Multiple Timescales and Modeling of Dynamic Bounce Phenomena in RF MEMS Switches

Ryan C. Tung  
*Purdue University, Birck Nanotechnology Center, rtung@purdue.edu*

Adam Fruehling  
*Purdue University, Birck Nanotechnology Center, soimems@purdue.edu*

Dimitrios Peroulis  
*Purdue University, Birck Nanotechnology Center, dperouli@purdue.edu*

Arvind Raman  
*Purdue University, Birck Nanotechnology Center, raman@purdue.edu*

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Abstract—Electrostatically operated RF-MEMS switches are known to suffer from discrete switch bounce events during switch closure that increase wear and tear and lead to increased switching times. Here, we use laser Doppler vibrometer to analyze the switch response of three types of cantilevered dc-contact switches at a 200 ns time resolution. We find that bounce events are multiple time scale events with distinct motion occurring at $10^{-1}$ and $10^{3}$ s timescales in effect high frequency bounces within a bounce. To understand the origin of this effect, we develop a multiple eigenmode model of a cantilever switch with electrostatics, repulsive and adhesive contact forces, and rarefied gas damping and find that the high frequency bounce arises from the transient excitation of the 2nd eigenmode of the cantilever structure of the RF-MEMS switch. This phenomenon not only describes the multiple time scales involved in bounce events, but also shows that the transient excitation of the second mode leads to complex drum roll like dynamics, leading to a series of closely spaced impacts in each actuation cycle. A careful study of the dependence of the phenomenon on contact stiffness and adhesion shows how the landing pad stiffness, adhesion, and actuation voltage in dc contact switches can increase or diminish repeated impacts during actuation.

Index Terms—RF MEMS, switch bounce, laser Doppler vibrometer (LDV), switch wear.

I. INTRODUCTION

MICROMECHANICAL RF MEMS switches can increase signal isolation, signal linearity, and shock resistance while decreasing power consumption and insertion losses [1], however, their reliability remains a concern [2]. While several key failure mechanisms can affect RF-MEMS switches [3], the contact wear due to repeated actuation is considered an important source. Switch bounce refers to the phenomenon where the switch, upon electrostatic actuation, fails to immediately close the air gap but rather suffers intermittent bounces before closing completely. This leads to a higher potential for electrical arcing, increased contact wear which may lead to premature stiction, and an overall lose of signal integrity [2–4].

The phenomenon of switch bounce has been observed since the fabrication of the earliest MEMS switches [5]. In fact its existence pre-dates MEMS and has been observed in macro-scale electromagnetic relays [6]. Since Petersen [5] reported microscale switch bounce in 1979 much work has been done to understand the nature of switch bounce in MEMS [7]. McCarthy et al. [8] developed a dynamic MEMS switch model using a finite difference scheme in both time and space: a Reynolds based squeeze film damping model was used and a piecewise linear contact stiffness was assumed. Switch bouncing was captured, but its origin and dependence on system parameters was not discussed. Decuzzi [9] et al. used a similar approach, but included an adhesive tip force in their model. Ostasevicius [10] used a 2D FEA model to study MEMS cantilever switch closure and bounce. Guo et al. [11] used a 3D nonlinear FEA model, along with a finite difference scheme in time to simulate MEMS switch closing events. Krylov and Maimon [12] used a Galerkin procedure to study the pull-in instability dynamics of a MEMS switch. All these works have helped improve the modeling of switch closing events, but the underlying physics to explain, predict bounces, and design against bounces remains elusive. Additionally, in [13], we present a new method to monitor in situ the bounce in RF MEMS switches.

In this work we investigate the origin of switch bounce through a compact multiple eigenmode model, incorporating electrostatics, contact forces, and squeeze film gas damping. Specifically we focus on DC-contact RF MEMS switches, with cantilevered structures whose contacts can be approximated by point contact models. Switches whose contact is better represented by area-type contact models, such as capacitive RF MEMS switches, will be neglected. In the latter cases, switch bouncing is highly suppressed due to large contact area, electrostatic, adhesive, and gas damping forces [14].

The origin of switch bounce is studied through the use of efficient reduced order models based on cantilever eigenmodes, thus avoiding the more computationally intensive approaches using spatial finite difference schemes, or FEM analyses [15]. This also allows for computationally efficient studies of parameter variation and optimal selection of system parameters. MEMS switch analysis based upon reduced order models generated from component eigenmodes of the system has been studied [16], [17] in the context of resonators, here we present a reduced order model that accurately explains the origin of multiple timescale switch bouncing in DC-contact RF MEMS switches. Experimental data are obtained using a laser Doppler vibrometer (LDV) on three distinct cantilevered RF MEMS devices. In addition we provide a parametric analysis
based on key non-dimensional parameters in an attempt to minimize switch bouncing, in contrast other researchers have attempted to mitigate switch bounce events through tuning of the input actuation voltage waveform [4], [18].

