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Understanding Valve Dynamics

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UNDERSTANDING VALVE DYNAMICS
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ABSTRACT
The objective of this paper, that deals with modeling dynamics of self-acting compressor valves, is not a creation of a model that produces charts and diagrams that perfectly agree with the experiment. Rather than that, the model is intended to provide understanding and deeper insight into the function of self-acted compressor valves. It also intends to provide input for designers before the compressor is designed and built. The equivalent one-degree-of-freedom system with several types of non-linearity is used. Each valve, suction or discharge, may have unlimited lift, or its lift may have rigid or flexible stop with or without a mass, and with or without a rebound. The friction force and the sticktion force are also considered. Over thirty design-parameters that affect function of each valve can be investigated. A compressible flow of gas through the valve is considered as well as the leakage of gas between the wall of the cylinder and the piston. Friction losses in bearings and due to the motion of the piston are also considered. The function of each valve starts from well-defined initial conditions. This approach enables investigation of individual parameters, and how each of them affects function of the valve, compressor capacity and coefficient of performance. Program is written in Visual Basic for Applications (VBA) as a macro in EXCEL spreadsheet.

MODELS OF SELF-ACTING VALVES
A proper function of compressor valves is essential to compressor performance. The difficulty in valve design arises from the interaction of the flow of compressible gas and a dynamics of a mechanical system, the valve. Both the non-linear thermodynamics and non-linear dynamics of flexible structures are involved. Despite the progress in computational techniques and mathematical modeling, the simplest models still give best result [9]. Thus, a single-degree-of-freedom mechanical system has been chosen to analyze function of compressor valves, case 1 through 5, and two-degree-of-freedom model of case 6 in Figure 1.

A reed valve is most common type of self-acting valve used in refrigeration and air-conditioning positive displacement reciprocating compressors. Some designs also use discus valves that are true single-degree-of-freedom systems. By inspecting available patent literature (too many to list all of them), one can identify several types of equivalent single-degree of freedom models of reed valves. While there are two types of suction reed valves, reed with a lift-limiter (stop) or a reed without a lift-limiter, the designs of discharge valves are more numerous. Figure 1 shows, schematically, six so far identified equivalent models of compressor valves.

Model 1: Is a clamped-free model of a reed valve or a discus valve.

Model 2: Is the model 1 of a reed valve with a stopper. When the reed hits the stop, the model changes to clamped-hinged type of beam, its mass and stiffness suddenly changes, \( k_1 \) becomes \( k_2 \) and \( m_1 \) becomes \( m_2 \).

Model 3: Is a valve with hard sop. The stop may be perfectly soft (coefficient of restitution equal to zero), or perfectly resilient (coefficient of restitution equal to one), or a real stop (with the coefficient of restitution \( 0 < k < 1 \)). The sticktion force can also be considered if applicable.

Model 4: Is a valve with mass-less stop. Mass of valve remains unchanged while stiffness suddenly changes from \( k_1 \) to \( k_1 + k_2 \). This model is adequate when, for example, a relatively heavy disc hits much lighter spring.

Model 5: Is a model of the valve that has flexible stop. A rigid frame supports both, the valve and the stop. The mass of the stop is comparable with the mass of the valve. The impact of both the masses can be ideal or real (1 \( \geq \) coefficient of restitution \( \geq 0 \)), or a sticktion force can exist between the valve and the stop for a short time interval.

Model 6: This model differs from the model 5 in that it has the valve supported by a flexible stop, which mass is comparable with the mass of the valve. The motion of the valve starts as a two-degree-of-freedom system. When the
valve hits the stop it may or it may not rebound (1 ≥ coefficient of restitution ≥ 0), or a sticktion force can exist between the valve and the stop for a short time interval. The stop may or may not be pushed against the plate (be preloaded).

![Diagrams showing lumped parameter models of compressor valves.](image)

Figure 1: Lumped parameter models of compressor valves.

**EQUATION OF MOTION**

The equation of motion of the valve and all the other supporting equations are similar to those presented in [2], [3], [4], [8],[9],[10], [11] with some modifications. The space available does not permit detailed derivations.

