Aspects of Two-Phase Flow Screw Compressor Modelling Part I: Leakage Flow and Rotor Tip Friction

D. Zaytsev
Delft University of Technology

C. A. Infante Ferreira
Delft University of Technology
ASPECTS OF TWO-PHASE FLOW SCREW COMPRESSOR MODELLING
PART I: LEAKAGE FLOW AND ROTOR TIP FRICTION

D. Zaytsev and C. A. Infante Ferreira
Delft University of Technology
Laboratory for Refrigeration and Indoor Climate Technology
Mekelweg 2, 2628 CD Delft, The Netherlands
Phone ++31 15 2786691, Fax ++31 15 2787204
E-mail D.Zaytsev@WbM.TU.Delft.nl

ABSTRACT

A one-dimensional leakage flow model for two-phase ammonia-water twin screw compressors is presented. The model takes into account viscous and acceleration forces and is based on three conservation equations and on the equation of state for homogenous NH₃/H₂O mixture. The governing equations are solved with a finite difference method. Numerical shooting is used to find such inlet flow velocity and position of the shock that the outlet boundary condition is fulfilled. Results of the solution are used for calculation of the shear stress and friction between rotors and housing. Comparison of the results of the proposed leakage flow model with the results of the isentropic converging nozzle model shows that the latter predicts up to two times higher mass flow rate. Calculations of the rotor tip friction force indicate that tip friction losses in the two-phase NH₃/H₂O compressor are negligibly small.

NOMENCLATURE

A – cross sectional area of the flow passage, m²
B – constant in Blasius’ equation
C_f – coefficient of friction
D_h – hydraulic diameter, m
F – friction force, N
h – specific enthalpy, J/kg
m – mass flow rate, kg/s
n – constant in Blasius’ equation
p – pressure, Pa
q – specific heat, J/kg
Re – Reynolds’ number
u – velocity, m/s
v – specific volume, m³/kg
x – coordinate, m
x₀ – concentration of ammonia in NH₃/H₂O mixture, kg/kg
Π – wetted perimeter, m
ρ – density, kg/m³
τ – shear stress, Pa

Subscripts
1 – pre-shock conditions
2 – post-shock conditions
f – friction
w – wall

INTRODUCTION

Research on two-phase flow compressors is carried on within the framework of the European Commission’s project “Development of Compression/Resorption Heat Pumps”. For industrial applications the most promising operational conditions of the heat pump require that the compression of ammonia-water mixture takes place from 2 to 16 bar in the two-phase region. The amount of liquid in the medium to be compressed varies approximately from 0.25% by volume at the suction to 2.5% by volume at the discharge. The isentropic efficiency of the (wet) compressor influences in large extend the efficiency of whole heat pump cycle. In order to improve the heat pump performance, it is important to gain understanding of the irreversible processes taking place during (wet) compression. Examples of such processes are leakage and friction.

This paper presents a model for calculation of leakage flow in twin screw ammonia-water compressors. Results of the solution are used for calculation of the shear stress and friction between rotors and housing. Since it is difficult to separate lubricants from the wet working fluid, the compression has to be oil-free. As a consequence,
higher flow velocities are expected in leakage passages, because the viscosity of ammonia-water mixtures is lower than the viscosity of oils.

The simplest leakage flow model widely used in simulations of screw compressors is the converging nozzle flow model with assumptions that the flow is isentropic and the pressure in the narrowest part of the leakage path is equal to the downstream cavity pressure (see for example Fujiwara et al [1974], Xiao et al [1986]). Such model is based on the balance of inertia and pressure forces, while viscous forces are neglected. The calculated flow rate is adjusted by a flow coefficient, which must be found experimentally. The model gives no information on wall shear stress, though such information is important for calculation of rotor tip friction.

Sangfors [1984] presented a one-dimensional leakage flow model for oil-injected compressors. The model takes into account the viscosity of the gas-oil mixture and the passage geometry, but neglects the influence of flow inertia. Since the expected velocities in the NH$_3$/H$_2$O leakage flow are higher than for oil, the inertia effects should also be considered in the leakage flow model for ammonia-water mixture.

Recently Prins and Infante Ferreira [1998] developed a one-dimensional model, which accounts both for inertia and viscous forces. Supersonic flow effects (chocking and normal shock) were also included in the model. The model was simplified by using the isentropic energy equation and the equation of state for the perfect gas.

