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Huiying Xu
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Report Number:
02-007

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COMPUTATIONAL MODELS FOR MECHANICAL DEFORMATION OF SOFT TISSUES OF THE CENTRAL NERVOUS SYSTEM

Yinlong Sun
Huiying Xu
Steven Teoh

Department of Computer Sciences
Purdue University
West Lafayette, IN 47907

CSD TR #02-007
February 2002
Computational Models for Mechanical Deformation of Soft Tissues of the Central Nervous System

Huiying Xu, Steven Teoh and Yinlong Sun
Department of Computer Sciences, Purdue University

Riyi Shi
Department of Basic Medical Sciences & Department of Biomedical Engineering, Purdue University

Abstract

Tissues of the central nervous system (CNS) such as spinal cord and brain are made of soft composite materials. These tissues can be traumatized by non-physiological loads with resultant morphological and functional deficits. Deformation of tissue subjected to these loads is the primary causal agent for tissue damage. The magnitude of deformation is determined by the load, material properties and boundary conditions on the tissue. The spatial and temporal variation of these parameters within a single subject complicates the analysis of the trauma and non-invasive elucidation of the resultant functional deficits. Computational methods, which relate microscopic tissue damage to global loads, are very useful to study in vivo tissue damage for measurable external loads. In particular, Finite Element Analysis (FEA) has a potential in predicting accurately the response of the soft tissues to trauma. This article provides a comprehensive review of common computational models in the mechanical analysis of spinal cord and brain. These include linear and nonlinear viscoelastic models, and poroelastic and poroviscoelastic models which are based on the biphasic theory.

Keywords: spinal cord, brain, computing method, finite element analysis (FEA), viscoelastic, non-linear viscoelastic, hyperelastic, poroelastic, poroviscoelastic, biphasic.
1. Introduction

Morphological damage to soft tissues of the spinal cord and brain by mechanical trauma (such as physical penetration of projectiles, slippage of vertebral discs and growth of tumors) often result in serious functional deficits and permanent paralysis. These tissues are capable of withstanding physiological mechanical loads. However, supra-physiological loads and consequent large deformations inflict severe morphological damage. Therefore, determination of the magnitude of deformation of the tissue subjected to trauma is of critical importance to the neurotrauma community. In order to predict local mechanical deformation of a given volume of nervous soft tissue to measurable global traumatic loads, the constitutive material behavior of the material and the boundary conditions of the tissue should be understood and quantified. At present, only scanty information is available to the researchers. Solving this problem maybe important not only to understanding the pathology of central nervous system injury, but also in the development of therapeutic interventions.

The lack of information about the local damage inflicted by supra-physiological loads is due to the challenges that confront an investigator due to the complexity of the central nervous system materials and structure. Firstly, spinal cord is a composite material with spatially varying material properties. It consists of the gray matter in the center and the white matter on the outside. The gray matter contains neuronal cell bodies, glial cells (such as astrocytes and microglia), and blood vessels. The white matter, which also contains astrocytes and blood vessels, is mainly composed of axons, which are covered with a white, insulating myelin layer. Secondly, the mechanical properties of the components of spinal cord, which are crucial to determining the global response to an external load, are not adequately known. Finally; additional complications are involved due to factors such as the rate of loading and boundary conditions. For example, the mechanical response of spinal cord material to a rapid deformation occurring during a projectile impact will be different from its response to a very slow deformation rate encountered with a growing tumor. Properties of the cerebrospinal fluid surrounding the cord, the nerve roots emanating from the cord, and the dimensions of the vertebral column that constitute the boundary conditions for the deforming cord, vary spatially.

Computational modeling and numerical methods have demonstrated their unique capabilities in solving complex boundary value problems. One such method is the finite element analysis (FEA) method used routinely by engineers in solving problems of continuum mechanics. This numerical tool is capable of handling complicated composite materials and boundary conditions. FEA can be used to predict the local deformation of soft tissues of the central nervous system if a realistic computational model, that encompasses applied physical loads, boundary conditions, and reasonable assumptions of the relevant material properties, can be developed from experiments. This review focuses on the computational material models used in the analysis of soft tissues like spinal cord and brain.

A number of theoretical models have been proposed to characterize the mechanical properties of spinal cord. Hung et. al. [Hung79, Hung81] developed a novel in vivo experimental method to measure the stress-strain relationship of the spinal cord of anesthetized cats. They found that the nonlinear viscoelastic behavior is reflected in the pronounced effects of strain (or loading) history on the stress-strain relationship. Bilston et. al. developed a quasi-linear
A viscoelastic model was used to describe the behavior of spinal cord and compared it with the hyperelastic model and the four-element viscoelastic fluid model [Bilston96]. They found that the quasi-linear viscoelastic model describes the material behavior adequately.

Recently, constitutive models for brain tissue were proposed [Mendis95, Miller97, Miller99, Miller00a]. These models were based on a strain energy function in a polynomial form with time-dependent coefficients. Miller et al. [Miller99] pointed out that the applicability of the model by Mendis [Mendis95] was restricted to the condition of very high strain-rate loading, while their model [Miller97] was suitable for low strain-rate loading. But the non-linear viscoelastic nature of the model causes problems in implementation of FEA. Hence, Miller et al. [Miller99] developed the hyperelastic, linear viscoelastic constitutive model that can be conveniently implemented in a commercial FEA software ABAQUS. Considering the tissue similarities between the spinal cord and the brain tissue, these models might also work for the spinal cords.

There are other soft tissue models that are based on experiments done in soft tissues outside of the central nervous system. One such model was based on the biphasic theory, which was first proposed by Mow et al. [Mow80]. This model treats the tissue as a two-phase immiscible mixture, consisting of an intrinsically incompressible solid phase (collagen and proteoglycan) and an intrinsically incompressible fluid phase (physiological medium). The tissue as a whole is compressible through exudation of the physiological fluid. The biphasic models include the biphasic poroelastic model and biphasic poroviscoelastic model. The latter takes into account the contribution of the intrinsic viscoelastic behavior of the solid phase to the viscoelastic behavior of the tissue. The finite element formulations have been developed based on these two models. Since the constituent elements of all soft tissues are similar, such material models can be effectively extended to the soft tissues of the CNS with slight modifications of parameters. The finite element formulations for the hyperelastic, viscoelastic models and biphasic models are shown in Fig. 1.1.

Fig. 1.1 Finite element formulation and theoretical models for soft tissue (such as spinal cord).

This report presents a comprehensive review of the theoretical models for spinal cord along with the numerical computational techniques. Section 2 briefly reviews the experimental
studies and related theoretical analysis of spinal cord injuries. Section 3 presents an introduction to the FEA method and ABAQUS software package. Section 4 reviews the mechanical models that address non-linear, inelastic material properties of spinal cord. Section 5 focuses on the constitutive models that are mainly applied to brain tissues. The biphasic poroelastic and poroviscoelastic models are described in section 6. Section 7 summarizes the entire report.

2. Experimental Studies

In the past, many investigators have attempted to characterize the mechanical response of spinal cord tissue (for example, refer to [Bain01, Blight88, Blight86, Blight91, Shi02, Maxwell91, McDonald99] for comprehensive reviews). Some of these investigations were adopted an engineering approach and tried to determine mechanical parameters of the spinal cord material. However, in spite of several years of research, several critical mechanical parameters have not been estimated conclusively and the data on the few that are available remain controversial. An example of such a controversy is one surrounding the tangential moduli of the white and gray matters of the spinal cord. Schneider et. al. [Schneider54] has suggested that gray matter has a relatively looser texture and less supportive strength than white matter. They regarded the gray matter as being softer than the white matter. However, Levine argued that gray matter was only slightly less rigid than white matter [Levine97], and did an FEA model of the spinal cord using this assumption. With the pipette aspiration method, Ozawa et al. [Ozawa01] measured the elastic moduli of both gray and white matters and concluded that there is no significant difference between them in spinal cord sections made in various orientations. However, Ichihara et al. [Ichihara01] demonstrated that the gray matter was more rigid (i.e. higher tangent modulus) and more fragile (i.e. lower elongation) than white matter. They applied their experimental data to computer simulation using FEA method, and found that simulation results agree with the magnetic resonance imaging data of the spinal cord under compression. This situation points out the need for more refined investigations for extraction of accurate continuum mechanical parameters of the spinal cord material.

Other researchers like Shi et al. [Shi96, Shi97, Shi99, Shi00, Shi02] have adopted a pathological approach to the response of spinal cord material to mechanical loads. They conducted a series of experiments to understand the pathology of spinal cord injury and comprehend the underlying mechanisms of functional damage and recovery, with the purpose of eventually developing therapeutic intervention. They have made detailed electrophysiological and morphological observations in spinal cord tissue (ventral white matter) subjected to compression and tensile (stretch) trauma. For stretch trauma [Shi02], they have observed three different types of conduction deficits exhibited by the traumatized ventral white matter—a immediate, spontaneously reversible component probably caused by the change in axolemmal permeability, an irreversible component caused by membrane damage and a therapeutically reversible component caused by the disruption of the myelin sheath. These components of electrophysiological damage are likewise seen in compression injuries [Shi96, Shi97, Shi99, Shi00]. Using a horse radish peroxidase assay, they located the distribution of axonal damage in the cross section of a stretch injured spinal cord ventral white matter. They found most of the morphological damage to be concentrated near the periphery. In contrast, compression injuries tend to produce damage in the interior of the spinal cord. They showed that there is no size
selectivity of initial injury for large axons in both stretch and compression injuries to spinal cord. These studies offer critical information for developing and verifying realistic models of the mechanical behavior of spinal cord.

