Near-Contact Gas Damping and Dynamic Response of High-g MEMS Accelerometer Beams

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Near-Contact Gas Damping and Dynamic Response of High-g MEMS Accelerometer Beams

Devon Parkos, Nithin Raghunathan, Ayyaswamy Venkattraman, Brett Sanborn, Weinong Chen, Dimitrios Peroulis, Member, IEEE, and Alina Alexeenko

Abstract—This paper introduces and experimentally validates a new model for near-contact gas damping of microbeams. The model is formulated based on numerical simulations of rarefied gas dynamics using the Boltzmann Ellipsoidal Statistical Bhatnagar-Gross-Krook (ES-BGK) equation. The result is compared with existing models by simulating the motion of beams under high-g acceleration. To experimentally validate the damping models, single crystal silicon MEMS gas-switches with cantilever microbeams of various lengths were utilized. The experimental measurements of beam dynamics under peak accelerations of approximately 50,000 g and acceleration ramp rates from 600 to 3,000 g/s are compared with simulations. Additionally, the damping coefficients are extracted from existing vibrational mode data, and the resulting values are compared to the various models. The new near-contact model was found to predict contact and release times within a root-mean-square deviation from experiment below 9 μs (±2%) and 7 μs (±5%) for contact and release events, respectively. The damping values for the vibrational modes away from contact were predicted within 35% error, showing a more consistent predictive capability than provided by earlier models.

Index Terms—Gas damping, acceleration measurement, modeling, beams.

I. INTRODUCTION

The high durability, low cost and weight of MEMS devices are ideal for high-g accelerometers [1], [2], specifically for use in measuring impact dynamics in industrial and military applications. The further development of MEMS with moving and contacting components requires accurate modeling of their dynamic response. However, due to the small size of MEMS, certain macroscale models do not accurately predict microscale behavior [3]. For instance, the surface area to volume ratio is substantially higher for MEMS. As a result, the aerodynamic forces drastically affect device motion, near contact and at low velocities. This effect is amplified as device approach contact with each other.

The effect of gas damping on contacting components also affects the longevity of device components. Increased gas damping immediately before contact drastically reduces the contact velocity. For the same reason, response time is greatly affected by gas damping effects near contact. It is therefore crucial to accurately model the aerodynamics, especially near contact, in order to accurately model the performance and reliability of contacting MEMS.

There are many types of MEMS devices, both with or without dynamic components. Examples of devices with only stationary elements include strain gauges, and pressure sensors [3]. MEMS devices with dynamic parts include gyroscopes, filters, and resonators [3]. Aerodynamic damping is of particular interest to devices with relatively fast motion or motion near contact with another surface. Devices with these characteristics include switches, relays, valves, shutters, pumps, and many other types of devices [3]. The aerodynamic damping addressed in this article pertains to a rectangular beam near contact with another surface. In particular a relatively thick beam that undergoes high levels of acceleration, where the forces due to aerodynamic damping play a significant role in determining contact velocities.

The response characteristics of MEMS devices show a strong dependence on the aerodynamic damping. While gas damping can be accurately described using a macroscopic approach when the flow is in the continuum regime, a microscopic approach is required when the characteristic dimensions of the flow decrease. The Knudsen number (Kn), defined as the ratio of mean free path [4] to a characteristic length, is used to determine when the continuum approach breaks down. The Reynolds equation is valid for small Knudsen numbers (Kn ≤ 0.01) in the continuum regime, whereas a modified slip boundary condition for the Reynolds equation is required in the slip flow regime. When Kn further increases resulting in the transitional regime, an approach based on the Boltzmann equation is necessary to accurately describe the behavior of the surrounding gas.

For the case of a rectangular microbeam near contact with another surface, the characteristic dimension of the flow is the
constant pressure, \( Kn \) beam widths (Fig. 1. Cross-section of beam near contact with another surface).

A number of gas damping models are based on solutions to the Boltzmann equation in its original or approximate form. Several of those models and the parameter ranges for which they were formulated are listed in Table I.

