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An Improved Moving Boundary Heat Exchanger Model with Pressure Drop

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ABSTRACT

A literature review indicates that almost all moving boundary heat exchanger models used in dynamic simulations of heat pumps rely on the common hypothesis that the refrigerant pressure drop is negligible. In fact, it is important to include the momentum balance in some applications, such as electronics cooling where microchannels are commonly used and significant pressure drop is observed and large-scale heat exchangers in solar thermal plants where tube length can be longer than several hundred meters. In addition, a comprehensive and robust switching approach is needed to handle transitions between different model states due to phase change. It is found that the current switching methods in the literature exhibit several shortcomings which may cause serious errors and stability issues when simulating cycling transients of vapor compression systems. The objective of this paper is to propose an improved moving boundary formulation that aims to fill in the above research gaps. Specifically, two different approaches are presented to account for the refrigerant pressure drop across the heat exchanger. A novel and comprehensive switching scheme is introduced to ensure smooth transition between different model representations under large disturbances. The proposed model is validated using measured data. The validation shows that the proposed heat exchanger model along with other supporting component models can reasonably capture the start-up transients of a flash tank vapor injection heat pump system.

1. INTRODUCTION

Transient simulation is a key step in the design and optimization of vapor compression cycles for first cost, operating costs and energy consumption. Heat exchangers are a major component in vapor compression systems and have a significant impact on the cycle transients. Transient heat exchanger models generally fall into one of two categories: namely phase-dependent moving boundary models and phase-independent distributed models. Moving boundary models are characterized by dividing the heat exchanger into control volumes, each of which exactly encompasses a particular fluid phase (superheated vapor, two-phase flow or subcooled liquid). The control volumes are separated by a moving interface where the refrigerant phase transition occurs between the single-phase regime and the two-phase regime. In contrast to the distributed models, the number of control volumes in moving boundary models may vary depending on phases currently present within the heat exchanger. The objective of these models is to capture the thermodynamic behavior inside these control volumes and the time-varying positions of phase boundaries.

Figure 1: Moving boundary heat exchanger model

In moving boundary models, as shown in Figure 1, each control volume is analyzed with the lumped parameter approach. To integrate the conservation equations over the control volume, it is necessary to make reasonable assumptions about the distribution of mass and energy inside the volume. A common practice is to assume a linear...
profile for the enthalpy along the length of each control volume so that the overall mass and energy storage inside control volume can be readily evaluated. Hence, mean enthalpy for the single-phase control volume can be computed as the arithmetic average of the instantaneous enthalpy that enters and leaves the control volume. However, a more sophisticated approach is needed to handle the case where the control volume is occupied by two-phase refrigerant. The two-phase mean void fraction model, originated by Wedekind and Stoecker (1968) to study the transient response of the mixture-vapor transition point in horizontal evaporating flow, can be applied to calculate the overall mass and energy of the two-phase flow. This concept, which has been demonstrated to successfully predict a variety of transient two-phase flow phenomena (Wedekine et al., 1978; Beck and Wedekine, 1981), greatly facilitates the development of various moving boundary models by possibly simulating the two-phase region in lumped fashion. However, a key simplification in these studies is that the time-varying effect of mean void fraction has been neglected because its change tends to be small during transients, and also because its time dependence is related to dynamic modes that are faster than the dominant system dynamics (Rasmussen, 2006). It turns out that this simplification may result in numerical discontinuities when single-phase zones appear or disappear (Zhang and Zhang, 2006). Therefore, some recent publications discard this simplification and include the time derivatives of mean void fraction in the model formulations (Zhang and Zhang, 2006; Eldredge et al., 2008; Li and Alleyne, 2010).

One of the distinctions between different moving boundary models lies in the assumption of the wall temperature at the boundary between neighboring regions. Unlike the assumption of a continuous distribution for the enthalpy along the heat exchanger, the corresponding tube wall temperature associated with each region is spatially independent and uniform within the region. Therefore, the wall temperature is discontinuous at the boundaries, which causes stability issues during simulation. Some commonly adopted methods to calculate the wall temperature at the boundaries can be summarized as follows: 1) the wall temperature at the boundary equals the wall temperature in the two-phase region (Rasmussen and Alleyne, 2004); 2) take the average of the temperatures of adjacent walls (Jensen, 2003); 3) the wall temperature at the boundary is chosen based on the boundary velocity (McKinley and Alleyne, 2008); and 4) a weighted mean temperature based on the length ratio of the neighboring regions (Zhang and Zhang, 2006). Of these four different formulations, the last two are more commonly used because they capture the underlying physics very well.