II. EXPERIMENTS

We begin with experiments performed with different DC-contact switches. These devices are single crystal silicon RF MEMS switches previously reported in [19], with gold to gold contact. A Polytec Microsystems Analyzer (MSA) 400 LDV was used to measure the response of the beams to various step voltage inputs. All data for this analysis was collected synchronously with the experiments conducted in [13] to ensure consistency.

Figure 1 shows scanning electron microscope images of the three types of switches tested. Type I switches refer to cantilevered switches with a contact pad fixed on three sides, Type II switches refer to cantilevered switches with a cantilevered contact pad, Type III switches refers to U-shaped cantilevered switches with a cantilevered contact. Additional information about the fabrication together with the details of the low noise capacitive measurements that were performed in parallel with laser-Doppler-vibrometry measurements are available by direct request from the authors, and have also been submitted for publication [13].

All measurements were conducted at atmospheric pressure and room temperature in air. The laser spot of the LDV was focused as close to the tip of the switches as possible, while avoiding the contact pad. Figure 1 also shows typical measurement data for the various switch types at different bias voltages, notice that the bouncing phenomenon is apparent in every switch measured. In Fig. 1(a) we see a slight change in the displacement over time caused by a constant velocity bias in the measurement, which we have attempted to remove in all cases. Note that the raw measurement data is velocity vs. time, which we integrate to provide displacement vs. time.

Figure 2 shows the measured response, sampled at 5.12 MHz, of a 240 μm long Type I RF switch actuated at 230 V, again the bouncing phenomenon is quite apparent. The inset picture in Fig. 2 shows a zoomed in portion of the measured response. Notice that the ratio between the time period of the fundamental bouncing, \( T_1 \), and the time period of the secondary bouncing, \( T_2 \), corresponds to approximately 6.3, which reflects the analytical frequency ratio of the second transverse eigenmode to the first, \( f_2/f_1 \), for a cantilevered beam.

To elucidate this phenomenon we utilize the continuous wavelet transformation [20]. Additionally, the resolution in the time-frequency plane is variable, as opposed to the short-time Fourier transform (STFT) which has a fixed time-frequency resolution. Here we have chosen to use the “Ricker” wavelet function for the analysis, which is proportional to the second Hermite function, due to its geometric similarity with the bouncing phenomenon. Other wavelets, such as the “Meyer” wavelet, can give similar results. Figure 3 shows the continuous wavelet transformation of the signal in Fig. 2, the high frequency oscillations of the second mode are clearly visible at each impact event in scales 1–3. Recall that the scale is inversely proportional to the frequency, larger scales represent low frequencies, while smaller scales represent higher frequencies. Figure 4 shows the evolution of the experimentally measured response and wavelet transformation of a typical Type I RF switch to increasing input bias voltages. The multiple timescale bouncing signature apparent in the wavelet data increases in intensity as the applied voltage bias is increased. As the voltage is further increased the duration of continued bouncing also increases. With additional voltage, not shown in Fig. 4, the double bouncing phenomenon transitions to triple bouncing. We now develop a compact model to explain this
The characteristic response of type I RF switch to an applied voltage measured by the Polytec MSA-400 LDV at a sampling rate of $f_s = 5.12$ MHz. Data reflects experiment for a $240 \mu m$ long cantilevered switch with cantilevered contact at an applied voltage equal to 230 V. Inset image shows an expanded view of the data, the bouncing phenomenon is clearly visible. The bouncing frequency corresponds to a ratio of approximately 6.3 time the fundamental frequency, indicating excitation of the second eigenmode of the RF switch.

Fig. 3. Continuous Wavelet transformation of data in Fig. 2 using the “Ricker” wavelet. The LDV displacement vs. time data is represented by the blue curve. The high frequency components of the signal can be seen in the range of scales from 1–3, notice that these components are stimulated at each contact event.

III. THEORY

The simple geometry of the cantilevered DC contact switches allows their dynamics to be modeled with the Euler-Bernoulli equation given by:

$$\rho A \ddot{w}(x,t) + EI \frac{\partial^4 w(x,t)}{\partial x^4} + C_f \dot{w}(x,t) = F_C + F_E, \quad (1)$$

where $w(x,t)$ is the transverse displacement of the beam, $\rho$ is the beam density per unit length, $A$ is the cross sectional area, $E$ is the elastic modulus, $I$ is the second moment of area, overdots represent differentiation with respect to time, $F_C$ represents contact forces, and $F_E$ represents electrostatic forces. The list of symbols used in the model description equations are summarized in Table I for brevity. A simplified diagram is shown in Fig. 5.

