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RESEARCH ARTICLE

Geometrical design of thin film photovoltaic modules for improved shade tolerance and performance
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ABSTRACT
Partial shading in photovoltaic modules is an important reliability and performance concern for all photovoltaic technologies. In this paper, we show how cell geometry can be used as a design variable for improved shade tolerance and performance in monolithic thin film photovoltaic modules (TFPV). We use circuit simulations to illustrate the geometrical aspects of partial shading in typical monolithic TFPV modules with rectangular cells, and formulate rules for shade tolerant design. We show that the problem of partial shading can be overcome by modifying the cell shape and orientation, while preserving the module shape and output characteristics. We discuss two geometrical designs with cells arranged in radial and spiral patterns, which (i) prevent the reverse breakdown of partially shaded cells, (ii) improve the overall power output under partial shading, and (iii) in case of spiral design, may additionally improve the module efficiency by reducing sheet resistance losses. We compare these designs quantitatively using realistic parameters and discuss the practical aspects for their implementation. Copyright © 2013 John Wiley & Sons, Ltd.

Supporting information may be found in the online version of this article

KEYWORDS
partial shading; thin film PV module; module design; module efficiency; sheet resistance

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1. INTRODUCTION
In the last decade, thin film photovoltaic technology has evolved from lab to commercial scale and is now an important segment of the photovoltaic market [1]. Moreover, TFPV promises to offer additional advantages in the growing building integrated photovoltaic sector [2]. This change has been driven by the advances in large scale manufacturing of TFPV, most notably the monolithic fabrication technique [3]. This approach involves successive deposition of contact and semiconductor layers on large area glass or flexible substrates, interspersed with scribing steps to form a series connected module, as shown in Figure 1 [4,5]. The resulting module configuration has thin rectangular cells next to each other, connected in series with metal interconnects (Figure 1). This monolithically integrated manufacturing process keeps throughput high and provides important cost advantage to TFPV technologies.

This monolithic fabrication, however, introduces a unique set of challenges for TFPV module operation under real world operating conditions. One such challenge arises in the case of partial shading of the modules by shadows cast by nearby objects and structures [6,7]. This problem is by no means limited to TFPV technologies, having been first observed for crystalline cells in space applications [8]. In the case of crystalline PV modules, however, the manufacturing process allows for incorporation of bypass diodes inside the module [9], or alternate wiring schemes for cells [10,11], which can mitigate the impact of shading. These approaches are not easily transferrable to monolithic TFPV modules, because the scribing-based interconnection scheme makes it difficult to integrate bypass diodes [12], or alternate wiring schemes [13]. Another aspect of partial shading unique to TFPV modules is related to the rectangular geometry of individual cells. The analysis of partial shading in monolithic TFPV using 2D circuit simulations was reported in [14]. It shows that the interplay between rectangular shadows and (series-
In this paper, we show that this monolithic fabrication technique can be adapted to create different cell geometries for improving shade tolerance, while preserving (even improving) module performance. These geometrical transformations, however, must ensure that (i) the series connection of cells is preserved, (ii) all cell areas must remain identical, and (iii) the rectangular module shape is unchanged. These design constraints ensure that the nominal module output remains unchanged after geometrical transformation of cell shape. We use 2D SPICE circuit simulation to compare various designs quantitatively for their resilience to partial shading, as well as change in their nominal efficiency.

We begin in Section 2 by describing the simulation framework, which will be used to assess the shadow effect, as well as module performance. Next, in Section 3, we discuss the shadow effect in typical TFPV modules with rectangular cells. We highlight the key reliability issues of partial shading and formulate the rules for shade tolerant design. In Section 4, we discuss two geometrical shade tolerant designs for TFPV modules and evaluate their reliability and performance under various shading scenarios. Next, in Section 5, we analyze the sheet resistance losses in these non-rectangular cells for assessing the efficiency of these new module designs. Finally, in Section 6, we will summarize the results and discuss the practical considerations of implementing the proposed designs.

