PORE SIZE DISTRIBUTION OF SANDY SOILS AND THE PREDICTION OF PERMEABILITY

C.H. Juang
Interim Report

PORE SIZE DISTRIBUTION OF SANDY SOILS
AND
THE PREDICTION OF PERMEABILITY

To: H. L. Michael, Director
    Joint Highway Research Project

From: R. D. Holtz, Research Engineer
    Joint Highway Research Project

August 13, 1981

Attached is an Interim Report on the HPR-1(19) Part II Research Study entitled "Effects of Pore Size Distribution on Permeability and Frost Susceptibility of Selected Subgrade Materials". This is the fourth report from this study and it covers Task E of the approved work plan. The author of the report is Mr. Charng-Hsein Juang who worked under the supervision of Prof. C. W. Lovell and myself. The report title is "Pore Size Distribution of Sandy Soils and the Prediction of Permeability".

The results of the study show the usefulness of pore size distribution parameters for characterizing the fabric of sandy soils. They are also useful for predicting the permeability of such soils. The author has developed a statistical prediction equation which uses the PSD parameters and which is shown to give an accurate estimate of permeability for compacted soils ranging from sands to clays.

The Report is a partial fulfillment of the objectives of the Study. Copies will be submitted to the IDOH and FHWA for their review, comment, and acceptance.

Sincerely yours,

R. D. Holtz
Research Engineer

RDH:cm

cc: A. G. Altschaeffl  G. K. Hallock  C. F. Scholer
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Interim Report

PORE SIZE DISTRIBUTION OF SANDY SOILS
AND
THE PREDICTION OF PERMEABILITY

by

Charng-Hsein Juang
Graduate Instructor in Research

Joint Highway Research Project

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and the

U.S. Department of Transportation
Federal Highway Administration

The contents of this report reflect the views of the author who is responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. The report does not constitute a standard, specification, or regulation.

Purdue University
West Lafayette, Indiana
August 13, 1981
**Title and Subtitle**

PORE SIZE DISTRIBUTION OF SANDY SOILS AND THE PREDICTION OF PERMEABILITY

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**Abstract**

The soils used in this study were a sand and two sand-clay mixtures. Kneading compaction was used to prepare samples of sand-clay mixtures while pluvial compaction was used to prepare compacted samples of sand. The testing program consisted of the permeability test and the pore size distribution (PSD) test. A technique for the preparation of PSD specimens of cohesionless sands has been developed. The mercury intrusion technique was used to conduct PSD test. Falling head tests under back pressure were used to measure permeability.

The pore size density function of the sands studied showed a single modal characteristic on a log diameter scale while that of sand-clay mixtures showed bimodal characteristics on the same scale. The influence of varying water content and compactive effort on the fabric of the sandy soils studied was characterized by their pore size distributions and pore size density functions.

A permeability model which related the pore size distribution to the permeability of soils has been proposed. The prediction of permeability by the proposed model was excellent (e.g. $R^2 = 0.98$) for both the author's and Garcia-Bengochea's (1978) data. Thus, the proposed permeability model can provide an excellent predictive tool for a rather wide range of compacted soils.
ACKNOWLEDGEMENTS

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The laboratory assistance provided by Mrs. Janet Lovell was extremely helpful. The secretarial work by Mrs. Edith Vanderwerp and thesis typing by Ms. Linda Gibson were greatly appreciated.

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Finally, very special thanks are extended to my wife, Anne, for her assistance in both laboratory and computer work, and many sacrifices during the course of this study; indeed this dissertation would not have been possible without her help.
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<tr>
<td>ASTM</td>
<td>American Society for Testing and Materials</td>
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<td>PSD</td>
<td>Pore Size Distribution</td>
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<td>SEM</td>
<td>Scanning Electron Microscopy</td>
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<tr>
<td>AASHTO</td>
<td>American Association of State Highway and Transportation Officials</td>
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SUMMARY

The soils used in this study were a sand and two sand-clay mixtures. Kneading compaction was used to prepare samples of sand-clay mixtures while pluvial compaction was used to prepare compacted samples of sand. The testing program consisted of the permeability test and the pore size distribution (PSD) test. A technique for the preparation of PSD specimens of cohesionless sands has been developed. The mercury intrusion technique was used to conduct PSD test. Falling head tests under back pressure were used to measure permeability.

Mathematical interpretations of PSD were given in detail. For the soils studied, both the pore size distributions and pore size density functions were presented. Pore size density function of sand studied showed a single modal characteristic on a log diameter scale while that of sand-clay mixtures showed bimodal characteristics on a log diameter scale. The influence of varying water content and compactive effort on the fabric of the sandy soils studied was characterized by their pore size distributions and pore size density functions.

