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ON STUDY OF PRESSURE PULSATION USING A MODIFIED HELMHOLTZ METHOD

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ABSTRACT

Pressure pulsation has a critical importance in the design of refrigerant compressor since it affects the performance by increasing over-compression loss, and it acts as a noise and vibration source. For the numerical analysis of pressure pulsation, quasi-steady flow equation has been used because of its easy manipulation derived from the pressure difference. By considering the dynamic effect of fluid, a new Helmholtz Resonator model was also proposed on the basis of the continuity and the momentum equations, which consists of necks and cavities in flow manifolds.

In this paper, a modified new Helmholtz Resonator is introduced to include the gas inertia effect due to the volume decrease in the cavity. Comparisons between this modified new Helmholtz calculations and experimental results show that it is necessary to include the gas inertia effect in predicting pressure over-shooting phenomena at an instant of valve opening state and this modified new Helmholtz model can describe the over-compression phenomena in compressor cylinder.

1. INTRODUCTION

1.1 General

A compressor has a significant role in the refrigeration and air-conditioning industry which uses the freon gas causing ozone destruction. Many studies therefore have been conducted for the improvement of a compressor performance and the replacement of freon gas.

In the design of a hermetic compressor, pressure calculation is essential since pressure pulsation affects the performance and noise of a compressor.

- The cause of pressure pulsation
  Unsteady flows in suction and discharge pipes are generated by the reciprocating action of the piston, aided by the rapid opening and closing of pressure actuated valves. These pressure fluctuations, in turn, affect valve displacements, cylinder pressure and instantaneous fluid flow rates. Pressure pulsation consists of a steady fluctuation due to the periodical motion of refrigerant gas during the suction and discharging process, and a transient pulsation due to the valve motion which controls the discharge of the compressed refrigerant gas.

- Definition and role of a cylinder
  Among hermetic compressors, a rolling piston type rotary compressor has a lot of components. The compressor cylinder is a chamber where a suction and compression cycle of a refrigerant gas occurs, and chamber volume varies according to compression process due to a roller rotation. Therefore, the state of a temperature and pressure of a refrigerant gas changes from low state to high. The volume of a compression chamber of cylinder is described by geometrical relations as a function of rotating angle.

1.2 The calculation method of pressure pulsation in a cylinder

A lot of studies for the pressure calculation of a compressor cylinder has been reported, namely a quasi-steady(QS), Classical Helmholtz Resonator(CHR), and New Helmholtz Resonator(NHR). These conventional methods are summarized below briefly for comparison with our proposed method(MNHR).

- Quasi-steady(QS) [Soedel, 1972]:
  The general calculation method of the pressure of a cylinder is a quasi-steady flow equation. This method calculates the mass flow rate due to the pressure difference between upstream and downstream sides, and it is
convenient to get the density and pressure without a calculation of fluid velocity. However, it is not sufficient for this method to describe a dynamic response of pressure pulsation since it disregard a dynamic behavior of fluid.

- **Classical Helmholtz Resonator (CHR) [Soedel, 1973]:**
  The CHR approximation was proposed as a new convenient approach. The gas is considered to have inertia only in the necks (or connecting passages) and to be inertialess in the cavities (or plenums) because of the relative differences of accelerating level. On the other hand, the gas is considered to be compressible in the cavities, but incompressible in the neck because of the relative differences in volume. One may, therefore, picture the acoustic model as consisting of incompressible plugs of gas in the necks, that oscillate like pistons on springs that are provided by the elasticity of the compressible gas in the cavities.

- **New Helmholtz Resonator (NHR) [Yee, 1983]:**
  Another simplified method is NHR, which has the same assumption of the CHR described above. However, CHR assumes that pressure and velocity amplitudes are small, and all densities are constant, and introduce bulk modulus of elasticity which represents a value of the density times the sound speed squared. But NHR distinguishes a difference of densities at control volumes, all densities are thus not constant. The density of a neck is also obtained by averaging those of cavities.

