Thermodynamic Analysis and Optimization of Cascade Condensing Temperature of a CO2(R744)/R404A Cascade Refrigeration System

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Thermodynamic Analysis and Optimization of Cascade Condensing Temperature of a CO₂(R744)/R404ACascade Refrigeration System

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ABSTRACT

Synthetic refrigerants are widely used in refrigeration applications. However, it is shown that such refrigerants have negative impacts on the ozone layer of atmosphere. Recently, natural refrigerants such as carbon dioxide and various hydrocarbon compounds are proposed to replace synthetic refrigerants in the industrial refrigeration systems. Carbon dioxide is one of the most promising and environment-friendly refrigerant solutions due to its thermo-physical properties, low ozone depletion value and low global warming potential.

In this study thermodynamic analysis of a two stage sub-critical cascade refrigeration system using CO₂ and R404a refrigerants in low temperature and high temperature cycles is presented. The energy and exergy analysis of the system and its components are performed to determine optimum operating conditions for condensing temperature of the cascade condenser and to maximize the coefficient of performance (COP) and second law efficiency of the system. The required equations are the mass, energy and exergy balances for the cascade refrigeration system. The optimum condensing temperature of the cascade condenser is computed at the first phase of the study. Then correlations are developed to maximize COP of the system according to condensing temperatures of both high and low temperature cycles.

Keywords: Carbon dioxide Refrigeration, Cascade Refrigeration system, Thermodynamic Analysis

1. INTRODUCTION

Synthetic refrigerants are widely used in refrigeration industry. However, it is shown that they have negative impact on the ozone layer. Recently, natural refrigerants such as carbon dioxide and various hydrocarbon compounds are proposed to replace synthetic refrigerants in industrial refrigeration systems. Carbon dioxide is one of the most promising and environment-friendly refrigerant solutions due to its superior thermo-physical properties, low ozone depletion index and global warming potential. CO₂ is also a non-toxic, non-explosive, easily available refrigerant.

The use of CO₂ in refrigeration systems is widely proposed to be refrigerant of low temperature cycle of a cascade system. There are plenty of numerical and experimental studies available in literature about cascade refrigeration systems using carbon dioxide as the refrigerant. In study of (Boalian, 2007) a mixture of R744/R290 is used as an alternative to R13 in high temperature cycle of a cascade refrigeration system. According to the different evaporator inlet temperatures for this mixed refrigerant, condensation pressures of high temperature cycle, cooling capacity of the system and effect on COP are examined to obtain optimum evaporation temperatures.

Lee at al.(2006) examined cascade system using carbon dioxide and ammonia as refrigerant for low temperature cycle and high temperature cycle, respectively. They developed a numerical model to maximize COP of the system
and minimize the exergy lost for different operating parameters such as evaporation, condensation temperatures and temperature differences in cascade condenser. They compared these results with experimental measurements.

Bansal and Jain (2007) calculated optimum condensation temperatures in cascade heat exchanger and the COP change for different standard operating conditions (SC) at the evaporator side using R744 as a refrigerant in the low temperature cycle and ammonia(R717), propane(R290), R1270 and R404A in the high temperature cycle. They performed energy analysis of the system only. They also examined the effects of sub cooling, superheating and mass flow rate on COP of the system.

Yilmaz et al. (2013) performed energy analysis of cascade refrigeration system using carbon dioxide as the refrigerant. The influence of operating parameters defined in ASHRAE standards including evaporating and condensing temperatures, temperature difference in the cascade heat exchanger, superheating and sub-cooling is examined. It is concluded that, an increase of evaporation temperature, superheating and sub cooling have positive effect on COP. In addition, increasing the condensation temperature R404A side and temperature difference in cascade condenser decrease the system COP.

Johansson (2009) showed that the cascade solution has the lowest COP compared to the other systems at the same cooling capacity and ambient conditions using CO2 as refrigerant. It is important to mention that this lower COP is due to the operating conditions and not necessarily due to the system solution. Moreover, in that work the supermarket refrigeration systems are modeled in EES. The model makes possible to observe and evaluate the system performance at different operating conditions.

Thermodynamic analysis of carbon dioxide-ammonia (R744-R717) cascade refrigeration system is performed by Getu and Bansal (2008). In their cascade system, the effect of different operating parameters (condensing, evaporating, sub cooling, superheating) on COP is determined. It is shown that an increase of superheating and condensing temperatures reduces the COP and increasing the level of sub cooling and evaporating temperature increases the COP.