Except Model 1 the mass and stiffness change abruptly when the valve hits the stop. That means the parameters in the equation of motion are functions of displacement.

\[
\begin{align*}
m_V (x_V) \cdot \ddot{x}_V + c_V \cdot \dot{x}_V + k_V (x_V) \cdot x_V &= F_{PV} + F_G - F_{\text{friction}} - F_{\text{sticktion}} - F_{V-P\text{reload}} \\
m_S \cdot \ddot{x}_S + c_S \cdot \dot{x}_S + k_S (x_S - x_{ST}) &= -F_{S-P\text{preload}}
\end{align*}
\]

Where is:
- \(x\) Lift \([m]\)
- \(k\) stiffness \([N.m^{-1}]\)
- \(m\) mass \([kg]\)
- \(F_P\) force due to pressure differential \([N]\)
- \(F_G\) force due to gas flow \([N]\)
- \(c\) coefficient of viscous damping \([N.s.m^{-1}]\)
- \(v,s\) index indicating valve or stop
- \(x_{ST}\) height of stop \([m]\)

The abrupt changes in the magnitude of mass of the valve and its stiffness that occur when the lift of the valve hits the stop have been already explained. The explanation of the remaining forces follows.

**Force of Gas Pressure**

The force \(F_P\) due to the pressure differential across the valve is

\[
F_P = \frac{\pi}{8} \left[ (p_2 - p_1) \cdot d_1^2 + (p_2 + p_1) \cdot d_2^2 \right]; \quad p_2 > p_1
\]

Where is
- \(d_1\) inner diameter of valve seat \([m]\)
- \(p_2\) pressure down the stream \([Pa]\)
Force of Gas Flow

The dynamic force of the gas flow is

\[ F_G = C_D \cdot A_P \cdot \rho_P \cdot \frac{w_p^2 - v^2}{2} \]  

(3)

Where is:
- \( C_D \): drag coefficient of valve
- \( \rho_P \): density if gas in the port [kg.m\(^{-3}\)]
- \( A_P \): area of the valve [m\(^2\)]
- \( w_P \): velocity of gas in the port [m.s\(^{-1}\)]

Velocity of gas

The velocity of gas in the port and within the seat is calculated in two steps. First, the velocity of frictionless compressible flow is calculated using [1]

Where \( p_1, \rho_1, \) and \( n_1 \) are the inlet pressure, inlet density, and inlet polytropic exponent respectively, and \( p_2 \) and \( w_2 \) are outlet pressure and outlet velocity respectively. Pressure loss due to gas friction is [1], [6]

\[ w_2 = \sqrt{\frac{2 \cdot p_1 \cdot n_1}{n_1 - 1} \cdot \frac{p_1}{\rho_1} \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n_1 - 1}{n_1}} \right]} \]  

(4)

\[ p_L = \frac{\lambda}{d} \cdot \frac{\rho \cdot w^2}{2}; \text{ where } \lambda = \frac{64}{Re} \text{ for } Re < 2320; \text{ } \frac{0.3164}{\sqrt{Re}} \text{ for } Re > 10^4 \]  

(5)

\[ \lambda = \frac{0.3164 \cdot \frac{64}{Re} - \frac{64}{2 \cdot 10^4}}{10000 - 2320} \times (Re - 2320) \text{ for } 2320 < Re < 10^4 \]

The Reynolds number \( Re \) is

\[ Re = \frac{w \cdot \rho}{\mu} \cdot 2 \cdot x \text{ for valve opening}; \text{ } Re = \frac{w \cdot \rho}{\mu} \cdot d_p \text{ for the port} \]  

(6)

Where is
- \( d_p \): diameter of the port [m]
- \( w \): velocity of gas

The corrected velocity of compressible gas with friction is calculated from [6]

\[ w_2 = \sqrt{\frac{2 \cdot n_1 \cdot p_1}{n_1 - 1} \cdot \frac{p_1}{\rho_1} \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n_1 - 1}{n_1}} \right]} \]  

(7)

Where is:
- \( \xi_i \): inlet resistance coefficients of the valve and the port
- \( \lambda_j \): friction resistance coefficient of the valve opening and the port

Velocity of gas calculated from equation (4) or (6) is reiterated using equation (5) and (6) one more time. The velocity of gas at the valve outlet can not exceed critical velocity \( w_c \). The gas velocity becomes critical when the pressure ratio \( r \) exceeds critical pressure ratio \( r_c \), that is when

\[ r_c = \left( \frac{p_2}{p_1} \right)_c = \left( \frac{2}{k+1} \right)^{\frac{1}{k-1}} \]  

(8)

\[ w_c = \sqrt{\frac{2 \cdot k \cdot p_1}{k+1} \cdot \frac{1}{\rho_1}} \]
The velocity of gas in the port and at the valve outlet is different except in the case when the valve lift is $x \geq \frac{d_P}{4}$, where $d_P$ is port diameter. The velocity of gas that strikes the valve is

$$w_v = \frac{A_p}{A_v} \frac{p_p - w_p}{p_v}; \quad A_v = \pi \cdot d_2 \cdot x \quad \text{is valve area}, \quad A_p = \frac{\pi \cdot d_P^2}{4} \quad \text{is port area} \quad (9)$$

