. He is the author or coauthor of more than 100 journal and conference papers, primarily in the area of modeling and simulation of photovoltaic devices and systems.

Richard Schwartz is a Professor Emeritus in the School of Electrical and Computer Engineering at Purdue University. He is a Fellow of the IEEE and received the IEEE William Cherry Award for his work on high concentration silicon solar cells.