Investigations that involve both principles of engineering and pathology would be ideal in understanding the fundamentals of the trauma of central nervous system injury. This is essentially a bioengineering approach to the study of neurotrauma. With the emergence of bioengineering as a separate entity, it is highly likely that such studies will be conducted in the near future.

3. Finite Element Analysis

The application of the finite element analysis (FEA) method for solution of engineering problems is commonly available in literature (for example, [Entwistle99]). In the FEA method, a volume of continuum (material or a field) is first divided into discrete but connected parts that are called finite elements. These elements are usually triangles or quadrilaterals in two-dimensional applications, or tetrahedral or parallelepipeds in a three-dimensional analysis. Each element can have either straight or curved boundaries. Adjacent elements touch without overlapping, and there are no gaps between the elements. The points at which the elements are connected are called nodes, located at the corners of the elements and sometimes along the sides. The process of dividing the continuous body into elements is called discretization. The resultant pattern is called a mesh. The selection of the mesh pattern and the element size depends on the specific problems. Commercial finite element packages can usually generate meshes automatically.

The next step is to establish the relation between the forces applied to the nodes of a single element and the resulting nodal displacements. These two quantities are linked by the stiffness matrix through the equation

\[(\text{nodal forces}) = [\text{stiffness matrix}] \times (\text{nodal displacements})\]

We can combine all the stiffness matrices of the individual elements into a single large matrix, which is the global stiffness matrix for the whole body. This combination process is called assembly. In most of the cases, we know the applied nodal forces and the nodal displacements are to be determined. Therefore, we need to invert the stiffness matrix equation into the form

\[(\text{nodal displacements}) = [\text{inverse stiffness}] \times (\text{applied nodal force})\]

This matrix equation can be solved for the nodal displacements. This is the most important part of the finite element analysis.

The process of setting-up the stiffness matrix has two stages. The first involves relating the strains in the element to the nodal displacements, and the second relating the nodal forces to the stresses in the elements.
For the first step, we need to assume the way in which the displacement at a point in the element varies with position. A simple assumption is that the displacement varies linearly with the position, which gives strains that are constant across the element. To analyze steeply varying stress patterns using this element, we need to divide the loaded body into a large number of elements in order to achieve the desired mesh where the stress difference between the adjacent elements is small enough. An equally accurate solution can be achieved using fewer elements and less computing time by adopting elements in which the displacement is assumed to vary with the position in the element as a higher-than-linear dependency, for example, quadratic and cubic functions of position. Higher-order variations lead to strains that vary across the element and hold out the possibility of securing a closer approximation to the exact strain. The relations that define the law of variation of particle displacement with position are called interpolations functions and are expressed in terms of what are called shape functions.

The second step is to relate the nodal forces to the stresses in the element. This requires the use of energy principles in the form of either the virtual work principle or the minimization of the total potential. These two principles are equivalent alternatives. The virtual work principle states that a necessary and sufficient condition for equilibrium of a particle is that the work done by forces on the particle is zero under any virtual displacement. The principle of minimization of total potential assumes that a mechanically loaded system adopts a configuration (local displacements) that has the least total potential.

A highly sophisticated, general-purpose software package that implements the finite element analysis method is ABAQUS, which was developed by Hibbitt, Karlsson, and Sorensen. It is designed primarily to model the behavior of solids and structures under external loading. ABAQUS includes the following features:

- Analysis capability for both static and dynamic problems.
- 2D and 3D modeling of continuum fields (solids, fluids and electromagnetic fields).
- Provides a very extensive element library, including a full set of continuum elements (beam elements, shell and plate elements).
- An advanced material library, including elastic and elastic-plastic solids, foams, concrete, soils, and piezoelectric materials.
- Sophisticated analysis to model complex problems involving solid body contact.
- Able to model a number of phenomena including vibrations, coupled fluid/structure interactions, acoustics, and buckling problems.

The ABAQUS finite element system includes:

- ABAQUS/Standard, a general-purpose finite element program;
- ABAQUS/Explicit, an explicit dynamics finite element program;
- ABAQUS/Pre, an interactive preprocessor that can be used to create finite element models and the associated input file for ABAQUS;
- ABAQUS/Post, an interactive postprocessor that provides contour plots, animations, x-y plots, and tabular output of results from the restart and results files written by ABAQUS/Standard and ABAQUS/Explicit.
ABAQUS is very easy to use because the input is organized around a few natural concepts and conventions. Since ABAQUS is based on a very sound theoretical framework, it has been widely used in academia and aerospace, automotive, microelectronics and petroleum industries.

4. Mechanical Models of Spinal Cord

The mechanical properties of a specific material are usually characterized in terms of the stress-strain relation. This relation can be illustrated through a simple one-dimensional tensile test, as shown in Fig. 4.1.

![Tensile specimen with gauges](Courtesy of Shames and Pitarresi)

The specimen to be tested is subjected to an axial force $F$ along its centerline. The gage 1 measures the change of length between two points, which is aligned in the direction of the load and originally is $L_1$. At the same time, the gage 2 measures the change in the distance between two points on line perpendicular to the loading direction. The original length of this line is $L_2$. The axial stress $\sigma_\perp$ is computed as $F/A_0$, where $A_0$ is the initial unstrained cross-sectional area of the specimen; the strain $\varepsilon_\perp$ is computed as the ratio $\Delta L_1/L_1$, where $\Delta L_1$ is the change of length obtained from the gage 1; the strain $\varepsilon_\parallel$ is computed by the ratio $\Delta L_2/L_2$, where $\Delta L_2$ is the change of length obtained from the gage 2.

The relationship of the axial stress $\sigma_\perp$ versus the axial strain $\varepsilon_\perp$ of the mild steel is shown in Fig. 4.2. The stress-strain diagram may be different for different materials, here the authors just want to give an example. It is noticeable that the stress is proportional to the strain at the early stage of the loading. This stress-strain relation is known as the Hooke’s law, the model describing such a deformation is called linear elastic model. In one dimension, Hooke’s law is given as

$$\sigma_\perp = E \varepsilon_\perp,$$  \hspace{1cm} (4.1)

where $E$ is the Young’s modulus of the material. In three dimensions, this law is given as

$$\sigma_\gamma = \sum_k \sum_l C_{\gamma l} \varepsilon_\mu,$$  \hspace{1cm} (4.2)
where \( \sigma_{ij} \) is the stress tensor, \( \varepsilon_{ij} \) is strain tensor, and \( C_{ijkl} \) is the elastic modulus tensor.

The linear elastic model is only suitable for the small deformation and it ceases at some critical point, which is called the proportional limit. In Fig. 4.2, the point A is the proportional limit. However, this point is difficult to be measured accurately. For the steel specimen considered, there is a stress level close to the proportional limit such that when the specimen is loaded beyond this level, then unload the specimen, the specimen cannot return back to its original length, some residual deformation is left, which is called the residual strain as shown in Fig. 4.2. The maximum engineering stress is at the point U, the value of the stress at this point is called the ultimate stress. Once the deformation of specimen passes this point, it will result in fracture of the material. For an elastic material, the maximum propagation speed of cracks is the speed of the elastic wave.

Viscoelasticity is a special type of nonlinear elastic behavior. Viscoelastic materials exhibit both viscous and elastic characteristics. Some common phenomena involving [Lakes99] of viscoelasticity are:
• if the stress is held constant, the strain increases with time (creep);
• if the strain is held constant, the stress decreases with time (relaxation);
• the efficient stiffness depends on the rate of applications of the load;
• if cyclic loading is applied, hysteresis (a phase lag) occurs, leading to a dissipation of mechanical energy;
• acoustic waves experience attenuation;
• rebound of an object following an impact is less than 100%;
• during rolling, friction resistance occurs.

The stress-strain relationship of the spinal cord shows viscoelastic behavior. Chang et al. used the linear and nonlinear viscoelastic models to fit the viscoelastic responses of the spinal cord of cats [Chang88]. They found that the linear viscoelastic model is capable of predicting the prolonged stress relaxation and recovery curve for small deformations. However, for larger deformation, the nonlinear viscoelastic model is needed for the viscoelastic response. An attempt in this direction was made by Bilston et al. In 1996, they developed the quasi-linear viscoelastic model and found that it fit the experimental data very well [Bilston96]. The quasi-linear viscoelastic model was originally proposed by Fung [Fung81]. These models are described in the following.

