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ASPECTS OF TWO-PHASE FLOW SCREW COMPRESSOR MODELLING
PART II: FRICTION BETWEEN ROTORS

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ABSTRACT

A model for friction between the rotors of a two-phase ammonia-water twin screw compressor is presented. The model calculates the torque from gas on each rotor, the contact force between the rotors and the liquid film thickness. It is considered that the contact force depends on the male rotor turning angle and on the axial coordinate, thus, is not constant along the contact line. Results of the film thickness calculation have shown that the process takes place in the boundary lubrication regime when the working mixture of ammonia and water is used for lubrication. Friction coefficient values for boundary lubrication conditions presented in the literature are used for friction force calculation. Impact of the rotor friction on the compressor isentropic efficiency is quantified.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>force, N</td>
</tr>
<tr>
<td>G</td>
<td>geometrical function, Eq.(9), m³</td>
</tr>
<tr>
<td>G̅</td>
<td>geometrical function, Eq.(10), m³</td>
</tr>
<tr>
<td>h</td>
<td>axial screw pitch, m</td>
</tr>
<tr>
<td>hₘᵢₙ</td>
<td>minimal film thickness, m</td>
</tr>
<tr>
<td>l</td>
<td>length of the power transmission section of the contact line, m</td>
</tr>
<tr>
<td>p</td>
<td>pressure, Pa</td>
</tr>
<tr>
<td>r</td>
<td>distance from axis to contact point, m</td>
</tr>
<tr>
<td>T</td>
<td>torque, N·m</td>
</tr>
<tr>
<td>w</td>
<td>contact load, N/m</td>
</tr>
<tr>
<td>z</td>
<td>axial coordinate, m</td>
</tr>
<tr>
<td>β</td>
<td>helix angle</td>
</tr>
<tr>
<td>ε</td>
<td>angle between radius and touching line at contact point</td>
</tr>
<tr>
<td>μ</td>
<td>coefficient of friction</td>
</tr>
<tr>
<td>ηₛ</td>
<td>isentropic efficiency</td>
</tr>
<tr>
<td>ϕ</td>
<td>rotor's turning angle</td>
</tr>
</tbody>
</table>

Subscripts

1 - male rotor
2 - female rotor
a - axial component
c - contact
f - friction
g - gas
n - normal in the xy - plane
r - radial component
t - tangential component

INTRODUCTION

A twin screw compressor is being developed for application in compression/resorption heat pumps operating with ammonia-water mixture. According to the requirements of the heat pump thermodynamic cycle the compression must be performed in the two-phase region. It was mentioned in the parallel paper (Zaytsev and Infante Ferreira [2000]) that since separation of oil from the wet working fluid will be difficult to realise, the compression has to be oil-free. One of the conclusions was that, due to the low viscosity of NH₃/H₂O mixtures, the losses from friction between the rotor tip and housing are negligible. At the same time, the compressor has to be lubricated by this low-viscous working mixture and substantial losses from rotor contact friction are expected.

A model for calculation of losses due to friction between the rotors is the subject of this paper. To obtain the friction force between the rotors, the contact force and the friction coefficient values must be known. For the male rotor driven compressor the contact force between the rotors can be found from the female rotor torque balance: the
torque from gas pressure on the female rotor is equal to the torque from the rotor contact force. The direction of the contact force is normal to the rotor surface at every contact point. The absolute value of the contact force depends on the rotor turning angle and on the axial coordinate, thus, is not constant along the contact line. The model proposed here takes this fact into account.

The paper will start with a brief description of the method for calculation of the torque from gas pressure. Then the contact force between the rotors will be derived from the balance of torques. Further the lubrication regime will be determined by means of lubricant film thickness calculation. Based on the calculated contact force and friction coefficient values for boundary lubrication presented in the literature, the friction losses will be calculated. Finally the decrease in the compressor isentropic efficiency due to friction will be quantified.

THE TORQUE FROM GAS PRESSURE

The gas pressure induces a torque on each rotor of the twin screw compressor. The torque value depends on the rotation angle and varies periodically. The torque can be calculated with the slice method proposed by Rinder [1979] and described in English by You et al [1995] and Stošić et al [1998].

