Unsteady State Analysis Of A Hermetic Reciprocating Compressor: Heat Transfer Inside the Cylinder And Valve Dynamics

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ABSTRACT
In this work, an unsteady state analysis of the compression cycle of a small hermetic reciprocating compressor for domestic refrigeration was carried out. The mass and energy balances were applied to the refrigerant inside the cylinder in order to determine the mass and temperature behaviour through the compression process. A specific one-dimensional model of the valves was developed to calculate the mass flow rates that were processed. The compression cycle unsteady state analysis was inserted into a traditional steady state model of the compressor to evaluate the combined effect of the heat transfer inside the cylinder and the valve dynamics on compressor performance and efficiency. The whole simulation code was validated against the experimental measurements carried out on an R134a commercial unit in a wide range of operative conditions.

INTRODUCTION
The performance of a small hermetic reciprocating compressor is greatly affected by the heat transfer inside the cylinder during the compression cycle and by the valve dynamics.

The evaluation of the heat transfer between the refrigerant and the cylinder wall requires an unsteady state analysis of the compression cycle as suggested by Todescat et al. 1992. This analysis may be complicated by the phase lag between the heat transfer and the temperature difference which is due to the simultaneous heat and work transfer as illustrated by Fagotti et al. 1998.

The study of valve dynamics involves the evaluation of the mass and stiffness parameters and the analysis of the unsteady state flow through the valves. The most relevant unsteady state effects in refrigerant flow through the valves are those of the inertia of the refrigerant stream, which produces a time delay between the pressure variation and the relative velocity change, and the work transfer between the refrigerant flow and the valve plate. The latter modifies the kinetic energy and, hence, the velocity of the refrigerant flow, as shown by Böswirth 1984 and 1990.

In this work, an unsteady state analysis of the compression cycle was inserted into a traditional steady state model of the compressor (Cavallini et al. 1996) in order to evaluate the combined effect of the heat transfer inside the cylinder and the valve dynamics on compressor performance and efficiency.

THEORETICAL MODEL AND COMPUTER CODE
The basic assumption behind the model was to consider the refrigerant flow through the compressor, except for the valves and inside the cylinder, as a one-dimensional steady state current. In this way it is possible to establish a steady state thermal balance to compute the temperatures and the heat and work flow rates for each component and for the overall system. The compressor was subdivided into six parts (shell, body, suction muffler, suction chamber, discharge chamber, discharge line) (see figure 1) and, for each of them, the mass and energy balances were established. The main irreversibilities inside the compressor (electric energy conversion losses, friction losses) were taken into account by suitable electrical and mechanical efficiencies.

The compression cycle, the refrigerant flow through the valves and the valves dynamics were analysed by an unsteady state approach. This allows the direct computation of the heat and work flow rates exchanged together with the mass flow rate processed, as well as also the behaviour of the main characteristic parameters during the compression cycle.
The behaviour of the refrigerant mass $M$ inside the cylinder was derived from the following mass balance:

$$\frac{dM}{d\tau} = m_s(\tau) - m_d(\tau) - m_l(\tau)$$  \hfill (1)

where $m_s$, $m_d$ and $m_l$ are the suction, discharge and leakage mass flow rate and $\tau$ is time.

The leakage mass flow rate was computed using the Jacobs 1976 model

$$m_l(\tau) = \rho(\tau) \pi D h^3 \Delta p(\tau) / [8 \mu(\tau) L]$$  \hfill (2)

where $\rho$ and $\mu$ are the density and dynamic viscosity of the refrigerant, $D$ is the cylinder diameter, $h$ piston to cylinder radial clearance, $L$ piston height and $\Delta p$ pressure difference between cylinder and shell.

The suction and discharge mass flow rates were computed using the following equation:

$$m_i(\tau) = \rho_i(\tau) A_i(\tau) u_i(\tau) \quad i = s, d$$  \hfill (3)

where $A_i$ and is the effective flow area and $u_i$ the effective refrigerant flow velocities in the suction and discharge valves (see figure 2).
The effective flow area was correlated to the valve plate lift $X_i(\tau)$ by the discharge coefficient $C_D$:

$$A_i(\tau) = \pi d_{si} C_D X_i(\tau) \quad i = s, d \quad (4)$$

where $d_{si}$ is the valve seat diameter. The discharge coefficient was computed in accordance with Böswirth 1982. The effective refrigerant velocity was correlated to the pressure losses through the valves by Böswirth 1984 and 1990 using two different relations derived from the Bernoulli equation:

$$\Delta p_i(\tau) = \frac{1}{2} \rho_i u_i^2(\tau) [1.5 + 0.03 d_{bi}/X_i(\tau)] \quad X(\tau) / d_{bi} \leq 0.15 \quad (5)$$

$$i = s, d$$

$$\Delta p_i(\tau) = \frac{1}{2} \rho_i u_i^2(\tau)/(1+\beta) + \rho_i J_i [u_i(\tau) X_i(\tau)]/d\tau + F_{PL_i}(\tau) d[X_i(\tau)/m_i(\tau)]/d\tau \quad X(\tau) / d_{bi} > 0.15 \quad (6)$$

where $d_{bi}$ is the thickness of the bearing surface of the valves, $\beta$ is a frictional coefficient which accounts for the inlet losses, $J_i$ is a geometrical parameter and $F_{PL_i}$ the fluidodynamic force on the valve plate. The first correlation (eq. 5), valid for the opening and closing phases, is the Bernoulli equation modified by a multiplicative term on the right hand side to account for the frictional losses. The second correlation (eq. 6), valid for open valve, is also obtained from the Bernoulli equation, corrected on the right side to account for the inlet frictional losses, the gas inertia effect and the unsteady state work transfer between the flow and valve plate. These two last terms represent the most significant unsteady state effects in refrigerant flow through the valves. The inertia of the refrigerant stream produces a time delay between the pressure variation and the relative velocity change, whereas the work transfer between the refrigerant flow and the valve plate modifies the kinetic energy and hence the velocity of the refrigerant flow. The interaction between the refrigerant and the valve plate was estimated using the Böswirth 1984 and 1990 model:

$$F_{PL_i}(\tau) = \Delta p_i(\tau) A_{si} \{1 + \frac{1}{2} A_{di}/A_{si} [1 - X_i(\tau)/(0.15 d_{bi})]\} \quad X_i(\tau) / d_{bi} \leq 0.15 \quad (7)$$

$$i = s, d$$

$$F_{PL_i}(\tau) = \frac{1}{2} \rho_i u_i^2(\tau) A_{pi} \quad X_i(\tau) / d_{bi} > 0.15 \quad (8)$$

where $A_{di}$ is the area of the bearing surface of the valves, $A_{si}$ is the area of the valve seat and $A_{pi}$ is the cross section area of the valve hole (port area). The first correlation (eq. 7) is valid during the opening and closing phases when the kinetic energy of the stream is negligible with regard to the frictional losses, whereas the second equation (eq. 8) is valid for the open valve when the kinetic energy of the flow is predominant.

The valves were simulated using a one-dimensional model governed by the following second order differential equation:

$$m_{vi} d^2 X_i(\tau)/d\tau^2 + b_{vi} dX_i(\tau)/d\tau + k_{vi} [X_i(\tau) - X_{pc,i}] = F_{PL_i}(\tau) \quad i = s, d \quad (9)$$

where $m_{vi}$ is the equivalent mass, $b_{vi}$ the damping and $k_{vi}$ the stiffness of the valve and $X_{pc,i}$ accounts for the preload.

The first law of thermodynamics was applied to the refrigerant inside the cylinder in order to determine its temperature behaviour through the compression process, as suggested by Todescat et al. 1992, in the form:

$$dT/d\tau = [1/M(\tau)c_v] \{q(\tau) + m_s(\tau)[h_s - h(\tau)] - T(\tau)(\partial p/\partial T)_v [dV/d\tau - V(\tau)[m_s(\tau) - m_l(\tau) - m_d(\tau)]\}] \quad (10)$$

where $T$, $p$, $v$, $h$ and $c_v$ are temperature, pressure, specific volume, specific enthalpy and specific heat capacity of the refrigerant inside the cylinder, while $q$ is the heat flow rate between the refrigerant and the cylinder wall and $V$ is the volume inside the cylinder. The refrigerant to cylinder wall heat transfer was computed using the equation of Annand et al. 1963 multiplied by a factor of three as suggested by Todescat et al. 1992:

$$q(\tau) = \alpha(\tau) A(\tau) [T_c - T(\tau)] \quad (11)$$
\[ \alpha(\tau) = 3 \times 0.7 \left[ \lambda(\tau) / D \right] \left[ \rho(\tau) u_p D / \mu(\tau) \right]^{0.7} \]  

(12)

where \( \alpha \) is the heat transfer coefficient, \( A \) is the heat transfer area, \( T_c \) is the cylinder wall temperature, \( u_p \) is the mean piston speed, \( \lambda \) is the thermal conductivity of the refrigerant.