**Cylinder Pressure**

Pressure in the cylinder is governed by the expression

$$p_C = p_{C0} \cdot \left( \frac{M_0 + \Delta M}{p_{C0} \cdot V_C} \right) ; \quad M_0 = A_{C} \cdot \rho_{C0} \cdot s_0; \quad \Delta M = \left( A_p \cdot p_p - M_{BB} \right) \cdot \Delta t \quad (10)$$

$$\dot{M}_{BB} = \beta_B \left( \frac{\pi \cdot (D_C - D_p)^3 \cdot (D_C + D_p) \cdot (p_C - p_S)}{96 \cdot \mu_{C or S} \cdot L_p} + \frac{(D_C - D_p) \cdot L_p \cdot \nu_p}{2} \right) \cdot p_{C or S} \quad (11)$$

Where is:

- $M_0$ initial mass of gas in the cylinder [kg]
- $L_p$ length of piston [m]
- $\Delta t$ is time interval [s]
- $\rho_{C or S}$ gas density [kg.m$^{-3}$]
- $\beta_B$ blow by coefficient
- $\mu_{C or S}$ dynamic gas viscosity [Pa.s]
- $D_C$ cylinder diameter [m]
- $p_S$ suction pressure [Pa]
- $D_p$ piston diameter [m]
- $p_C$ cylinder pressure [Pa].

In equation (10) and (11), index $C$ means cylinder, index 0 means initial state of gas in the cylinder; index $P$ means port, and $M_{BB}$ is mass flow rate of gas that escapes from the cylinder (during compression), or enters the cylinder (during suction) through the gap between the piston and the wall of the cylinder. The blow by coefficient is the decimal fraction of piston circumference open to the gas flow [5].

Density of gas in the cylinder is calculated from

$$\rho_C = \rho_{C0} \cdot \left( \frac{p_C}{p_{C0}} \right)^{\frac{1}{n_0}} \quad (12)$$

Where

- $\rho_{C0}$ is initial density of gas in the cylinder [kg.m$^{-3}$]
- $p_{C0}$ is initial pressure of gas in the cylinder [Pa]

**Friction Force**

The design of discharge valve considered in this paper has spring that has free shape close to the arc of a circle. When the valve opens, the arc is stretched and the ends of the spring rub against the frame. Using principle of virtual work, the friction force is derived as

$$F_{Friction} = F_F = \nu \cdot k \cdot \left[ h - (x_0 + x) \right] \cdot \sqrt{\frac{3 \cdot h - (x_0 + x)^2}{3 \cdot L^2 - 16 \cdot \left[ h - (x_0 + x) \right]^2}} \quad (13)$$

Where is:

- $F_F$ friction force [N]
- $\nu$ coefficient of friction
- $x_0$ initial compression of spring [m]
- $k$ linear spring constant [N.m$^{-1}$]
- $L$ length of the arc of unloaded spring [m]

**Sticktion force**

The magnitude of sticktion force depends on the geometry of the valve seat and the geometry of the oil film. There is two possible geometry of the valve seat. The first one is a flat valve approaching (or separating from) a flat seat that is parallel with the valve. The second one is a flat valve that approaches (or separates from) a parallel valve seat that has a doughnut shape cross-section. One needs to consider the difference between the force of approach and the force of separation because the initial thickness of oil film for the valve separation depends on the force that pushes
the valve against the seat. The sticktion force of approach starts when the valve that is approaching the valve seat touches the surface of the oil film. The oil is pushed out of the valve gap. On the other hand, when the valve starts to separate from the valve seat oil has to flow into the opening gap. Two situations can happen. The oil film is relatively thick, and there is enough oil flowing into the valve gap; the force of separation will be the same as the force of approach. Or, the oil film is relatively thin, and the quantity of oil in the valve gap remains constant during the valve separation (oil starved gap) the force of separation is different from the force of approach. It is assumed the thickness of oil film on the seat and on the valve is the same, and that both together constitute the total thickness of oil film when the valve goes into the contact with the seat or the stop.