Another key to simplifying moving boundary models is the assumption of the uniform pressure distribution throughout the heat exchanger given the fact that the pressure loss results in minor influences on the system performance compared to heat transfer. Although it causes deviations from real systems, this simplification is universally applied because the equation of momentum is not required in the analysis and thus modeling complexity can be substantially reduced. A couple of exceptions are found in Tian and Li (2005), Tian et al. (2005) and Yebra et al. (2005). In Tian’s work, static pressure loss is assumed to concentrate at the end of each region. However, this approach only works fine when there is no creation or destruction of fluid phases, otherwise, a mathematical discontinuity in pressure will be experienced. While following the staggered grid scheme and finite volume method, Yebra et al. (2005) presented a general moving boundary model containing the momentum conservation. But their work can be further enhanced with four control volume analysis.

There are a variety of model reduction techniques to further simply the moving boundary paradigm. Common practices include neglecting the dynamics in the superheated region and lumping the wall and fluid temperature to set them equal to each other (Jensen, 2003). These model reduction techniques are often applied for different applications.

As stated previously, the number of available fluid phases in heat exchangers may vary under large disturbances. Fluid phases can form or disappear. For instance, the condenser may experience transitions twice during the start-up operation, vapor → vapor & two-phase → vapor, two-phase & liquid. Early models often have a fixed number of zones and hence they are suitable for modeling large transients. Recent research efforts for this type of models are focused on providing switching schemes to ensure smooth transition between different model representations when fluid phases appear or disappear (Li and Alleyne, 2010). However, these type of models are inherently not as robust as distributed models (Bendapudi et al., 2008) because of the resulting potential numerical failures associated with model transition, such as chattering and numerical singularities.

A thorough review of literature indicates that the existing moving boundary models (Grald & MacArthur, 1992; He et al., 1996; Willatzen et al., 1998; Pettit et al., 1998; Jensen & Tummescheit, 2002; Leducq et al., 2003;
Rasmussen, 2006; Zhang & Zhang, 2006; Diaz, 2007; Kumar et al., 2008; McKinley & Alleyne, 2008; Bendapudi et al., 2008; Eldredge et al., 2008; Liang et al., 2010; Li & Alleyne, 2010; Cecchiniato & Mancini, 2012) have the following limitations:

1. Most models do not account for pressure drop. In fact, it is important to include the momentum balance in some applications, such as electronics cooling where microchannels are commonly used and significant pressure drop is observed in micro-scale heat exchangers (Kandlikar et al., 2006), and large-scale heat exchangers in solar thermal plants (the length can be more than several hundred meters) where pressure drop must be considered.

2. Models are not able to work under cyclic conditions except the one presented by Li and Alleyne (2010). Moreover, even in Li & Alleyne’s work, there is still room for improvement. More details are given in the section for the switching criteria.

3. A clear distinction between these models lies in whether the time-varying effect of mean void fraction is taken into account or not. Based on this, these models can be classified into two categories: the category including mean void fraction in the state vector and the category including the outlet enthalpy in the state vector. Both approaches have advantages and disadvantages.

When mean void fraction is a state variable, mass can be well conserved during model switching. However, it needs to solve an implicit transcendental equation to obtain the outlet enthalpy. That is the reason why Thermosys (McKinley & Alleyne, 2008; Li & Alleyne, 2010) uses huge look-up tables correlating mean void fraction, the refrigerant pressure, and the outlet vapor quality before conducting the simulation in order to improve computational efficiency. However, establishing such huge look-up tables is very tedious and time-consuming since they are refrigerant dependent and application specific.

When the outlet enthalpy is a state variable instead, there is no need to solve an implicit transcendental equation. However, the refrigerant mass cannot be conserved during mode switch.

In order to address the above research gaps, this paper aims at presenting an improved moving boundary model that has the following features:

1. A comprehensive and robust switching approach handling transitions between different model representations.

2. Be capable of accounting for refrigerant pressure drop across the heat exchanger.

3. Both mean void fraction and outlet enthalpy are state variables. By doing this, refrigerant mass can be conserved and the need to solve the transcendental equation is eliminated at the same time.