4.1. Linear and Nonlinear Viscoelastic Models

For uniaxial loading, the stress-strain relationship of linear viscoelastic model [Chang88] is given as

\[ \sigma(t) = \int_0^t G(t - \tau) \frac{\partial \varepsilon(\tau)}{\partial \tau} \, d\tau, \quad (4.3) \]

where \( G(t) \) is known as the relaxation function. It can be related to the initial and transient components of the relaxation compliance by

\[ G(t) = G(0) + \Delta G(t). \quad (4.4) \]

The substitution of Eq. (4.4) into Eq. (4.3) gives

\[ \sigma(t) = G(0) \varepsilon(t) + \int_0^t \Delta G(t - \tau) \frac{\partial \varepsilon(\tau)}{\partial \tau} \, d\tau. \quad (4.5) \]

In three dimensions, this model [Lakes99] can be described as

\[ \sigma_{ij}(t) = \int_0^t G_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} \, d\tau. \quad (4.6) \]

Where \( i, j, k, l = 1,2,3 \). Many practical materials are approximately isotropic (their properties are independent of directions). For isotropic materials, the constitutive equation is

\[ \sigma_{ij} = \lambda \varepsilon_{ii} \delta_{ij} + 2\mu \varepsilon_{ij} \quad (4.7) \]

in which \( \lambda \) and \( \mu \) are the two independent Lamé elastic constants, \( \delta_{ij} \) is the Kronecker delta (1 if \( i = j \); 0 if \( i \neq j \)), and \( \varepsilon_{ii} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \). Engineering constants such as Young's modulus \( E \), shear modulus \( G \), and Poisson's ratio \( \nu \) can be extracted from these tensorial constants as

\[ G = \mu. \quad (4.8) \]
\[
E = \frac{G(3\lambda + 2G)}{(\lambda + G)}, \quad (4.9)
\]
\[
\nu = \frac{\lambda}{2(\lambda + G)}. \quad (4.10)
\]

In isotropic viscoelastic materials, Eq. (4.6) can be rewritten as
\[
\sigma_y(t) = \int_0^t G_2(t - \tau) \delta_y \frac{\partial \varepsilon_y(\tau)}{\partial \tau} d\tau + \int_0^t 2G_1(t - \tau) \frac{\partial \varepsilon_y(\tau)}{\partial \tau} d\tau. \quad (4.11)
\]

As to the nonlinear viscoelastic constitutive equation, very few equations have been used to describe it. The nonlinear viscoelastic constitutive equation was developed by Schapery [Schapery69] to analyze the in-vivo measurement of nonlinear viscoelastic properties of the spinal cord. The equation can be written as
\[
\sigma(t) = g_0 G(t_0) \varepsilon(t) + g_1 \int_0^t \Delta G(\phi' - \phi) \frac{\partial g_2 \varepsilon(\tau)}{\partial \tau} d\tau, \quad (4.12)
\]
where
\[
\phi - \phi' = \int_0^t \frac{d\tau}{\alpha \varepsilon}. \quad (4.13)
\]
The functions \(g_0\), \(g_1\), and \(g_2\) represent the nonlinear strain-dependent viscoelastic material properties, and the coefficient \(\alpha \varepsilon\) is the time scale factor. When \(\varepsilon_0\) is very small, the viscoelastic behavior becomes linear and \(g_0\), \(g_1\), \(g_2\), and \(\alpha \varepsilon\) approach unity, Eq. (4.12) reduces to Eq. (4.3).

The more general equation for nonlinear viscoelasticity has been proposed [Lakes99]. In one dimension for small strain, in the modulus formulation,
\[
\sigma(t) = \int_0^t E(t - \tau) \frac{\partial \varepsilon}{\partial \tau} d\tau + \int_0^t \int_0^t E_2(t - \tau_1, t - \tau_2) \left( \frac{\partial \varepsilon}{\partial \tau_1} \right) \left( \frac{\partial \varepsilon}{\partial \tau_2} \right) d\tau_1 d\tau_2 + \ldots, \quad (4.14)
\]
and the compliance formulation can be written as
\[
\varepsilon(t) = \int_0^t J_1(t - \tau) \frac{\partial \sigma}{\partial \tau} d\tau + \int_0^t \int_0^t J_2(t - \tau_1, t - \tau_2) \left( \frac{\partial \sigma}{\partial \tau_1} \right) \left( \frac{\partial \sigma}{\partial \tau_2} \right) d\tau_1 d\tau_2 + \ldots. \quad (4.15)
\]
Here, \(\tau_1\) and \(\tau_2\) are time variables of integration.

### 4.2. Quasi-Linear Viscoelastic Models

Experiments [Hung79, Hung81, Bilston96] demonstrated that the spinal cord samples exhibit a nonlinear stress-strain response, and the history of strain affects the stress. Consider a cylindrical specimen subjected to a tensile load. If a step increase in elongation is imposed on the specimen, the stress developed will be a function of time as well as of the strain. The history of the stress response, called the relaxation function, denoted as \(Y(t, \varepsilon)\), is assumed to be of the form
\[
Y(t, \varepsilon) = G(t) \sigma^*(\varepsilon), \quad G(0) = 1, \quad (4.16)
\]
in which \(G(t)\), a normalized function of time, is called the reduced relaxation function, and \(\sigma^*(\varepsilon)\) is called the elastic response. Assume that the stress response to an infinitesimal change
in strain, \( \delta \varepsilon (t) \), superposed on a specimen in a state of strain \( \varepsilon \) at an instant of time \( \tau \), is, for \( t > \tau \):

\[
G(t-\tau) \frac{\partial \delta \varepsilon (\varepsilon(t))}{\partial \varepsilon} \delta \varepsilon (\tau).
\]  

(4.17)

Suppose that Boltzmann’s superposition principle applies, such that

\[
\sigma(t) = \int_0^t G(t-\tau) \frac{\partial \delta \varepsilon (\varepsilon(t))}{\partial \varepsilon} \delta \varepsilon (\tau) \, d\tau.
\]  

(4.18)

That is, the stress at time \( t \) is the sum of contributions of all the past changes, each governed by the same reduced relaxation function.

Furthermore, if we assume that the strain history starts at time \( t = 0 \), and the stress is zero before \( t = 0 \), then Eq. (4.18) can be rewritten as

\[
\sigma(t) = \sigma'(0)G(t)+ \int_0^t G(t-\tau) \frac{\partial \delta \varepsilon (\varepsilon(t))}{\partial \varepsilon} \delta \varepsilon (\tau) \, d\tau
\]

\[
= \int_0^t G(t-\tau) \frac{\partial \delta \varepsilon (\varepsilon(t))}{\partial \varepsilon} \delta \varepsilon (\tau) \, d\tau.
\]  

(4.19)

The elastic response function is that used by Woo et al. [Woo93] to describe the response of ligaments:

\[
\sigma'(\varepsilon) = A(e^{B\varepsilon} - 1).
\]  

(4.20)

The reason for Bilston et al. to use this equation is that the general form of this stress-strain behavior predicted by this equation is similar to the experimental data. If \( B \varepsilon \to 0 \), we might use the assumption \( e^{B\varepsilon} = 1 + B\varepsilon \), so the elastic response function becomes \( \sigma'(\varepsilon) = AB\varepsilon \), it is really linear. However, the average values for \( B \) and the peak strain \( \varepsilon_p \) in the reference.[Bilston96] are given as \( <B> = 25.9 \) and \( <\varepsilon_p> = 0.105 \), so we know that we cannot use this assumption. Therefore, we cannot approximately regard the stress-strain relationship as the linear behavior; it is quasi-linear.

Fung [Fung81] suggested to use the following form for a generalized reduced relaxation function, \( G(t) \):

\[
G(t) = \frac{\sum C_i \tau_i^t}{\sum C_i \tau_i},
\]  

(4.21)

where \( \tau_i \) is the relaxation time and \( C_i \) is the constant.

Bilston et al. used the strain history as:

\[
\varepsilon(t) = \begin{cases} 
\dot{\varepsilon}_0 t & 0 < t < t_1 \\
\dot{\varepsilon}_0 t_1 & t \geq t_1
\end{cases}
\]  

(4.22)

This results in a strain rate of

\[
\dot{\varepsilon}(t) = \dot{\varepsilon}_0 (1 - h(t-t_1)),
\]  

(4.23)
where \( h(t - t_1) \) is the Heaviside step function. The differential of the elastic response with respect to the strain can be obtained from Eq. (4.20) as

\[
\frac{\partial \sigma^e}{\partial \varepsilon} = AB e^{B \varepsilon}.
\]  

(4.24)

Therefore, the stress responses are obtained as

\[
\sigma(t) = \int \left( \sum \frac{C_i e^{-t}}{C_i} AB e^{B \varepsilon} \dot{\varepsilon}_0 d \tau \right) = AB \dot{\varepsilon}_0 \sum G_i \left[ \frac{e^{B \varepsilon_0} - e^{-t \varepsilon_0}}{t \varepsilon_0 + B \dot{\varepsilon}_0} \right],
\]

\[0 < t < t_1;\]

\[
\sigma(t) = \int \left( \sum \frac{C_i e^{-t}}{C_i} AB e^{B \varepsilon} \dot{\varepsilon}_0 d \tau \right) = AB \dot{\varepsilon}_0 e^{t \varepsilon} \sum G_i \left[ \frac{e^{t \varepsilon_0} - e^{-t \varepsilon_0}}{t \varepsilon_0 + B \dot{\varepsilon}_0} \right],
\]

\[t \geq t_1.\]

Here

\[
G_i = \frac{C_i}{\sum C_i}.
\]

(4.26)

These relaxation times, \( \tau_i \), cannot, however, be interpreted as actual relaxation constants, because the fitting of the exponentials is not a mathematically unique process.

5. Constitutive Models

With the development of computational mechanics, the hyperelastic, viscoelastic model is developed for the FEA of brain tissue. The hyperelastic, viscoelastic constitutive model can be regarded as the combination of the hyperelastic model with viscoelastic model.