According to the slice method the screw rotors are divided in thin slices by a set of planes normal to the rotor axes. Each slice has the same geometry but different angular position relative to the rotor axis. Each slice has infinitely small thickness $dz$. The torque from gas is integrated first for one slice and then the contributions of all other slices are summated. This procedure is performed for one cavity to obtain the torque inducted by gas enclosed in the cavity. Since the angular shift between the cavities is known, the gas torque for the whole rotor can be obtained by summation of the effects of each cavity.

CONTACT FORCE CALCULATION METHOD

The torque from gas pressure on the female rotor is balanced by the torque arising from the rotor contact force. Direction of the torque transfer through the contact line is determined by the sign of the female rotor gas torque. If the torque induced by gas on the female rotor is positive (against the direction of rotation), the compensation torque is transferred from the male to the female rotor through the contact line. Otherwise, if the torque from gas on the female rotor is negative (in the direction of rotation), this excessive torque is transferred through the contact line from the female to the male rotor.

In this study the torque transfer from the female to the male rotor is considered. The contact force between the rotors is normal to the rotors' surface at the contact point. It can be expressed as a vector sum of three components: the tangential, the radial and the axial force component as shown in Fig. 1.

\[
dF_c^2 = dF^2 + dF_r^2 + dF_a^2 = dF_{a2}^2 + dF_{a2}^2.
\]

From the vector triangles (Fig. 1) one can obtain

\[
dF_c^2 = dF_{r2}^2 \left( \cos^2 \beta_2 + \tan^2 \beta_2 \right).
\]

Angle $\beta_2$ is the helix angle of the female rotor at radius $r_2$ and it can be calculated as

\[
\tan \beta_2 = 2\pi r_2 / h_2,
\]

where $h_2$ is the axial pitch length of the female rotor.

The torque transferred per slice due to the contact force is

\[
1 \text{ In this paper under term "gas" is understood the compressor working medium, i.e. ammonia-water mixture.}
\]
Substituting the tangential force component from Eq. (4) into Eq. (2) the relation for the contact force within one slice is obtained

\[ dT_{c2} = dF_{c2}r_2. \] (4)

Both \( dF_c \) and \( dT_{c2} \) are functions of the rotational angle and the axial coordinate. Neither \( dF_c \) nor \( dT_{c2} \) are known. Known is only that the torque from gas pressure on the whole female rotor is equal by absolute value to the torque from the contact force. To integrate Eq. (5) the contact force (load) distribution along the contact line must be known.

The proposed way to find the contact force distribution is based on the following considerations. The torque from gas tends to drive the female rotor, so that the rotor makes an infinitesimal rotation \( d\phi_2 \), which results in deformation of the male rotor. Assumption of the method is that the contact load \(^1\) is proportional to the deformation.

The direction of deformation, as well of contact force, coincides with the common normal to the rotors' surfaces at the contact point. As the female rotor turns on angle \( d\phi_2 \), the contact point C moves to point T (Fig. 1). The displacement is

\[ CT = r_2d\phi_2. \]

---

\(^1\) The contact load is defined as contact force per unit of contact line length \( w = dF/dl \)

---

Fig. 1. a) Contact force components
b) Deformation components
The normal deformation \( CN \) is the projection of \( CT \) on the normal to the rotor surface and can be found from the rectangular parallelepiped shown in Fig. 1b.

\[
CN = r_2 d\varphi_2 \cos \left( \arctan \sqrt{\tan^2 \varepsilon_2 + \tan^2 \beta_2} \right)
\]

and according to the assumption of the method, the normal contact load for one slice is

\[
w_c = dF_c / dl_c = k_2 r_2 d\varphi_2 \cos \left( \arctan \sqrt{\tan^2 \varepsilon_2 + \tan^2 \beta_2} \right),
\]

where \( k_2 \) is an unknown constant.