- **Modified New Helmholtz Resonator (MNHR) [Ma, 1998]:**
  This method is a new proposed calculation approach, to consider a gas inertia effect due to the volume decrease of this cylinder, which is not included in these conventional methods mentioned above (QS, CHR, NHR).

### 2. FORMULATION FOR A DEFORMABLE VOLUME

For the calculation of cylinder pressure using control volume approach, a model of deformable and moving control volume is introduced since the volume of cylinder structure is varying during compressor operation.

#### 2.1 The motion equation in a deformable control volume

The fundamental laws governing the motion of a fluid are generally Lagrangian descriptions; however, when a particular group of particles are not fundamentally interested in, Eulerian descriptions are used, which define a region in space called a control volume and observe the fluid flowing through it. Therefore we introduce a mathematical description of fluid motion equation on a deformable and moving control volume, from this Eulerian point of view [Potter, 1975].

Let $N_{sys}$ be the extensive property in system. It would be calculated by integrating its corresponding intensive property $\eta$ over the volume of interest. The time rate of change of $N_{sys}$ is

$$\frac{D(N_{sys})}{Dt} = \int \eta \rho dV \lim_{\Delta t \to 0} \frac{N_{sys}(t + \Delta t) - N_{sys}(t)}{\Delta t}$$

(1)

The deformable control volume differs from the fixed control volume in that the control boundary is allowed to move, as for a reciprocating piston. A system including control volume moves and deforms from arbitrary time $t$ to $t + \Delta t$, and this relation and defining terms for the following derivation are presented in Fig. 1. Relative velocity effects between a system and a control volume are described when both move and deform. The system boundaries move at velocity $\vec{U}$, and the control surface moves at velocity $\vec{u}_b$. The relative velocity $\vec{u}_r$ thus occurs between the fluid system and the control surface.

The system and control volume are coincident at time $t$. At time $t + \Delta t$, the system will have translated and deformed and control volume will also have translated and deformed, but differently from the system. Referring again to Fig. 1, equation (1) above therefore may be written as

$$\frac{D}{Dt}(N_{sys}) = \frac{d}{dt} \left( \int \eta \rho dV \right) + \int \eta \rho (\vec{u}_r \cdot \vec{n}) dA$$

(2)

Where, $\vec{u}_r$ represents the velocity of the fluid relative to control volume between the control volume and system at time $t + \Delta t$, $\vec{n}$ always points out of the control volume. This equation (2) is often referred to as the Reynolds
transport theorem. The time-rate-of-change term of the basic system-to-control-volume transformation may be reformulated to more easily account for the rate of deformation. Consider a generalized control volume at time \( t \) and let it deform, assuming the position at time \( t + \Delta t \). The time-rate-of-change term is reformulated as follows:

\[
\frac{d}{dt} \int_{cv} \rho \nu dV = \int_{cv} \frac{\partial}{\partial t} (\rho \nu) dV + \int_{cs} \rho (\ddot{u}_b \cdot \ddot{n}) dA
\]  

(3)

Where, \( \ddot{u}_b \) is the velocity of the control surface during the time from \( t \) to \( t + \Delta t \), and indicates the motion of control volume. For a non-deformable control volume, \( \ddot{u}_b \) is everywhere zero and the control volume is fixed in space.

The equation (2) above can be arranged by substituting equation (3) into.

\[
\frac{D(N_{sys})}{Dt} = \int_{cv} \frac{\partial}{\partial t} (\rho \nu) dV + \int_{cs} \rho (\ddot{u}_b \cdot \ddot{n}) dA
\]  

(4)

Where, \( \ddot{u} \) is the total velocity of the fluid with respect to the chosen reference frame in Fig. 1, \( \ddot{u} = \ddot{u}_r + \ddot{u}_b \), during the time from the control volume at \( t \) to the system at \( t + \Delta t \), and thus represents the boundary velocity of system. To get the motion equation, choose \( \eta = \ddot{u} \) as dividing a momentum with a mass, then the equation (4) above is transformed as

\[
\sum F = \int_{cv} \frac{\partial}{\partial t} (\rho \ddot{u}) dV + \int_{cs} \rho \ddot{u} (\ddot{u}_b \cdot \ddot{n}) dA + \int_{cs} \rho \ddot{u} (\ddot{u}_r \cdot \ddot{n}) dA
\]  