Utilizing the developed model, the start-up transients of a flash tank vapor injection heat pump system are investigated and validated using experimental data. For the sake of brevity, this paper will not present the modeling details of other components in the system. Interested readers are referred to Qiao et al. (2012 & 2014).

2. MATHEMATICAL MODELS

In this section, two different approaches (Figure 1) are proposed to account for pressure drop in moving boundary models. The conventional moving boundary models without pressure drop use the inlet mass flow rate, inlet enthalpy and outlet mass flow rate as the boundary conditions. However, in the proposed models, the outlet pressure is used in place of the outlet mass flow rate to maintain model consistency. Readers are referred to Qiao et al. (2014) for details.

For brevity, we only show the formulations for the V_TP_L mode of the condenser model in the paper. The formulations of other modes of the condenser and the evaporator can be derived similarly.

**Approach #1:**
Assume that refrigerant pressure is uniform within the heat exchanger. The pressure drop is concentrated at the outlet of the heat exchanger. In this case, only one additional equation is required to account for the global momentum balance. In this approach, the state vector is defined as

\[
\begin{bmatrix}
    p \\
    \bar{\gamma} \\
    L_1 \\
    L_2 \\
    T_{w,1} \\
    T_{w,2} \\
    T_{w,3}
\end{bmatrix}
\]

*Superheated region*

\[
A_c L_1 \frac{d\bar{\gamma}}{dt} + A_c \left( \bar{\gamma} - \rho_g \right) \frac{dL_1}{dt} = \bar{m}_{r,in} - \bar{m}_{r,12}
\]

\[
A_c L_2 \left( \bar{\gamma} \frac{dh_f}{dt} - \frac{dp}{dt} \right) + A_c \rho_g \left( \bar{\gamma} - h_g \right) \frac{dL_2}{dt} = \bar{m}_{r,12} \left( h_{r,in} - \bar{\gamma} \right) - \bar{m}_{r,12} \left( h_g - \bar{\gamma} \right) + q_{r,1}
\]

\[
\frac{dT_{w,1}}{dt} = \frac{q_{a,1} - q_{r,1}}{c_{p,w}A_{w,1}L_1} + \frac{T_{w,2} - T_{w,1}}{L_2} \frac{dL_1}{dt}
\]

*Two-phase region*

\[
A_c \left( L_2 - L_1 \right) \left( 1 - \bar{\gamma} \right) \frac{d\rho_f}{dp} + A_c \left( \rho_g - \rho_f \right) \frac{d\bar{\gamma}}{dt} = A_c \left( \rho_g - \rho_f \right) \left( \frac{dL_2}{dt} - \bar{\gamma} \frac{dL_2}{dt} \right)
\]

\[
A_c \left( L_2 - L_1 \right) \left( 1 - \bar{\gamma} \right) \frac{d\left( \rho_f h_f \right)}{dp} + A_c \left( \rho_g h_g - \rho_f h_f \right) \frac{d\bar{\gamma}}{dt} = A_c \left( \rho_g h_g - \rho_f h_f \right) \left( L_2 - L_1 \right) \frac{d\bar{\gamma}}{dt}
\]

\[
= \bar{m}_{r,12} \left( h_{r,12} - \bar{\gamma} \right) - \bar{m}_{r,23} \left( h_g + q_{r,2} \right)
\]

\[
\frac{dT_{w,2}}{dt} = \frac{q_{a,2} - q_{r,2}}{c_{p,w}A_{w,2}L_2} + \frac{T_{w,2} - T_{w,1}}{L_2} \frac{dL_1}{dt} + \frac{T_{w,3} - T_{w,2}}{L_2} \frac{dL_2}{dt}
\]

Early models (He et al., 1995; Willatzen et al., 1998; Jensen & Tummescheit, 2002) generally neglect the time-varying effect of mean void fraction \( \bar{\gamma} \) and hence the term including \( \frac{d\bar{\gamma}}{dt} \) in Equation (4) & (5) is cancelled out. Later on, McKinley & Alleyne (2008) and Li & Alleyne (2010) found that it is advantageous to include this term for switching purposes by establishing a first-order filter on the difference between \( \bar{\gamma} \) and \( \bar{\gamma}_{total} \) (the equilibrium value for complete condensation or evaporation). One of the drawbacks of this approach is that \( \bar{\gamma} \) is never strictly equal to \( \bar{\gamma}_{total} \) and there is an inevitable delay for \( \bar{\gamma} \) to catch up with \( \bar{\gamma}_{total} \). Moreover, the following transcendental equation needs to be solved for \( x_{o,wp} \) (if the outlet state is two-phase) to determine the refrigerant enthalpy leaving the heat exchanger.