The hyperelastic model [ABAQUS95] has the following features:

- Isotropic and nonlinear;
- Valid for materials that exhibit instantaneous elastic response up to large strains (such as rubbers, solid propellant, or other elastomeric materials);
- Requires that geometric nonlinearity be accounted for during the analysis step, since it is intended for finite-strain applications.

Materials that undergo large elastic deformations are often characterized as hyperelastic materials. Typically, such materials have a bulk modulus that is several orders of magnitude larger than the shear modulus, and they tend to deform in an incompressible manner [Mendis95]. The mechanical response of hyperelastic response materials has been rigorously analyzed in the past, most notably in the pioneering work done by Mooney [Mooney1940].
5.1. Hyperelastic Nonlinear Viscoelastic Model

Constitute models of a hyperelastic material can be defined by incorporating a Strain Energy Density Function (SEDF) to link stresses and strains. The SEDF is defined as a function of the three invariants of strain, \( I_1, I_2, \) and \( I_3 \). The strain invariants \([\text{Green68}]\) are defined in terms of the principal stretch ratios \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) as

\[
\begin{align*}
I_1 &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \\
I_2 &= 2\lambda_1^2\lambda_2^2 + \lambda_3^2\lambda_1^2 + \lambda_2^2\lambda_3^2, \\
I_3 &= \lambda_1^2\lambda_2^2\lambda_3^2. \\
\end{align*}
\]  

(5.1)

The principle stretch ratios are defined as the lengths of the sides of an initially undeformed unit cube oriented in the directions of the principal strains. A well-known and widely used form for the SEDF is the Mooney-Rivlin SEDF, defined in its most general form for incompressible materials \([\text{Mendis95}]\) as

\[
U = \sum_{i=0}^{N} \sum_{j=0}^{N} C_{ij} (I_1 - 3)^i (I_2 - 3)^j,
\]  

(5.2)

where the value of the index \( N \) governs the complexity of the polynomial dependence of \( U \) on \( I_1 \) and \( I_2 \). Typically \( N \) takes the values of 1 or 2, values greater than 2 are rarely used. A simple form for \( N = 1 \) was used by Mooney \([\text{Mooney40}]\) and Mendis \([\text{Mendis95}]\), and Miller et al. \([\text{Miller97, Miller99, Miller00a}]\) used the value 2.

In a conventional finite element formulation for solid elements, the stress tensor is derived directly from the kinematic variables. Severe numerical difficulties are encountered for incompressible or nearly incompressible materials because small volumetric strains cause large volumetric stresses due to the high effective bulk modulus. Thus the hydrostatic part of the stress tensor is very sensitive to computed fluctuations in the hydrostatic strain, which leads to numerical instability in the finite element simulation \([\text{Mendis95}]\). Some alternative finite element formulations were developed to solve this problem. One is the split energy formulation, which was developed by Malkus et al. \([\text{Malkus78}]\) and used by Ward \([\text{Ward82}]\) in finite element models of the brain. The split energy formulation separately derived the finite element equations from the deviatoric and volumetric strain energy components \([\text{Mendis95}]\). The split energy element was the first instance where a nontraditional finite element procedure was employed in soft tissue finite element modeling. The ABAQUS uses a mixed formulation, in which the element uses a separately interpolated hydrostatic stress field, in addition to the (linear or quadratic) interpolation of the nodal displacements within the element. The element formulation involves introducing of the incompressibility constraints equation by use of the Lagrange multiplier in the variational statement of the internal energy of the element.

Mendis et al. \([\text{Mendis95}]\) considers the cylindrical brain tissue specimens, the uniaxial stress is given as

\[
\sigma = \frac{F}{A} = \frac{A_0}{A} \frac{\partial U}{\partial \lambda_1} = \lambda_1 \frac{\partial U}{\partial \lambda_1} = 2C_{10} \left( \lambda_1^2 - \frac{1}{\lambda_1} \right) + 2C_{01} \left( \lambda_1 - \frac{1}{\lambda_1} \right). \\
\]  

(5.3)
Here $F$ is the uniaxial force, $A_o$ is the cross sectional area of the uniaxial specimen in the undeformed state, and $A$ is the cross sectional area under the deformed state. The elastic stress, derived from the strain energy function $U$, is a nonlinear function of strain. For the viscoelastic materials, the strain energy density function can be interpreted as an energy density function that depends on the time history of the strain invariants. The dependence of the energy density function on the time history of the strain invariants can be implemented by defining relaxation functions for the coefficients $C_{10}$ and $C_{10}^s$, akin to the definition of a stress relaxation function in classical linear viscoelasticity. The time-dependent coefficients of the energy function [Mendis95] is a Prony series defined as follows,

$$ C_y = (C_y^s + \sum_{k=1}^{N} (C_y^i - C_y^s) \cdot e^{-\tau/k}) \cdot h(t), \tag{5.4} $$

where $C_y^s$ is the steady-state value of the coefficient (determined previously from the quasi-static test) and $h(t)$ is the Heaviside step function. The strain energy density due to time varying strain invariants is expressed in terms of a convolution integral involving the strain invariants and the relaxation energy coefficients as follows

$$ U(t) = \int_0^t [C_{10}(t - \tau) \frac{d}{d\tau} I_1(\tau) + C_{01}(t - \tau) \frac{d}{d\tau} I_2(\tau)] d\tau. \tag{5.5} $$

Similarly the viscoelastic stress in a uniaxial test can be expressed using the relaxation coefficients of the stored energy function as follows

$$ \sigma(t) = \int_0^t [2C_{10}(t - \tau) \frac{d}{d\tau} \left( \lambda_1^2(\tau) - \frac{1}{\lambda_1(\tau)} \right) + 2C_{01}(t - \tau) \frac{d}{d\tau} \left( \lambda_1(\tau) - \frac{1}{\lambda_1^2(\tau)} \right)] d\tau. \tag{5.6} $$

For uniaxial compression at a constant strain rate, $\lambda_1(\tau)$ in Eq. (5.6) increases linearly with time. Using this condition it was not possible to evaluate the above convolution integral while retaining the Prony series form for $C_{10}(t)$ and $C_{01}(t)$ given in Eq. (5.4). Therefore standard curve fitting protocols cannot be applied to determine the parameters of the assumed forms of $C_{10}(t)$ and $C_{01}(t)$. Instead they are determined by simulating Eq. (5.6) so that the predicted stresses for all four strain rates approximate the experimentally determined stresses as best as possible.

The applicability of the model by Mendis is restricted to very high strain-rate loading conditions, Miller et al. developed the hyperelastic, nonlinear viscoelastic model for the viscoelastic material under compression at low strain rates.

The strain energy function that is used by Miller et al. [Miller97] is given as

$$ W = \sum_{i+j=1}^{N} C_y (J_1 - 3)^i (J_2 - 3)^j, \tag{5.7} $$

where the strain invariant are:

$$ J_1 = \text{Trace}[B], \quad J_2 = \frac{J_1^2 - \text{Trace}[B^2]}{2J_3}, \quad J_3 = \sqrt{\det B} = 1. $$

$B$ is a left Cauchy-Green strain tensor. Actually Eq. (5.7) is the same as the Eq. (5.2) if $B$ is defined as...
For infinitesimal strain conditions, the sum of constants \( C_{01} \) and \( C_{10} \) have a physical meaning of one-half of the shear modulus, it is given as

\[
\frac{\mu_0}{2} = C_{01} + C_{10}.
\]

(5.8)

The energy dependence on strain invariants only comes from the assumption that brain tissue is initially isotropic.

In experiments conducted the deformation was orthogonal, and here the left Cauchy-Green strain tensor has only diagonal components:

\[
B = \begin{bmatrix}
\lambda_1^2 & 0 & 0 \\
0 & \lambda_2^2 & 0 \\
0 & 0 & \lambda_3^2
\end{bmatrix}
\]

(5.9)

where \( \lambda_i \) is a stretch in vertical direction.

In such case, taking \( J_1 = \lambda_1^2 + 2\lambda_1^{-1} \) and \( J_2 = \lambda_2^2 + 2\lambda_2^{-1} \), allows computation of the only non-zero Lagrange stress components from the simple formula

\[
\sigma_2 = \frac{\partial W}{\partial \lambda_2}.
\]

(5.10)

To model the viscoelasticity of the brain tissue, the time-dependent coefficients of the strain energy function are given as

\[
C_y = C_{y0} + \sum_{k=1}^{N} C_{yk} e^{-\theta/\tau_k},
\]

(5.11)

where \( \tau_k \) are the characteristic times. The energy function is presented as

\[
W = \int \left\{ \sum_{i,j=1}^{N} C_{ij}(t-\tau) \frac{d}{d\tau} [(J_1 - 3) (J_2 - 3)^i (J_2 - 3)^j] \right\} d\tau.
\]

(5.12)

Substitution of Eq. (5.12) into Eq. (5.10) yields

\[
\sigma_2 = \int \left\{ \sum_{i,j=1}^{N} C_{ij}(t-\tau) \frac{d}{d\tau} \frac{\partial}{\partial \lambda_z} ((J_1 - 3) (J_2 - 3)^i (J_2 - 3)^j) \right\} d\tau.
\]

(5.13)

Where \( N \) takes the value 2, it is different from the value 1 used by Mooney [Mooney40] and Mendis et al. [Mendis95]. The hyperelastic, nonlinear viscoelastic model developed by Miller et al. is similar to the model developed by Mendis et al. However, Miller et al. pointed out that the some experimental data was incorrectly used by Mendis et al.
5.2. Hyperelastic, Linear Viscoelastic Model

Although the model [Miller97] can be used to analyze the viscoelasticity of brain tissue, the nonlinear viscoelasticity causes problems in its finite element implementation. Miller et al. [Miller99] developed the hyperelastic, linear viscoelastic model of brain tissue, which could be immediately used with ABAQUS software.