2. SIMULATION FRAMEWORK

In order to study the effect of partial shading in realistic systems, we consider a string of series connected modules (each with an external bypass diode [15]) connected to a string inverter, as shown in Figure 2(a). The number of modules in the string is adjusted to obtain the desired string DC output voltage, which is assumed to be kept fixed at all times [16]. We will evaluate the string output power, as well as the voltage across shaded cells, when one of the modules in the string is partially shaded. For the partially shaded module, we consider rectangular shadow at the bottom left corner (see Figure 2(b)), and all possible shade configurations are explored by varying the shadow dimensions \( L_{\text{sh}} \times W_{\text{sh}} \) from 0 (no shade) to the module dimensions \( L_{\text{module}} \times W_{\text{module}} \) (fully shaded). It is assumed that the shaded region only receives diffused light so that the light intensity in the shaded region is 20\% of the normal sunlight intensity [17].

2.1. Analysis of shadow effect

For exact assessment of the effect of a partial shadow covering only a part of cell width (as in Figure 2(b)), a full 2D circuit simulation of the module is required [18]. We find, however, that even when only a part of cell area is shaded, the voltage across the cell width is essentially constant; as shown in the illustrative simulation with full 2D analysis of shadow provided in Section S1 of the Supporting Information. Therefore, an accurate analysis of shadow effect is possible using a simplified 1D equivalent circuit of the partially shaded module, shown schematically in Figure 2(c). This technique is also suitable for simulating the shadow effects for arbitrary cell shapes of shade tolerant designs, as discussed in Appendix A. Simulation results comparing the 2D and 1D simulation approaches are given in Section S1 of the Supporting Information, and additional details can be found in [14]. This approach allows us to calculate the individual cell voltages, \( V_{\text{cell}} \), for all the cells in the partially shaded module (see Figure 2(c)) and to find the minimum cell voltage, \( V_{\text{cell}}^{\text{min}} \), for different shading scenarios.

The objective of a shade tolerant design is to restrict \( V_{\text{cell}}^{\text{min}} \) to small values and prevent catastrophic reverse breakdown of shaded cells [19,20]. From this circuit simulation, we also calculate the string output power, \( P_{\text{string}} \), for various shading scenarios. The shade tolerance of the designs will be evaluated by comparing the \( V_{\text{cell}}^{\text{min}} \) and \( P_{\text{string}} \) values for each design under all possible shading scenarios.

In order to compare the module designs quantitatively, we chose single junction a-Si:H technology as a reference [21]. We use a physical equivalent circuit representation for the a-Si:H cells including the voltage-dependent collection [22], generation-dependent recombination [23], non-Ohmic shunt [24], and diode current components, including the reported breakdown voltage for the cells [25]. The material parameters and details of the subcell equivalent circuit were published earlier [14,18]. For simplifying the geometrical transformations, we have assumed that the module is a square, with \( L_{\text{module}} = W_{\text{module}} = 128 \text{ cm} \) and \( N_{\text{series}} = 128 \) (although for a typical module \( L_{\text{module}} = 104 \text{ cm} \), \( W_{\text{module}} = 120 \text{ cm} \), with \( N_{\text{series}} = 104 \) [21]). This slight modification makes no difference to the conclusions of this work. With this module geometry and cell parameters, six modules connected in a string produce an output voltage of 535 V DC, with all modules operating at their maximum.

![Figure 1. Schematics of the side view of a TFPV module, showing the laser scribes (P1/P2/P3) and various layers used for creating the series connections (top), and the top view of the resulting series connected (arrows show direction of current flow) cells showing the rectangular cells forming the module bot.](image-url)
power points. We consider the situation when one of the modules is partially shaded and evaluate the worst case voltage developed across cells inside the shaded module, as well as the string output power under each shading scenario.

2.2. Sheet resistance analysis

Although the equivalent circuit approach using SPICE is sufficient for evaluating the shadow effect, a second (complementary) simulation approach is necessary to calculate the power dissipation in the sheet conductors of cells with arbitrary geometry. We need to consider the 3D current flow (shown for typical rectangular TFPV cell in Figure 2(d)), showing the current entering from the right side of the bottom metal contact, then the bulk current flow in the semiconductor, and finally collected at the left edge of the top TCO contact.