**Sticktion Force of Approach - Flat Seat - Oil-Filled Gap**

When flat plate approaches parallel an annular valve seat, and both are covered with oil film, the squeeze film force is [7]

\[ F_s = F_A = C \cdot \frac{v}{x^3}; \quad C = \frac{3 \cdot \pi \cdot \mu \cdot \left( d_2^2 - d_1^2 \right)}{32 \left( \frac{d_2^2 + d_1^2}{\ln d_2 - \ln d_1} \right)} \]

Where is

- \( F_s \)  sticktion force [N]
- \( d_1 \)  inner diameter of seat [m]
- \( d_2 \)  outer diameter of seat [m]
- \( \mu \)  viscosity of oil [Pa.s]

Sticktion force of approach is inversely proportional to the third power of instantaneous thickness of oil film.

**Sticktion Force of Separation - Flat Seat - Oil-Filled Gap**

In this case, the sticktion force of separation is the same as the force of approach. Equation (14) applies.

**Sticktion Force of Separation - Flat Seat - Oil-Starved Gap**

This is the case of raised seat with a recess on both sides of the seat or a seat with trepan.

\[ F_s = C \cdot \frac{v}{x^3}; \quad C = \frac{3 \cdot \pi \cdot \mu \cdot \left( r_B^4 - r_A^4 \right)}{2 \left( \frac{r_B^2 - r_A^2}{\ln r_B - \ln r_A} - 1 \right)} \]

\[ r_A = \frac{d_1}{4} \left( 1 + \frac{x_0}{x} \right); \quad r_B = \frac{d_1}{4} \left( 1 - \frac{x_0}{x} \right) \]

\[ x = \text{thickness of oil film [m]} \quad x_0 = \text{initial thickness of oil film [m]} \]

By inspecting equation (15), one can see, the force of separation is inversely proportional to the seventh power of instantaneous thickness of oil film.

**Sticktion Force - Doughnut Shaped Seat**

In this case, there is a little difference, that can neglected, between the oil-filled and oil-starved gap because the oil film on the seat follows the curvature of the seat. The sticktion force of approach is the same as the sticktion force of separation

\[ F_s = \frac{v}{\sqrt{x^3}}; \quad C = \frac{3 \cdot \sqrt{2} \cdot \pi^2 \cdot \mu \cdot \sqrt{r_S}}{D_S} \]

Where

- \( D_S \)  is diameter of contact circle [m]
- \( r_S \)  radius of seat [m]

**Valve pre-load**

Frequently, valves are assembled, intentionally or because of tolerance chaining, so that the valve is pushed against its seat. It is believed that this force helps to improve sealing and prevents rebound of the valve from its seat. On the other hand it also delays valve opening that results in bigger spike on the cylinder pressure. In any case, the impact of this force on the function of the valve has to be taken into consideration.
Valve Rebound
In the case the valve is not parallel with the seat when it approaches it, such as when the edge of a suction valve hits the piston, or the valve hits a sharp projection on the stop, the sticktion force will not exist, and a rebound should be considered instead. This will change velocity of valve. The after the impact velocities of colliding masses are

\[
v_{1a} = \frac{(m_1 - e \cdot m_2) \cdot v_{1b} + m_2 \cdot v_{2b} \cdot (1 + e)}{m_1 + m_2}, \quad v_{2a} = \frac{m_1 \cdot v_{1b} \cdot (1 + e) + (m_2 - e \cdot m_1) \cdot v_{2b}}{m_1 + m_2}
\]

Where is:
- \(m\) mass [kg]
- \(e\) coefficient of restitution
- \(v\) velocity [m.s\(^{-1}\)]

Indexes 1 and 2 indicate first and second mass and their velocities. Index a means after impact; index b means before impact.

SOLVING EQUATION OF MOTION
Although more sophisticated methods of solution of a system of differential equations of second order with non-linearity and abrupt change in parameters are available [x]; the Euler method is used. Its matrix form is

\[
x_{i+1} = A_i^{-1} \left[ \Delta t^2 \cdot f_i + A_2 \cdot x_i + A_3 \cdot x_{i-1} \right]; \quad A_1 = \left[ M + \frac{\Delta t}{2} \cdot C \right], \quad A_2 = \left[ 2 \cdot M - \Delta t^2 \cdot K \right],
\]

\[
A_3 = \left[ \frac{\Delta t}{2} \cdot C - M \right]
\]

The first step of solution, and every time the non-linearity is accoutered, is found from initial conditions

\[
x_i = B_i^{-1} \left[ \Delta t^2 \cdot f_0 + B_2 \cdot \dot{x}_0 + A_1 \cdot x_0 \right]; \quad B_1 = \frac{1}{2} \cdot M, \quad B_2 = \Delta t \cdot \left[ 2 \cdot M - \Delta t \cdot C \right]
\]

Where is
- \(M\) mass matrix
- \(C\) damping matrix
- \(K\) stiffness matrix
- \(f\) vector of forces
- \(x\) vector of displacements
- \(t\) time [s]

Index i indicates current time instant, index 0 indicates initial state.