For rectangular beams, the Knudsen number can be defined as

\[
Kn = \lambda / g = (\sqrt{2} m v_{th}^2 / k_B T) \lambda / g \approx \lambda / g \sqrt{\pi d} \approx 0.35 \lambda / g (1)
\]

where the variables \( T_{ref} \) and \( T \) are the reference and actual fluid temperatures, and \( \lambda \) is a fluid dependent coefficient. The parameter \( d_{ref} \) represents the reference diameter for the gas with number density \( n \).

Gas damping models specify a function for the damping coefficient, which is defined in Eq. 2. The variable \( Cf \) represents the damping coefficient and \( F_D \) represents the damping force per unit length. The velocity of the beam, \( dy/dt \), can vary with lengthwise location.

\[
C_f = \frac{F_D}{dy/dt} \quad (2)
\]

This quasi-steady function of velocity assumes the damping force is linearly dependent upon the beam velocity. Nonlinear effects increase with velocity and are assumed to be negligible for conditions in this work, based on results from Chigullapalli et al. [5].

Sadd and Stiffler [6] presented an adaptation of the Reynolds equation, formulated for oscillatory motion in the continuum regime at low squeeze numbers. A modified version of this model that accounts for large displacements and rarefied effects was presented by Rebeiz (Equation 3.15 in [3]), using a viscosity correction factor derived by Veijola et al. [7].

Veijola and Turowski [8], [9] presented another model based on the Reynolds equation that accounted for the inertia of the fluid flow. Gallis and Torczynski [10] developed a damping model based on the direct simulation Monte Carlo (DSMC) method, which is shown in Eq. 3.

\[
C_f = \frac{\mu}{(g/w)^3} (1 + 6 \chi Kn)^{-1} (1 + 6 \eta (g/w) + 12 \zeta (g/w)^2) \quad (3)
\]

In this equation, \( \mu \) represents viscosity and \( Kn \) represents the Knudsen number. The variables \( h \) and \( w \) represent the beam height and width respectively, while \( g \) represents the gap size between the beam and the surface, which is assumed to vary only along the length. The coefficients \( \chi, \eta, \) and \( \zeta \) were calculated as functions of the Knudsen number by Gallis and Torczynski [10], and are given by the following relations:

\[
\begin{align*}
\chi &= \frac{1 + 8.834 Kn}{1 + 5.118 Kn} \\
\eta &= \frac{0.634 + 1.572 Kn}{1 + 0.537 Kn} \\
\zeta &= \frac{0.445 + 11.20 Kn}{1 + 5.510 Kn}
\end{align*}
\]


\[
C_f = \frac{A (u/g)^{\eta} h}{1 + B (u/g)^{2\eta} (Kn)^{\nu}} \quad (5)
\]

The values for \( A \) and \( B \) used in Eq. 5 are 10.39 and 1.374, respectively. The values for \( c, d, \) and \( e \) are 3.100, 1.825, and 0.9660, respectively.

All of these damping models were derived for use in MEMS devices that do not involve contact and hence were based on a finite range for the ratio \( g/w \). As a result when \( g/w \) tends to zero, such as near contact, these models diverge. As can be seen from Eqs. 3 and 5, as the value of \( g \rightarrow 0 \) the model diverges due to an exponent of around 3 for the \( g/w \) term in the expressions for the damping coefficient.

Numerous gas damping models assume quasi-steady flow behavior. The error due to this assumption will be negligible for cases where the time scale for information in the flow to propagate is significantly smaller than the time scale for the beam to move. Consider the ratio of these two time scales to be

\[
\tau_{flow} / \tau_{motion} = \frac{1}{g (dy/dt)} \quad (6)
\]

where the root mean square of the thermal speed in equilibrium, \( \nu_{th} \), is defined as follows:

\[
\nu_{th} = \sqrt{\frac{3 k_B T}{m}} \quad (7)
\]

In this expression, \( k_B \) represents the Boltzmann constant and \( m \) represents the molecular mass of the gas. Eq. 7 shows that for sufficiently slow velocities, unsteady effects will be negligible. This was shown by Chigullapalli et al. [5] to be accurate for velocities up to 2.5 m/s upward and 1.5 m/s downward for 120 \( \mu \)m wide rectangular beams at a gap size of 3.52 \( \mu \)m. This corresponds to time scale ratios below 8% and 5% for upward and downward directions, respectively. For the cases in this paper, this ratio was found to be below 8% for the entire motion of the beam, indicating negligible error is incurred from assuming quasi-steady flow.