The model presented in this paper is developed for the ammonia-water mixture with use of the non-isentropic energy equation. The model is formulated with the following assumptions:

- The working mixture is homogenous
- The flow is adiabatic (but non-isentropic)
- If supersonic velocity is achieved, transition back from supersonic to subsonic flow takes place with a normal shock wave. The thickness of the shock wave is zero.

**GOVERNING EQUATIONS**

The model is based on three conservation laws.

**Mass conservation**

\[
dm = 0 \iff \frac{dp}{\rho} = \frac{du}{u} + \frac{dA}{A}.
\] (1)

**Momentum conservation**

\[
d(pu) + Adp + \Pi dx = 0.
\] (2)

The average wall shear stress is expressed as

\[
\tau = \frac{C_f}{4} \frac{pu^2}{2},
\] (3)

where the coefficient of friction \( C_f \) is calculated with Blasius' equation

\[
C_f = B \text{Re}^{-n}.
\] (4)

The empirical constants in Blasius' equation are \( B = 0.316 \) and \( n = 0.25 \).

After substitution of equations (3) and (4) into Eq (2) the momentum conservation equation can be rearranged as

\[
dp = \frac{m}{A} du - B \text{Re}^{-n} \frac{pu^2}{2} \frac{dx}{D_h},
\] (5)

where \( D_h = \frac{4A}{\Pi} \) is the hydraulic diameter of the leakage passage.

**Energy conservation**

\[
dh = -udu + \delta q.
\] (6)
Heat from the friction work performed by the moving wall (rotor’s surface) is found as
\[ \delta q = \frac{u_w dF_w}{m}, \]  
(7)
here \( u_w \) is velocity of the wall and \( F_w \) is the wall friction force.

In narrow passages the friction force acting on each of the two walls is equal to half of the friction force acting on the flow
\[ dF_w = 0.5 \cdot A \cdot dp_f, \]  
(8)
where \( dp_f = B \frac{Re^{-n} u^2}{2} \frac{dx}{D_h} \) is the pressure drop due to friction.

To integrate the three conservation equations an additional equation is required. This is the equation of state, which can be written for ammonia-water mixture as
\[ \rho = \rho(p, h, x_0). \]  
(9)
Itard [1994] has developed a model for calculation of the thermodynamic properties of the ammonia-water mixture, based on the Trepp/Ziegler equation of state. Function (9) is evaluated using Itard’s model.

Three conservation equations (1), (5) and (6) and the equation of state (9) form a closed system with four unknowns: \( du, dp, dh \) and \( dp \). Taking into account relations \( \frac{v}{\rho} = \frac{1}{A} \) and \( \frac{m}{\rho} = \frac{u}{A} \), an equation with one unknown \( du \) can be derived from this system. The equation reads:
\[ v \left( p - \frac{u}{v} du - B Re^{-n} \frac{u^2}{2v} \frac{dx}{D_h} \right) \left( h - u du + \delta q \right) x_0 = \left( \frac{v + \frac{du}{u} + \frac{v}{A} \frac{dA}{A} }{u} \right) = 0. \]  
(10)
Further, differentials of the parameters of state \( dp, dh \) and \( dp \) are found from the conservation equations as functions of \( du \). Equations (1), (5), (6) and (10) are the model’s governing equations. They are set to a final difference scheme. Equation (10) is solved with Newton-Raphson algorithm for each final segment. The boundary conditions are the thermodynamic state of the fluid at the inlet of the leakage passage and the pressure at the passage outlet.

**SHOCK RELATIONS**

The first attempt to solve the governing equations in the whole computational domain failed when high ratios of inlet to outlet pressure where chosen. With such pressure ratios the flow of two-phase NH\(_3\)/H\(_2\)O mixtures becomes supersonic. Since transition from supersonic to subsonic flow is connected to a shock, which is a form of discontinuity, the differential governing equations are not valid at the transition point. Nevertheless, the conservation laws in integral form are still valid. So far, the leakage path can be subdivided on the pre-shock and post-shock computational domains, while the change of flow parameters across the shock is determined with the integral conservation equations.