The damping coefficient, $C_f$, is estimated using the compact model of Guo and Alexeenko [21]. This model accurately captures the sub-continuum squeeze film damping between a vibrating microcantilever and nearby wall, and is based on the numerical solution to the Boltzmann kinetic equation. $C_f$ is given by [21]:

$$C_f = \frac{A \left( \frac{g}{\lambda} \right)^c h}{1 + B \left( \frac{g}{\lambda} \right)^d \left( \frac{g_s}{\lambda} \right)^e}, \quad (2)$$

where $g$ represents the gap distance between the beam and nearby wall, $\lambda$ is the mean free path of the gas, $b$ is the beam width, $A$, $c$, $d$, and $e$ are fitting coefficients. For this problem there are two relevant gaps, $g_0$ and $g_s$, the gap between the beam and bias bridge and the gap between the beam and the bottom substrate, respectively. Additionally, we assume
that damping forces due to each gap can be superimposed. With this assumption we replace $g$ in Eq. 2 with $g_0 - w(x, t)$ for damping caused by the gap $g_0$ and $g_s + w(x, t)$ for damping caused by the gap $g_s$.

The contact force experienced at the tip of the beam is given by:

$$
F_C = -k_s [g_c - w(L, t)] \delta(x - L) H[(w(L, t) - g_c)] \\
- k_s [g_c - w(L, t)] \delta(x - L) \\
\times [H[w(L, t) - (g_c - g_d)] - H[w(L, t) - g_c]] \\
\times H[-\dot{W}(L, t)],
$$

(3)

$\delta$ is the Dirac delta function and $H$ is the Heaviside step function,

$$
H[n] = \begin{cases} 
0, & n < 0, \\
1, & n \geq 0. 
\end{cases}
$$

Equation 3 represents a hysteretic linear contact force model. This model will be used to explore the role of adhesion due to direct solid contacts or due to the formation of a capillary neck under elevated humidity conditions. Figure 6 shows a diagram of the force model used, which has been experimentally observed in many cases [22], [23]. As the switch comes into contact at $g_c$ it experiences the linear spring with stiffness $k_s$, as the switch leaves contact it experiences an additional force proportional to $k_s$ for a distance $g_d$. This force model is meant to qualitatively simulate the overall contribution of van der Waals forces, the meniscus or capillary force, and forces due to chemical bonds [23].

The electrostatic force per unit length applied to the beam from the bias bridge is given by:

$$
F_E = \frac{1}{2} V^2 \frac{\partial C}{\partial w},
$$

(4)

where $C$, accounting for fringe field corrections, is given by [24]:

$$
C = \epsilon_0 \left( \frac{b}{g_0 - w(x, t)} + 0.77 + 1.06 \left( \frac{b}{g_0 - w(x, t)} \right)^{\frac{3}{2}} \\
+ 1.06 \left( \frac{h}{g_0 - w(x, t)} \right)^{\frac{3}{2}} \right),
$$

(5)

where $\epsilon_0$ is the free space permittivity. In order to model the voltage ramp applied to the device in an experimental setting we replace the constant $V$ with $V(t) = V_{dc}(1 - e^{-\frac{t}{\tau_V}})$. Where $\tau_V$ represents the voltage ramp time constant, and $V_{dc}$ is the desired constant voltage bias applied. To aid in the solution of Eq. 1 the electrostatic force is represented as a $4^{th}$ order Taylor series expansion in the form of:

$$
F_E = a_0 + a_1 w + a_2 w^2 + a_3 w^3 + a_4 w^4.
$$

(6)

This approximation is accurate to within 5% of the analytical expression across the entire calculation range. The damping factor $C_f$ is also represented in a $2^{nd}$ order Taylor series for each gap, in the form of:

$$
C_f = \gamma_0^2 + \gamma_1^2 w + \gamma_2^2 w^2.
$$

(7)
This approximation has a mean error of 5% across the entire calculation range. \( i \) is an index that represents which gap the damping coefficient is modeling. The coefficients of the expansion will be dependent on the gap being modeled, for instance \( \gamma^0_0 \) represents the first Taylor coefficient for damping caused by the gap \( g_0 \) and \( \gamma^i_0 \) represents the first Taylor coefficient for damping caused by the gap \( g_s \).