Assuming that the metal sheet resistance is negligible (see Appendix B), we can solve the continuity equation for 2D current flow $J_{xy}$ in the top TCO contact only, with photocurrent $J_{ph}$ injected at all points of the sheet conductor (see Figure 2(e)) so that $\bar{V}_{xy} J_{xy} = J_{ph}$. Writing $J_{xy} = \sigma \bar{E}_{xy} = J_{ph} = \bar{E}_{xy}/R_{\Omega}$, with the sheet resistance $R_{\Omega}$ of the TCO layer and $\bar{E}_{xy} = -V_{xy} \phi$ in terms of potential $\phi$, we obtain

$$V_{xy}^2 \phi = -J_{ph} R_{\Omega}$$

This equation can be solved numerically with appropriate boundary conditions, and the solution can be used to calculate the resistive power dissipation in the TCO layer as

$$P_{dis} = \int_{A_{cell}} \bar{J}_{xy} \bar{E}_{xy} dS = \int_{A_{cell}} |\bar{E}_{xy}|^2 / R_{\Omega} dS$$

where $\int_{A_{cell}} dS$ denotes the surface integral over the entire area of the cell $A_{cell}$. Additional details regarding the numerical solution setup can be found in Appendix B. With these tools, we are now ready to examine the shadow behavior and module efficiency for various module types. We begin by analyzing conventional rectangular module geometry.

3. SHADOW EFFECT AND DESIGN RULES

In this section, we analyze the geometrical aspects of shadow effect in typical TFPV modules with rectangular cells and outline the design rules for shade tolerant design.

3.1. Typical module with rectangular cells

Figure 3 shows the results of the SPICE simulation analyzing the effect of partial shading, by plotting the worst case reverse stress $V_{\text{min}}^{\text{cell}}$ at the cell level and the string power output $P_{\text{string}}$ for all possible shadow sizes. Each point on the color plot represents the effect of a shadow of certain size, and the color denotes the corresponding $V_{\text{min}}^{\text{cell}}$ (Figure 3(a)), or $P_{\text{string}}$ (Figure 3(b)). The simulation shows that although the worst case reverse stress occurs for small wide shadows (e.g., along the bottom edge of the module), the external bypass diode turns on for only a fraction of shading scenarios with large shadows (highlighted by dashed polygon in Figure 3(a), (b)) and does not prevent the worst case reverse stress [14]. Some interesting insights about the shadow effect are also apparent from the plots. First, note that a symmetric edge shadow (with all cells shaded equally, marked magenta in Figure 3(a)) causes no reverse stress and relatively small loss of output power. On the other hand, an asymmetric shade at the edge (marked red in Figure 3(a)) causes reverse breakdown of shaded cells and reduces
The fundamental insight from the previous analysis is the observation that a good overlap of a rectangular shadow with rectangular cell is the cause of worst case shadow stress, and although shadows are generally rectangular (buildings, poles, etc.), the cells need not be. Their shape can be modified in a way that reduces the probability of perfect overlap between a rectangular shadow and a non-rectangular cell.

4. SHADE TOLERANT DESIGN

The simplest geometry that satisfies the design constraints outlined in the previous section is formed by modifying each rectangular cell into two triangular half-cells and arranging them in a radial pattern, as shown in Figure 4 (a). The current flow patterns in two types of half-cells are also shown for comparison. In this arrangement, the terminals need to be put in diagonally, as shown in the schematic, and the current flows in a curved path dictated by the series connection. Note that the triangle dimensions and orientations can be chosen to ensure that all the cells are of equal area, the number of series connected cells is the same, and the square module dimensions are preserved (see Section S3 of Supporting Information for details on how to generate the radial geometry).

The new design can now be assessed using the same SPICE simulation framework (see Appendix A for details). The results of the simulation are shown in Figure 4(b), which shows the color plot of $V_{\text{min}}^\text{cell}$ under partial shading, for all different shadow sizes for the radial design. We find that the $V_{\text{min}}^\text{cell}$ value for the radial design is always above $-4.7 \text{ V}$, thus preventing any permanent damage from partial

output power dramatically. This is because, in the asymmetric case, the fully illuminated cells continue to drive the current through the shaded cells and push them in reverse bias. As the number of shaded cells in the asymmetric case increases, however, the stress on individual cells is reduced ($V_{\text{min}}^\text{cell}$ becomes less negative), because the reverse voltage is equally divided among the shaded cells.

3.2. Geometrical design rules for shade tolerance

On the basis of these observations, we can formulate a set of design rules for a shade tolerant design of a TFPV module. These can be summarized as follows:

1. The strong difference in effect of symmetric versus asymmetric shading suggests that a shade tolerant design must be free from this orientation dependence.
2. The worst case of thin asymmetric shadow must be avoided to prevent permanent damage.
3. Simultaneous shading of multiple cells distributes the reverse bias, and if it can be utilized by the new design, permanent damage to shaded cells can be averted.