The polynomial strain energy function of the hyperelastic, linear viscoelastic model [Miller99] is given as

\[ W = \int \left\{ \sum_{i+j=\eta} C_{i+j} \left( 1 - \sum_{k=1}^{n} g_k (1 - e^{-u_r / \tau_k}) \right) \right\} \times \frac{d}{d\tau} \left[ (J_1 - 3)^i (J_2 - 3)^j \right] d\tau \]  

(5.14)

where \( \tau_k \) is the relaxation times, \( g_k \) are the relaxation coefficients, \( N \) is the order of polynomial in strain invariants and it takes the value 2 in this model, \( J_1 \) and \( J_2 \) are defined as the same as those of Eq. (5.7).

The only non-zero Lagrange stress components can be computed from the simple formula as

\[ \sigma_{\alpha\beta} = \frac{\partial W}{\partial \lambda_{\alpha\beta}} = \int \left\{ \sum_{i+j=\eta} C_{i+j} \left( 1 - \sum_{k=1}^{n} g_k (1 - e^{-u_r / \tau_k}) \right) \right\} \times \frac{d}{d\tau} \left[ \frac{\partial}{\partial \lambda_{\alpha\beta}} \left( (J_1 - 3)^i (J_2 - 3)^j \right) \right] d\tau . \]  

(5.15)

Miller et al. demonstrated that the agreement between the hyperelastic, linear viscoelastic model and experimental data was very good for fast and medium loading speeds, but it was worse match for slow loading speed, which resulted from a different character of the stress-strain curve than that for medium and fast loading speeds. This fact was taken into account in the nonlinear model [Miller97] by setting parameters \( C_{200} = C_{300} = 0 \) and by suitable choice of \( C_{20k} \) and \( C_{30k} \). Such flexibility is not possible in the linear model. In the hyperelastic, linear viscoelastic model the shape of stress-strain curve does not depend on the strain rate.

Miller et al. [Miller99] concluded that the main advantage of the linear viscoelastic model compared to the nonlinear viscoelastic model was that the large deformation, linear viscoelastic model could be immediately applied to larger scale finite element computations by directly using ABAQUS commands HYPERELASTIC to describe instantaneous elasticity of the tissue, and VISCOELASTIC to account for time dependent tissue behavior. Moreover, the hyperelastic, linear viscoelastic model required only four material parameters to be identified, too fewer than the nonlinear model.

6. Biphasic Theory

It is widely accepted that soft connective tissues such as tendon, ligament, intervertebral disc, articular cartilage and meniscus are multiphasic materials, i.e., a mixture of collagen/elastin...
fibrils, water, glycoaminoglycans, glycoproteins and cells [Suh91]. Mow et al. [Mow80] developed the biphasic model, which used the theory of mixtures developed by Truesdell et al. [Truesdell60], Bowen [Bowen76, Bowen80], and others. Mow et al. assumed that cartilage tissue was a mixture of two immiscible constituents: an incompressible solid matrix and an incompressible interstitial fluid. This model has been used to accurately describe both the stress distribution and interstitial fluid within the cartilage tissue under various loading conditions [Armstrong84, Hou89, Mak87, and Mow84]. Moreover, the interaction between the solid and fluid phases has been identified to be responsible for the apparent viscoelastic properties in the compression of hydrated soft tissue [Mow84]. The biphasic model has been extended to include non-linear behaviors such as strain dependent permeability [Holmes85, Lai80] and finite deformation [Holmers86, Mow86]. Lai et al. [Lai89] presented an extended version of the theory, called the triphasic model, to include the dissolved solute concentration which was known to be responsible for the Donnan osmotic pressure and swelling effect of the soft tissue.

Some finite element formulations based on the biphasic model have been developed. Spilker et al. [Spilker90] presented a penalty finite element formulation of the linear biphasic mixture equations for soft tissues, and used that formulation to develop a 4-node axisymmetric biphasic element with solid and fluid phase displacement/velocity as nodal degrees of freedom. This formulation was extended to include the non-linearity to strain-dependent permeability [Spilker88]. An alternate mixed-penalty formulation of the linear biphasic equations was presented by Spilker and Maxian [Spilker90-2, Suh91]. In this approach, the penalty form of the continuity equation is introduced into the weighted residual statement and the pressure is independently interpolated. Mak [Mak86a, Mak86b] proposed the biphasic poroviscoelastic theory by taking into account the contributions from the intrinsic viscoelasticity of the solid matrix. Recently, Suh et al. [Suh98] developed the biphasic poroviscoelastic model using two different algorithms, the continuous and discrete spectrum relaxation functions for the viscoelasticity of the solid matrix. Zhu et al. [Zhu01] used this model to analyze the traumatic brain injury.
6.1. Biphasic Poroelastic Model

6.1.1. Conservation of Mass

This section follows the reference [Mow80]. Let the two constituents of the binary mixture be denoted by $s$ and $f$. Each constituent is assigned a fixed but otherwise arbitrary reference configuration and a motion. Thus

$$x = x^s(X^s, t) \text{ and } x = x^f(X^f, t),$$

where $X^s$ and $X^f$ are material coordinates, $t$ is the time, and $x$ is the spatial position occupied at time $t$ by the particles $X^s$ and $X^f$. In the continuum theory of a mixture, a spatial point in the mixture is simultaneously occupied by all constituents. Thus, at each point, four densities are defined: the true densities and the apparent densities of each of the two constituents. If $dV^s$ and $dV^f$ are the true volumes of the solid mass $dm^s$ and fluid mass $dm^f$, respectively, then the true densities are $\rho^s = dm^s/dV^s$ and $\rho^f = dm^f/dV^f$. On the other hand, the apparent densities are $\rho_s = dm_s/dV$ and $\rho_f = dm_f/dV$, where

$$dV = dV^s + dV^f.$$  

From Eq. (6.2) and the definition of the densities, we obtain the following relationship between these four densities

$$\rho^s(t) \rho^f(t) + \rho^s(t) \rho^f(t) = \rho^s(t) \rho^f(t).$$  

The density of the mixture at $x$ at time $t$ is given by

$$\rho = \rho^s + \rho^f.$$  

The velocities of the particles $X^s$ and $X^f$ are given by

$$v^s = \partial x^s(X^s, t)/\partial t |_{X^s}$$

and

$$v^f = \partial x^f(X^f, t)/\partial t |_{X^f}.$$  

The mean velocity of the mixture, i.e., the barycentric velocity, is defined by

$$v = (\rho^s v^s + \rho^f v^f)/\rho.$$  

The axiom of balance of mass for each constituent leads to the following local balance equations in the absence of chemical reactions

$$(\partial \rho^s/\partial t) + div(\rho^s v^s) = 0$$

and
Assume that the organic solid matrix and the interstitial fluids are by themselves intrinsically incompressible. These assumptions are analytically stated by the following equations:
\[ \rho^s(t) = \rho^s(0) \quad \text{and} \quad \rho^f(t) = \rho^f(0) \]. With these conditions, Eq. (6.3) becomes
\[ \rho^s(t) \rho^s(0) + \rho^f(t) \rho^f(0) = \rho^s(0) \rho^f(0) \].

By taking the partial time derivatives of Eq. (6.8) and making use of Eq. (6.7), we obtain
\[ \text{div} \ \mathbf{v}^f + \alpha \text{div} \ \mathbf{v}^s + \alpha [\mathbf{v}^s \cdot \nabla \rho^s \cdot \text{grad} \ \rho^f + \left( \frac{\rho^s(0)}{\rho^f(0)} \right) \mathbf{v}^s \cdot \text{grad} \ \rho^f ] = 0 \].

where \( \alpha = \frac{\rho^f(t) \rho^f(0)}{\rho^f(t) \rho^f(0)} = \frac{d V^s}{d V^f} \). An alternate form of equation (6.9) can be obtained by using Eq. (6.8) to eliminate \( \rho^f \),
\[ \text{div} \ \mathbf{v}^f + \alpha \text{div} \ \mathbf{v}^s + \alpha [\mathbf{v}^s - \mathbf{v}^f] \cdot \text{grad} \ \ln \ \rho^s ] = 0 \]
as the continuity equation relating \( \mathbf{v}^f \), \( \mathbf{v}^s \) and \( \rho^s \). Finally, the summation of equations (6.7a) and (6.7b) yields the continuity equation for the mixture as a whole, it is given as
\[ \frac{D \rho}{Dt} + \rho \text{div} \ \mathbf{v} = 0 \]
where
\[ \frac{D \rho}{Dt} = \frac{\partial \rho}{\partial t} + \mathbf{v} \cdot (\text{grad} \ \rho) \].