The goal of this paper is to develop and validate a higher fidelity aerodynamic damping model for cantilever microbeams near contact with a surface. In Parkos et al. [13], preliminary work was performed in extending the existing Gallis-Torczynski model to be valid for \( g/w = 0 \) but applicable only
TABLE I
SUMMARY OF INTENDED PARAMETER RANGES FOR SQUEEZE-FILM DAMPING MODELS AND EXPERIMENTS. MODELS THAT DO NOT ACCOUNT FOR THE THICKNESS OF THE BEAM ARE DENOTED BY N/A.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>(w/h)</th>
<th>(b/g)</th>
<th>(K_{n2} = \lambda/g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsteady Reynolds Eq. [4], [6]</td>
<td>N/A</td>
<td>N/A</td>
<td>≤ 88.6</td>
</tr>
<tr>
<td>RE with Flow Inertie [7], [8]</td>
<td>N/A</td>
<td>N/A</td>
<td>0.014 - 0.027</td>
</tr>
<tr>
<td>RE with Slip Condition [9]</td>
<td>2.5 - 30</td>
<td>≤ 1</td>
<td>0.0085 - 1.16</td>
</tr>
<tr>
<td>ESRGK Thin-Beam Model [10]</td>
<td>10 - 18</td>
<td>≤ 1</td>
<td>0.05 - 50</td>
</tr>
<tr>
<td>Near-Contact Model (This Work)</td>
<td>1 - 5</td>
<td>10 - 2000</td>
<td>0.027 - 1.1</td>
</tr>
</tbody>
</table>

![Image]

The remainder of the paper is organized as follows. Section II describes the MEMS high-g switch and the experimental approach to characterize its dynamic response, including the circuitry and the testing method. Section III describes the modeling approach used for both the g-switch dynamics and determining the improved near-contact damping model. Section IV presents the results including comparison with experiments and the corresponding discussion with Section V summarizing the conclusions.

II. EXPERIMENTAL METHOD

A. DEVICE DESCRIPTION

To experimentally verify the developed damping model, single crystal silicon (SCS) g-switches which were previously designed for detecting high-g accelerations from 20,000 g to 40,000 g [2] were used, where 1 g is approximately 9.8 m/s². The dynamic response of the device under such applied profiles needs to be well understood and this is largely dependent on the near contact interactions. The cantilevers were designed with different spring constants for different acceleration levels of up to 40,000 g. These are designed using a rudimentary second order differential equation of motion [3].

For example, a cantilever corresponding to a beam length of 527.5 μm, width of 98.7 μm, and height of 20.0 μm for a total deflection of 2 μm has a spring constant of 462 N/m and triggers around 40,000 g.

The silicon cantilevers were fabricated on silicon-on-insulator (SOI) wafers with device and handle layer resistivity of (<0.01 Ω-cm). The devices were fabricated using the same process developed in Raghunathan et al. [2], utilizing standard micromachining techniques. Figure 2 shows pictures of 705, 578.8, and 527.5 μm long cantilevers fabricated using this process. These cantilevers have respective widths of 20.3, 68.6, and 98.7 μm and share a 20 μm height.

All indicated measurements were taken using the Olympus OLS-3000 LEXT confocal microscope, which provides a theoretical x-y uncertainty of 10 nm.

Fig. 2. SEM images of typical cantilevers described in Section II-A. From top to bottom they are 705.0, 578.8, and 527.5 μm long. Geometrical measurements have been conducted using the Olympus OLS-3000 confocal microscope that provides a x-y uncertainty of 10 nm [14].

B. EXPERIMENTAL EVALUATION

The experimental evaluation of the fabricated devices involves the following, 1) electrical measurement setup and 2) acceleration evaluation setup.

1) ELECTRICAL MEASUREMENT SETUP: A block diagram schematic of the electric measurement setup used in all experiments is shown in Figure 3. In all conducted experiments, four
cantilevers of the same physical dimensions were connected in parallel. Their combination was then connected through a resistor ($R_s$) to an Agilent E3630A power supply [15] set at 5 V. The contact event was then detected by the change in resistance when the beams make contact with the substrate. This is recorded as a voltage drop from 5 V onto the Tektronix TDS3014B oscilloscope [16], which is synchronized with the applied acceleration input.