It is easy to see that if the rectangular cells of TFPV modules could be arranged radially (like the blades of a fan), the worst case shadow stress can be reduced, because a rectangular shadow will now cover small areas of multiple cells. This, however, cannot preserve the series connection or the rectangular module shape. Fortunately, the monolithic fabrication allows us to change the cell shape in a way that will satisfy all these constraints and preserve the module shape and output characteristics at the same time.

Figure 3. (a) Schematic of a typical TFPV module with rectangular cells. Arrows show the direction of current flow in $N_{\text{series}}$ series connected cells, each with area $A_{\text{cell}}$ (see bottom schematic for 3D current flow pattern at cell level). 2D color plots of (b) minimum cell voltage $V_{\text{min}}^\text{cell}$ and (c) string output power $P_{\text{string}}$ for all possible rectilinear shadows on a typical rectangular module. Each point on the plot corresponds to a shadow of length $L_{\text{sh}}$, and width $W_{\text{sh}}$, and the color denotes the worst case reverse stress $V_{\text{min}}^\text{cell}$ (color bar in V), or power output $P_{\text{string}}$ (color bar in W). The dashed polygon highlights the cases where external bypass is on. Schematic in (a) also defines symmetric (magenta) and asymmetric shading (red); the corresponding $V_{\text{min}}^\text{cell}$ and $P_{\text{string}}$ are highlighted with arrows on respective plots.
Also note that the radial symmetry ensures that there is no difference between symmetric and asymmetric shading scenarios (evident from the schematic in Figure 4(a)). The corresponding $P_{\text{string}}$ values under different shading scenarios for this design are shown in Figure 4(c), showing marked improvement for smaller shadows, and values are comparable for large shadows. Moreover, Figures 4(b) and 4(c) show that for this radial design, only large shadows which are less probable cause any significant output power loss or reverse stress, but the more probable smaller shadows are rendered practically harmless. Thus, we see that the radial design can significantly improve the shade tolerance of the module.

So far, we have compared the shade tolerance of different designs subjected to shadows that are anchored to the edge of the module, as would be typical for shadows cast by nearby modules or other objects [26]. It is apparent from the schematics in Figure 4(a), however, that a thin long shadow, placed along the axis of any of the radial cells would be comparable with the worst case (asymmetric) shading for rectangular cells (see Figure 3). Such diagonal shadows are improbable in a properly installed system [2], but it does reflect a limitation of the radial design. Section S2 in the Supporting Information explores various worst case scenarios for different shadow positions and orientations for the three designs and illustrates this limitation of the radial design. Moreover, even this improved shade tolerance of radial design is achieved at a cost of higher resistive losses in the triangular cells. This is because the cells have to be wider near the base of the triangle, thereby increasing the path length for the photocurrent. A detailed analysis and comparison of the resistive losses is presented in Section 5.

In order to reduce the resistive losses, we note that the average cell width should be reduced so that photocurrent flows over shorter distances before being collected. This must be carried out while keeping the cell area constant, which will be possible by making the cell shape longer and thinner. And, the problem posed by diagonal shadows can be avoided if the cells themselves are non-rectilinear so that the asymmetric shading will never arise. We show next that both these objectives are achieved by a spiral arrangement of curvilinear cells.

### 4.2. Spiral design with curvilinear cells

Figure 5(a) shows the schematic of the spiral design, with the same $N_{\text{series}}$ series connected cells, with the curved positive and negative terminals. Each cell is a concave polygon with varying curvature, constructed so that their areas are identical, while preserving module shape and size. Details regarding constructing this geometry are given in the Section S4 of Supporting Information. These curved cells can be considered stretched and twisted forms of the triangles used in the radial design, arranged within the same rectangular module. Therefore, the current flow direction is also similar, as shown in Figure 5(a).