A permeability model which related the pore size distribution to the permeability of soils has been proposed. The prediction of permeability by the proposed model was excellent (e.g. $R^2_a = 0.98$ for
both the author's and Garcia-Bengochea's (1978) data. In other words, the proposed permeability model can provide an excellent predictive tool. Thus a functional relationship between the PSD and the permeability of compacted soils is clearly demonstrated.
CHAPTER I INTRODUCTION

1.1 FABRIC AND ITS ROLE IN GEOTECHNICAL ENGINEERING

The characterization of soil fabric is a very important aspect of geotechnical engineering. It can lead to better understanding and prediction of engineering properties of soils. The term "soil fabric" has widely been used without a common definition, e.g., Terzaghi and Peck (1948), Mitchell (1956), Scott (1963), and Oda (1972). However, the following definition given by Brewer (1964) is preferred:

"Soil fabric is the physical constitution of a soil material as expressed by the special arrangement of the solid particles and associated voids."

Studies at Purdue University, e.g., Diamond (1970, 1971), Bhasin (1975), Reed (1977) and García-Bengochea (1978) among others, have shown that the fabric of compacted clayey soils can be identified by their pore size distribution (PSD) and that some engineering properties of clayey soils can be quantitatively correlated with their PSD. Moreover, the engineering properties of sandy soils are also believed to be affected by their fabric characteristics, e.g., Oda (1972, 1976), Mahmood (1973), and Mitchell, et al. (1976). Figure 1-1 illustrates the concept of sand fabric. Two different arrangements of particles could have identical void ratios and grain size distributions, but their compressibility characteristics would be obviously different (Leonards, as cited by Holtz and Kovacs, 1981). Similarly,
Figure 1-1  Schematic Diagram of Sand Fabric (After Leonards, as cited by Holtz and Kovacs, 1981)
the permeability, liquefaction properties, and the static strength of sands are strongly affected by sand fabric.

1.2 OBJECTIVE OF THIS STUDY

This study is part of a research project entitled "The Effects of Pore Size Distribution on Permeability and Frost Susceptibility of Selected Subgrade Materials". The specific objectives are:

1) to determine the fabric characteristics of compacted sand and sand-clay samples by their PSD as measured by the mercury intrusion technique; and

2) to develop a theoretical permeability model which correlates the permeability with the PSD of the selected soils.
2.1 PORE SIZE DISTRIBUTION

2.1.1 Theoretical Background of Pore Size Distribution and Pore Size Density Function

The pore system has an extremely complex geometry. Therefore, to characterize a soil fabric by means of pore size and its distribution is a rather abstract concept. In fact, there is no clear definition of pore size and therefore its distribution. In the mercury intrusion technique (see Section 2.1.2) a cylindrical pore shape model was assumed, i.e., the circular diameter was considered to be its corresponding pore size.

Based on Washburn's (1921) equation:

\[
P = -\frac{4T\cos\Theta}{d}
\]  

(2-1)

where \( P \) = absolute pressure being applied

\( T \) = surface tension of mercury

\( \Theta \) = contact angle between mercury and the pore wall

\( d \) = the size of pores which can be intruded at the pressure \( P \); called the apparent pore diameter.

With the assumption of constancy of the surface tension and contact angle, Equation (2-1) implies that as pressure increases, mercury will be intruded into progressively smaller pores. For each pressure increment, mercury is forced into the accessible soil pores of a diameter
larger than or equal to that which is calculated by Equation (2-1) at the corresponding pressure. Meanwhile the cumulative volume of pore space at any corresponding pressure is also recorded. Then a PSD can be generated (see Equation 2-6d and its interpretation).

Since there is no clear physical meaning of pore size, Scheidegger (1957) proposed a probabilistic approach to define a pore system. With slight modification to Scheidegger's ingenious idea and the pioneering work of Drake and Ritter (1945) and Garcia-Bengochea (1978), the PSD and pore size density function can be derived as follow:

Assume X is a random variable denoting a pore diameter at any point within the pore space. It is the diameter of the largest sphere which contains this point and remains wholly within the pore space. Thus, a "pore diameter" can be assigned to each point in the pore space, and a PSD can be defined by determining what fraction of pore space has a pore diameter between x and x + dx. (Scheidegger, 1957)

Pore size distribution can be expressed as

\[ F_X(x) = \int_x^\infty f_X(x) \, dx \quad (2-2a) \]

where \( F_X \) = PSD of the soil being considered

\[ f_X = \text{the probability density function of pore size} \]

and

\[ \int_0^\infty f_X(x) \, dx = 1 \quad (2-2b) \]

The probability of pores with diameters ranging from x to x + dx is \( f(x) \, dx \). Now denoting the volume of pores having diameters between x and x + dx as \( dV \), we have

\[ dV = s(x) \cdot f_X(x) \, dx \quad (2-3) \]
where \( s(x) \) = a shape factor and is a function of \( x \) (i.e., it depends on the pore shape model chosen).