2) **Acceleration Evaluation Setup:** The acceleration evaluation tests were performed using an modified Hopkinson bar test setup similar to the one described in Raghunathan et al. [2]. Hopkinson pressure bar techniques have been employed to evaluate the performance of piezoresistive accelerometers used in penetration experiments [17], [18]. Since there are no standard calibration methods for high-g accelerometers [17], evaluation of accelerometers prior to use in a penetration experiment is required for accurate penetration data [19]. Using such techniques, acceleration loads up to 200,000 g with pulse durations of 150 to 550 $\mu$s and rise times as short as 50 $\mu$s can be produced. [19], [18].

During an experiment, a maraging steel striker (C350) bar of 12 inches in length is fired by a gas gun toward the 144 inch long aluminum (7075-T6) incident bar. Impact of the striker on a copper (C11000) pulse shaper fixed to the end of the incident bar produces a nondispersive elastic stress pulse in the incident bar. This then interacts with the tungsten package containing the MEMS device mounted on the end of the incident bar. Pulse shaping is an essential component of the experimental technique that ensures the tungsten package is in dynamic equilibrium. The requirement of dynamic equilibrium ensures that the response of the tungsten package can be accurately approximated as rigid-body motion. Once the compressive stress pulse reaches the MEMS package, an impedance mismatch between the package and bar causes the package to separate from the end of the bar when the compressive pulse becomes tensile. The fly-away package is then caught in a foam catcher after a few inches of travel.

The elastic deformation of the bar during the experiment is measured using strain gages (Vishay WK-13-125EZ-10C) [20] mounted on the incident bar. The strain gages are connected to a Wheatstone bridge network in conjunction with a Tektronix A0A-400A differential probe [21] and the resistive imbalance caused by the elastic deformation is recorded on a Tektronix TDS-3014B oscilloscope [16]. One-dimensional wave propagation analysis is used along with Newton’s second law to calculate the acceleration profile seen by the MEMS package [19].

The fabricated dies were packaged and wired bonded in a ceramic package. This was then soldered onto a printed circuit board (PCB) and placed inside the hollowed tungsten disk filled with a potting material to prevent any movement or vibrations during impact [2]. This was connected to the electrical setup described in the previous section and the packaged sample was subjected to acceleration profiles created using the Hopkinson bar setup. The schematic of the modified Hopkinson bar apparatus is shown in Fig. 4. A picture of the experimental setup with the different sections of the setup are shown in Fig. 5.

### III. Modeling Approach

#### A. Beam Dynamics Model

The g-switches used in the experiments are modeled as cantilever beams using the Euler-Bernoulli beam theory. The governing equation for the motion of the beam as a function of time and location on the beam is given by

$$\rho A \frac{\partial^2 y}{\partial t^2} + E I \frac{\partial^4 y}{\partial z^4} + C \frac{\partial y}{\partial t} = f_s(z, t),$$

where $y(z, t)$ is the deflection at location $z$ on the beam measured from the fixed end and time $t$, $\rho$ is the density of the material, $A$ is the cross-section area, $E$ is the Young’s modulus, $I$ is the moment of inertia, $C$ is the damping coefficient, and $f_s(z, t)$ is the external force acting on the beam.
\textit{f}_{\text{ext}} \textit{is} the external force per unit length, which is only due to acceleration in the current work. As a result, the external force term simplifies to \textit{f}_{\text{ext}}(t). The beams used in this work had a rectangular cross-section and hence \textit{A} = \textit{wh} and \textit{I} = \textit{wh}^3/12 where \textit{w} and \textit{h} are the width and height of the beams.

For a given acceleration profile from experiments, Eq. 8 is solved numerically as follows. The differential equation is discretized using a finite element approach. Backwards differencing is used for time and central differencing is used for space. The time integration is performed using an implicit scheme. Fixed-free boundary conditions were applied. For more details concerning the method and the finite difference equations used for the simulation, see [22]. The beam was assumed to be undeflected and at rest at \textit{t}=0 providing the initial conditions to compute the response of the beams.