Since at an arbitrary rotation angle the infinitesimal rotation \( d\varphi_2 \) is the same for any female rotor slice (the rotor is assumed to be stiff), the term \( d\varphi_2 \) can be included in the constant

\[
k_2 = \tilde{k}_2 d\varphi_2,
\]

which yields the relation for the contact force within one slice

\[
dF_c = k_2 r_2 \cos \left( \arctan \sqrt{\tan^2 \varepsilon_2 + \tan^2 \beta_2} \right) dl_c
\]

(8)

After assigning Eq. (5) to (8) the torque from rotor contact force can be integrated for one cavity

\[
T_{cavity,c2} = k_2 \int_0^{l_{cavity,c2}} r_2^2 \cos \left( \arctan \sqrt{\tan^2 \varepsilon_2 + \tan^2 \beta_2} \right) \frac{dl_c}{\sqrt{1/\cos^2 \varepsilon_2 + \tan^2 \beta_2}} = k_2 G(\varphi_1).
\]

(9)

where \( l_{cavity,c} \) is the length of the power transmission section of the contact line within one cavity. The integral in Eq. (9), denoted as \( G(\varphi) \), is a rotation angle dependent function of the compressor geometrical parameters.

Summation of the torques from the rotor contact force for all cavities yields

\[
T_{rotor,c2} = k_2 \left( \frac{2\pi}{m} \sum_{i=0}^{\varphi_1 - \varphi_{\text{min}}} G\left( \varphi_1 - \frac{2\pi}{m} i \right) + \frac{2\pi}{m} \sum_{i=1}^{\varphi_1 - \varphi_{\text{max}}} G\left( \varphi_1 + \frac{2\pi}{m} i \right) \right) = k_2 \tilde{G}(\varphi_1),
\]

(10)

where \( \varphi_{\text{min}} \) denotes the angle at which the formation of the new cavity contact line starts and \( \varphi_{\text{max}} \) denotes the angle at which the cavity contact line length decreases to zero.

The torque balance for the female rotor is written as

\[
T_{rotor,g2} = -T_{rotor,c2}.
\]

(11)

By substitution of Eq. (11) into Eq. (10) we obtain the value of \( k_2 \)

\[
k_2 = -T_{rotor,g2} / G(\varphi_1).
\]

(12)
Finally, the contact force for a slice is determined from Equations (8) and (12) as a function of the rotation angle and axial coordinate

\[ dF_c = \frac{-T_{\text{rotor},g_2}^{2}(\phi_1)}{G(\phi_1)} r_2(\phi_1, z) \cdot \cos \left( \arctan \sqrt{\tan^2 \left[ \beta_2(\phi_1, z) \right] + \tan^2 \left[ \beta_1(\phi_1, z) \right]} \right) d_c . \] (13)

COEFFICIENT OF FRICTION

Most of the refrigeration screw compressors are oil-injected. They operate under elastohydrodynamic lubrication conditions. The oil separation from dry gas after compression causes no difficulties, but if the working medium is a two-phase mixture, an oil-free compression is desirable. In that case the compressor has to be lubricated by the liquid contained in the two-phase working mixture.

The friction coefficient depends on lubrication conditions in the contact zone, especially on the lubricant film thickness. Jacobson [1991] described a method for calculation of the minimum elastohydrodynamic film thickness. Based on solution of the Newtonian elastohydrodynamic problem, the minimal film thickness for linear contact is written as

\[ h_{\text{min}} = 3.07 \alpha^{0.57} R^{0.4} (\eta_0)^{0.71} \left( \frac{1}{\eta_0} \right)^{0.03} w^{0.11} \] (14)

where \( \alpha \) is the pressure-viscosity coefficient of the lubricant, 1/Pa; \( R \) – effective contact radius, m; \( u \) – the mean surface velocity, m/s; \( \eta_0 \) – lubricant dynamic viscosity at the atmospheric pressure, Pa·s; \( E' \) – effective elastic modulus of contacting surfaces, Pa and \( w \) – the contact load, N/m.