Ahamed et al. (2011) performed exergy analysis of the vapor compression refrigeration cycle. The influence of the condensing and evaporating temperatures on exergy losses, pressure losses, second law efficiency and COP are examined. It is observed that maximum exergy losses occur in compressors and exergy losses increase with the increase in suction and discharge temperature of the compressor. It is reported that for better performance of the system, compressor discharge and suction temperature should be 65°C and 14 °C, respectively.

Thermo economic and exergy analysis CO2/NH3 cascade refrigeration system are performed by Rezayan and Behbahaninia (2011).The optimum working parameters for CO2/NH3 system has been searched. They optimize the cascade system and obtain savings in annual cost.

In this study, thermodynamic analysis is performed for a two stage sub-critical cascade refrigeration system in which CO2 and R404A are refrigerants in high and low temperature cycles, respectively. The optimum condensing temperature of cascade condenser based on various systems design parameters are determined. In addition, the operating conditions to maximize the COP and minimize the exergy destruction of the system are examined.

2. CASCADE REFRIGERATION SYSTEM

The cascade system consists of the low temperature and high temperature cycles, in which CO2 and well-known R404A are used as refrigerants, respectively. Figure 1 shows schematic diagram of the two-stage cascade refrigeration system. High temperature cycle contains a R404A compressor, a water-cooled condenser, an expansion valve and a cascade condenser corresponding to evaporator of the cycle which is a heat exchanger. On the other hand, the low temperature cycle consists of the same components as in high temperature cycle however the compressor and cascade condenser are replaced by an evaporator and CO2 compressor. The heat transfer between two cycles occurs through a cascade heat exchanger. In the cascade heat exchanger, CO2 condenses and R404A evaporates so that there is heat transfer from CO2 side to R404A side.
In a conventional refrigeration system there are two temperature levels, namely the evaporating temperature, $T_E$, and the condensing temperature, $T_C$. For a designed system, these temperature levels depend on the temperatures of the conditioned space and the ambient conditions. However, in a cascade system there are four temperatures levels; the two additional temperatures being the condensing temperature of the low temperature system, $T_{CAS, C}$, and the evaporating temperature of the high temperature system, $T_{CAS, E}$. The intermediate temperatures for a given operating condition depend on the system design and ideal temperature levels can be determined through the use of optimization. Temperature difference cooling space temperature ($T_F$) and evaporation temperature of carbon dioxide ($T_E$) is 5°C.

### 3. MATHEMATICAL MODEL AND OPTIMIZATION OF THE SYSTEM

#### 3.1 Mathematical Model

The mathematical model of the cascade system is developed based on first and second law of thermodynamics. Mass, energy and exergy balance equations are derived for both low and high temperature cycles. Then, coefficient of performance and second law efficiencies are computed for various operating conditions. The operating conditions of the system are determined from ANSI/ASHRAE 33(2000) standards for evaporation temperatures which are listed in Table 1.

<table>
<thead>
<tr>
<th>Standard Condition (SC)</th>
<th>Room Temperature ($^\circ$C)</th>
<th>Evaporation Temperature ($^\circ$C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC1</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>SC2</td>
<td>0</td>
<td>-8</td>
</tr>
<tr>
<td>SC3</td>
<td>-18</td>
<td>-25</td>
</tr>
<tr>
<td>SC4</td>
<td>-25</td>
<td>-31</td>
</tr>
<tr>
<td>SC5</td>
<td>-34</td>
<td>-40</td>
</tr>
</tbody>
</table>

Following assumptions are taken into consideration in order to perform the thermodynamic analysis of two-stage cascade refrigeration system.

- isenthalpic expansion of refrigerants in expansion valves,
- isentropic compressor efficiencies of 0.80 both low and high temperatures cycles,
Numerical analysis is carried out using Engineering Equation Solver (EES) software. EES is a general equation-solving program that can numerically solve coupled non-linear algebraic and differential equations. The program is also used to perform optimization studies, linear and non-linear regression and generate plots.

### 3.2 Optimization Studies

Optimization studies are conducted to obtain optimum operating conditions which maximize the system performance. The COP of the system is optimized according to condensing temperature of cascade condenser (T_{CAS,C}) and condensation temperature of high temperature cycle (T_C). The evaporation temperature (T_E) is assumed to be constant in case studies. EES software with linear regression method is applied in optimization studies.