The pressure behaviour during the compression cycle was computed from the temperature and specific volume behaviour by the equation of state of the refrigerant:

\[ p = f(T, v)_{R134a} \]  

(13)

The compression power was estimated by the following correlation:

\[ W(\tau) = \left[ p(\tau) - p_s \right] \frac{dV}{d\tau} \]  

(14)

where \( p_s \) is the suction pressure.

The whole unsteady state analysis of the compression cycle was represented by a system of nine equations in the variables temperature \( T \), pressure \( p \), mass \( M \), effective velocities through the valves \( u_t \) and \( u_{dt} \), valves plate lifts \( X_s \) and \( X_d \), interactions between the refrigerant flow and the valves plate \( F_{PLs} \) and \( F_{PLd} \). The system included two first order and two second order differential equations which were integrated using a finite difference approach.

The simulation computer code had an iterative structure. Input values were set for the boundary conditions (surroundings temperature, suction refrigerant temperature and pressure, discharge pressure) and the characteristic parameters of the hermetic unit (geometrical data, refrigerant, electric motor speed, electrical and mechanical efficiencies). Guess values were assumed for the refrigerant mass flow rate, refrigerant temperature at the inlet of the cylinder, for the temperature of the gas recirculated inside the shell and for the temperature of the cylinder. Then a thermal balance was established for each compressor component and the unsteady-state analysis of the compression cycle was carried out. The refrigerant properties was calculated using REFPROP 5.1. This way, the characteristic temperatures inside the compressor and along the refrigerant flow were computed together with the refrigerant mass flow rate, the heat flow rates and the electric power input. The guess temperatures and mass flow rate were compared to the calculated ones and further iterations were carried out until convergence was reached. The final output results included the refrigerant mass flow rate and outlet temperature, the temperatures inside the hermetic unit, the heat flow rate, the electric power input, and the trend of the characteristic parameters during the compression cycle.

**RESULTS AND COMPARISON WITH EXPERIMENTATION**

The theoretical model was validated against the experimental measurements carried out on a commercial unit, a R134a hermetic reciprocating compressor for domestic refrigerators with 6 cm³ swept volume.

The equivalent mass and stiffness for the valves of the commercial unit were obtained by a dynamic FEM analysis in ANSYS 5.5.1: the suction and discharge valves were simulated by a mesh consisting of 1601 and 1287 elements type SHELL 63 respectively. The FEM model was used to compute the first natural frequency \( \omega_{ni} \) and the stiffness \( k_{vi} \) of the valves: these parameters were used to estimate the equivalent mass by the following equation:

\[ m_{vi} = k_{vi} / \omega_{ni}^2 \]  

(15)

Table 1 gives the values computed for the suction and discharge valves.

<table>
<thead>
<tr>
<th></th>
<th>( k_{vi}[N/m] )</th>
<th>( \omega_{ni}[Hz] )</th>
<th>( m_{vi}[g] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suction valve</td>
<td>808</td>
<td>394</td>
<td>0.1318</td>
</tr>
<tr>
<td>Discharge valve</td>
<td>2783</td>
<td>528</td>
<td>0.259</td>
</tr>
</tbody>
</table>

**Table 1.** Stiffness, natural frequency and equivalent mass of the valves calculated by the FEM analysis.
Figure 3. Experimental measurement of the frequency response of the suction valve.

The FEM analysis was validated against the experimental measurement of the valves' frequency response, reported on figures 3 and 4, which show the first natural frequency at 377 Hz for the suction valve and at 517 Hz for the discharge valve. The fair agreement between the experimental and calculated natural frequencies allows a use of the calculated equivalent mass and stiffness with sufficient confidence.

Figure 4. Experimental measurement of the frequency response of the discharge valve.
Table 2. Comparison between experimental and calculated parameters.