(5)

2.2 Application of motion equation on compressor model

The equivalent model, piston-cylinder of compressor as a deformable control volume, is introduced in Fig. 2. This PC model can be interpreted as consisting of two cavities, namely the cylinder \( V_1 \) and the muffler \( V_2 \), and of two necks, namely the clearance volume \( V_3 \) and the muffler discharge hole \( V_4 \). Subscripts denote the component position of cavities and necks. \( V \) means a cavity, \( u \) is velocity, \( \rho \) is density and \( A \) is an effective area. At this PC model, volume of cavity \( V_1 \) is not constant due to movement of rolling piston which has velocity \( u_p \).

In this paper, in order to include the gas inertia due to decreasing the volume, the motion equation in the cavity \( V_1 \) has to be rearranged using equation (5).

\[
\ddot{u}_b = u_p, \hspace{1cm} \ddot{u}_r = u_2
\]  

(6a, b)

where, \( u_p \) is the velocity of a rolling piston in a rotary compressor, and is calculated by dividing the time derivative of a volume with a section area of the roller.

Our new proposed motion equation including the gas inertia term therefore can be obtained by substituting equations (6a, b) into equation (5). The revision of equation (5) gives

\[
\frac{\partial}{\partial t} (\rho \ddot{u}) V + \rho \ddot{u} (\ddot{u}_b \cdot \ddot{n}) A + \rho \ddot{u} (\ddot{u}_r \cdot \ddot{n}) A = \sum F
\]  

(7)

First and second terms in equation (7) are described using equation (6a, b) in case of cavity \( V_1 \) at PC model.

\[
\frac{\partial}{\partial t} (\rho \ddot{u}) V = \left[ \rho \frac{1}{2} \left( |u_p + u_2| \right) \right] V_1
\]  

(8a)

\[
\rho \ddot{u} (\ddot{u}_b \cdot \ddot{n}) A_1 = \rho \left[ \frac{1}{2} |u_p + u_2| \right] V_1
\]  

(8b)
where, $A_1$ is sectional area of roller, $\bar{u}_r A_1 = \dot{V}_1$.

The third term in equation (7) can also be represented as below, since this term indicates the exit of a cavity $V_1$ which does not have the moving velocity of control volume boundary, $\bar{u}_r = u_2$.

$$\rho \bar{u}(\bar{u}_r \cdot \bar{n})A_2 = \rho u_2 u_2 |A_2|$$

(9)

The equation of motion in the cavity $V_1$ can be represented as below, since this term indicates the exit of a cavity $V_1$ which does not have the moving velocity of control volume boundary, $\bar{u}_r = u_2$.

$$\rho \bar{u}(\bar{u}_r \cdot \bar{n})A_2 = \rho u_2 u_2 |A_2|$$

(9)

<The equation of motion in the cavity $V_1$>

By applying conservation of momentum to the first control volume $V_1$, the equation (7) can be written as

$$\frac{d}{dt} \left\{ \rho \bar{V}_1 \left[ \frac{1}{2} |u_p + u_2| \right] + \rho u_2 u_2 |A_2| \right\} = (P_1 - P_{r1})A_2$$

(10)

where, $P_{r1}$ is the entrance pressure of a neck $V_2$ in PC model, and $P_1 - P_{r1}$ means therefore the pressure change across the exit of a control volume $V_1$.

However, the conventional NHR method describes the motion equation in the cavity $V_1$ as another different equation instead of equation (10), since conventional NHR method states that the effect of a gas inertia is negligible on the assumption that the volume of a cavity is larger than that of neck.

$$\rho u_2 u_2 |A_2| = (P_1 - P_{r1})A_2$$

(11)

<The equation of motion in the neck $V_2$>

Since the gas in the neck is considered to be incompressible according to NHR assumption, it can be treated as a rigid body for a given time step, and Newton’s Second Law of motion may be applied in the neck $V_2$.

$$\frac{d}{dt} (\rho u_2 |u_2|) = (P_{r1} - P_3)A_2$$

(12)

This description of a neck is the same since the assumption of MNHR and NHR methods are coincident.