### 4. RESULTS

The mathematical model described above is implemented in EES in order to evaluate the COP and exergy loss of both overall system and its components. The performance and second law efficiencies of the cascade system are computed and depicted for several conditions. Operating conditions for the system are chosen based on standards mentioned in Table 1. Three of these standards which are supposed to represent upper, intermediate and lower limits of operating conditions are taken into consideration for case studies. The evaporation temperatures for low temperature cycle are considered to be 0°C, 25°C, -40°C, corresponding to SC1, SC3 and SC5 conditions in Table 1. Correspondingly, depending on evaporation temperatures, the cascade condensation temperatures of the low temperature cycle (T_{CAS,C}) are varied between 5°C/30°C, -5°C/25°C and -35°C/5°C, respectively. The temperature difference in cascade condenser between the high and low temperature cycles, ΔT, is assumed to be 5°C for initial cases then it is also varied. The condensing temperatures for high temperature cycle are varied from 25°C to 45°C in case studies. The evaporator cooling capacity is kept constant to be 10 kW.

### 4.1 The effect of T_{CAS,C} on exergy loss, system performance

Figure 2&3 display the effect of T_{CAS,C} on the total exergy loss and exergy loss of each system component at specified operating conditions (T_E = 40°C and T_E = 40°C and ΔT = 5°C). Figure 3 indicates that increasing the T_{CAS,C} decreases the total exergy loss rate of the overall system. While T_{CAS,C} increases, the amount of exergy loss...
Figure 3. The effect of $T_{CAS,C}$ on the total exergy loss and exergy loss of system components (SC5 case)

The total exergy loss and exergy loss of system components for specified conditions corresponding to SC3 conditions ($T_C=40 \, ^\circ C$ and $T_E=-25\, ^\circ C$ and $\Delta T=5 \, ^\circ C$) are plotted in Figure 5. In this figure increasing $T_{CAS,C}$ decreases the total exergy loss rate of the overall system. There is a minimum value for total exergy loss about 0$^\circ$C of $T_{CAS,C}$. A similar trend is obtained for exergy loss of system components as in SC5 case. Figure 6 indicates the effect of $T_{CAS,C}$ on the COP and second law efficiency of the system for SC3 case. The COP and the second law efficiency profile of the system result in a maximum about $T_{CAS,C}=0 \, ^\circ C$ where exergy loss is the lowest.

Finally, The SC1 condition is examined in terms of system performance and exergy loss. The operating conditions for this case are chosen as such $T_C=40 \, ^\circ C$, $T_E=0 \, ^\circ C$ and $\Delta T=5 \, ^\circ C$. The temperature of the condensing side of cascade condenser, $T_{CAS,C}$, is varied between 5 $^\circ C$ and 30$^\circ C$. The results are shown in Figure 7 and 8. It is observed that increasing the $T_{CAS,C}$ slightly increases the COP and exergy loss however after 10 $^\circ C$ system performance parameters decrease almost exponentially. Therefore 10 $^\circ C$ can be assumed as the optimum operating temperature.
Figure 5. The effect of $T_{\text{CAS,C}}$ on the total exergy loss and exergy loss of system components (SC3 case) for this case. In Figure 6, the increase in exergy loss of low temperature cycle components by increasing $T_{\text{CAS,C}}$ dominates the system performance.

Figure 6. The influence of $T_{\text{CAS,C}}$ on the COP and second law efficiency of the system (SC3 case)

Figure 7. The effect of $T_{\text{CAS,C}}$ on the total exergy loss and exergy loss of system components (SC1 case)
Figure 8. The influence of $T_{CAS,C}$ on the COP and second law efficiency of the system (SC1 case)

4.3 The effect of $\Delta T$ in Cascade Condenser on system performance

The effect of level of the temperature difference in cascade condenser on the system COP and second law efficiency is also examined for SC3 case where CO$_2$ evaporation and cascade condenser condensation temperatures are assumed to be -25 °C and -10 °C, respectively. In Figure 9, the distribution of system COP and second law efficiency versus $\Delta T$ which is varied from 2°C to 10°C are plotted for cascade condenser. It is found that increasing the temperature difference decreases the COP and second law efficiency of system.

Figure 9. The effect of temperature difference of cascade condenser on system performance

4.4 The effect of both $T_C$ and $T_{CAS,C}$ on system COP

The influence of both $T_C$ and $T_{CAS,C}$ on system COP is examined in contour plots in Figure 10 and Figure 11. Condensation temperatures, $T_C$, are varied between 25 °C and 45 °C and $T_{CAS,C}$ is varied between 10 °C and -20 °C that corresponds to SC3 conditions. Figure 10 shows that the maximum COP is observed in the region where $T_{CAS,C}$ is about -5 °C and $T_C$ is 25 °C. The corresponding iso-COP contour lines are depicted in Figure 11 in which the region of the maximum COP is observed better.