From the schematic in Figure 5(a), it is apparent that this spiral design retains the advantages of the radial design in terms of shade tolerance. This is validated from the circuit simulation result in Figure 5(b), which shows that the worst case $V_{\text{cell}}^{\text{min}}$ from shading in this case is limited to $-4.2$ V and the overall number of cases that result in reverse bias is also reduced. Section S2 of the Supporting Information shows some additional results for different shadow positions and orientations, which affirm the robust shade tolerance of the spiral design in more general shading scenarios. Figure 5(c) also shows the improvement in string output...
power for the various shading scenarios, which also shows that external bypass diode is activated only for very large shadows. Therefore, with the spiral design, it may be possible to avoid the external bypass diode altogether, enabling a truly monolithic TFPV module, and eliminate the considerable reliability issues associated with the external bypass diodes [27]. Moreover, from the schematic in Figure 5(a), it is apparent that because of the curved cell shape, the asymmetric shading problem cannot arise for rectilinear shadows of any orientation. Finally, we will show in Section 5 that the curvilinear cell shape also reduces the overall resistive power loss compared with the rectangular cells and improves module efficiency.

5. MODULE EFFICIENCY

In this section, we evaluate how a change in cell shape affects the normal (no shade) operating performance of the module. We demonstrate that it is possible to achieve shade tolerance without a tradeoff in module efficiency, because of reduced sheet resistance losses in non-rectangular cells of the shade tolerant designs. We also calculate and compare the dead area losses in different designs to estimate the overall efficiency change for new designs.

5.1. Sheet resistance loss calculation

In order to compare sheet resistance losses in cells with different geometries, we solve Equation (1) for all three geometries and calculate the power dissipation per unit area in the sheet conductors. To make a realistic comparison, we assume typical values for a-Si:H technology in our simulations, namely $R_{\text{TCO}}^{\text{cell}} = 10 \Omega$/$\text{sq}$ [21] and $J_{\text{ph}}^{\text{cell}} = 15$ mA/$\text{cm}^2$ [28]. A more detailed discussion about the numerical calculation of the power dissipation in sheet resistors, and its relation to previous approaches, is provided in Appendix B.

Color plots at the top of Figure 6 show the power dissipation per unit area in the TCO layer, obtained from the numerical solution to Equation (1). The plots show the simulation results for the submodule schematics shown in Figures 3(a), 4(a), and 5(a), respectively. For the rectangular geometry, the power dissipation profiles are identical for all cells. The dissipation per unit area increases monotonically toward the current collecting (top) edge of each cell. This is because the regions near the current collecting edge carry the total current generated in the area below, resulting in higher power loss per unit area (see Appendix B for details). Integrating over the rectangular cell area using Equation (2), we get total resistive power dissipation per cell as $P_{\text{dis}}^{\text{rec}} = 96$ mW.

For the two shade tolerant designs, the $P_{\text{dis}}$ values will be different for each cell, because the shape and orientation (and hence the current flow pattern) are different for each cell. As shown in the color plots in Figure 6, the power dissipation per unit area in wider regions is higher than that in the thinner regions, as the current collecting edge near the wider areas collects more photocurrent. As a consequence, the wider triangles in the radial design (close to the horizontal and vertical axes) dissipate twice the power compared with the rectangular cells, whereas the dissipation in the thin diagonal cells is almost equal to the rectangular case (compare the $P_{\text{dis}}^{\text{rad}}$ values with $P_{\text{dis}}^{\text{rec}}$ from the plot in Figure 6).

This geometry dependence in resistive power dissipation is even more interesting in the curvilinear cells of the spiral design, where the thinner regions near the center
dissipate less power compared with the wider areas toward the edges. Moreover, the pattern in $P_{\text{spi}}$ across different cells is also the same, and the longer diagonal cells dissipate less power compared with wider cells near the middle. Overall, however, the cells in radial design dissipate less power in sheet resistance compared even to the rectangular cells (see the $P_{\text{spi}}$ values in the plot in Figure 6). Correspondingly, the total resistive loss in the spiral module is less than that in rectangular case. This reduction in $P_{\text{spi}}$ is due to the fact that the perimeter of these curvilinear polygonal cells is larger than the rectangular cells, while $A_{\text{cell}}$ is the same. Therefore, the width of each cell is smaller (on an average) compared with rectangular cells, which reduces the overall power dissipation.