Denote conditions before the shock (supersonic) with subscript 1 and after the shock (subsonic) with subscript 2. The integral conservation equations have a simple form:
\[ v_2 = v_1 \frac{u_2}{u_1}, \]  
(11)
\[ p_2 = p_1 + \frac{u_1}{v_1} (u_1 - u_2), \]  
(12)
\[ h_2 = h_1 + 0.5 \left( u_1^2 - u_2^2 \right). \]  
(13)
The friction terms are omitted according to the assumption that the shock wave thickness is zero. Combining equations (11) – (13) with the equation of state (9), a relation can be derived for the post-shock velocity

\[
\left[ p_1 + \frac{u_1}{v_1} (u_1 - u_2) \right] \left( h_1 + \frac{u_1^2}{2} - \frac{u_2^2}{2} \right) x_0 - v_1 \frac{u_2}{u_1} = 0.
\]

Equation (14) has two solutions. One solution is trivial \( u_2 = u_1 \); the other solution, which is of interest, is found with the bisection method by bracketing the root between \( 0 \) and \( u_1 \). The position of the shock is determined by the outlet pressure as described in the next Section.

**NUMERICAL SOLUTION STRATEGY**

A numerical shooting method is used to find such inlet flow velocity and position of the shock that the calculated outlet pressure is equal to the pressure prescribed by the outlet boundary condition. The main steps are:

1. Bracket the inlet velocity \( u_{in} \) between zero and the speed of sound.
2. Set trial inlet velocity to the midpoint of the interval.
3. Integrate the governing equations.
   - If the flow velocity remains subsonic until the outlet, compare the calculated outlet pressure with the prescribed one. When the calculated value is larger, set minimum of the interval equal to the trial velocity value; otherwise, set maximum of the interval equal to the trial velocity, in this case there will be no shock. If during the integration the flow velocity reaches the sonic barrier, stop the integration and set maximum of the interval equal to the trial velocity.
4. Repeat Steps 2 and 3 until the interval is smaller than the prescribed accuracy.
5. Output the mass flow rate.
6. If the speed of sound was achieved, fix position of the sonic point and integrate the governing equations further (downstream the sonic point). Otherwise, go to Step 12.
7. Compare the calculated outlet pressure with the prescribed one. When the calculated value is larger, the flow remains supersonic until the outlet, the shock is outside the computational domain, go to Step 12; otherwise, bracket position of the shock between the sonic point and the passage outlet.
8. Set trial position of the shock to the midpoint of the bracketed interval and determine conditions behind the shock using relations (11) – (14).
9. Integrate the governing equations in the subsonic region between the shock and the outlet.
10. Compare the calculated outlet pressure with the prescribed one. If the calculated pressure is higher, shock takes place downstream the trial position, set minimum of the interval to the trial position. Otherwise, shock takes place upstream the trial position, set maximum of the interval to the trial position.
11. Repeat Steps 8 – 10 until the difference between the calculated and prescribed outlet pressure is less than the prescribed accuracy.
12. Output the outlet enthalpy and wall friction force, which is found by integration of equation (8).

If the flow outgoing the compressor control volume is evaluated, only the leakage mass flow rate is of interest and the algorithm is interrupted after Step 5. When the incoming flow is evaluated, the enthalpy of the flow coming to the control volume should also be known and the algorithm proceeds until the last step.

**RESULTS AND COMPARISON WITH ISENTROPIC MODELS**

Figure 1 schematically shows the geometry of the leakage passage between the female rotor lobe tip and the compressor housing. Figure 2 presents examples of calculated pressure and velocity profiles. The results were obtained for a homogenous two-phase NH\(_3\)/H\(_2\)O mixture with ammonia concentration of 0.35 kg/kg. The inlet temperature is 150 °C and liquid mass fraction at the inlet is 0.7.
After the passage entrance, in segment AB, the flow velocity rises because of the decreasing cross sectional area. At the same time the pressure drops, mainly due to the flow acceleration. In the central part of the passage (segment BC), where the cross sectional area is constant, the pressure decreases because of the flow viscosity. The pressure reduction causes a specific volume increase, which according to the mass conservation law leads to further increase in velocity. Higher velocity results in higher friction and in higher rate of pressure reduction. With moderate pressure ratios (downstream pressures of 10 bar or higher), the velocity reaches its maximal value at point C, where the pressure is minimal. In segment CD the passage area increases and in case of subsonic flow the velocity falls down. The pressure rises because of the flow deceleration. With high pressure ratios (downstream pressure less than 10 bar) the flow becomes supersonic after point C and accelerates further until the shock point. In such cases the downstream pressure determines the position of the shock and has no influence on the flow parameters before the shock point. The high velocities attained indicate that in a leakage model for two-phase ammonia-water mixture the inertia forces cannot be neglected.