Recalling Equation (2-1) and noting that the diameter is \( x \), we have

\[
P_x = \text{constant} \tag{2-4a}
\]

or

\[
dx = -\frac{x}{P} \, dP \tag{2-4b}
\]

Substituting into Equation (2-3), we have

\[
dV = -s(x) \cdot f_x(x) \cdot dP/P \cdot x \tag{2-5a}
\]

or

\[
f_x(x) = \frac{P}{x \cdot s(x)} \cdot \frac{d(V_t - V)}{dP} \tag{2-5b}
\]

where \( V_t \) = total void volume that can be intruded by mercury

\( V_t - V = \) a measurement of pore volume with diameter equal to or greater than \( x \)

\( P = \) corresponding pressure needed to intrude mercury into pores with diameter \( x \).

The terms \( x, f_x(x), s(x) \) are the same as defined previously in Equations (2-2) and (2-3). Note that the total void volume that can be intruded by mercury is not necessarily the same as that existing in the specimen. Therefore the pore size density functions based on different definitions of \( V_t \) are different from each other (see Section 4.3.2). Also note that \( s(x) \) is the problem in determining \( f_x(x) \) of Equation (2-5b).

One way to treat it is to introduce the volume pore size density function, \( f_{vX}(x) \) as follows:
Substituting Equation (2-5b) into Equation (2-6a), we have

\[ f_{vx}(x) = s(x) \cdot f_x(x) \]  

(2-6a)

Substituting Equation (2-4b) into Equation (2-6b), we have

\[ f_{vx}(x) = - \frac{d(V_t - V)}{dP} \]  

(2-6b)

Substituting Equation (2-4b) into Equation (2-6b), we have

\[ f_{vx}(x) = - \frac{d(V_t - V)}{dx} \]  

(2-6c)

Integrating Equation (2-6c) gives

\[ F_{vx}(x) = V_t - V \]  

(2-6d)

where \( F_{vx}(x) \) = the cumulative intruded volume which is directly recorded in the mercury intrusion technique.

So far, the above derivations and definitions provide the theoretical development of pore size distribution and its density function.

For any pore diameter which can be calculated from Equation (2-1) with each corresponding pressure being recorded, \( F_{vx}(x) \) can be obtained from the volume intrusion record. Hence a PSD [i.e., \( F_{vx}(x) \) vs. \( x \)] can be generated. The pore size density function, \( f_{vx}(x) \), can then be determined from Equation (2-6c).

The pore size distribution defined above is in terms of pore volume. In many cases, it is convenient to express the PSD in terms of pore volume per gram of specimen. It is also a standard presentation of the PSD test as specified by an ASTM standard being developed by Subcommittee D18.14. Moreover, some researchers prefer to express the PSD in dimensionless terms (e.g., a percentage) for the purpose of convenient mathematical treatment. The following expressions provide a theoretical
basis for this consideration.

\[ f_{px}(x) = \frac{P}{V_t} \cdot \frac{d(V_t - V)}{dP} \]  

(2-7a)

where \( f_{px}(x) \) = density function of pore diameter with percentage as its ordinate

and

\[ F_{px}(x) = \int_x^\infty f_{px}(x) \, dx \]  

(2-7b)

where \( F_{px}(x) \) = cumulative distribution of pore size (for diameter \( \geq x \)) with percentage as its ordinate; this is simply called percent greater by volume.

Furthermore, Equation (2-6b) can also be modified to the following expression:

\[ f_{nx}(x) = \frac{P}{x \cdot V_s} \cdot \frac{d(V_t - V)}{dP} \]  

(2-8)

where \( V_s \) = volume of soil specimen

\( f_{nx}(x) = \) density function of pore diameter with a relative frequency scale such that

\[ \int_0^\infty f_{nx}(x) \, dx = n \]

where \( n \) = porosity of specimen.

The multiplication of \( f_{nx}(x) \) by a constant \( c \) (see Section 4.3.2) is called "porosity frequency" by Garcia-Bengochea (1978) and White (1980).

A detailed discussion of the presentation of the PSD test results on the soils studied is given in Section 4.3.2.
2.1.2 The Measurement of Pore Size Distribution and Its Limitation

The measurement of the PSD of a soil is a technique which allows the engineer to indirectly examine the soil fabric. Methods available for the determination of PSD are as follows:

a) Indirect determination from representative grain size distributions by applying probabilistic theory (e.g. Muruta and Sato, 1969);

b) Indirect determination by scanning electron microscopy (SEM) techniques with probabilistic theory;

c) Capillary suction technique;

d) Mercury intrusion technique.

Detailed discussion of all these techniques except (a) are given by Bhasin (1975) and Lowell (1979). Bhasin (1975) had also pointed out that the mercury intrusion technique is the most suitable method for the measurement of the PSD of soils.

The concept of forcing mercury into material pores to determine their sizes was first proposed by Washburn (1921). This led to the development of mercury intrusion technology which is based on the principle that the surface tension of a non-wetting liquid will oppose the entry of the liquid into a small pore of a material. Washburn stated that this opposition could be overcome by external pressure, and that the external pressure required was inversely proportional to the pore diameter, as indicated by Equation (2-1).