Two-variable optimization studies to maximize the system performance are also held using EES to obtain optimum operating conditions. The condensing temperature of cascade condenser temperature ($T_{CAS,C}$ and condensation temperature of high temperature cycle $T_C$) are chosen to be independent variables while the evaporation temperatures in high temperature cycle are kept constant in case studies. The correlations are developed based on these two parameters. The linear regression is applied in optimizations. The correlations are obtained for three studied standard condition cases which are 0 °C, -25 °C and -40 °C evaporation temperature conditions.
The correlations obtained for the three cases, corresponding to SC1, SC3 and SC5 cases, are as follows:

\[ \text{COP}_{SC1} = 7.244 - 0.0228T_{CAS,C} - 0.1023T_C \quad R^2 = 96.55\% \]
\[ \text{COP}_{SC3} = 3.192 + 0.0027T_{CAS,C} - 0.0413T_C \quad R^2 = 98.17\% \]
\[ \text{COP}_{SC5} = 2.325 + 0.0056T_{CAS,C} - 0.0281T_C \quad R^2 = 99.26\% \]

5. CONCLUSION

In this study, thermodynamic analysis is performed for a two-stage sub-critical cascade refrigeration system which uses CO\(_2\) and R404A refrigerants in high and low temperature cycles, respectively. A mathematical model is developed and then implemented in EES software. The aim is to examine influence of different operating conditions on system performance and determine the optimum working conditions to maximize the COP and thereby minimize the exergy destruction of the system. The standards of ASHRAE are taken into account in determining the operating conditions. Two-variable optimization correlations are also derived at the final stage of this work.

The influence of cascade condenser temperature is initially examined for high, moderate and low evaporation temperature conditions corresponding to SC1, SC3 and SC5 cases, respectively. It is observed that increasing \(T_{CAS,C}\) decreases overall exergy loss and increases both the COP and the second law efficiency of the system within the
studied temperature range. The optimum $T_{CAS,C}$ is observed about 5 °C for SC5 case. The SC3 case has been studied similarly. Similar trends have been observed however a minimum about 0 °C is observed in the COP of the system. An optimum temperature of 10 °C is obtained for SC1 case conditions.

The effect of ambient temperature conditions corresponding to the R404A condenser temperature, $T_C$, is investigated by varying it between 25 °C to 45 °C. It is found that the exergy loss of the overall system increases by increasing $T_C$ although no change in exergy loss of low temperature cycle components. The heat transfer in condenser of R404A cycle is concluded to be the main source of exergy loss of overall system for this case study.

The temperature difference in the cascade condenser which is directly related to size of it is also varied. It is found that increasing the temperature difference decreases the COP and second law efficiency of system for constant evaporation temperature of -25 °C.

The correlations to obtain the COP of the system have been also derived with two variables, namely $T_{CAS,C}$ and $T_C$. The COP of the system can be calculated with a high certainty using these equations. The region for maximum COP of the system for SC3 case is found to be -5 °C and 25°C for $T_{CAS,C}$ and $T_C$, respectively.

**NOMENCLATURE**

| T | Temperature $(°C)$ |
| W | Work $(kW)$ |
| $\dot{S}_{\text{gen}}$ | Entropy Generation $(kW/K)$ |
| $\dot{X}_{\text{Lost}}$ | Rate of Exergy Lost $(kW)$ |
| COP | Coefficient of Performance |
| h | Specific Enthalpy $(kJ/kg)$ |
| HTC | High Temperature Circuit |
| LTC | Low Temperature Circuit |
| $m$ | Mass Flow Rate $(kg/s)$ |
| $\eta$ | Efficiency |
| P | Pressure $(kPa)$ |
| R744 | Carbondioxide |
| $\dot{Q}$ | Heat Transfer Rate $(kW)$ |
| s | Specific Entropy $(kJ/(kg K))$ |
| SC | Standard Conditions |
| $\Delta T$ | Temperature Difference $(°C)$ |

**Subscripts**

E | Evaporator |
C | Condenser |
CAS | Cascade Heat Exchanger |
II | Second Law |
F | Cooling Space |
max | Maximum |
opt | Optimum |

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