5.2. Efficiency comparison

In order to compare the efficiencies of different shade tolerant designs, we must account for the increased “dead area” due to longer scribe lines in the new designs. We can calculate the total length of scribes required for each design numerically. Using this estimate of dead area, along with calculation of the total power dissipation for different $R_{\text{TCO}}$ values, we determine the exact change in module efficiency as a result of shade tolerant design. For a-Si:H technology considered here with 9.8% cell efficiency (lab scale) [28], the absolute change in module efficiency ($\Delta \eta = \eta_{\text{design}} - \eta_{\text{rect}}$) is shown for radial (dashed lines in Figure 7) and spiral (solid lines in Figure 7) designs, as function of sheet resistance and dead region width. As expected, the module efficiencies for the radial design are always lower, owing to higher sheet resistance as well as dead area loss. For the spiral design, however, the efficiency change is mostly positive, as the improvement in sheet resistance losses effectively compensates the active area loss due to longer scribe lines. Specifically, Figure 7 highlights that the efficiency change is positive for the spiral design for current state of the art processes, where the dead region width is 250 $\mu$m [29], and TCO sheet resistance is 10 $\Omega$/sq. [21]. Furthermore, advanced techniques such as pointwise interconnection [29] are directly applicable to the shade tolerant designs and will further increase the efficiency gain for spiral designs by reducing the dead area loss even more.
In the previous sections, we have demonstrated that a geometrical approach to module design can not only alleviate the problem of partial shading in TFPV modules but can also enhance the overall module performance. We believe this approach toward module design is practically viable and offers attractive improvements without requiring significant tradeoffs. We would like to emphasize that while the calculations presented here were carried out for a typical a-Si:H solar cell and module, the conclusions are equally valid for all monolithic TFPV technologies. For other TFPV technologies, the exact cell characteristics including the dark and light IV behavior and reverse breakdown voltages would change. Consequently, the exact values of $V_{\text{min}}$ will change depending on the reverse characteristics and breakdown voltage for the particular technology. The general conclusions about the shadow size and orientations with respect to cell shape, however, will remain unchanged for any other thin film technology.

From a practical standpoint, laser scribing is the most suitable technique for manufacturing of these non-conventional cell geometries. Laser scribing has been used extensively for thin film Si technologies [30,31], and it is being actively developed for other polycrystalline TFPV technologies [5,32,33]. We believe that it should be possible to adapt the scribing methods for creating the proposed non-rectangular cell geometries, possibly through a combined motion of the laser head and rotation of the platform carrying the module. For the spiral design, a curvilinear bus bar will also have to be used, which may be accomplished by cutting the metal ribbon in desired shape. A potential cause of concern is that the longer scribe lines in radial and spiral designs may result in increased edge shunting. It has been shown for optimized laser scribing process, however, the edge shunts are not a major concern [34], and the random shunt formation across the cell surface is the dominant shunting mechanism [35].

Finally, we note that geometrical design in different guises has been used for improving the photovoltaic performance on different levels. The prominent examples of this include various light trapping schemes (at the cell level) [36] and the recent 3DPV approach to module arrangement (at the module level) [37]. We feel that the shade tolerant design proposed in this work is in the same vein and utilizes geometry in a unique way to address an important reliability and performance issue in TFPV modules.

### 7. CONCLUSIONS

In this paper, we propose a new geometrical design approach for TFPV modules, which provides a novel method for improving their shade tolerance and overall efficiency. We illustrate the geometrical aspects of partial shading and show how it can be overcome by breaking the symmetry in cell shape and orientation. We also demonstrate how the new cell geometries can reduce the power dissipation in the sheet conductor, using full 2D analysis for sheet resistance loss. We provide the spiral design as a realization of this design approach, which achieves the shade tolerance, as well as improved performance for typical parameter values. We also survey the practical aspects associated with implementation of this approach and find that the state of the art instrumentation is fully capable of implementing these designs, without requiring any retooling or incurring significant extra cost.

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**APPENDIX A**

In order to simulate the behavior of a partially shaded module, we use a 1D equivalent circuit. We have shown in Ref. [14] that even for a shadow covering only a part of the cell area, the voltage developed across the shaded cell is quite uniform. Therefore, we can simplify the simulation of partially shaded module by creating a 1D equivalent circuit of $N_{series}$ series connected cells. The photocurrent of each of these cells is determined by the amount of area shaded $A_{sh}$ and the photogeneration current in the shaded area ($J_{ph,sh}$). This method is applicable for any arbitrary cell or shadow shape; therefore, for $N_{series}$ cells of a module with arbitrary shape and orientation $C_i$ and shadow of a given size and shape $S$, we need to find their intersection and shaded areas as

$$A_{sh,i} = a(C_i \cap S)$$  \hspace{1cm} (A1)

where $a(\cdot)$ denotes the area of a given shape and the intersection determines the region of cell $C_i$ covered by the shadow $S$ (see schematic in Figure A1(a)). Now, we can calculate the current output of each cell by using photogeneration in shaded ($J_{ph,sh}$) and unshaded regions ($J_{ph}$) as

$$J_{sh,i} = J_{ph,sh}A_{sh,i} + J_{ph}(A_{cell} - A_{sh,i})$$  \hspace{1cm} (A2)