### 6.1.2. Balance of Momentum

This section follows the reference [Mow80]. The axiom of linear momentum for each constituents, together with the balance of mass equation, leads to the following local equation of motion
\[ \rho^s \left( D^s \mathbf{v}^s / Dt \right) = \text{div} \ \mathbf{\sigma}^s + \rho^s \mathbf{B}^s + \mathbf{\pi}^s \]  
and
\[ \rho^f \left( D^f \mathbf{v}^f / Dt \right) = \text{div} \ \mathbf{\sigma}^f + \rho^f \mathbf{B}^f + \mathbf{\pi}^f \],
where \( D^s / Dt \) and \( D^f / Dt \) are time derivatives keeping \( X^s \) and \( X^f \) fixed, respectively, \( \mathbf{\sigma}^s \) and \( \mathbf{\sigma}^f \) are partial stresses, \( \mathbf{B}^s \) and \( \mathbf{B}^f \) are body force per unit mass and \( \mathbf{\pi}^s \) and \( \mathbf{\pi}^f \) are the momentum supplies. \( \mathbf{\pi}^s \) and \( \mathbf{\pi}^f \) account for the local interaction between the constitution \( s \) and \( f \), therefore they must satisfy
\[ \mathbf{\pi}^s = -\mathbf{\pi}^f \].

### 6.1.3. Constitutive Equation

Under the conditions of infinitesimal strains, Mow et al. [Mow80, Mow85] showed that the linearized relationship between the deformation field of the intrinsically incompressible, porous, permeable solid matrix and the apparent stress on the solid phase is
\[ \mathbf{\sigma}^s = -\alpha \mathbf{\varphi} + \lambda_1 \text{tr} \mathbf{e} + 2\mu \mathbf{e} + \lambda_2 \text{div} \ \mathbf{v}^s \mathbf{I} + 2\mu \mathbf{D} + 2K \epsilon \Gamma \]
and the relationship between the flow field of the intrinsically incompressible viscous fluid and the apparent stress on the fluid phase is
\[ \sigma^s = -p I + \lambda^s \text{div} \, \nu^s I + 2\mu I D + 2K_c \Gamma. \]  

(6.16)

Here, \( \nu^s \) is the velocity field of the solid phase; \( \epsilon \) and \( D \) are the infinitesimal strain tensor and the rate of deformation tensor, respectively; \( \Gamma \) is the spin tensor of the solid phase relative to the fluid phase; \( p \) is the pressure; \( \nu^f \) and \( D^f \) are the fluid phase and rate of deformation tensor, respectively; \( \lambda^s, \mu^s \), and \( \lambda^f, \mu^f \) are the isotropic elastic and viscoelastic moduli, respectively, of the solid matrix; and \( \lambda^f \) and \( \mu^f \) are the bulk and dynamic viscosities of the interstitial fluid. The constant \( K_c \) represents a diffusive couple interaction between the solid phase and the fluid phase.

The conditions of intrinsic incompressibility for the solid phase and fluid phase of the mixture are expressed by the equations

\[ \rho^s(t) = \rho^s(0) \]  

(6.17a)

and

\[ \rho^f(t) = \rho^f(0). \]  

(6.17b)

Assume that each phase is homogeneous, Eq. (6.10) becomes

\[ \nabla \cdot (\nu^s + \alpha \nu^s) = 0 \]  

(6.18a)

or

\[ \nabla \cdot (\alpha^s \nu^s + \alpha^f \nu^f) = 0 \]  

(6.18b)

where

\[ \alpha^s = \frac{\nu^s}{\nu}, \]

\[ \alpha^f = \frac{\nu^f}{\nu}, \]

and

\[ \alpha = \frac{\alpha^s}{\alpha^f}. \]

Since each phase is incompressible, we can obtain \( \alpha = \nu^s(0)/\nu^f(0) \). The momentum supplies \( \pi^s \) and \( \pi^f \) are given as

\[ \pi^s = -\pi^f = b \nabla \epsilon^s - K (\nu^s - \nu^f). \]  

(6.19)

Here \( K \) is the diffusive drag coefficient, and \( b \) is a measure of the capillary force within the interstitium. For slow flow, \( K \) is related to the permeability by the expression \([\text{Lai80}]\)

\[ K = \frac{1}{(1 + \alpha)^2 k}, \]  

(6.20)

where \( k \) is permeability. Mow and co-workers \([\text{Mow80, Mow82, Lai80}]\) have shown that the permeability of the tissue decreases exponentially with the magnitude of compression. This is given by as expression of the form

\[ k = k_0 \exp(M I^s), \]  

(6.21)

where \( k_0 \) and \( M \) are constants, \( I^s \) is the first invariant of the infinitesimal strain tensor, \( k_0 = 0(10^{-15}) m^4/N \cdot s \), and \( M \) ranges from 1 to 20.
So far we have obtained many parameters in this model, which brings inconvenience for application. Mow and co-workers proposed a linear biphasic model by assuming:

1. \( \lambda' = \mu' = 0 \) (i.e. the solid phase is linearly elastic);
2. \( \lambda_f = \mu_f = 0 \);
3. the permeability force coefficient \( b = 0 \);
4. \( K_c = 0 \);
5. \( \alpha = V_s/V_f = \text{constant} \).

Therefore, Eqs. (6.15), (6.16) and (6.19) can be simplified into

\[
\sigma' = -\alpha \rho I + \bar{\sigma}', \quad (6.22)
\]

and

\[
\pi' = -\pi' = -K(v^s - v^f), \quad (6.24)
\]

respectively. Here \( \bar{\sigma}' \) is the effective solid stress tensor and \( \bar{\sigma}' = \lambda'_s \text{tr}(e)I + 2\mu'_s e \).

By neglecting the inertia terms and body forces, Eqs. (6.13) and (6.14) can be simplified into

\[
\nabla' \cdot \sigma' + \pi' = 0, \quad (6.25a)
\]

and

\[
\nabla' \cdot \sigma^f + \pi^f = 0, \quad (6.25b)
\]

respectively. Eqs. (6.18) and (6.22-25) are the basic equations for the biphasic model.

6.2 Finite element formulation of biphasic poroelastic model

Spilker et al. [Spilker90] developed the finite element formulation corresponding to the linear biphasic poroelastic theory using the Galerkin weighted residual method. In this approach, a weighted residual statement is constructed using the momentum equations and traction boundary conditions (natural boundary conditions) for the solid phase and for the fluid phase, and the penalty method is used to the continuity equation, and thereby eliminate the pressure as an independent field variable.

Consider the domain \( \Omega \) with the boundary \( \Gamma \) to have total volume \( V \) with a fluid volume \( V_f \) and solid volume \( V_s \), the boundary conditions [Spilker90] are given as

\[
u' = \bar{u}' \quad \text{on} \quad \Gamma_{u}', \quad (6.26a)
\]

\[
\nu_f = \bar{v}' \quad \text{on} \quad \Gamma_{\nu}', \quad (6.26b)
\]

\[
\sigma' = \bar{\sigma}' \quad \text{on} \quad \Gamma_{\sigma}', \quad (6.26c)
\]

\[
p = \bar{p} \quad \text{on} \quad \Gamma_{p}', \quad (6.26d)
\]

where \( \mathbf{u}' \) is the vector of solid displacement components, \( \Gamma_{u}' \), \( \Gamma_{\nu}' \), \( \Gamma_{\sigma}' \), and \( \Gamma_{p}' \) are the portions of the boundary, \( \Gamma \), on which solid displacements, fluid velocities, solid tractions, and fluid pressures, respectively, are prescribed as \( \bar{u}' \), \( \bar{v}' \), \( \bar{\sigma}' \) and \( \bar{p} \).

Let \( \mathbf{w}' \), \( \mathbf{h}' \), \( \mathbf{w}' \) and \( \mathbf{h}' \) be the arbitrary admissible weighting functions for the solid and fluid phases, which are \( C^0 \) continuously over \( \Omega \) and zero on \( \Gamma_{u}' \) and \( \Gamma_{\nu}' \), respectively (i.e. satisfy boundary conditions equivalent to the homogeneous part of the essential boundary conditions).
The momentum equation and natural boundary conditions [Spilker90] are introduced into the weighted residual statement to give:

$$\int_\Omega \mathbf{w} \cdot (\nabla \mathbf{\sigma} + \mathbf{p} \nabla \mathbf{p}) d\Omega + \int_{\partial \Omega} \mathbf{h} \cdot (\mathbf{\sigma} - \mathbf{\sigma}^r) d\Gamma = 0$$  \hspace{1cm} (6.27a)

for solid phase, and

$$\int_\Omega \mathbf{w}^f \cdot (\nabla \mathbf{\sigma}^f + \mathbf{p} \nabla \mathbf{p}) d\Omega + \int_{\partial \Omega} \mathbf{h}^f \cdot \mathbf{n} (\mathbf{\bar{p}}^f - \mathbf{p}^r) d\Gamma = 0$$  \hspace{1cm} (6.27b)

for fluid phase, where \( \mathbf{n} \) is an outward normal to the boundary.

The continuity equation (6.18) is introduced via the penalty method in which the pressure, \( \mathbf{p} \), is related to solid and fluid velocities by the equation

$$\nabla \cdot (\mathbf{\phi}^s \mathbf{v}^s + \mathbf{\phi}^f \mathbf{v}^f) + \frac{\mathbf{p}}{\mathbf{\beta}} = 0$$  \hspace{1cm} (6.28a)

or

$$\mathbf{p} = -\mathbf{\beta} \nabla \cdot (\mathbf{\phi}^s \mathbf{v}^s + \mathbf{\phi}^f \mathbf{v}^f),$$  \hspace{1cm} (6.28b)

where \( \mathbf{\beta} \) is the penalty parameter.