<table>
<thead>
<tr>
<th>Input Data</th>
<th>$T_e = -35^\circ$C</th>
<th>$T_e = -23.3^\circ$C</th>
<th>$T_e = -10^\circ$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor inlet temperature (°C)</td>
<td>38</td>
<td>35.8</td>
<td>34.2</td>
</tr>
<tr>
<td>Compressor suction pressure (kPa)</td>
<td>66</td>
<td>114.8</td>
<td>200.5</td>
</tr>
<tr>
<td>Compressor discharge pressure (kPa)</td>
<td>1483</td>
<td>1484</td>
<td>1486</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Electric power input (W)</td>
<td>92.5</td>
<td>100.7</td>
<td>137.7</td>
<td>141.8</td>
<td>204.7</td>
<td>203.5</td>
</tr>
<tr>
<td>Refrigerant mass flow rate (kg/h)</td>
<td>1.316</td>
<td>1.122</td>
<td>3.016</td>
<td>2.779</td>
<td>6.009</td>
<td>5.601</td>
</tr>
<tr>
<td>Cylinder inlet temperature (°C)</td>
<td>97.3</td>
<td>98.6</td>
<td>98.9</td>
<td>97.9</td>
<td>94.9</td>
<td>96.1</td>
</tr>
<tr>
<td>Cylinder outlet temperature (°C)</td>
<td>150.6</td>
<td>144.9</td>
<td>156.7</td>
<td>152.5</td>
<td>145.5</td>
<td>147.5</td>
</tr>
<tr>
<td>Compressor outlet temperature (°C)</td>
<td>85.5</td>
<td>94.2</td>
<td>102.1</td>
<td>102.2</td>
<td>111.9</td>
<td>109.4</td>
</tr>
<tr>
<td>Recirculated gas temperature (°C)</td>
<td>86.2</td>
<td>85.4</td>
<td>93.3</td>
<td>90.0</td>
<td>92.7</td>
<td>92.9</td>
</tr>
<tr>
<td>Cylinder average temperature (°C)</td>
<td>100.1</td>
<td>105.6</td>
<td>109.7</td>
<td>109.2</td>
<td>110.9</td>
<td>111.6</td>
</tr>
</tbody>
</table>

The commercial unit was tested in a calorimetry rig for the measurement of the refrigerating capacity in accordance with the ASHRAE Standard. During each test, the following parameters were measured: electric power input, refrigerating capacity, evaporation and condensation pressures, gas temperature at inlet and at outlet of the compressor, refrigerant temperature at outlet of the condenser and at outlet of the evaporator. The compressor was also equipped with several copper-constantan thermocouples to measure the temperatures of its components and of the refrigerant in different positions. Table 2 gives the comparison between the experimental data measured and the values calculated by the simulation program under the same operative conditions. Three different operative conditions were considered in accordance with the ASHRAE standard: evaporation temperatures -35°C, -23.3°C and -10 °C, respectively. The comparison shows a fair agreement between the calculated and experimental temperatures inside the compressor, whereas the result is less satisfactory for electric power and refrigerant mass flow rate.

The output of the simulation model also includes the characteristic parameters' trend during the compression cycle. Figure 5 shows the compression cycle on the pV diagram under the different operative conditions studied: it is possible to observe the compression phase, the discharge phase with the characteristic pressure fluctuation, the re-expansion of the refrigerant in the clearance volume and, finally, the suction phase.

![Figure 5. Compression cycle on the pV diagram.](image-url)
Figure 6. Refrigerant temperature behaviour during the compression cycle.

Figure 6 shows the temperature trend: during the compression phase, the refrigerant temperature increases until the opening of the discharge valve, when the temperature starts to decrease due to the heat exchange with the cylinder wall. The temperature decrease continues during the whole discharge and re-expansion phases until the opening of the suction valve, when the temperature again increases due to the mixing with the new entering refrigerant. Figures 7 and 8 show the change of heat and work flow rates under the different operative conditions studied. The heat transfer rate from the refrigerant to the cylinder wall increases during the compression phase due to the enhancement in the heat transfer coefficient and temperature difference between refrigerant and cylinder wall.

Figure 7. Heat transfer flow rate between refrigerant and cylinder wall during the compression cycle.
Then it rapidly decreases in the discharge and re-expansion phases until it reverses, from cylinder to gas, in the suction phase. The compression power increases, in absolute value, during the compression phase due to the increased differential pressure on the piston; then, it decreases in the discharge phase reaching zero at the top dead centre (no volume variation), then becoming positive during the re-expansion phase. The subsequent suction phase involves a small power input.

CONCLUSIONS

In this work a computer code for the unsteady state analysis of a small hermetic reciprocating compressor for domestic refrigeration was presented. The model was able to evaluate the refrigerant mass flow rate, the electric power input, the heat flow rates, the temperatures inside the hermetic unit and the characteristic parameters' trend during the compression cycle. The computer code was validated against the experimental measurements carried out on a R134a commercial unit in a wide range of operative conditions: a fair agreement has been found between predicted and measured performance. The model can be easily adapted to different compressor geometry and different operative fluids; therefore, it can be a useful tool for design and development purposes.

REFERENCES