![Figure A1](image-url)
for the module with \( N_{\text{series}} \) solar cells with different photocurrent output (see Figure A1(b)), each of which is represented by an appropriate equivalent circuit as determined by the technology under consideration (a-Si:H in this case [23]). We assume all cells have identical \( I-V \) characteristics, with the photocurrent as the only varying parameter, as determined by the amount of shading. We can simulate this partially shaded module, with external bypass diodes, in the string topology (Figure 2(a)) using SPICE and obtain the operating voltage of each cell inside the partially shaded module, for any given shadow dimension. The minimum of these cell voltages (\( V_{\text{mp}}^{\text{cell}} \)) is calculated for all possible shading configurations. This value is compared for different designs, as a measure of its shade tolerance. From this simulation, we also obtain the DC power output of the string, for different shading conditions, and can identify when external bypass will turn on to clamp the loss in power output. Note that in the circuit simulation, the series resistances connecting all cells are kept constant for all three designs. Although this is not strictly the case for non-rectangular cells, it has negligible impact on shadow effects. This is because the current flow in the sheet conductors in the radial and spiral designs is two-dimensional. Therefore, a single net resistance for a whole cell cannot be used, and we must use a full continuity equation solution to determine the resistive dissipation in non-rectangular cell geometries, as discussed in Appendix B.

**APPENDIX B**

In order to analyze current flow in a cell of arbitrary shape, we must consider the 2D continuity equation for current in the TCO and metal layers. This can be carried out by solving a set of coupled continuity equations for both 2D sheet conductors so that

\[
\begin{bmatrix}
\nabla_x J_{xy}^{\text{TCO}} \\
\nabla_y J_{xy}^{\text{metal}}
\end{bmatrix} = \begin{bmatrix}
J_{\text{ph}} \\
- J_{\text{ph}}
\end{bmatrix}
\quad (B1)
\]

Here, \( \nabla_{xy} \) is the divergence in 2D, \( J_{xy} \) is the sheet current per unit width (A/cm), and \( J_{\text{ph}} \) is the photocurrent density in A/cm\(^2\), which is being injected in plane at all points on the TCO or metal. The negative sign of \( J_{\text{ph}} \) denotes current exiting the metal layer into the solar cell, and the positive sign reflects current entering the TCO layer from the solar cell. The local magnitude of the photocurrent is a function of local potential difference between the TCO and metal; that is, \( J_{\text{ph}} = f(\phi_{\text{metal}}^{\text{TCO}}) \), where \( \phi_{\text{metal}}^{\text{TCO}} \) is the voltage difference between the two contacts and \( f(\cdot) \) stands for the solar cell \( I-V \) characteristics. This system of coupled partial differential equations can be simplified considerably, however, if we assume the metal to be far more conductive than the TCO. Now, the metal layer can be assumed equipotential, and we only need to solve one continuity equation for the TCO layer, as

\[
\nabla_{xy} J_{xy} = J_{\text{ph}}
\quad (B2)
\]

A further simplification is possible, if we assume \( J_{\text{ph}} \) to be constant in the voltage range of interest and set it equal to \( J_{\text{ph}}^{\text{mpp}} \) at the maximum power point (\( J_{\text{ph}}^{\text{mpp}} \)) of an ideal cell (15 mA/cm\(^2\) for the a-Si:H technology considered). Note that these simplifications have very little effect on the accuracy of the calculation of sheet resistance loss. Moreover, the error is the same across all cell geometries and will not affect the comparison of different cell geometries.