Substitute Eq. (6.28) into Eqs. (6.22) and (6.23), and the resulting equations and Eq. (6.24) are substituted into Eq. (6.27). After applying the divergence theorem, the weighting function \( \mathbf{h}^s \) can be related to \( \mathbf{w}^s \), and \( \mathbf{h}^f \) can be related to \( \mathbf{w}^f \), producing the weak form of the weighted residual statements for the solid and fluid phases. The finite element weak form is obtained by subdividing the continuum into elements of domain \( \Omega_n \) and boundary \( \Gamma_n \). Assume that the field variables in the weak forms are \( C^0 \) continuous, the finite element weak form of the weighted residual statements may be expressed [Spilker90] as follows:

$$\sum_n \left\{ \int_\Omega (\nabla \mathbf{w}^s)^T \cdot [\mathbf{\phi}^s \nabla \cdot (\mathbf{\phi}^f \mathbf{v}^f + \mathbf{\phi}^f \mathbf{v}^f) + \lambda_n \mathbf{e}^s \mathbf{1} + 2\mu_n \mathbf{e}^s \mathbf{1}] d\Omega + \int_{\partial \Omega} \mathbf{h}^s \cdot \mathbf{n} d\Gamma \right\} = 0$$  \hspace{1cm} (6.29a)

and

$$\sum_n \left\{ \int_\Omega (\nabla \mathbf{w}^f)^T \cdot [\mathbf{\phi}^f \nabla \cdot (\mathbf{\phi}^f \mathbf{v}^f + \mathbf{\phi}^f \mathbf{v}^f) + \lambda_n \mathbf{e}^f \mathbf{1}] d\Omega - \int_{\partial \Omega} \mathbf{h}^f \cdot \mathbf{n} d\Gamma \right\} = 0$$  \hspace{1cm} (6.29b)

where \( A : B = \text{trace}(A \cdot B^T) \).

Within a typical element, the solid phase displacement and fluid phase velocity are interpolated in terms of corresponding set of nodal values in the form

$$\mathbf{u}^s = \mathbf{N}^s \mathbf{d}^s, \mathbf{v}^s = \mathbf{N}^s \mathbf{v}^s, \mathbf{v}^f = \mathbf{N}^f \mathbf{v}^f,$$  \hspace{1cm} (6.30a)

where \( \mathbf{d}^s, \mathbf{v}^s, \) and \( \mathbf{v}^f \) are nodal displacements and velocities, respectively, for the typical element \( n \), and \( \mathbf{N}^s \) and \( \mathbf{N}^f \) are matrices of \( C^0 \) interpolation functions. For our applications, we utilize the same order of approximation for the solid and fluid phases. Thus we assume that \( \mathbf{N}^s = \mathbf{N}^f = \mathbf{N} \) in Eq. (6.30a). In the Galerkin method, the weighted functions are defined in terms of the same interpolation functions, so that

$$\mathbf{W}^s = \mathbf{Nw}^s \text{ and } \mathbf{W}^f = \mathbf{Nw}^f,$$  \hspace{1cm} (6.30b)
where \( w_i^f \) are arbitrary coefficients of the weighting functions for the elements. By applying the linear strain-displacement relations to the displacement interpolations, the strains of the solid phase, \( \varepsilon^s \), and the dilatation, \( \varepsilon^d \), can be related to element nodal displacements \( d_n \) in the form

\[
\varepsilon^s = B d_n^s \quad \text{and} \quad \varepsilon^d = m \varepsilon^s = m B d_n^s,
\]

where \( m \) is a Kronecker delta operator. Define \( d_n, v_n \) and \( w_n \) as

\[
d_n = \begin{bmatrix} d_n^s \\ d_n^f \end{bmatrix}, \quad v_n = \begin{bmatrix} v_n^s \\ v_n^f \end{bmatrix}, \quad w_n = \begin{bmatrix} w_n^s \\ w_n^f \end{bmatrix}.
\]

Substitution of Eq. (6.30) into Eq. (6.29), the expression \([\text{Spilker90}]\) is obtained as

\[
\sum_n w_n^T \left[ \beta c_n^1 + c_n^2 \right] v_n + \sum_n w_n^T \begin{bmatrix} k_n & 0 \\ 0 & 0 \end{bmatrix} d_n = \sum_n w_n^T \begin{bmatrix} f_n^s \\ f_n^f \end{bmatrix},
\]

where the element matrices are defined by

\[
c_n^1 = \begin{bmatrix} (\phi^s)^2 c^3 & \phi^s \phi^s c^2 \\ \phi^s \phi^s c^3 & (\phi^s)^2 c^3 \end{bmatrix},
\]

\[
c_n^2 = \begin{bmatrix} c^4 & -c^4 \\ -c^4 & c^4 \end{bmatrix},
\]

\[
c_n^3 = \int_{\Omega_n} B^T D_s B d\Omega,
\]

\[
c_n^4 = \int_{\Omega_n} \mathbf{k}^T \mathbf{N} d\Omega,
\]

\[
k_n = \int_{\Omega_n} \left( \lambda \mathbf{B}^T \mathbf{D}_s \mathbf{B} + \mu \mathbf{D}_s \mathbf{B} \right) d\Omega,
\]

\[
f_n^s = \int_{\Gamma_n} \mathbf{N}^T \mathbf{\sigma}^s d\Gamma,
\]

\[
f_n^f = \int_{\Gamma_n} \mathbf{N}^T \mathbf{\phi}^f \mathbf{p} d\Gamma,
\]

\[
D_1 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

and

\[
D_2 = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.
\]
Standard assembly operations can be performed to sum up the element contributions. The element degrees of freedom are related to the continuum degrees of freedom using a Boolean matrix $A_n$ in the form

$$d_n = A_n d, \quad v_n = A_n v, \quad w_n = A_n w,$$

where $d$, $v$ and $w$ are the continuum displacement, velocity and weighting vectors, respectively. For the arbitrary $w \neq 0$, this produces the following system of coupled first order differential equations for the complete biphasic continuum [Spilker90]

$$Cv + Kd = F,$$

where $C = [\beta C^1 + C^2]$, and $C^1$, $C^2$, $K$ and $F$ are the assembled matrices for the continuum.

### 6.3 Biphasic poroviscoelastic model

Compared to biphasic poroelastic model, the biphasic poroviscoelastic model considers the contribution of both the frictional interaction between the solid and fluid phases (fluid flow-dependent) and the intrinsic viscoelastic behavior of the solid phase to the viscoelastic behavior of the tissue (fluid flow-independent). Based on the biphasic poroelastic model, the biphasic poroviscoelastic model incorporates an integral type linear viscoelastic representation into the effective solid stress term [Mak86a, Mak86b, Suh98, Suh99] as

$$\tilde{\sigma} = B_1 \left[ \int_0^\infty G(t-\tau) \frac{\partial}{\partial \tau} \text{tr}(e) d\tau \right] + 2\mu \int_0^\infty G(t-\tau) \frac{\partial e^s}{\partial \tau} d\tau,$$

where $B_1$ and $\mu$ represent the intrinsic elastic bulk modulus and the intrinsic elastic shear modulus, respectively, of the solid phase; $e^s$ represents the deviator strain tensor of the solid phase, i.e., $e^s = e - \text{trace}(e)I$, where $e$ is the strain tensor of the solid phase.

Assume that both bulk and deviatoric terms are equally governed by a single relaxation function $G(t)$, which can be expressed in terms of a continuous relaxation spectrum $S(\tau)$ as [Suh98]

$$G(t) = 1 + \int_0^\infty S(\tau)e^{-\mu_\tau} d\tau,$$

where

$$S(\tau) = \begin{cases} \frac{c}{\tau} & \text{for } \tau_1 \leq \tau \leq \tau_2 \\ 0 & \text{for } \tau < \tau_1, \tau > \tau_2 \end{cases} \quad (6.38)$$

Here, $c$ is the magnitude of the relaxation power spectrum, and $\tau_1$ and $\tau_2$ are short-term and long-term relaxation time constants, respectively. This is called continuous spectrum model.