Perhaps the primary limitation of the mercury intrusion technique is that it does not always measure the diameter of the actual pore
but rather the diameter of the channel leading into the pore, which obviously could be smaller. As a result of "ink-bottle-pore effect", discussed by Ritter and Drake (1945), Orr (1970), and Ahmed (1971) and the other Purdue researchers, the measured pore size is called the limiting pore diameter or called apparent pore diameter (e.g., by a proposed ASTM standard D18.14). An attempt has been made by Cebeci, et al. (1978) to evaluate the arrangement of ink-bottle pores. However, there is still no way to obtain the "true" pore size distribution. This problem is of minor importance in the case of flow through porous materials, since the pore channels the water has to pass through are probably equivalent to those for mercury.

The second problem with mercury intrusion technology is that cylindrical pores are assumed. Bhasin (1975) pointed out that this assumption may be fairly reasonable since the difference in calculated PSD's due to different assumptions of pore shapes are relatively small (i.e., within an order of magnitude) when compared with the range of pore sizes that are possible (i.e., up to five orders of magnitude).

Another limitation is that mercury intrusion porosimetry, in common with many other methods, is only capable of intruding pores that are open to the outside of a soil specimen. The volume of any pores that are completely enclosed by surrounding solids cannot be determined. Moreover, the method will only determine the volume of intrudable pores that have an apparent diameter corresponding to a pressure within the pressuring range of the testing instrument.

Further discussion of the assumptions and sources of error of the mercury intrusion technique, including the concern of whether or not
the soil structure breaks down during intrusion, the assumptions of constant contact angle and surface tension values, non-reaction of the mercury with the soil, and incompressibility of the soil fabric, among others, is found in Ritter and Drake (1945), Rootare (1968), Winslow (1969), Diamond (1970), Bhasin (1975), Garcia-Bengochea, et al. (1980), Kenney (1980), Reed, et al. (1980), and a proposed ASTM standard, subcommittee D18.14.

2.1.3 The Applications of Pore Size Distribution

To predict the permeability of porous materials is one of the early applications of the PSD. As is well known, seepage occurs only through the pores, and what is directly related with the permeability are the characteristics of pores themselves. Moreover, permeability is extremely sensitive to subtle changes in soil structure and these changes are not accurately reflected by the bulk volumetric pore parameters (e.g., the porosity or void ratio of the sample). This led to the attempts to use PSD to predict the permeability of porous material. Among those are Childs and Collis-George (1950), Marshall (1958), Millington and Quirk (1959), Kunze, et al. (1968), Murota and Sato (1969), Klock, et al. (1969), Green and Corey (1971), Jackson (1972), and Garcia-Bengochea, et al. (1979). However, only the last reference clearly demonstrated the usefulness of the PSD measurements on interpreting changes in soil fabric and predicting how these changes might affect the permeability of soils.

Another application of PSD is to predict the frost heave rate (or amount) in soils. References include Csathy and Townsend (1962), Zoller (1973), Gaskin and Raymond (1973), and Reed, et al. (1979).

Finally, based on PSD studies at Purdue University during the past decade, an understanding of the mechanism of compressibility and that of liquefaction of sandy soils may be possible.

2.2 PERMEABILITY MODEL

2.1.1 Relation of Permeability to Pore Geometry

Since permeability is a characteristic physical property of a porous medium, it would seem only reasonable to assume that it is related in some functional way to certain measureable properties of the soil pore geometry, e.g., porosity, pore size distribution, internal surface area, etc. However, numerous attempts to discover a functional relation of universal applicability have so far met with disappointing results (Hillel, 1980).

Perhaps the simplest approach is to seek a correlation between permeability and porosity (or void ratio). However, Scheidegger (1957) and Hillel (1980), among others, concluded that there is no simple correlation between permeability and porosity because of the strong dependence of flow rate on the width, continuity, shape, and tortuosity of the conducting channels. Thus, a medium composed of numerous small pores with a high total porosity is likely to exhibit a lower saturated permeability than a medium composed of fewer large pores with a lesser porosity.

Prediction of permeability from grain size distribution has also been used for a long time (Hazen, 1911). Such predictive equations
have had only limited success for coarser soils and have proved to be unsatisfactory for fine-grained soils. Efforts to refine this approach by introducing empirical grain shape and packing parameters have not won general acceptance (Hillel, 1980).

Numerous theoretical models have been introduced to represent porous media by a set of relationships that are amenable to mathematical treatment. Basically, there are two main groups of those models. The first is based on a generalization of Kozeny's approach (Carman, 1936) for saturated and unsaturated porous media. This theory is based on the concept of a hydraulic radius, i.e., a characteristic length parameter presumed to be linked with the hypothetical channels to which the porous medium is thought to be equivalent. The derivation and critique of this theory can be found in Scheidegger (1957).

The second group employed the capillary model using the PSD to calculate permeability. Scheidegger (1957) gave a comprehensive review of these models, including the straight capillaric, parallel, serial, and branching models. He pointed out that, in general, natural porous media are extremely disordered, so that it seems a rather poor procedure to represent them by something which is intrinsically ordered. He therefore suggested that the preferred model of a porous medium should be based on statistical concepts.