Calculations with Eq (14) for lubrication with an ammonia-water liquid film have shown that the minimum film thickness is of the order of magnitude of \( 10^{-9} \) – \( 10^{-8} \) m. This is less than the surface roughness. It means that contact between solid asperities occurs and boundary lubrication regime takes place. In that case the contact lubrication mechanism is governed by the physical and chemical properties of the boundary surface film. The film is a result of interaction between lubricant and solid. Schipper [1988] specifies the fundamental mechanisms of interaction as physical adsorption, chemical adsorption and chemical reaction. Besides the boundary film properties, the surface elastic properties and the surface topography determine the frictional characteristics. Jacobson [1991] indicated the surface roughness, the wavelength and slope of the surface asperities as important factors.

Because of the variety of features involved, the friction under the boundary lubrication conditions is less understood than the elastohydrodynamic lubrication. Some data on friction coefficients are found in the literature. Schipper [1988] estimates the coefficient between 0.1 and 0.4. According to data of Hamrock [1994] the value of friction coefficient for boundary lubrication conditions varies between 0.06 and 0.2, which is approximately hundred times larger than for elastohydrodynamic lubrication.

EFFICIENCY LOSSES DUE TO ROTOR FRICTION

The friction force between the rotors is calculated as a product of the contact force and friction coefficient. For one slice the friction force is

\[ dF_f = \mu dF_c . \] (15)

The friction force for each rotor is directed against the relative surface velocity along the common touching line. Figure 2 shows the friction force \( dF_f \) and its radial and tangential components \( dF_p \) and \( dF_t \) for both rotors.

The tangential component of the friction force is determined for both rotors as
\[ dF_{f1} = dF_f \sin \varepsilon \]  \hspace{1cm} (16)

The torque from friction for the male rotor is
\[ dT_{f1} = \eta dF_{f1} \]  \hspace{1cm} (17)

and for the female rotor
\[ dT_{f2} = -\eta dF_{f2} \]  \hspace{1cm} (18)

After substitution of Eq. (15) into (16) and then into Eqs. (17) and (18) the frictional torques for the male and female rotors become
\[ dT_{f1} = \mu \sin(\varepsilon_1) dF_c, \]  \hspace{1cm} (19)
\[ dT_{f2} = -\mu \sin(\varepsilon_2) dF_c. \]  \hspace{1cm} (20)

Angle \( \varepsilon_2 \) can be as positive as negative. The sign of angle \( \varepsilon_2 \) determines the sign of the frictional torque at the female rotor.

![Friction force components](image)

**Fig. 2. Friction force components**

When the friction force values are obtained for all slices, the torque from friction can be integrated, first within one cavity and then for the whole rotors. Afterwards, the friction power on the compressor shaft is obtained.

The power consumed by the compressor shaft is the sum of the gas and friction power. The compressor isentropic efficiency is determined as ratio of the isentropic compression power to the shaft power.

\[ \eta_{is} = \frac{P_{is}}{P_g + P_f} \]  \hspace{1cm} (21)

In Fig. 3 the isentropic efficiency \( \eta_{is} \) is plotted against the friction coefficient \( \mu \) for the compressor specified in the parallel paper, Zaytsev and Infante Ferreira [2000] (isentropic compression power 45.5 kW).
The poor lubrication properties of NH$_3$/H$_2$O mixture lead to operation in the boundary lubrication regime. The results in Fig. 3 indicate that under boundary lubrication conditions the losses from friction between rotors are substantial. Possible ways to decrease losses are selection of profiles with minimal torque transfer between the rotors and/or use of surface material (coating) with low friction coefficients. These measures will also reduce the wear rate.

CONCLUSIONS

A model for calculation of mechanical losses from friction between rotors in a twin screw compressor has been presented. It was found for a simulated compressor that the isentropic efficiency drops almost linearly from 0.82 to 0.7 when the friction coefficient increases from 0 to 0.5. Due to the low viscosity of ammonia-water mixture, boundary lubrication conditions take place in the rotor contact region. This increases the friction coefficient in comparison to oil lubrication. As a result, losses from rotor friction become unacceptable and friction-reducing techniques are required.

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