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Abstract: We present a new approach to subwavelength optical confinement, based on hyperbolic media in planar Fabry-Perot geometry. Unlike higher-order resonance modes in indefinite metamaterial cavities, the predicted resonance corresponds to 0th-order mode and can be observed in planar systems. Our approach combines subwavelength light confinement with strong radiative coupling, enabling a practical planar design of nanolasers and subwavelength waveguides.

OCIS codes: (160.3918) Metamaterials; (160.1190) Anisotropic optical materials.

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1. Introduction

Hyperbolic optical metamaterials, highly anisotropic nanostructured composites with opposite signs of the dielectric permittivity in two orthogonal directions, represent one of the most active areas of current research [1–19]. These systems show a wide range of new physical phenomena – from extreme light localization in subwavelength waveguides [2] and resonant cavities [3], to enhanced radiative energy transport [4, 5] and quantum-electrodynamics phenomena [6, 7], and offer many intriguing applications – from far-field imaging with the resolution beyond the diffraction limit [8–10] to new stealth technology [11].

Generally, this behavior relies on two important aspects of the hyperbolic media – the ability to support propagating waves with the wavenumbers unlimited by the frequency [2] (leading to a broadband "super-singularity" [12] in the phonic density of states), and all-angle negative refraction [2,13–16]. In particular, high-wavenumber modes in hyperbolic media allow for light confinement beyond the diffraction limit and enhanced light-matter interactions in "indefinite" resonant cavities – three-dimensional subwavelength resonators formed by a hyperbolic metamaterial (HMM) [3]. However, these are inherently high-order modes with exponentially small radiative coupling in a high-loss environment, making practical applications of such devices a challenging problem.

There is however another aspect of the hyperbolic media which so far received relatively little attention, but which could be just as important: when a given interface of the hyperbolic medium shows positive refraction, the transmitted wave accumulates negative phase, leading to a zeroth-order Fabry-Perot resonance at subwavelength thickness – similarly to resonators based on simultaneously negative permittivity and permeability [20]. As opposed to high-order hyperbolic modes studied earlier [3], such a resonator is refractively coupled to the surrounding medium, and therefore allows an arbitrary degree of radiative coupling (by e.g. forming thin metallic "mirrors" of the required thickness on its surfaces).

In the present work, we introduce the concept and present the theoretical description of the hyperbolic 0th-order resonator. Our approach can combine arbitrary subwavelength resonator dimensions with strong radiative coupling, leading to possible applications in nanolasers and deep subwavelength waveguides.

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2. Zeroth-order trasmission resonance in a Fabry-Perot hyperbolic resonator

Consider a plane electromagnetic wave incident on a planar interface between an isotropic dieletric medium (e.g. air) and a hyperbolic medium with negative in-plane dielectric permittivity component $\varepsilon_x < 0$ and positive normal permittivity component $\varepsilon_y > 0$ (Fig. 1(a)). For a TM (p-polarized) incident plane wave (represented by the magnetic field $\vec{\mathbf{B}} = B_0\ \hat{\mathbf{z}}\ e^{\ i\ (k_x x + k_y y - \omega t)}$ with the wavevector $\mathbf{k} = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}}$ and the angular frequency ω), the associated electric field $\mathbf{E}$ and the Poynting vector $\mathbf{S}$ can be expressed as

$$\vec{\mathbf{E}} \quad = \quad B_0\ \frac{c}{\omega}\left[\frac{k_x}{\varepsilon_y}\hat{\mathbf{y}} - \frac{k_y}{\varepsilon_x}\hat{\mathbf{x}}\right]\exp\left(ik_x x + ik_y y - i\omega t\right), \tag{1}$$

$$\vec{\mathbf{S}} \quad = \quad |B_0|^2\ \frac{c^2}{4\pi\omega}\ \left[\frac{k_x}{\varepsilon_y}\hat{\mathbf{x}} + \frac{k_y}{\varepsilon_x}\hat{\mathbf{y}}\right]. \tag{2}$$