The equation of motion is then non-dimensionalized as follows:

\[
\dot{\omega} = \frac{\omega}{g_c} \quad \hat{x} = x, \quad \tau = \frac{t}{T_0}, \quad T_0 = \frac{2\pi}{\omega_1},
\]

where \( \omega_1 \) is the first natural frequency of the beam and is given by:

\[
\omega_1 = \beta_1^2 \sqrt{\frac{EI}{\rho AL^4}}, \tag{8}
\]

where \( \beta_1 \) is the first root of the dispersion relation for the beam. The equation of motion is then re-written as:

\[
\ddot{\omega}(\hat{x}, \tau) + \frac{T_0^2 E I}{\rho A L^4} \dot{\omega} \dot{\hat{x}} + \frac{C_f(\omega) T_0}{\rho A} \ddot{\omega} = \frac{T_0^2}{\rho A g_c} (\dot{\hat{F}}_C(w) + \dot{\hat{F}}_E(w)), \tag{9}
\]

primes denote differentiation with respect to \( \hat{x} \).

We then discretize the transverse displacement using as a functional basis the eigenmodes of a free cantilevered beam, which constitute a complete set for the transverse displacement field:

\[
\hat{w}(\hat{x}, \tau) = \sum_{n=1}^{\infty} q_n(\tau) \phi_n(\hat{x}), \tag{10}
\]

where \( \phi_n(\hat{x}) \) represents the \( n^{th} \) transverse bending mode of the cantilevered beam, \( q_n(\tau) \) represents the temporal component of the solution. The modes are normalized such that \( \phi_n(1) = 1 \) [25]. Using the relation \( \phi_n^{\infty}(\hat{x}) = \frac{\rho A L^4}{EI} \omega_n^2 \phi_n \), Eq. 9 becomes:

\[
\sum_{n=1}^{\infty} \ddot{q}_n \phi_n + T_0^2 \sum_{n=1}^{\infty} q_n \omega_n^2 \phi_n + \frac{C_f(q_n) T_0}{\rho A} \sum_{n=1}^{\infty} \ddot{q}_n \phi_n = \frac{T_0^2}{\rho A g_c} (\dot{\hat{F}}_C(q_n) + \dot{\hat{F}}_E(q_n)), \tag{11}
\]

here we have left \( C_f, \dot{\hat{F}}_C, \) and \( \dot{\hat{F}}_E \) unexpanded for clarity. A Galerkin projection [26] is then performed on the above equation, in which the inner product is taken with each respective mode shape to obtain \( n \), coupled 2\textsuperscript{nd} order ordinary differential equations in \( q_n \). Through modal orthogonality one can show that:

\[
\int_0^1 \phi_i \phi_j d\hat{x} = 0 \quad \text{for} \quad i \neq j. \tag{12}
\]

This system is then decomposed into a set of coupled first order differential equations for solution in an ordinary differential equation solver such as MATLAB [27].

The coupled nonlinear ordinary differential equations resulting from the Galerkin projection procedure for \( n = 2 \) are provided in Appendix A. These coupled equations are then solved numerically, from which they can readily be re-dimensionalized by inserting appropriate parameters gathered from measurement.

Table II provides the parameter values used in the simulation. As indicated by Table II the parameter values used were: based on nominal parameter values, measured experimentally, or empirically tuned. For the gas damping model we have used room temperature and an ambient pressure of 1 \( \text{atm} \). The thickness of the beam was calculated from the measured first natural frequency and Eq. 8 using nominal values for the remaining parameters. The frequencies of the first and second transverse bending modes were obtained experimentally using a ring-down analysis [28]. The contact stiffness and voltage time constant were empirically tuned, to match the experimental data.

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
<th>Value</th>
<th>Obtained</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 )</td>
<td>First Transverse Bending Frequency</td>
<td>98 kHz</td>
<td>Measured</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>Second Transverse Bending Frequency</td>
<td>627 kHz</td>
<td>Measured</td>
</tr>
<tr>
<td>( f_3 )</td>
<td>Third Transverse Bending Frequency</td>
<td>1.72 MHz</td>
<td>Nominal</td>
</tr>
<tr>
<td>( g_0 )</td>
<td>Gap Between Beam and Bias</td>
<td>7 ( \mu m )</td>
<td>Nominal</td>
</tr>
<tr>
<td>( g_s )</td>
<td>Gap Between Beam and Contact</td>
<td>1.24 ( \mu m )</td>
<td>Measured</td>
</tr>
<tr>
<td>( g_c )</td>
<td>Gap Between Beam and Substrate</td>
<td>2.5 ( \mu m )</td>
<td>Nominal</td>
</tr>
<tr>
<td>( V )</td>
<td>Actuation Voltage</td>
<td>230 V</td>
<td>Measured</td>
</tr>
<tr>
<td>( b )</td>
<td>Beam Width</td>
<td>20 ( \mu m )</td>
<td>Nominal</td>
</tr>
<tr>
<td>( h )</td>
<td>Beam Thickness</td>
<td>4.19 ( \mu m )</td>
<td>Measured</td>
</tr>
<tr>
<td>( L )</td>
<td>Beam Length</td>
<td>240 ( \mu m )</td>
<td>Nominal</td>
</tr>
<tr>
<td>( E )</td>
<td>Young’s Modulus</td>
<td>160 GPa</td>
<td>Nominal</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Beam Density</td>
<td>2330 ( \text{kg/m}^3 )</td>
<td>Nominal</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Voltage Time Constant</td>
<td>0.6 ( \mu s )</td>
<td>Tuned</td>
</tr>
<tr>
<td>( k_c )</td>
<td>Contact Stiffness</td>
<td>833 kN/m</td>
<td>Tuned</td>
</tr>
<tr>
<td>( P )</td>
<td>Ambient Pressure</td>
<td>101.325 kPa</td>
<td>Nominal</td>
</tr>
</tbody>
</table>
IV. SIMULATION RESULTS