\[
\bar{\gamma} = \frac{c}{\left( x_{o,wp} - x_{i,wp} \right) (c-1)} \ln \left[ \frac{x_{i,wp} \left( c-1 \right) - c}{x_{o,wp} \left( c-1 \right) - c} \right] + \frac{1}{c-1}
\]

where
Equation (7) is derived by using Zivi void fraction model and assuming a linear refrigerant enthalpy profile in two-phase region.

In the proposed model, Equation (8) is used to calculate \( \frac{d\overline{\gamma}}{dt} \). When the outlet state is single-phase, \( \frac{\partial x_{o,\text{sp}}}{\partial p} \) and \( \frac{\partial x_{o,\text{sp}}}{\partial h_{o,\text{sp}}} \) are both zero since \( x_{o,\text{sp}} \) is either one or zero. When the outlet state is two-phase, \( h_{o,\text{sp}} \) is actually equal to \( h_{r,\text{out}} \) (\( h_{r,\text{out}} \) is a state variable being calculated by integration). Another advantage of applying Equation (8) is that there is no delay in calculating \( \overline{\gamma} \).

\[
\frac{d\overline{\gamma}}{dt} = \left( \frac{\partial \overline{\gamma}}{\partial c} \frac{dc}{dp} + \left( \frac{\partial \overline{\gamma}}{\partial x_{i,\text{sp}}} \right)_{h_{i,\text{sp}}} \frac{\partial x_{i,\text{sp}}}{\partial p} + \left( \frac{\partial \overline{\gamma}}{\partial x_{o,\text{sp}}} \right)_{h_{o,\text{sp}}} \frac{\partial x_{o,\text{sp}}}{\partial p} \right) \frac{dp}{dt} + \left( \frac{\partial \overline{\gamma}}{\partial h_{i,\text{sp}}} \right)_{c_{i,\text{sp}}} \frac{dh_{i,\text{sp}}}{dt} + \left( \frac{\partial \overline{\gamma}}{\partial h_{o,\text{sp}}} \right)_{c_{o,\text{sp}}} \frac{dh_{o,\text{sp}}}{dt}
\]

Since \( \overline{\gamma} \) is a state variable and can be integrated through Equation (8), there is no need to use Equation (7) to calculate \( \overline{\gamma} \) in Equation (4) & (5) anymore. Equation (7) is used to calculate the partial derivatives in Equation (8) only.

**Subcooled region**

\[
A_c \left( L_1 - L_2 \right) \frac{d\overline{\rho}_3}{dt} - A_c \left( \overline{\rho}_3 - \rho_f \right) \frac{dL_3}{dt} = \dot{m}_{r,23} - \dot{m}_{r,\text{out}}
\]

\[
A_c \left( L_3 - L_2 \right) \left( \overline{\rho}_3 \frac{d\overline{h}_3}{dt} - \frac{dp}{dt} \right) - A_c \rho_f \left( \overline{h}_3 - h_f \right) \frac{dL_3}{dt} = \dot{m}_{r,23} \left( h_f - \overline{h}_3 \right) - \dot{m}_{r,\text{out}} \left( h_{r,\text{out}} - \overline{h}_3 \right) + q_{r,3}
\]

\[
\frac{dT_{w,3}}{dt} = \frac{q_{w,3} - q_{r,3}}{c_{p,w} A_c \rho_w \left( L_1 - L_2 \right)} + \frac{T_{w,3} - T_{w,2}}{L_3 - L_1} \frac{dL_3}{dt}
\]

**Momentum balance**

Since the pressure is assumed to be uniform within the entire heat exchanger, an equation accounting for global momentum balance is required to calculate the mass flow rate leaving the heat exchanger. Recall that the outlet pressure is the boundary condition.