Figure 1(b) shows the directions of the phase velocity (corresponding to the wave vector $\vec{\mathbf{k}}$) and the energy flow (given by the Poynting vector $\vec{\mathbf{S}}$) with respect to the iso-frequency curve of the medium. As refraction at a planar surface preserves the tangential (in-plane) momentum k_x and the direction of the energy flux of the transmitted wave $\vec{\mathbf{S}}_t$ is away from the boundary, for $\varepsilon_x < 0$ and $\varepsilon_y > 0$ the refraction is formally *positive* [2, 13, 14, 21]. However, in this case the wave vector $\vec{\mathbf{k}}_t$ lies in the opposite direction, and a wave propagating in this hyperbolic medium ($\varepsilon_x < 0$, $\varepsilon_y > 0$) will accumulate a *negative* phase – with dramatic consequences for interference and transmission resonances in this system.

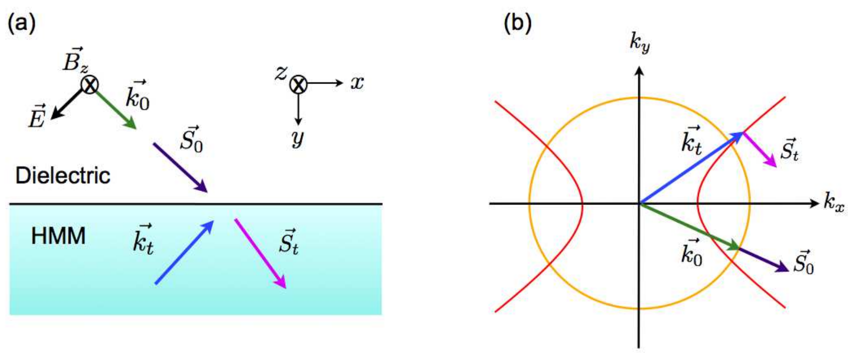

Fig. 1. (a) The phase velocity and Poynting vector in regular dielectric ($y < 0$) and in hyperbolic medium ($y > 0$) with negative in-plane permittivity component ($\varepsilon_x < 0$). The wave vector $\vec{k}_0$ and Poynting vector $\vec{S}_0$ in an isotropic dielectric medium are collinear, while in HMM $\vec{k}_t$ and $\vec{S}_t$ are in two different directions. (b) The iso-frequency curves for an isotropic dieletric medium (circle) and an HMM (open hyperbolic curve) with $\varepsilon_x < 0,\ \varepsilon_y > 0$. The corresponding wave vectors ($\vec{k}_0$, $\vec{k}_t$) and Poynting vectors ($\vec{S}_0$, $\vec{S}_t$) are also shown. Note that refraction from the dielectric into such hyperbolic medium is only possible if the radius of the circle (proportional to the corresponding dielectric permittivity) exceeds the smallest wavenumber supported by the hyperbolic medium.

In a planar Fabry-Perot resonator formed by a "core" (of thickness D) bounded by two thin conducting layers (barriers) with negative permittivity ε_m (Fig. 2(a)), the transmission peak positions can be approximated by

$$k_n D + \phi_r \quad = \quad \pi m, \tag{3}$$

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where k_n is the normal component of the wavenumber in the core, $m = 0, 1, \ldots$ is an integer, and ϕ_r is the reflection phase at the core–conducting layer interface. For a regular dielectric, Eq. (3) implies that $m > 0$ and such Fabry-Perot resonances can only be observed if the thickness D of the dielectric core is on the order of or larger than the light wavelength λ_0.