A statistical flow model, as shown in Figure 2-1, was first presented by Childs and Collis-George (1950) and later modified by Marshall (1958) and Millington and Quirk (1959). Conceptually, their approach rests on the assumptions that (1) a soil contains distinct pores of various sizes which are randomly distributed in space, and (2) when adjacent sections of the soil are brought into contact, the
Figure 2-1  Schematic Representation of a Sectioned Soil
(After Hillel, 1980)
overall quantity of flow across the section depends statistically on
the number of pairs of interconnected pores and geometrically on their
configurations. The quantity of flow of each pair of interconnected
pores is determined by the narrower of the two pores.

The original equation by Childs and Collis-George (1950) was

$$k = c \frac{\rho R}{\mu} \sum_s \sum_\delta \delta^2 f(s) \, ds \, f(\delta) \, d\delta$$

(2-9)

where

$k =$ permeability

$c =$ a matching constant

$\rho =$ density of water

$g =$ gravitational acceleration

$\mu =$ coefficient of absolute viscosity of water

$f(s) \, ds =$ the partial pore space occupied by pores with radii

of $s \rightarrow s + ds$

$f(\delta) \, d\delta =$ the partial pore space of pores with radii of

$\delta \rightarrow \delta + d\delta$

$R =$ maximum pore radius

Note that although theoretically the calculation based on
Poiseuille's law does not require a matching factor, in actual prac-
tice a matching factor is needed to adjust the computed and measured
permeability at full saturation. The method of calculation was sub-
sequently improved by Marshall (1958), Millington and Quirk (1959),
Kunze et al. (1968), Green and Corey (1971), and Jackson (1972).

Basically these developments are different only by some minor arbi-
trary factors and the ways of calculation. Moreover, Garcia-Bengochea
(1978) presented a somewhat different equation based on regression
analysis of his data as follows:
\[ k = c_s \cdot PSP^b \]  
(2-10a)

where

\[ b = \text{parameter of regression} \]

\[ c_s = \text{a constant, called shape factor} \]

and

\[ \text{PSP} = \sum \sum \tilde{d}^2 f(d_i) f(d_j) \]  
(2-10b)

\[ \tilde{d} = \text{smaller of } d_i \text{ and } d_j \]

\[ f(d_i), f(d_j) = \text{volumetric frequency of occurrence of pores with} \]
\[ \text{diameter of } d \rightarrow d + \Delta d; \text{ conceptually it equals} \]
\[ f(s)ds \text{ or } f(\delta)d\delta \text{ in Equation (2-9)}. \]

As noted by Garcia-Bengochea, et al. (1979), as \( b \rightarrow 1 \), Equation (2-10) becomes the Child and Collis-George's (1950) model. Since experimentally \( b \) ranges from 1.67 to 4.95, Equation (2-10) is indeed an empirical equation.

2.2.2 Proposed Permeability Model

From the previous discussion, it is clear that Child and Collis-George's (1950) model and its modifications are more generally applicable than those based on earlier models. However, an arbitrary matching factor or regression parameter, as appears in Equation (2-9) and Equation (2-10) respectively, is necessary to reduce or to eliminate the deviations between predicted and measured permeability. One of the objectives of the present study is to propose a modified model which eliminates the deviation without the necessity of an arbitrary factor or parameter.

Considering a homogeneous soil having interconnected pores which are randomly distributed in space, the probability of pores of diameter
$x + x + dx$ is $f(x) \, dx$, where $f(x)$ is the pore size density function of the soil. Note that no assumption is needed for $f(x)$. However, in the present study, $f(x)$ is defined by Equation (2-7a). The term $f(x) \, dx$, represents the ratio between the pore volume occupied by pores of diameter $x + x + dx$ and the total pore volume. Note that if a cross section through soil mass is considered, the above volumetric ratio is equal to the areal ratio between the pore area of diameter $x + x + dx$ and the total pore area.

Now, consider a soil column with a thickness $\Delta y$ and two cross sections with an identical pore size density function (see Figure 2-2). The probability of pores of diameter $x_i + x_i + dx_i$ on cross section $i$ being connected to pores of diameter $x_j + x_j + dx_j$ on cross section $j$, $P(x_i, x_j)$, has the following extreme cases:

1. For $\Delta y \gg x$, the connections between pores on the two cross sections can be reasonably assumed to be completely random. Hence

$$P(x_i, x_j) = f(x_i) \, f(x_j) \, dx_i \, dx_j \quad (2-11)$$

2. For $\Delta y \rightarrow 0$, the connections between pores considered are completely correlated. Hence

$$P(x_i, x_j) = f(x_i) \, dx_i \text{ or } f(x_j) \, dx_j \quad (2-12)$$

For case 1, following the similar derivation by Childs and Collis-George (1950) or Garcia-Bengochea (1978), we have

$$k = \frac{\gamma n^2}{32 \mu} \int_0^\infty \int_0^\infty \frac{z^2}{x} f(x_i) \, f(x_j) \, dx_i \, dx_j \quad (2-13)$$
Random variable, $X_i$
with a density function, $f_i(x_i)$

Random variable, $X_j$
with a density function, $f_j(x_j)$

$f_i = f_j$

Figure 2-2  Schematic Representation of a Flow Region
where \( \gamma = \) unit weight of water

\( \mu = \) coefficient of absolute viscosity of water

\( n = \) porosity of soil

\( \tilde{X} = \) equivalent diameter of capillary element.