\section*{B. Aerodynamic Damping Model}

In order to obtain \textit{C}_f, as a function of the surrounding gas conditions, the flow around the beam was simulated using the Boltzmann ellipsoidal statistical Bhatnagar-Gross-Krook (ESBGK) equation [12], shown below:

\begin{equation}
\frac{\partial \textit{f}_{\text{o}}}{\partial \textit{t}} + \textit{u} \cdot \nabla \textit{f}_{\text{o}} = \frac{\textit{f}_{\text{o}} - \textit{f}}{\tau} \tag{9}
\end{equation}

In this expression, \textit{f} represents the velocity distribution function. The molecular velocities \textit{u}, \textit{v} are in the \textit{x}, \textit{y} directions, respectively. The variable \textit{τ} represents the mean time between collisions, which is the inverse of collision frequency. The equilibrium distribution function, \textit{f}_{\text{o}} is given by:

\begin{equation}
\textit{f}_{\text{o}} = \frac{n}{\sqrt{(2\pi)\text{det}(\text{T})}} \exp\left(-\frac{1}{2} \frac{\text{det}(\text{T})^{-1}}{\tau} \right). \tag{10}
\end{equation}

\begin{equation}
\text{T} = \frac{1}{3\text{tr}(<\text{u} \otimes \text{u}>)}. \tag{11}
\end{equation}

The variable \textit{R} is the specific gas constant, and \textit{u}' represents the thermal velocity given by

\begin{equation}
\text{u}' = \text{u} - \text{u}_0, \tag{12}
\end{equation}

where the variable \textit{u} represents the molecular velocity and \textit{u}_0 is the mean velocity. The operations \text{det} and \text{tr} denote the determinant and trace of the corresponding matrix. The bracket notation used in Eq. 11 represents the following operation, representing integration across all velocities.

\begin{equation}
<\text{u}> = \int_{\text{u}=-\infty}^{\infty} \text{f}(\text{u})\text{d}\text{u}. \tag{13}
\end{equation}

The solution was generated using a finite volume, discrete ordinate solver [11] for a specified region surrounding the beam. Due to the symmetry of the beam, only the right half of the cross-section was analyzed. A symmetry boundary condition was applied to the left side of the domain. A diffuse-reflection wall boundary condition was used for the beam and the contact surfaces. A macroscopic average velocity was prescribed such that it moved away from the contact surface. Pressure inlet conditions were applied for the top and right side of the domain, and the domain’s limits were chosen to be three times the size of the beam to mitigate the effect of these inlet conditions. The mesh was chosen to have 140 grid points in each direction, with the distribution more concentrated near the beam. A sample computational mesh is shown in Fig. 6. For more information on the exact method used to solve the equation, see Guo et al. [11].

The solution converged for the chosen spatial and velocity meshes, predicting the damping force acting on the beam within 5% of the final value. Nonlinear effects were examined and the velocity was found to be sufficiently low to assume linear behavior for the damping force with respect to velocity.

After the solution converges, the damping coefficient was extracted using Eq. 2, where the aerodynamic damping force per unit length is given by

\begin{equation}
F_d = \int_{-w/2}^{w/2} \left( \sigma_{\text{xx}}(x, y) - \sigma_{\text{yy}}(x, y) \right) dx + 2 \int_{y=0}^{h} \sigma_{\text{xy}}(w/2, y) dy \tag{14}
\end{equation}

As shown in Eq. 2, the damping coefficient can be calculated by dividing the net aerodynamic force by the chosen beam velocity (0.1 m/s). In Eq. 14, \textit{σ}_{\text{ij}} represents an element of the stress tensor (referred to as the pressure tensor in gas kinetic theory [4]). Unlike the scalar quantity thermodynamic...
Fig. 8. Comparison of simulated pressure profiles along the bottom surface of the beam for 50 \( \mu \text{m} \) wide / 20 \( \mu \text{m} \) thick beam moving downward at 0.1 m/s for gap sizes of 0.1 and 1 \( \mu \text{m} \).

As seen in Eq. 14, the total aerodynamic force per unit length is the integral of the vertical component of pressure, taken around sides of the beam. This yields the coefficient for a given gap size and set of beam dimensions at a given Knudsen number, calculated using Eq. 1. By repeating this process for different values of \( w/g \) and \( Kn \), an extension of the Gallis-Torczynski for large values of \( w/g \) was generated.