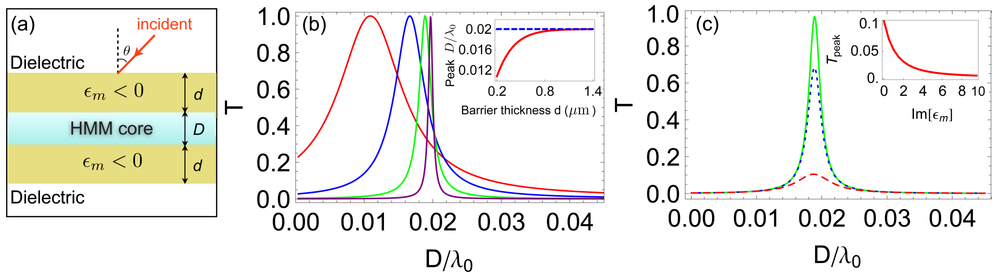

Fig. 2. (a) The schematics of a Fabry-Perot resonator with a hyperbolic metamaterial core. Conducting layers ($\varepsilon_m < 0$) serve as reflective "mirrors" and the wave is incident from the dielectric at the angle θ. (b) Transmission coefficients for a lossless HMM core, with different conducting layer thicknesses: $d = 200$ nm (red curve), 400 nm (blue curve), 600 nm (green curve) and 800 nm (violet curve). The dielectric permittivities of the HMM core are $\varepsilon_x = -2$ and $\varepsilon_y = 1$, and the dielectric permittivity in the conducting barriers is $\varepsilon_m = -10$. The free-space incident wavelength is $\lambda_0 = 10\ \mu$m and incident angle $\theta = 60^\circ$. The inset shows the resonance position as a function of conducting layer thickness, with the dashed blue line corresponding to Eq. (4). (c) Transmission coefficients for the resonator with a lossy HMM core, with $\varepsilon_x = -2 + 0.002i$, $\varepsilon_y = 1 + 0.001i$ (solid green curve), $\varepsilon_x = -2 + 0.02$, $\varepsilon_y = 1 + 0.01i$ (dotted blue curve), and $\varepsilon_x = -2 + 0.2i$, $\varepsilon_y = 1 + 0.1i$ (dashed red curve), while the conducting layer thickness d in this example is 600 nm. The inset shows the variation of the peak transmission with the loss in the metal barries $\mathrm{Im}[\varepsilon_m]$, for the core permittivity $\varepsilon_x = -2 + 0.2i$, $\varepsilon_y = 1 + 0.1i$.

However, if the resonator core is formed by a hyperbolic medium with negative in-plane permittivity, we find that $k_n < 0$, and the resonance condition Eq. (3) allows a zeroth-order ($m = 0$) resonance. When the metal barrier thickness d is on the order of or larger than the corresponding penetration depth, for this resonance we obtain

$$\left.\frac{D}{\lambda_0}\right|_{\text{res}} = \frac{1}{2\pi\sqrt{\varepsilon_x - \frac{\varepsilon_x}{\varepsilon_y}\varepsilon_d(\sin\theta)^2}} \mathrm{Arg}\left[\frac{-\varepsilon_m\sqrt{\varepsilon_x - \frac{\varepsilon_x}{\varepsilon_y}\varepsilon_d(\sin\theta)^2} + \varepsilon_x\sqrt{\varepsilon_m - \varepsilon_d(\sin\theta)^2}}{\varepsilon_m\sqrt{\varepsilon_x - \frac{\varepsilon_x}{\varepsilon_y}\varepsilon_d(\sin\theta)^2} + \varepsilon_x\sqrt{\varepsilon_m - \varepsilon_d(\sin\theta)^2}}\right], \quad (4)$$

which leads to transmission resonances at deeply subwavelength geometry. This behavior is illustrated in Figs. 2(b) and 2(c), where the transmission is shown as a function of D (in units of λ_0) calculated for the parameters typical for mid-IR hyperbolic metamaterials based on semiconductor components [16]. Figure 2(b) corresponds to the lossless limit, showing the shift of resonance position with the change of the conducting barrier thickness d. The inset in panel (b) illustrates the resonance position as a function of d, with the dashed blue line corresponding to the approximation of Eq. (4). As expected, with the increase of the conducting barrier thickness d, the resonance peak position is quickly approaching the theoretical prediction by Eq. (4). Figure 2(c) incorporates the effect of material losses in both the hyperbolic metamaterial core (see the main panel of Fig. 2(c)) and in the surrounding metal barriers (inset to Fig. 2(c)), as substantial absorption is inherent to all existing types of hyperbolic metamaterials. While absorption

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certainly reduces the transmission intensity, Fig. 2(c) clearly shows a pronounced 0th-order peak, with its position virtually unchanged with respect to the corresponding "lossless" value.