Now, we use the TCO sheet resistance to write \( J_{xy} = \sigma_{xy} \tilde{\tau}_{xy} \), for sheet conductor with conductivity \( \sigma \) and the in-plane electric field \( \tilde{\tau}_{xy} \). Also, using the relation \( \sigma_{xy} = 1/R_{e} \) in 2D, we can write the equation in terms of contact sheet resistance as \( \nabla_{xy} \tilde{\tau}_{xy} = J_{\text{ph}} R_{e} \). Finally, writing in terms of voltage using \( \tilde{\tau}_{xy} = -\nabla_{xy} \phi \), we have

\[
-\nabla_{xy}^2 \phi = J_{\text{ph}} R_{e}
\quad (B3)
\]

In this setup, current is injected at all points of the TCO and exits at one of the edges, which is connected to the metal contact of the next cell. Our assumption that the metal is highly conductive ensures that the voltage at the current collecting edge is edge is kept constant to \( V_{\text{mp}}^{\text{cell}} \) (Dirichlet condition), while all other boundaries are at open circuit condition (Neumann condition), as shown in the schematic in Figure B1(a). With these boundary conditions, Equation (B3) can be solved numerically, for any arbitrary 2D geometry with a finite element PDE solver, and the total power dissipation can be calculated using Equation (2). This formulation is similar to the one used for crystalline cells [38,39] and is a generalization of the piecewise circuit approach used for rectangular solar cells [40,41].

![Figure B1](image)

**Figure B1.** (a) Schematics of the cell shapes for the different designs with the current collecting edge highlighted in black and the direction of current flow shown by the arrows. (b) The distribution of resistive power dissipation per unit area for the different cells obtained from numerical solution of Equation (2) (color bar in mW/cm\(^2\)).

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It can be shown easily that for the rectangular cells, the solution to Equation (B3) reproduces the results of the piecewise approach. As shown in the schematic in Figure B1, for the rectangular cell, there is no current flow in the horizontal (\(x\)) direction. Therefore, we can write the solution to Equation (B3) in 1D as

\[
\phi(x, y) = -\frac{J_{ph} R_c}{2} y^2 + k_1 y + k_2 \quad (B4)
\]

where \(k_1\) and \(k_2\) are the constants to be determined by the boundary conditions. From Equation (B4), the electric field can be calculated by \(\vec{E} = -\nabla \phi = (J_{ph} R_c y + k_1) \hat{y}\). Applying open circuit boundary condition at \(y = 0\), we have \(\vec{J}(x, 0) = \vec{E}(x, 0) = 0\), which yields \(k_1 = 0\). Now, we can find the power dissipation over the cell area using Equation (2) as

\[
P_{\text{dis}} = \frac{1}{R_c} \int_{x=0}^{x=w_{\text{cell}}} \int_{y=0}^{y=l_{\text{cell}}} J_{ph}^2 R_c^2 y^2 \, dy \, dx = \frac{J_{ph}^2 R_c}{3} \frac{l_{\text{cell}}^3}{w_{\text{cell}}} \quad (B5)
\]

This is exactly equal to the result obtained by taking a limit on the piecewise equivalent circuit approach in [40,41]. We also use this analytical calculation to check our numerical simulation for the rectangular case so that the simulations can be used reliably for more complicated geometries.

Figure B1(b) shows the power dissipation per unit area for the cells with rectangular, triangular, and polygonal geometries, respectively. From these plots, we can see that the power dissipation is dominated by the regions near the current collecting edge (connected to the metal contact). This is because the sheet conductor near the contact carries most of the current generated within the cell area and hence dissipates most of the power. This is very apparent for the triangular cells, where the cell width is larger toward the outer edges, and the corresponding power dissipation is also higher (Figure B1(b)). This asymmetry in dissipation profiles is exploited in the spiral design to reduce overall resistive loss. As the cell shape is elongated while keeping the area constant (Figure B1(a)), the cross sectional width is reduced and each point of the current collecting edge collects current from a smaller region that reduces local power dissipation (see Figure B1(b)). This reduces the overall resistive power loss in these curvilinear cells by a significant amount. This can also be seen qualitatively from Equation (B5), which shows that power dissipation \(P_{\text{dis}}\) has a quadratic dependence on \(l_{\text{cell}}\).

In the triangular geometry, where the average cross sectional distance is between 0 to 2\(l_{\text{cell}}\), the wider regions dissipate four times as much as the thinner regions, causing higher power dissipation in total. This problem is averted for the spiral design because the cells are longer and thinner and the average cross-sectional distance stays below \(l_{\text{cell}}\), which suppresses the total dissipation.