Equation (2-13) is essentially equal to Equation (2-9) except that no matching factor is provided. This equation is also called the Marshall model by Garcia-Bengochea (1978).

For case 2, in a similar manner, we have

\[
k = \frac{\gamma n^2}{32 \mu} \int_{0}^{\infty} x^2 f(x) \, dx \tag{2-14}
\]

Equation (2-14) is called the capillary model by Garcia-Bengochea (1978).

Since we are concerned with the effect of pore geometry changes on the permeability, it is more relevant to consider \( \Delta y \) to be of the same order of magnitude as the pore diameter. Therefore the connections between pores on the two cross-sections are partially correlated.

A reasonable assumption to account for the partially correlated connections is that pores of diameter \( x_i \rightarrow x_i + dx_i \) on cross-section \( i \) have a greater probability of being connected to pores of the same diameter (i.e., \( x_i \rightarrow x_i + dx_i \)) on cross-section \( j \), than that of being connected to the other different-sized pores on cross-section \( j \).

Based on this idea, a connection rule is illustrated in Figure 2-3.

The probability of pores of diameter \( x_i \rightarrow x_i + dx_i \) on cross-section \( i \) being connected to pores of diameter \( x_j \rightarrow x_j + dx_j \) on cross-section \( j \) is

\[
P(x_i, x_j) = g(y, x_i, x_j) \cdot f(x_i) \cdot f(x_j) \, dx_i \, dx_j \tag{2-15}
\]
Figure 2-3 Family of Governing Functions
and
\[
k = \frac{Y_n^2}{32\mu} \int_0^\infty \int_0^\infty z^2 \cdot g(y, x_i, x_j) \cdot f(x_i) \cdot f(x_j) \, dx_i \, dx_j \tag{2-16}
\]
where \( g(y, x_i, x_j) \) = connecting function and \( y = \) the third directional variable (length).

If we also consider a correction factor, \( t(y, x_i, x_j) < 1 \), to account for the tortuosity of flow, then the permeability becomes
\[
k = \frac{Y_n^2}{32\mu} \int_0^\infty \int_0^\infty z^2 \cdot g \cdot t \cdot f(x_i) \cdot f(x_j) \, dx_i \, dx_j \tag{2-17}
\]

Based on the above development, we now introduce a governing function \( G(x_j) \) to account for \( g \) and \( t \) as follows (see Figure 2-3)
\[
G(x_j) = e^{-\left(\frac{x_j - x_i}{V}\right)^2} \tag{2-18}
\]
where \( V = V(x_j) \)

= an undetermined function of pore size to account for the third directional variable and the tortuosity of flow.

Note that pore size density functions \( f(x_i) \) and \( f(x_j) \) are identical. And random variables \( X_i \) and \( X_j \) are symmetrical in Equation (2-18).

For any pores of diameter \( x_i + x_i + dx_i \), the governing function is bell-shaped and symmetrical with a mean value of \( x_i + x_i + dx_i \), as shown in Figure 2-3. Therefore Equation (2-18) is a family function and each member of the family is determined by a pair of parameters, i.e., \( X_i \) and \( V \).

Based on the above discussion, Equation (2-17) can be refined as follows:
\[
k = \frac{Y_n^2}{32\mu} \int_0^\infty \int_0^\infty z^2 \cdot G(x_j) \cdot f(x_i) \cdot f(x_j) \, dx_i \, dx_j \tag{2-19}
\]
Equation (2-19) is the proposed permeability model which has the following characteristics:

(1) a probabilistic basis;

(2) no arbitrary matching factor required; and

(3) a tortuosity factor being considered.
CHAPTER III TESTING PROGRAM AND EXPERIMENTAL PROCEDURES

3.1 TESTING PROGRAM

The testing program was designed to accomplish the objectives mentioned in Section 1.2. The flow chart of the program is shown in Figure 3-1. The soils were selected in order to extend the results of the previous permeability-pore size distribution study at Purdue University by Garcia-Bengochea (1978), i.e., to extend the prediction model to wider ranges of permeability and to characterize different fabrics of sandy soils. The compaction variables considered in this study are water content, energy level, and compaction method.