**IV. RESULTS AND DISCUSSION**

**A. Aerodynamic Results**

In this subsection, the details of the formulation of the near-contact damping model as an extension to the Gallis-Torczynski model is presented. The Boltzmann model equation with the ESBGK collision operator as described in Section III-B is used. Once the simulation reaches a steady-state, the pressure forces on the beam were integrated to obtain the net damping force and hence damping coefficient for the specified conditions. Figure 7 compares contours of velocity for gap sizes of 0.1 \( \mu \text{m} \) and 1 \( \mu \text{m} \). The beam width for these cases was chosen as 50 \( \mu \text{m} \), corresponding to \( w/g \) values of 500 and 50 respectively. The simulations were both performed for atmospheric pressure and corresponding to \( Kn = 0.54 \) and 0.054 respectively. The gas under the beam was accelerated to velocities higher than the beam velocity with the maximum velocities occurring at the edge of the beam. As shown in Fig. 7, the relatively short amount of time the air has to adjust to the beam results in an elevated pressure, an effect that increases tremendously near contact with the surface.

Figure 8 shows a comparison of the pressure variation on the bottom surface of the beam for both gap sizes with the pressure being significantly higher for the 0.1 \( \mu \text{m} \) gap. In order to formulate the near-contact damping model, simulations were performed for several cases of \( g/w \) and \( Kn \), the two non-dimensional parameters that determine the damping coefficient \([10],[11]\). The value of \( Kn \) was varied from 0.027 to 1.1 while \( w/g \) ranged from 10 to 1360.

It was observed that the damping coefficient predicted by ESBGK simulations was slightly lower than that predicted by the Gallis-Torczynski model even for large \( g/w \) values. To ensure that the unified near-contact damping model is completely based on ESBGK simulations, a correction factor of 0.84 was used to scale the Gallis-Torczynski model, which is equivalent to scaling the viscosity by the same factor (\( \mu_{G} = 0.84\mu_{gas} \)). In order to formulate the near-contact extension to the Gallis-Torczynski damping model using the performed simulations the near-contact damping is assumed to be of the form

\[
C_f = A_1 (g/w)^a + A_2
\]  

for \( g/w \) values less than a certain cut-off value \( x_c \). Here, \( a \) is a free parameter that is computed using the ESBGK simulations while \( A_1 \) and \( A_2 \) are functions of \( Kn \) and are computed by enforcing that at \( g/w = x_c \), the near-contact extension and the Gallis-Torczynski model in its original form have the same value of the damping coefficient and its derivative with respect to \( g/w \). Using a least-squares approach and the ESBGK data points, the value of \( x_c \) and \( a \) were determined as 0.0197 and
Table II

Average Percent Error for Models Compared with ESBGK Simulation Data for Several Different Beam Widths.

<table>
<thead>
<tr>
<th>Model</th>
<th>20 μm</th>
<th>50 μm</th>
<th>68 μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsteady Reynolds Eq. [6], [3]</td>
<td>183.2%</td>
<td>110.6%</td>
<td>76.75%</td>
</tr>
<tr>
<td>RE with Slip Condition [10]</td>
<td>435.8%</td>
<td>38.11%</td>
<td>3.524%</td>
</tr>
<tr>
<td>Near-Contact Model</td>
<td>18.16%</td>
<td>13.82%</td>
<td>11.83%</td>
</tr>
</tbody>
</table>

The values of the coefficients \( A_1 \) and \( A_2 \) were obtained as:

\[
A_1 = \frac{-3\mu_G}{(1+6\chi KN)}\left(\frac{1}{x^4_c} + \frac{4\eta}{x^3_c} + 4\zeta/x^2_c\right) a^{-1} x^a c A_1 \tag{18}
\]

\[
A_2 = \frac{\mu_G}{(1+6\chi KN)}\left(\frac{1}{x^3_c} + 6\Omega/x^2_c + 12(z/x_c) - x^a A_1 \right)
\]

Figure 9 summarizes the data obtained from ESBGK simulations in the near-contact domain thereby comparing it to the model formulated in this work and the original Gallis-Torczynski model. The variation of the damping coefficient as a function of gap size is shown for beams of three different widths. For a given width and pressure, changing the gap size changes both the ratio \( g/w \) and \( Kn \). The near-contact model uses a form similar to the Gallis-Torczynski model for large values of \( g/w > 0.0197 \) and Eq. 17 for \( g/w < 0.0197 \) and can be seen to agree very well with the ESBGK data points. The maximum error is about 34.3% while the average error is only 14.6%. Table II contains the average percent error for each model compared to the ESBGK data points.