Here we present the simulation results obtained from the previously derived model. We first begin with the case where adhesion is negligible, $g_a = 0$. For each simulation a bias voltage is simulated with an exponential rise to the desired DC value. Zero displacement and zero velocity initial conditions are used. The coupled differential equations are then numerically solved.

Figure 7 shows the simulated transient dynamics of the switch vs. the experimentally measured data for a Type I device. Here we have conducted three simulations for $n = 1$, $n = 2$, and $n = 3$, where $n$ is the number of degrees of freedom in the $n$ DOF reduced order model.

For $n = 1$ the fundamental period of bouncing is accurately captured, however, the multiple impact events are not present. The $n = 1$ system lacks sufficient degrees of freedom to quickly rebound at the stiff contact and bounce, and thus remains in contact until the trajectory of the periodic motion is reversed and the contact spring has fully relaxed. For $n = 2$ we see that the intermittent contact phenomenon is apparent, and it is due to the excitation of the second eigenmode of the structure. For $n = 3$ the results are qualitatively similar to $n = 2$. The added complexity of a 3 DOF reduced order model does not provide new information to describe the multiple impact phenomenon observed in the experimental data, and henceforth we utilize the $n = 2$ model.

It can be seen that in Fig. 7 the simulations for $n \geq 2$ agree very well with the experimentally measured data. Note that for the $n = 3$ simulation the third transverse bending mode frequency was analytically calculated, rather than experimentally measured, this is because this frequency is beyond the bandwidth of the LDV detector (1.5 MHz).

We now repeat the above $n = 2$ simulation, with a much larger contact stiffness, in an attempt to elucidate the multiple timescale bounce phenomenon and demonstrate that it is caused by the excitation of the second eigenmode. Figure 8 shows the calculated total response, along with the individual modal responses. The total response shows the characteristic bounce phenomenon. Examining the response of the 1st mode the fundamental period of bouncing is apparent, along with the individual contact events of the switch, indicated by the saturation of amplitude of the 1st mode as contact is made. Examining the 2nd mode we see that at each contact event apparent in the 1st mode the second mode responds sharply, as if excited by an impulse. The impulsive excitation of the 2nd eigenmode of the RF switch is in fact, the basis of the multiple timescale bounce phenomenon.

Now that the origin of the multiple timescale bounce phenomenon has been identified, a parametric analysis using key non-dimensional parameters of the system is performed to understand this phenomenon. This parametric analysis is easily afforded by the compact 2 DOF reduced order model, which is computationally inexpensive. Similar analysis performed using FEA would be computationally expensive and complex.

The key non-dimensional parameters are as follows, the non-dimensional electrostatic force, $\tilde{F}_E$, given by:

$$\tilde{F}_E = \frac{12\pi^2 k_0 L}{s^2 \beta_1^2 k_c},$$

and the non-dimensional contact stiffness, $\tilde{k}$, given by:

$$\tilde{k} = \frac{12\pi^2 k_s L}{\beta_1^2 k_c}.$$
thought of as the ratio of electrostatic force to elastic restoring force of the RF switch. The non-dimensional contact stiffness, \( \tilde{k} \), is the ratio of the contact stiffness to the static stiffness of the RF switch. The static stiffness, \( k_c \) is given as:

\[
k_c = \frac{3EI}{L^3}.
\]