3. Zeroth-order resonance in a multilayer HMM core

In contrast to 3D optical metamaterials where unit cell size is on the order of the wavelength in the component materials [22–25], planar metamaterials take advantage of fabrication technologies such as molecular-beam epitaxy [26] and chemical-vapor deposition [27] leading to the unit size on the scale of less than one-tenth of the wavelength [16]. As a result, such planar hyperbolic metamaterials (especially in the mid-IR range [16]) generally closely approach the predictions of the effective medium theory (EMT) [28] based of the effective dielectric permittivity tensor in terms of its conductive (permittivity ε_c) and dielectric (permittivity ε_d) components. In particlar, the tangential (in-plane) permittivty (ε_n) and normal permittivty (ε_τ) for a planar HMM with 1:1 conducting and dielectric layer thickness ratio can be expressed by

$$\varepsilon_n(\omega) = \frac{\varepsilon_c(\omega)+\varepsilon_d(\omega)}{2}, \tag{5}$$

$$\varepsilon_\tau(\omega) = \frac{2\varepsilon_c(\omega)\varepsilon_d(\omega)}{\varepsilon_c(\omega)+\varepsilon_d(\omega)}. \tag{6}$$

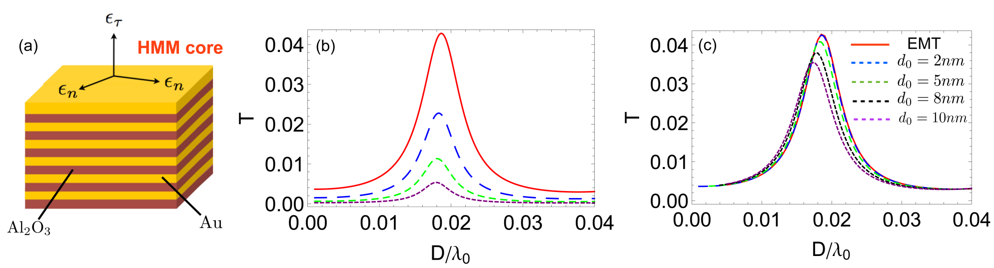

Fig. 3. (a) The schematic structure of a 1:1 alternating Au-Al_2O_3 multilayer HMM resonator core with the single layer thickness $d_0 \ll \lambda_0$. (b) Transmission coefficients using the HMM core in (a) with different metal barrier thicknesses: $d = 30$ nm (solid red curve), 35 nm (blue dashed curve), 40 nm nm (green dashed curve) and 45 nm nm (violet dashed curve). The free-space incident wavelength is $\lambda_0 = 4$ μm and the incident angle $\theta = 60^\circ$. (c) Comparison of the calculated transmission coefficients using the EMT and T-matrix methods. The EMT result is shown by solid red curve and the T-matrix results are shown by dashed curves with different single layer thicknesses of HMM core: $d_0 = 2$ nm (dashed blue), $d_0 = 5$ nm (dashed green), $d_0 = 8$ nm (dashed black), and $d_0 = 10$ nm (dashed violet). In this example, the metal barrier thickness d is set as 30 nm.

However, even in this limit, the fact that the total thickness of the resonator is also subwavelength and may thus only contain ~ 10 individual layers, can still put the predictions of the EMT in question. To address this issue, we also calculate the transmission coefficient of the system using the T-matrix method, taking into account the actual geometry of the metamaterial using layer cells – see Figs. 3(b) and 3(c). Our results clearly confirm the existence of 0th-order transmission resonance even for a deeply subwavelength resonator core dimension ($D \approx 0.02$ λ_0) and a relatively large metal barrier thickness d = 50 nm. Furthermore, the agreement with our theoretical description remains not only qualitative but quantitative.

Note that the zeroth-order resonance described in the current work can only be observed in the HMM core with a negative in-plane permittivity ε_n. In the opposite case of $\varepsilon_n > 0$ and $\varepsilon_\tau < 0$, the system does not support such zeroth-order resonance.

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4. Conclusions

In conclusion, we have presented a new approach to subwavelength light confinement – based on a novel zeroth-order resonance due to negative phase velocity in hyperbolic metamaterials. Our approach is robust to losses, is compatible with planar semiconductor metamaterial technology, and when necessary allows strong radiative coupling.

5. Acknowledgements

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