The value of damping coefficient predicted by the near-contact model was compared with existing models. As \( g \to 0 \), the value of \( Kn \to \infty \) and both coefficients \( A_1 \) and \( A_2 \) tend to 0. Since the power \( a \) is greater than -1, the overall quantity \( A_1(w/g) a \) goes to zero. As a result, the damping coefficient tends to 0 as \( g \to 0 \) for the near-contact model as opposed to the divergence detected in existing models. This behavior is crucial to computing near-contact dynamics accurately as will be demonstrated in the following subsection.

B. Microbeam Dynamics Results

This subsection presents results for beam dynamics computed using the near-contact damping model formulated in the previous subsection. These results are compared with existing damping models. The numerical predictions were validated with experimental data for the response of MEMS high-g switches tested using the experimental set-up described in Section II. The cantilevers were subjected to acceleration profiles with varying ramp rates and their responses were recorded. Figure 10(a) shows a typical response of a 705 μm long cantilever when subjected to an acceleration profile with a peak load of 55,000 g. Figure 10(b) shows a typical response of a 527.5 μm long cantilever when exposed to a profile with a peak acceleration of 53,000 g with a ramp rate of 2800 g/μs.

The g-switches are designed to trigger after a set high-g acceleration threshold has been exceeded, resulting in a resistance decrease due to contact and the resulting voltage drop observed using the setup in Fig. 3. This causes the step-like behavior in the system response (Fig. 10), while the spurious modes can be attributed to contact bouncing. The response times of the devices under the prescribed acceleration level are extracted and categorized into contact and release times. The contact time is defined as the time taken for the switch to make its first contact with the substrate and is detected by a drop in the voltage from about 5 V to about 0 V. The release time is defined as the time taken for the switch to release from the substrate and is indicated by an increase in voltage from about 0 V to about 5 V.

Using the acceleration profile corresponding with each experiment, the motion of the beam is simulated using the method presented in Section III. This procedure was repeated using different damping models. A comparison of the results...
The damping force at the beam tip for each model is shown in Fig. 12(a) and (b), corresponding to the 705 μm and 527.5 μm long beams. Figure 13(a) shows a comparison of the times predicted for contact and release for the beam using each of the damping models (for the 705 μm long beam).

The time the beam takes to initially contact will be nearly unaffected by contact effects, but the release time will be slightly delayed by stiction. These compared times neglect the influence of contact effects on motion of the beam. All of the contact times are within 20 μs to the experimentally observed value, and the release times are within 15 μs. This indicates that aerodynamic damping has a relatively small effect on the motion of the beam. The root mean square (RMS) errors for each model are listed in Table III. The RMS error is defined to be

$$E_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (t_{\text{exp},i} - t_{\text{model},i})^2}$$
TABLE III
RMS Error Summary for the Cantilever Beam Experiments.

<table>
<thead>
<tr>
<th>Beam Length (μm)</th>
<th>Model</th>
<th>Contact Time Error [μs]</th>
<th>Release Time Error [μs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>705</td>
<td>Unsteady Reynolds Eq. [6], [9]</td>
<td>8.96</td>
<td>4.97</td>
</tr>
<tr>
<td></td>
<td>RE with Slip Condition [9]</td>
<td>7.58</td>
<td>6.02</td>
</tr>
<tr>
<td></td>
<td>ENB/GE Thin-Beam Model [10]</td>
<td>5.30</td>
<td>5.40</td>
</tr>
<tr>
<td></td>
<td>Near-Contact Model</td>
<td>8.82</td>
<td>5.68</td>
</tr>
<tr>
<td>527.5</td>
<td>Unsteady Reynolds Eq. [6], [9]</td>
<td>2.91</td>
<td>6.63</td>
</tr>
<tr>
<td></td>
<td>RE with Slip Condition [9]</td>
<td>50.77</td>
<td>13.15</td>
</tr>
<tr>
<td></td>
<td>ENB/GE Thin-Beam Model [10]</td>
<td>No Contact</td>
<td>No Contact</td>
</tr>
<tr>
<td></td>
<td>Near-Contact Model</td>
<td>6.26</td>
<td>6.59</td>
</tr>
</tbody>
</table>