Each of the non-dimensional parameters are varied independently, and the previously discussed simulation is performed. To vary the non-dimensional electrostatic force in experiments one would vary the applied bias voltage. To vary the non-dimensional contact stiffness independently of the non-dimensional electrostatic force one would increase the contact stiffness by increasing the thickness of the contact pad, for example. For each simulation the number of bounces in the first contact period is recorded. Figure 9 shows the results of the numerical simulations. “Zero bounces” refers to the situation in which the electrostatic force is insufficient to cause the switch to close permanently. “n” bounces, where \( n \) is an integer, refers to the number of bounces recorded in the first contact event. “Permanent contact” refers to the situation in which the beam closes with no bounces, and remains closed. We can see from Fig. 9 that for contact stiffnesses large in comparison to the RF switch stiffness (large \( \tilde{k} \)) we have an increased propensity for multiple bouncing events, up to four bounces for the parameter space studied. It is evident that for permanent contact to occur, and thus no bouncing, it is favorable to have cantilever stiffnesses on the same order as, or greater than, the contact stiffness and to keep the electrostatic forces large compared to cantilever elastic restoring forces. A similar simulation was performed using \( n = 3 \) eigenmodes for the reduced order model, in a smaller subset of the parameter space. Figure 10 shows the results. The “permanent contact” area remains unaffected, however the organized structure of the bouncing phenomenon appears to break down as higher eigenmodes are excited.

Exploring the same parameter space we now examine the static contact force applied by the spring, \( k_s \), at steady state equilibrium, for \( n = 2 \) eigenmodes in the reduced order model. To do this Eq. (15) and Eq. (16) are solved with all time dependence removed. That is, the static solutions are obtained for a given parameter pair, \( F_E \) and \( \tilde{k} \). Once the steady state displacements are obtained, the resulting steady state spring force can be calculated. Figure 11 shows the results of the calculations. It can be seen that in the desired “permanent contact” area, seen in Figs. 9 and 10, the static contact forces are low, compared to the remaining values in the parameter space. This indicates that the “permanent contact” region is still the optimal choice for efficient operation. It was surmised that the static contact forces in the permanent contact area would be quite large, and adversely effect switch lifetime. Here we see that the contact forces in this area are quite small, compared to the remaining parameter space. By operating in the “permanent contact” area we also reduce the dynamic forces caused by intermittent bouncing, which will be large compared to static contact forces. Additionally, switch settling time is reduced, due to the mitigation of overall bouncing.

Finally we discuss the effects of surface adhesion to the switch dynamics. It is known that as a MEMS switch is actuated in low or zero current conditions the overall adhesion forces will increase [3]. Here we have run simulations for the parameters given in Table II, but have added the adhesion force discussed in Eq. 3. Mathematically, for \( n = 2 \), it can be represented as:

\[
F_C = -k_s[g_c - (q_1 + q_2)]\delta(x - L)H[(q_1 + q_2 - g_c)]
-k_s[g_c - (q_1 + q_2)]\delta(x - L) \times [H(q_1 + q_2 - (g_c - g_a)) - H(q_1 + q_1 - g_c)]
\times H[-q_1],
\]

(14)

\( g_a \) defines the region in which the adhesive force exists. We have used \( q_1 \) to control the hysteretic behavior of the force. If the cantilever tip is within the contact adhesion region and \( q_1 \)
Fig. 11. Static contact force from the spring, $k_s$, for the non-dimensional parameter space studied in Fig. 9. It can be seen that the “permanent contact” area in Fig. 9 results in low static contact forces.

Fig. 12. Effect of hysteretic adhesion force on bounce duration. As the adhesion force is increased, the bounce duration decreases and thus the overall switch settling time decreases.

We have chosen three cases to simulate: no adhesive forces, $g_a = 1 \text{ nm}$, and $g_a = 10 \text{ nm}$. Here an increase in $g_a$ represents an increase in the adhesion force, and these values are on the order of what can be observed in experimentally obtained atomic force microscope (AFM) force-distance curves [23].

Figure 12 shows the results. It is clear that as the adhesive forces increase the overall duration of bouncing near the fundamental period decreases. As the adhesive force increases further all bouncing will cease. The overall magnitude and frequency of the smaller timescale bounces is weakly affected.

Here we have presented simulation results and experimentally measured data for DC-contact RF MEMS switches. A parametric study was performed to investigate the effect of system parameters on switch bounce. These results would be applicable for similar MEMS devices with cantilevered structures whose contacts can be approximated by point contact models. Devices with area-type contact, such as capacitive RF MEMS switches, would not be suitable to this analysis due to the nature of the contact mechanics.