For a single-degree-of-freedom system, the matrixes in equation (18) through (19) are replaced by scalar values.

MODELING SUCTION VALVE
The dynamic modeling of suction valve (SVD) starts from the top-dead-center, and it assumes the clearance volume is filled with gas that has pressure equal to the pressure in the discharge plenum and the temperature that is equal to the compression temperature.

Figures 2, 3, 4, and 5 show valve lift, cylinder pressure, valve velocity and velocity of gas in the valve opening.

![Figure 2: Lift of unrestricted cantilever beam type reed valve](image1)

![Figure 3: Suction pressure](image2)
respectively on the three cases of model 1 from Figure 1. Case number one is a valve close to the real design. Its frequency is 390 Hz and static stiffness is 510 N/m. Case number two is a lighter valve that has frequency of 82 Hz and static stiffness 133 N/m. Case number three is heavier valve with frequency of 87 Hz and static stiffness 900 N/m. In all the three cases valve closes at the BDC, and the quantity of gas that enters the cylinder is the same. Thus, the capacity of the compressor with any of those three valves will be same. The difference will be in the compressor noise due to differences in the mass of the valve and the velocity, magnitude and frequency content of pressure pulsation, and velocity of gas within the valve seat.

Figures 6 and 7 show the impact of oil on the valve lift and cylinder pressure. Presence of oil on the valve seat increases lift and pressure drop. Delayed valve opening and delayed valve closing decreased volumetric efficiency by 7.6%.

Figure 8 shows effect of flexible stop on valve closing, and Figure 9 shows effect of oil and valve rebound on valve closing.
MODELING DISCHARGE VALVE

The model of discharge valve (DVD) starts from at bottom dead center, and it assumes the cylinder is fully filled with the gas that has pressure and temperature equal to those in the suction plenum. Altogether, thirty-two parameters have an impact on discharge valve opening and closing, capacity of the compressor, its COP, and the noise of the compressor. This is too many to show in this paper. Some of them have smaller, some of them greater effect on the compressor's capacity, COP, and noise. Some combinations of parameters are more beneficial than the other ones. Three representative valves are shown in Figures 10 through 17.

The natural frequency of the valve is 1157 Hz, static stiffness is 3700 N/m, and the fraction of critical damping is 0.05. The natural frequency of soft stop is 318 Hz, static stiffness is 20kN/m, and the fraction of critical damping is 0.05. The height of stop is 1 mm. The dry friction of the valve spring has relatively small effect, unless the coefficient of friction is considered unrealistically high, say 0.5 or more.

Figure 10 and 11 show almost negligible difference between hard stop and soft stop without oil on the valve lift and cylinder pressure. In addition, the capacity of the compressor and COP were the same.

Figure 12 and 13 show impact of valve rebound (30% energy recuperation) on the valve lift of the same valve as in Figure 11 and 12. While the rebound from the stop has negligible impact on capacity, COP and cylinder pressure, the rebound from the seat decreases compressor's capacity by 2.5% due to the flow-back of the gas.

Figure 14 and 15 show impact of oil on the valve with hard or soft stop. While oil sticktion degraded capacity by 4.7%, COP by 4.3 %, and increased cylinder pressure by 216 kPa of the compressor with hard stop, the impact of oil sticktion with soft stop was slightly positive. Figure 16 and 17 shows that two valves with different natural frequencies may be designed to work properly on the same compressor. This contradicts traditional thought that only one valve with a unique frequency can work properly. In this case, the natural frequency of the second valve is 686 Hz and static stiffness 1300 N/m. Natural frequency of the stop is the same as in previous cases, as well as the height of stop. The effect of oil is negative resulting in 1.7-% drop in capacity. Due to much smaller stiffness of valve, the oil sticktion has greater effect on the motion of the valve.
CONCLUSION

The valve dynamics software SVD and DVD proved to be an efficient tool that helps to understand behavior of self-acting compressor valves, and how numerous parameters affect capacity, COP and noise of the compressor. The impact of some of valve design parameters on capacity, COP, and noise of the compressor has been positively identified and practically verified.

REFERENCES

[2] Boswirth Leopold: "Theoretical and Experimental Study on Valve Flutter", 1990 International Compressor Conference at Purdue, Purdue University, West Lafayette, Indiana, USA.