TABLE IV
Average Percent Error for Models Compared with Experimental Data for Vibrational Modes from Lee et al. [11].

<table>
<thead>
<tr>
<th>Beam Length (μm)</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>Unsteady Reynolds Eq. [6], [9]</td>
<td>38.55%</td>
<td>62.02%</td>
</tr>
<tr>
<td></td>
<td>RE with Slip Condition [9]</td>
<td>28.53%</td>
<td>27.13%</td>
</tr>
<tr>
<td></td>
<td>Near-Contact Model</td>
<td>27.84%</td>
<td>31.85%</td>
</tr>
<tr>
<td>500</td>
<td>Unsteady Reynolds Eq. [6], [9]</td>
<td>114.92%</td>
<td>133.93%</td>
</tr>
<tr>
<td></td>
<td>RE with Slip Condition [9]</td>
<td>24.43%</td>
<td>20.19%</td>
</tr>
<tr>
<td></td>
<td>Near-Contact Model</td>
<td>24.47%</td>
<td>23.60%</td>
</tr>
</tbody>
</table>

where E is the error, N is the number of experiments, and the t_i values represent the measured and predicted times for each experiment. Several experiments for each beam size were conducted, wherein the beam was exposed to acceleration loads similar to those shown in Fig. 10.

Figure 13(b) shows the contact and release times for the 527.5 μm long beam. The width of this beam is much larger than the 705 μm long beam, increasing the role of aerodynamic damping. As seen in the figure, the over-prediction of damping results in several of the models preventing contact completely. The only two models that accurately predict the beam’s motion are the Unsteady Reynolds Equation [6], [3] and the Near-Contact model presented in this work. Table III lists the RMS error for the prediction of contact and release times.

It was found that a 5% uncertainty in the input acceleration yielded an average error of 1.0 μs for prediction of contact time and 0.56 μs error for the predicted release time for the 705 μm beam. Similarly, for the 527.5 μm beam, a 5% acceleration uncertainty yielded an average contact time error of 3.7 μs and a release time error of 2.8 μs.

While the Sadd-Stiffler model [6], [3] correctly predicts the times for the contact and release events, there are inaccuracies when the beam is further from contact. Data for the vibration of a rectangular cantilever beam near a surface was presented by Lee et al. [23]. Figure 14(b) shows a comparison of several damping models with the data for the 300 μm beam presented. The measurements were made for a rectangular cantilever beam 18 μm wide and 2.25 μm thick for each of the first three vibrational modes. As seen in the figure, the Sadd-Stiffler model deviates significantly from both the other models and the measured points. The percent errors for each vibrational mode are listed in Table IV. For the longer beam, the differences are even more pronounced. Figure 14(a) shows...
A new model of microscale aerodynamic damping for the near-contact regime has been formulated based on Boltzmann-ESBGK simulations of rarefied gas flow around moving microbeams. The coupled fluid-structure, dynamics simulations with the new model accurately predict the contact time for microscale cantilever beams under high-g external acceleration. The observed difference between the measured and predicted release times for the cantilever beams can be attributed to adhesion after contact. In addition, the new model accurately predicts the values observed by Lee et al. [23].

V. CONCLUSION

A new model of microscale aerodynamic damping for the near-contact regime has been formulated based on Boltzmann-ESBGK simulations of rarefied gas flow around moving microbeams. The coupled fluid-structure, dynamics simulations with the new model accurately predict the contact time for microscale cantilever beams under high-g external acceleration. The observed difference between the measured and predicted release times for the cantilever beams can be attributed to adhesion after contact. In addition, the new model accurately predicts the values observed by Lee et al. [23].

REFERENCES


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