In another way, the reduced relaxation function [Suh98] can be written as a series of combinations of the discrete relaxation functions such as

$$G(t) = G_\infty + \sum_{i=0}^{N} G_i e^{-\mu_i},$$

where $G_\infty = G(\infty)$; $G_i$ is the discrete moduli; $\tau_i$ is the discrete relaxation time. This is discrete spectrum model.
The finite element formulation of the biphasic poroviscoelastic \cite{Suh98, Suh99} was extended from Eq. (6.35) as

\[ Cv(t) + Kd(t) = F(t) \]  

(6.40)

where

\[ d(t) = \int G(t - \tau)v(\tau)d\tau. \]  

(6.41)

In case of the continuous spectrum relaxation function, the Simpson rule and a first-order approximation of velocity \cite{Suh98} can be used to express Eq. (6.41) at time \( t + \Delta t \) as

\[ d(t + \Delta t) = \int G(t + \Delta t - \tau)v(\tau)d\tau + \alpha_1 v(t) + \alpha_2 \Delta v, \]  

(6.42)

where

\[ \alpha_1 = \frac{\Delta t}{6n} \left\{ G(0) - G(t) + 2 \sum_{i=1}^{n} G \left( \frac{n-i+1}{n} \Delta t \right) + 4 \sum_{i=1}^{n} G \left( \frac{2n-2i+1}{2n} \Delta t \right) \right\}, \]

\[ \alpha_2 = \frac{\Delta t}{6n} \left\{ G(0) + \frac{1}{n} \left[ \sum_{i=1}^{n} (2i-1) G \left( \frac{n-i+1}{n} \Delta t \right) + 2 \sum_{i=1}^{n} (2i-1) G \left( \frac{2n-2i+1}{2n} \Delta t \right) \right] \right\}. \]

The predictor-corrector iteration algorithm \cite{Suh98} can then be obtained as follows:

**Predicator:**

\[ \hat{v}(t + \Delta t) = v(t) \]  

(6.43)

\[ \hat{d}(t + \Delta t) = \int G(t + \Delta t - \tau)v(\tau)d\tau + \alpha_1 v(t) \]  

(6.44)

**Solver:**

\[ (C + \alpha_2 K) \Delta v = F(t + \Delta t) - C\hat{v}(t + \Delta t) - K\hat{d}(t + \Delta t) \]  

(6.45)

**Corrector:**

\[ v(t + \Delta t) = \hat{v}(t + \Delta t) + \Delta v \]  

(6.46)

\[ d(t + \Delta t) = \hat{d}(t + \Delta t) + \alpha_2 \Delta v \]  

(6.47)

When the discrete spectrum representation of the relaxation function, Eq. (6.39), is used, Eq. (6.41) can be written as \cite{Suh98}

\[ d(t) = G_d(t) + \sum_{i=0}^{N_d} G_i d_i(t), \]  

(6.48)

where \( d(t) = \int v(\tau)d\tau \) and \( d_i(t) = \int v(\tau)e^{-(t-\tau)\zeta_i}d\tau \).

Eq. (6.38) at time \( t + \Delta t \) can be written as \cite{Suh98}

\[ d(t + \Delta t) = G_d(t) + \sum_{i=0}^{N_i} G_i d_i(t)e^{-\zeta_i t} \]

\[ + \{ G_d + \sum_{i=0}^{N_i} G_i [(1 - \zeta) e^{-\zeta_i t} + \zeta_i] \} v(t) \Delta t + \zeta G(0) \Delta v \Delta t, \]  

(6.49)

where \( \zeta \) denotes the integration parameter of the modified trapezoidal integral rule, which can be assigned a value in the range of \( 0 \leq \zeta \leq 1 \). Eq. (6.44) of the precious predicator-corrector iteration algorithm can then be replaced by a recursive algorithm \cite{Suh98} as follows:
\[ \dot{d}(t + \Delta t) = G_{ae}[\dot{d}(t) + v(t)\Delta t] + \sum_{j=0}^{N_d} G_{rj}(t) e^{-\Delta t/r_j} + v(t)\Delta t[(1 - \zeta) e^{-\Delta t/r_{s}} + \zeta]. \] (6.50)

This formula allows the hereditary integral to be calculated simply by using values only from the time step immediately previous to the current time step and thereby significantly saves computing time and memory.

Suh and Bai [Suh98] pointed out that the continuous spectrum model required extensive function time and CPU memory; the discrete spectrum model could reduce the computational time and CPU memory considerably; the two models provided very similar results in predicting the overall tissue behavior, except the discrepancy in predicting the creep and the stress relaxation of articular cartilage due to the slight difference in defining their respective relaxation functions.

Miller [Miller98] pointed out that biphasic poroelastic model was not suitable for the unconfined compression experiment because both phases moved in a similar way so that the relative velocity of the phases was close to zero, and the dependence on loading velocity could not be reflected in the model. Suh and DiSilvestro [Suh99] found that the biphasic poroviscoelastic model could provide an excellent prediction of the short-term as well as the long-term stress relaxation response of the unconfined compression experiment of articular cartilage, however, the biphasic poroelastic model could only provide an excellent prediction of the slow relaxation characteristic of the long-term response of the experiment. Suh and DiSilvestro [Suh99] concluded that, while the long-term viscoelastic response of soft connective tissues was mainly governed by the fluid flow-dependent biphasic viscoelasticity, the short-term viscoelastic response was primarily associated with the fluid flow-independent intrinsic viscoelasticity of the tissue matrix; the fluid flow-independent viscoelasticic features of the solid matrix needed to be taken into consideration, especially when the tissue underwent a considerable deviatoric deformation.

Suh and DiSilvestro [Suh99] pointed out that some limitations associated with the biphasic poroviscoelastic model assumptions of the present study deserved further discussion. First, the constitutive model was based on the assumption of linearity in the range of infinitesimal deformations. The assumption of linearity had been considered acceptable for similar strain range in previous studies, although finite deformation models had shown to improve model simulations in many cases. Nonlinear viscoelastic modeling including large deformation would be investigated in future studies. Secondly, the present model also assumed the isotropic, homogenous material properties of hydrated biological tissue. Most of hydrated biological soft tissues, such as muscle, tendon, and ligament, had a highly organized fibrous structure in parallel to the principle loading direction. Finally, the large number of the biphasic poroviscoelastic model parameters could cause a technical difficulty in obtaining unique parameter identification from a single experimental data. While each parameter held its distinctive physical interpretation, the overall viscoelastic behavior of soft tissue most likely represented a complex coupling phenomenon of all of these physical parameters. Therefore, a more accurate parameter identification of the biphasic poroviscoelastic model might require a combination of several independently distinctive experiments, each of which was mostly representative of each model parameter.
7. Summary

The experimental results demonstrated that the mechanical behavior of spinal cord under axial tensile loading was characterized by a highly nonlinear response and significant stress relaxation. The linear viscoelastic, nonlinear viscoelastic, and quasi-linear viscoelastic models were developed to analyze the mechanical behavior of spinal cord. It was shown that the quasi-linear viscoelastic model could fit the mechanical behavior of the spinal cord adequately.

Considering the similarity between the brain tissue and the spinal cord, the models describing the mechanical properties of the brain tissue are described, which consist of the hyperelastic, nonlinear viscoelastic and hyperelastic, linear viscoelastic models based on the strain energy function. The hyperelastic, linear viscoelastic model is preferred because there are a number of advantages compared to the hyperelastic, nonlinear viscoelastic model. The main advantage is that it can be immediately applied to larger scale finite element computations by directly using ABAQUS software package. Compared to the biphasic theory, it can be used for larger deformation criteria.

Another important model that can be extended to analyze the mechanical properties of the spinal cord is the biphasic, poroviscoelastic model, which is based on the biphasic theory. This model exactly characterizes the deformation behavior of the hydrated biological soft tissue and it provides a way to understand how the interaction of two phases inside the tissue results in the deformation. The biphasic theory considers the hydrated biological soft tissue composed of two phases, one is the incompressible porous solid phase, another is the incompressible interstitial fluid phase. Using the weighted residual method with a penalty treatment of the continuity equation, the finite element formulation for the biphasic, poroviscoelastic model was developed. The disadvantage of this model is that it might be limited for the small deformation criteria. The brain tissue constitutive models and biphasic models are listed in Table 7.1.

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Conditions</th>
<th>Example materials</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hyperelastic, nonlinear viscoelastic model</td>
<td>Combination of hyperelastic model with nonlinear viscoelastic model</td>
<td>Isotropic materials, nonlinear stress-strain relationship without the change of time, large deformation, high strain-rate loading</td>
<td>Brain tissue</td>
<td>Mendis95</td>
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<tr>
<td>(Mendis's model)</td>
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<tr>
<td>Hyperelastic, nonlinear viscoelastic model</td>
<td>Combination of hyperelastic model with nonlinear viscoelastic model</td>
<td>Isotropic materials, nonlinear stress-strain relationship without the change of time, large deformation, low strain-rate loading</td>
<td>Brain tissue</td>
<td>Miller97</td>
</tr>
<tr>
<td>(Miller’s model)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hyperelastic, linear viscoelastic</td>
<td>Combination of hyperelastic model</td>
<td>Isotropic materials, nonlinear stress-strain relationship</td>
<td>Brain tissue, Liver and kidney</td>
<td>Miller99, Miller00a,</td>
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<tr>
<td>Model</td>
<td>Description</td>
<td>Constitutive Model</td>
<td>Reference</td>
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<tr>
<td>Biphasic poroelastic model</td>
<td>The tissue is modeled as a two phase immiscible mixture, consisting of intrinsically incompressible solid phase and an intrinsically fluid phase. The tissue is compressible through exudation of fluid phase.</td>
<td>Tissue composed of solid phase and fluid phase, incompressible solid and fluid phases, inviscid fluid</td>
<td>Articular cartilage, Skin Mow80, Mow85, Holmes85, Spilker90, Oomens87</td>
<td></td>
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<tr>
<td>Biphasic poroviscoelastic model</td>
<td>Combination of biphasic poroelastic model with viscoelastic model</td>
<td>Tissue composed of solid phase and fluid phase, incompressible solid and fluid phases, inviscid fluid, linear viscoelastic tissue</td>
<td>Articular cartilage, Brain tissue Suh98, Suh99, Zhu01</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1. Brain tissue constitutive models and biphasic models.
References


Lai80 W. M. Lai, and V. C. Mow, "Drag induced compression of articular cartilage during a penetration experiment", Biochimica et Biophysica Acta, Vol. 17, 1980, pp. 11-123


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