The pore size distributions (PSD) of the selected soils were determined by the mercury intrusion method. For clay-sand mixtures, following the permeability test, the PSD specimens were trimmed and dried by a freeze-drying technique and were made ready for the PSD test. For pure sand, the problem of determining its pore size distribution is in sampling, i.e., how to get a PSD specimen without destroying its fabric. In the present study, this problem has been solved by using a resin as a bonding material. A small amount, 0.5% by weight, of Varcum 1364 phenolic resin made by Reichhold Chemicals, Inc. was mixed with sand before compaction. After compaction the entire sample and mold were placed in the oven at a temperature of 170°C for 1 h. The sample was then removed and placed in a desiccator several hours until it cooled. The oven-dried compacted samples were found to have
Selection of Soils

Selection of Compaction Variables

Sand

Yes

No

Kneading Compaction

Addition of Resin

Pluvial Compaction

Permeability Tests

Sampling and Freeze-Drying

Oven Drying

PSD Tests

1) The Fabric of Sandy Soils Characterized by PSD

2) The Correlation between PSD and Permeability

Figure 3-1 Flow Chart of the Testing Program
about 5% shrinkage in volume but no change in total weight. Winslow (1980) stated that no chemical reaction between sand and powdered phenolic resin occurs during oven-drying because the resin only acts as a glue. Moreover, the results of the PSD tests on specimens trimmed from these oven-dried samples are reproducible. Therefore this technique is believed to be the best way to obtain PSD specimens of pure sand. During some preliminary work, the minimum amount of resin which provides sufficient bonding and workability was found to be 0.5% by weight of sand.

The following sections describe the soils studied, compaction methods, permeability tests, specimen drying methods, and the PSD tests.

3.2 SOILS STUDIED

Sand and sand-clay mixtures used in this study have been arbitrarily called Soils A, B, and C, respectively. The specifications of these three soils are given in Table 3-1. The clay used is Edgar plastic kaolin with the following index properties (Garcia-Bengochea, 1978): specific gravity of solids = 2.65, liquid limit = 59%, plasticity index = 22%, and the clay fraction (% less than 2 μm) = 76%. The sand used is a uniform Ottawa sand. The grain size distribution curves and the classification properties of Soils A, B, and C are shown in Figure 3-2 and Table 3-1 respectively.

3.3 COMPACTION

3.3.1 Pluvial Compaction for Soil A

For Soil A, 0.5% powdered phenolic resin was mixed with Ottawa sand before compaction. Pluviation was used to compact the sample in a standard Proctor compaction mold (ASTM D698) with a diameter of
<table>
<thead>
<tr>
<th>Soil</th>
<th>Contents</th>
<th>LL</th>
<th>PI</th>
<th>% finer than No. 40 sieve</th>
<th>% finer than No. 200 sieve</th>
<th>Soil Classification (AASHTO)</th>
<th>Unified</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100% Ottawa sand</td>
<td>-</td>
<td>NP</td>
<td>24</td>
<td>0</td>
<td>A-1-b(0)</td>
<td>SP</td>
</tr>
<tr>
<td>B</td>
<td>90% Ottawa sand 10% Kaolin</td>
<td>16</td>
<td>3</td>
<td>42</td>
<td>10</td>
<td>A-1-b(0)</td>
<td>SP-SC</td>
</tr>
<tr>
<td>C</td>
<td>70% Ottawa sand 30% Kaolin</td>
<td>34</td>
<td>18</td>
<td>54</td>
<td>30</td>
<td>A-2-6(1)</td>
<td>SC</td>
</tr>
</tbody>
</table>
102 mm and 116 mm high. Figure 3-3 illustrates the scheme of the pluviation. The sand-resin mixture was poured from a 1000 ml flask through a nozzle made from rubber stopper containing one hole of either 6.8 or 3.8 mm diameter. The sand-resin mixture was free to fall to the surface of the sample while the flask was rotated at a rate of approximately 15 rpm. The density of the sample obtained was a function of the intensity of particle rain, which, in turn, depended on hole size, and the dropping height. In the present study, the sample was made in five layers with height of drop for each layer being 108 mm. In order to keep the same height of drop, the position of flask was adjusted for each layer. The choice of dropping height and hole size depended on the density of the compacted sample desired. For this study, they were selected by trial and error.

3.3.2 Kneading Compaction for Soils B and C

An electrically driven semi-automatic kneading compactor shown in Figure 3-4 was used to compact the samples in a standard Proctor compaction mold. The compaction mold was also used as the permeameter cell in order to reduce the possibility of seepage along the walls of the mold. The kneading compaction procedures described by Garcia-Bengochea (1978) were used in this study with the exception of the definition of the energy level. In the present study, the density level was defined by the standard and modified Proctor compaction as specified in ASTM D698 and D1557. The gage pressure set on the kneading compactor at each water content was selected by trial and error in an attempt to obtain the same density as indicated on the Proctor compaction curves. In Garcia-Bengochea's (1978) study, the gage pressure
Height of Drop, $h_d$

$$h_d = h_s - h_t - h_b - h_i$$

Where

- $h_s$ = adjustable height
- $h_i$ = height of sand being filled after $(i-1)^{th}$ filling; $h_{i=1} = 0$
- $h_t, h_b$ = constants, as shown in figure

Figure 3-3  Schematic Diagram of Pluviation System
Figure 3-4 Kneading Compactor
was set to the same value for each water content to define an energy level, as shown in Figure 3-5. The reasons for the present approach are (1) the density-water content curves for different compaction methods may not have the same shape, and (2) the Proctor compaction curve is more common in soils engineering practice for specifying compaction energy. Following the present procedure, calibration curves for converting gage pressure to the actual foot pressure are not necessary. Hence the problem of variation of calibration curves by seasonal changes of temperature and humidity, as noted by Garcia-Bengochea (1978), has been eliminated.