The equation of motion on MNHR approach

Our proposed MNHR approach gives the following equation by adding equation (10) in the cavity $V_1$ and equation (12) in the neck $V_2$.

$$\frac{d}{dt} \left\{ \rho \bar{V}_1 \left[ \frac{1}{2} |u_p + u_2| \right] + \rho u_2 u_2 |A_2| + \frac{d}{dt} (\rho u_2 |u_2|) \right\} = (P_1 - P_3)A_2$$

(13)

A limiting condition is that $u_2$ has to be less than or equal to the speed of sound corresponding to the upstream temperature.

The equation of motion on NHR approach

NHR arrange the following motion equation by adding equation (11) and equation (12).

$$\frac{d}{dt} (\rho u_2 |u_2|) + \rho u_2 u_2 |A_2| = (P_1 - P_3)A_2$$

(14)

The differences between our proposed MNHR and conventional NHR are; MNHR contains the density $\rho_1$, the volume $V_1$ and time derivative of volume $\dot{V}_1$ in a compressor cylinder. And the velocity $u_p$ and acceleration $\dot{u}_p$ of a rolling piston also included. However, conventional NHR ignored these parameters.
The continuity equation of the cavity $V_1$ and $V_3$:

Applying the conservation of mass for the cavity $V_1$ and $V_3$, gives

\[
\frac{d}{dt}(\rho_1 V_1) + \rho_2 u_2 A_2 = 0 \tag{15a}
\]

\[
\frac{d}{dt}(\rho_3 V_3) - \rho_2 u_2 A_2 + \rho_4 u_4 A_4 = 0 \tag{15b}
\]

The equation of motion in the neck $V_4$:

If the same process is applied to the neck $V_4$, by adding the momentum equation of the cavity $V_3$ to that of the neck $V_4$, and by assuming negligible compressible effect of gas, a motion equation is obtained as follows:

\[
\frac{d}{dt}(\rho_4 V_4 |u_4|) + \rho_4 u_4 |u_4| A_4 = (P_3 - P_2) A_4 \tag{16}
\]

where, $P_2$ indicates the ambient or the final pressure of a terminal point in last control volume of PC model.

The density of a neck $V_2$ and $V_4$:

Conventional CHR implies that the density of the assumed incompressible plug of gas is equal to the mean flow density. All densities are therefore replaced by a constant value since CHR assumes that amplitude of pressure and velocity is small. However, NHR states the density of the neck is equal to an average of the densities in the two adjoining cavities in PC model.

\[
\rho_2 = \frac{(\rho_1 + \rho_3)}{2}, \quad \rho_4 = \frac{(\rho_3 + \rho_3)}{2} \tag{17a, b}
\]

The pressure of a cavity $V_1$ and $V_3$:

To relate pressure to mass changes and heat transfer, the first law of thermodynamics could be used. However, compressor designers have traditionally used the polytropic process, since the two descriptions are equivalent if one assumes that the heat transfer plus convected energy is proportional to external work.

2.3 Role and existence of a compressor valve

To describe the fluid flow during operation in an actual compressor, a valve motion must be taken into account, because a gas pulsation or unsteady flow has a relation with the periodical valve motion. A valve in a compressor controls the flow of refrigerant gas: When the pressure of compressed gas is higher than that of discharged gas, it opens and discharges a gas; and it closes in opposite case. A valve protects the incoming back-flow of a discharged gas stayed at the muffler during the compression stage. In an actual valve motion, there is a time delay due to the interaction between the valve motion and the fluid flow around valve. A lot of papers describing the valve motion under the influence of fluid flow were presented, with special attention to the stopper or the retainer [Ma, 1996, 1998].