V. Conclusion

In this paper we have discussed the origin of the multiple timescale bounce phenomenon in RF MEMS cantilevered switches. In summary a compact analytical model was developed and numerical simulations were conducted to explain the multiple timescale bounce phenomenon in RF MEMS, and develop strategies to mitigate this phenomenon. A LDV was used to experimentally measure the transient behavior of RF MEMS switch closing for three types of devices. It was determined that multiple timescale bouncing is the result of the intermittent excitation of the 2nd eigenmode of the system during contact events. A non-dimensional parameter study was conducted revealing that in order to mitigate bouncing it is favorable to have cantilever stiffnesses on the same order as, or greater than, the contact stiffness and to keep the electrostatic forces large compared to cantilever elastic restoring force. Additionally, the role of contact adhesion was studied and shown to decrease overall bounce height and duration with increasing adhesion force. We expect that the mathematical models and analysis methods described here will find use to optimize or better understand the bounce and bounce related wear in RF-MEMS switches.

APPENDIX

APPENDIX A

Here we present the fully expanded coupled nonlinear ordinary differential equations resulting from the Galerkin projection procedure, for $n = 2$.

The expression for the first mode is (15):

$$\ddot{q}_1 \int_0^1 \phi_1^2 \, d\hat{x} + T_0 \dot{q}_1 \phi_1^2 \int_0^1 \phi_1^2 \, d\hat{x} + \frac{T_0}{\rho A} \left[ q_1 (\gamma_0^0 + \gamma_0^1) \int_{a_D^{\beta_D}} \phi_1^3 \, d\hat{x} + (\gamma_1^0 + \gamma_1^1) \right] \times \left( q_1 q_1 \int_{a_D^{\beta_D}} \phi_1^3 \, d\hat{x} + \dot{q}_2 q_1 \int_{a_D^{\beta_D}} \phi_1^2 \phi_2 \, d\hat{x} \right)$$
\[ + q_1 q_2 \int_{a_D^D}^{\beta_D} \phi_1^3 d\hat{x} + q_2 q_2 \int_{a_D^D}^{\beta_D} \phi_1^3 d\hat{x} \]
\[ + (\gamma_1^0 + \gamma_1^1) \left( q_1 q_1 \int_{a_D^E}^{\beta_E} \phi_1^3 d\hat{x} + q_2 q_2 \int_{a_D^E}^{\beta_E} \phi_1^3 d\hat{x} \right) \]
\[ + q_2 q_2 \int_{a_D^E}^{\beta_E} \phi_1^3 d\hat{x} + 2 q_1 q_1 \int_{a_D^E}^{\beta_E} \phi_1^3 d\hat{x} \]
\[ + 2 q_2 q_2 \int_{a_D^E}^{\beta_E} \phi_1^3 d\hat{x} \]
\[ = \frac{T_0^2}{\rho A} \left( k_s (1 - q_1 - q_2) \int_0^1 \phi_1 \delta(\hat{x} - 1) d\hat{x} \right) H [q_1 + q_2 - 1] \]
\[ + \frac{1}{2} \left( V_{dc} (1 - e^{-\frac{\alpha_E}{\beta_E}}) \right)^2 \frac{T_0^2}{\rho A \sigma_0} \left[ a_0 \int_{a_E}^{\beta_E} \phi_2 d\hat{x} \right] \]
\[ + a_2 \left( q_1 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} + q_2 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} \right) \]
\[ + a_2 \left( q_1 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} + q_2 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} \right) \]
\[ + a_3 \left( q_1 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} + q_2 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} \right) \]
\[ + a_4 \left( q_1 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} + q_2 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} \right) \]
\[ + a_5 \left( q_1 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} + q_2 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} \right) \]
\[ (16) \]

The expression for the second mode is (16):
\[ \hat{q}_2 \int_0^1 \phi_2^2 d\hat{x} = \frac{T_0^2}{\rho A} \left( k_s (1 - q_1 - q_2) \int_0^1 \phi_1 \delta(\hat{x} - 1) d\hat{x} \right) H [q_1 + q_2 - 1] \]
\[ + \frac{1}{2} \left( V_{dc} (1 - e^{-\frac{\alpha_E}{\beta_E}}) \right)^2 \frac{T_0^2}{\rho A \sigma_0} \left[ a_0 \int_{a_E}^{\beta_E} \phi_2 d\hat{x} \right] \]
\[ + a_2 \left( q_1 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} + q_2 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} \right) \]
\[ + a_3 \left( q_1 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} + q_2 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} \right) \]
\[ + a_4 \left( q_1 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} + q_2 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} \right) \]
\[ + a_5 \left( q_1 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} + q_2 \int_{a_E}^{\beta_E} \phi_1^2 \phi_2 d\hat{x} \right) \]

Here we have written the equations in compact form, for integrals involving the damping terms $\gamma_1^0$ and $\gamma_1^1$. The integral bounds $a_E$ and $\beta_E$ take the values $a_E = a_D = 0.25$ and $\beta_E = \beta_D = 0.75$ when the integrals are multiplied by terms $\gamma_i$. Additionally, $a_E = 0.25$ and $\beta_E = 0.75$. This revision of integral boundaries accounts for the fact that the electrode bias bridges only partially covers the RF MEMS switch.