Fig. 1. Leakage path geometry

Fig. 2 Pressure and velocity profiles calculated for viscous flow

In order to determine the influence of viscosity on the flow pattern, the leakage flow was evaluated with the same input data as in the previous example, while the friction term in the momentum equation was set to zero. The calculation results are shown in Figure 3.

Fig. 3. Pressure and velocity profiles calculated for inviscid flow
In the case of inviscid flow even small pressure differences across the converging/diverging passage result in supersonic flow with a shock wave. In segment AB (Fig. 1) the flow accelerates up to the speed of sound and the pressure drops due to flow acceleration. In the central part of the passage (segment BC) the pressure and velocity remain constant. If there were no shock in segment CD, the flow velocity would decrease there and the outlet pressure would be equal to the pressure at the inlet, because the inviscid flow is reversible. Lower pressure at the outlet is only possible when the flow is irreversible. That is why in the case of inviscid flow even a small pressure difference causes a shock, which is irreversible. The outlet pressure has no influence on the mass flow rate.

In Figure 4 the mass flow rate results obtained with the one-dimensional model (viscous and inviscid case) and with an isentropic converging nozzle model are compared. The results were obtained with upstream pressure of 16 bar and the rotor tip sealing line length of 0.2 m. The converging nozzle model, when compared with the one-dimensional viscous flow model, predicts the same trend but overestimates 1.6 – 2 times the leakage mass flow rate. The one-dimensional inviscid model can hardly be used, because it predicts the flow rate without any relation to the downstream pressure. However, with large ratios upstream to downstream pressure, it predicts the same flow rate as the converging nozzle model. This is logic, because both models neglect friction and the flow velocity in the narrowest part of the leakage path is restricted by the speed of sound.

![Fig. 4 Calculated mass flow rate as function of downstream pressure](image)

**POWER LOSS DUE TO ROTOR TIP FRICTION**

The friction force on the rotor tip can be found by integration of Eq (8). The frictional torque and power loss due to friction then can be obtained. Implementation of the model for NH₃/H₂O two-phase flow compressors has shown that the rotor tip friction losses are negligibly small in comparison to the losses in oil-injected compressors (see Fleming et al [1994]). The calculated tip friction power equals to 75 W, while the total compression power is 56 kW. The power losses were calculated for the female rotor tip only.

Results of power calculations were obtained for a compressor with asymmetric rotor profile as described by Sakun [1970]. The main geometrical and operating conditions of the compressor are as specified in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Compressor geometry and operating conditions</th>
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<tbody>
<tr>
<td>Length/diameter ratio, mm/mm</td>
</tr>
<tr>
<td>Distance between the rotors' axes, mm</td>
</tr>
<tr>
<td>Number of lobes male/female</td>
</tr>
<tr>
<td>Built-in volume ratio</td>
</tr>
<tr>
<td>Wrap angle of the male rotor, deg</td>
</tr>
<tr>
<td>Average clearance, mm</td>
</tr>
<tr>
<td>Shaft speed (male rotor), rpm</td>
</tr>
<tr>
<td>Inlet pressure, bar</td>
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<tr>
<td>Inlet temperature, °C</td>
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<tr>
<td>Outlet pressure, bar</td>
</tr>
<tr>
<td>NH₃/H₂O overall concentration, kg/kg</td>
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</tbody>
</table>

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CONCLUSIONS

Neither inertia nor viscous forces should be neglected in two-phase flow leakage models for ammonia-water mixture. Calculations for a converging/diverging passage geometry have shown that supersonic flow velocities are achievable in two-phase NH₃/H₂O leakage flow, when the compressor operates with relatively high pressure ratios. Comparison of the results of the proposed model with the results of a converging nozzle model has shown that the latter predicts up to two times higher mass flow rates.

Calculation of shear friction force between the rotor tip and compressor housing has shown that unlike oil-lubricated compressors the tip friction losses in the simulated two-phase NH₃/H₂O compressor are negligibly small. At the same time, the low viscosity of ammonia-water mixtures can result in substantial losses from friction between the rotors. This is the subject of a parallel paper (Zaytsev and Infante Ferreira [2000]).

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REFERENCES