As suggested by Brasz and Koenig (1983), dynamic pressure waves are of minor importance for a heat transfer analysis and they can be neglected while still preserving the physical integrity. Therefore, we can obtain the following equation. The pressure drop is entirely due to the frictional loss.

\[
(p - p_{\text{out}}) A_c = \sum_{i=1}^{N} F_{w,i}
\]
The pressure loss within each region can be calculated using empirical correlations based on the average of inlet and outlet mass flow rates.

**Approach #2:**

Assume that refrigerant pressure varies linearly within the entire heat exchanger. Similarly, only one additional equation is required to account for the global momentum balance. In this case, the state vector is defined as 

\[ \begin{bmatrix} p_{in} \ y \ h_{r, out} \ L_1 \ L_2 \ T_{w, 1} \ T_{w, 2} \ T_{w, 3} \end{bmatrix} . \]

The mean pressure of each region is defined as the pressure at the midpoint of the corresponding region, i.e.

\[ p_1 = p_{in} - \frac{p_{in} - p_{out}}{L_1} \frac{L_1}{L_i} = \frac{L_1}{2} \]

\[ p_2 = p_{in} - \frac{p_{in} - p_{out}}{L_1 + L_2} \frac{L_1 + L_2}{L_i} = \frac{L_2}{2} \]

\[ p_3 = p_{in} - \frac{p_{in} - p_{out}}{L_2 + L_3} \frac{L_2 + L_3}{L_i} = \frac{L_3}{2} \]

(13)

The mean pressure in individual regions is used to calculate the related refrigerant properties. For instance, mean refrigerant density of the superheated region should be calculated using \( p_1 \) and \( \bar{h}_1 \).

Pressure at boundaries is required to calculate the density at the bubble point and dew point, i.e. \( \rho_b \) and \( \rho_f \).

\[ p(L_1) = p_{in} - \frac{p_{in} - p_{out}}{L_1} L_1 \]

\[ p(L_2) = p_{in} - \frac{p_{in} - p_{out}}{L_2} L_2 \]

(14)

The governing equations are essentially the same as those in Approach #1 and thus not repeated here for brevity.

**Figure 1:** Two different approaches to calculate pressure drop

**Switching criteria**

As pointed out earlier, the switching methods presented in the literature have flaws and may cause serious instabilities when it comes to the simulation of cycling transients. This paper proposes a consistent, comprehensive and robust switching scheme to handle transitions between different model representations. All the possible model modes of the studied system under cycling conditions and the switching criteria between different modes are shown.
in Figure 3. The minimum threshold $\varepsilon_{\text{min}}$ can be adjusted based on the application. The pseudo length is determined based on the mass conservation as follows:

$$L_{\text{pseudo}} = \frac{\rho_{\text{new zone}} - \rho_{\text{adjacent zone}}}{\rho_{\text{new zone}} - \rho_{\text{adjacent zone}}} L_{\text{adjacent zone}}$$

(15)

Figure 2: Moving boundary condenser model

Figure 3: Modes of moving boundary heat exchanger models of the studied FTVI system
3. CASE STUDY AND SIMULATION RESULTS

Utilizing the developed models based on Approach #1, the start-up transients of a flash tank vapor injection (FTVI) heat pump system is investigated. Detailed description about the system configuration and the modeling techniques for other components are presented in Qiao et al. (2012 & 2014). All the results are presented in Figure 4. It can be seen that the simulation results agree with experimental data. Since all the simulation results are obtained by conducting a similar analysis to what is performed in Qiao et al. (2014) except that the proposed moving boundary heat exchanger models are used in the presented paper, detailed explanations for the following figures will not be provided for brevity. Readers are referred to Qiao et al. (2014).
Figure 4: Start-up transients: (a) pressure response; (b) condenser heat load response; (c) suction superheat response; (d) injection superheat response; (e) temperature response; (f) simulated refrigerant charge distribution and liquid height in flash tank

4. CONCLUSIONS

An improved moving boundary heat exchanger model for use in dynamic simulations is proposed in the paper. Specifically, two different approaches are presented to account for the refrigerant pressure drop across the heat exchanger. A comprehensive switching scheme is introduced to ensure smooth transition between different model representations under large disturbances. A validation case based on experimental data is presented, showing that the proposed heat exchanger model along with other supporting component models can reasonably capture the start-up transients of a flash tank vapor injection system. Future work includes the investigation of system transients under shutdown and other operating conditions.

NOMENCLATURE

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