The main object of this paper is to propose new calculation method by considering a gas inertia effect due to decreasing volume of a cavity, which happens at a compressor cylinder in an actual operation of compressor. The effect of valve motion affecting fluid flow is therefore ignored, only to compare pressure calculation method mentioned above. We assume that fluid flow is generated from only pressure difference between the upstream and the downstream sides, and not affected by an interference due to valve motion. Time delay in fluid motion due to a valve displacement is thus not included in this calculation process. For the modeling of an ideal valve existence of PC model, an ideal valve can be defined with a comparison of pressures between a cavity $V_1$ and $V_3$. If $P_1$, pressure of a cavity $V_1$, is less than or equal to $P_3$, pressure of a cavity $V_3$, then fluid does not flow. Contrary, $P_1$ is greater.
than $P_3$, then fluid flows and this case indicates the valve open state.

3. EXPERIMENT

A pressure pulsation under actual compressor operating condition is measured, to compare our proposed MNHR with other conventional methods: QS and NHR. A test jig of a modified compressor in Fig. 3 which has pressure transducers inside of a compressor shell was fabricated for sensor installation at each part: motor upper and lower part, muffler inside and cylinder inside part. Photo. 1 shows the external view of this test jig of the modified compressor. To measure the practical value of pressure under actual operating compressor with a refrigerant gas as a working fluid, a facility named a calorimeter was utilized, which measures compressor performance and has secondary refrigerant system as an auxiliary cycle.

Initial and boundary conditions follow the ASHRAE(American Society of Heating, refrigeration and Air-conditioning Engineers) compressor test condition in case of R22 freon gas. For the numerical comparison, a cylinder $V_1$ has an initial values of the pressure $P_1$ and density $\rho_1$ at suction side, an ambient boundary values of discharge side has also $P_2$ and $\rho_5$ as indicated at PC model.

4. RESULTS AND DISCUSSIONS

Experimental and numerical results are obtained for comparison of pressure calculation methods between the MNHR and conventional methods: QS and NHR. Pressure oscillation of the cavity 1 is shown in Fig. 4. The present method describes the over-compression phenomena, at the instant of valve opening of a cavity 1, which is not obtained with other conventional methods: QS and NHR. Validity of the new proposed MNHR method is assured by comparing the calculation results to the experimental result as presented in Fig. 4. Only our MNHR method follows the over-shooting phenomena at an instant of valve opening depicted by the experimental result. This means that our proposed present method has a unique characteristic in predicting pressure variation in cavity 1 due to volume decrease according to piston movement of PC model in Fig. 2. This difference between MNHR and conventional methods (QS and NHR) are due to our new assumption that the gas inertia has to be included in cavities of classical Helmholtz resonator model when the volume of cavities such as a compressor cylinder decreases due to a rolling-piston movement.

5. CONCLUSIONS

The pressure pulsation in refrigerant compressor has been studied by various methods. A new pressure calculation method is proposed to include the gas inertia due to a decreasing of volume of capacity in the conventional Helmholtz resonator model by a rolling piston movement. The comparisons with an experimental result show that propose MNHR is better than other conventional QS or NHR in predicting pressure over-shooting phenomena at an instant of valve opening state.

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Soedel W. 1972 Purdue University Short Course Text Introduction to Computer Simulation of Positive Displacement Compressor.
system \((t)\) and control volume \((t)\) - (iir n)dA/\(\Delta t\)

- Element volume from \((6)\)

\[ - (\vec{u}_r \cdot \vec{n})dA\Delta t \]

- Element volume from \((5)\)

\[ (\vec{u}_r \cdot \vec{n})dA\Delta t \]

- Velocity polygon

\[ \vec{u} \]: velocity of fluid element

\[ \vec{u}_r \]: relative velocity with respect to control-volume boundary

\[ \vec{u}_b \]: velocity of control-volume boundary

Fig. 1 The system and the deformable control volume

Fig. 2 PC(Piston - Cylinder) model as a deformable control volume
Fig. 3 Schematic diagram of the modified compressor for measuring of pressure pulsation and valve lift.

Photo. 1 Test jig of modified compressor.

Fig. 4 Comparison of pressures in volume 1 of PC model.