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REFERENCES


Ryan C. Tung received the B.S. degree in mechanical engineering from the University of Nevada, Reno, NV, USA, in 2006, and the M.S. and Ph.D. degree in mechanical engineering from Purdue University, West Lafayette, IN, USA, in 2008 and 2012, respectively.

He is currently a National Research Council Post-Doctoral Fellow at the National Institute of Standards and Technology, Boulder, CO, USA. His current research interests include the dynamics of contact resonance atomic force microscopy.

Adam Fruehling received the bachelor’s and Ph.D. degrees in electrical engineering from Purdue University, West Lafayette, IN, USA, in 2005 and 2012, respectively.

He is currently working as a MEMS Designer with Texas Instruments, Plano, TX, USA, with a focus on new technology development.

Dimitrios Peroulis (S’99–M’04) received the Ph.D. degree in electrical engineering from the University of Michigan, Ann Arbor, MI, USA, in 2003. He has been with Purdue University, West Lafayette, IN, USA, since August 2003, where he is currently leading a group of graduate students on a variety of research projects in the areas of RF MEMS, sensing and power harvesting applications as well as RFID sensors for the health monitoring of sensitive equipment. He has been a PI or a co-PI in numerous projects funded by government agencies and industry in these areas. He has been a key contributor on developing very high quality (Q > 1000) RF MEMS tunable filters in mobile form factors. He has been investigating failure modes of RF MEMS and MEMS sensors through the DARPA M/NEMS S&T Fundamentals Program, Phases I and II) and the Center for the Prediction of Reliability, Integrity and Survivability of Microsystems funded by the National Nuclear Security Administration. He received the National Science Foundation CAREER Award in 2008. His students have received numerous student paper awards and other student research-based scholarships. He is a Purdue University Faculty Scholar. He has received ten teaching awards, including the 2010 HKN C. Holmes MacDonald Outstanding Teaching Award and the 2010 Charles B. Murphy Award, which is Purdue University’s highest undergraduate teaching honor.

Arvind Raman is a Professor of mechanical engineering and a University Faculty Scholar with Purdue University, West Lafayette, IN, USA. His current research interests include applied nonlinear dynamics, nanomechanics, and fluid-structure interactions. His group has significantly advanced the understanding of complex dynamics in nanotechnology applications, such as atomic force microscopy and micro- and nano-electromechanical systems, in gyroscopic systems for data storage and manufacturing, in electronics cooling, and in biomechanics. He has mentored 15 Ph.D. students, coauthored more than 100 peer-reviewed journal articles, held visiting positions with the Universidad Autonoma de Madrid, Madrid, Spain, University of Oxford, Oxford, U.K., and Darmstadt University of Technology, Darmstadt, Germany, and secured funding from the NSF, NIH, NASA, NASA, and several national and international industrial sponsors. He is a Fellow of the ASME and a recipient of the Gustus Larson Memorial Award from the ASME, a Kellely fellowship (Wadham College, Oxford), College of Engineering Outstanding Young Investigator Award, and the NSF CAREER Award. He has pioneered the use of cyber-infrastructure in the AFM community for research and education through advanced simulation tools and online classes, which are used by thousands around the world and led College of Engineering strategic initiatives for global engagement in Latin America.

Arvind Raman is a Professor of mechanical engineering and a University Faculty Scholar with Purdue University, West Lafayette, IN, USA. His current research interests include applied nonlinear dynamics, nanomechanics, and fluid-structure interactions. His group has significantly advanced the understanding of complex dynamics in nanotechnology applications, such as atomic force microscopy and micro- and nano-electromechanical systems, in gyroscopic systems for data storage and manufacturing, in electronics cooling, and in biomechanics. He has mentored 15 Ph.D. students, coauthored more than 100 peer-reviewed journal articles, held visiting positions with the Universidad Autonoma de Madrid, Madrid, Spain, University of Oxford, Oxford, U.K., and Darmstadt University of Technology, Darmstadt, Germany, and secured funding from the NSF, NIH, NASA, NASA, and several national and international industrial sponsors. He is a Fellow of the ASME and a recipient of the Gustus Larson Memorial Award from the ASME, a Kellely fellowship (Wadham College, Oxford), College of Engineering Outstanding Young Investigator Award, and the NSF CAREER Award. He has pioneered the use of cyber-infrastructure in the AFM community for research and education through advanced simulation tools and online classes, which are used by thousands around the world and led College of Engineering strategic initiatives for global engagement in Latin America.