After mixing and a one day curing period, the soil was placed in the Proctor mold and compacted in five layers of approximately equal thickness. Thirty tamps of the compaction foot were applied for each layer. Prior to compaction of each layer except the first one, the previous layer was scarified, and then fresh soil was added. The scarification of the layers prevented separation between the layers. Once the compaction procedure was completed, the collar of the mold was removed and the soil sample was trimmed flush with the top of the Proctor mold.

3.4 PERMEABILITY TESTS

Basically, the permeability testing equipment and procedures used in this study followed Garcia-Bengochea (1978).

For soil A, the compacted sample in the Proctor mold was connected to the saturation and permeation device after oven drying. For soils B and C, the sample in the mold was connected to the saturation and permeation device directly after completion of compaction process.
Permeability Values at 20°C in cm/sec x 10^{-6} appear within the parentheses.

S9H

Permeability measured for S9H:
- 100% Saturation
- 85% Saturation

LEGEND
- ○ Kneading Compaction, 8.5 psi
- ● Kneading Compaction, 8.5 psi Permeability Measured
- □ AASHTO T99-61 Dynamic Hammer
- △ Kneading Compaction, 40 psi
- ▲ Kneading Compaction, 40 psi Permeability Measured

Figure 3-5: Compaction Curves for 90% Silt and 10% Kaolin
(After Garcia-Bengochea, 1978)
The saturation and permeation device is a falling head permeameter system with provisions for applying both vacuum and back pressure. The apparatus shown in Figure 3-6 consists of three air pressure regulators, one air pressure gage, two standpipes and air pressure and vacuum lines. The two permeameters are nearly identical except for the inside diameters of the standpipes. The permeameter used for measuring the more permeable soils has 12.7 mm inside diameter (I.D.) standpipes, while the other has 6.3 mm I.D. standpipes. The standpipes can be pressurized simultaneously from one or independently from two regulators. Meter sticks were used to record the water elevation heads. The device is designed for both saturation and permeation. A permeameter cell is shown in Figure 3-6.

For soil C, a vacuum of about 34.5 kPa (5 psi) absolute pressure was placed on the top of the sample while the bottom end was under approximately 1 m of water head. This condition was maintained for about 10 hours which allowed de-aired water to flow upward through the sample and allowed air bubbles to escape. For soils A and B, due to their relatively high permeabilities, the above vacuum-saturation process was accomplished within an hour.

After vacuum saturation, back pressure was applied in increments of 34.5 kPa (5 psi) to both ends of the sample. Each increment was maintained for 10 minutes. When the back pressure reached 345 kPa (50 psi), it was maintained for about 15 hours.

In the above saturation process, no B-check is possible to assure that the sample is fully saturated. However, as noted by Garcia-Bengochea (1978), the vacuum saturation consistently resulted in a saturation level of at least 95% for the clay-silt mixtures. There is no measurement of the degree of saturation after vacuum saturation. Accord-
Figure 3-6 Saturation and Permeation Device and Permeameter Cells
ing to Black and Lee (1973), the back pressure required to reach a 99% saturation is about 300 kPa (44 psi) for initial degree of saturation of 95%. The corresponding time required to reach equilibrium is about 4 hours. For the saturation procedure described above, it is believed the degree of saturation is at least 99%, which is appropriate for the practical purposes of permeability measurement. To check this inference, a sample was tested following the above procedures except that a back pressure of 483 kPa (70 psi) was applied during the saturation and permeation phases. No measureable difference was found between the measured permeabilities.

At the completion of the saturation procedure, the falling head permeability tests were conducted. The direction of flow was from top to bottom across the sample. A initial head difference of about 100 cm or 45 cm was created between the two standpipes while a 345 kPa (50 psi) back pressure was maintained on both standpipes. Readings of the elevation heads in the water columns, time, and air temperature was recorded.

Following permeation, the sample was dissected to check for uniformity and trimmed for freeze drying and pore size distribution tests. As noted by Garcia-Bengochea (1978), the back pressure on the sample should not be released too rapidly in order to prevent possible sample disturbance. Thus each decrement of 34.5 kPa (5 psi) was maintained for 15 minutes. When atmospheric pressure was reached, the sample was left in the mold for at least 2 hours before dissection and specimen trimming.
3.5 SPECIMEN PREPARATION AND DRYING AND PSD TESTS

For Soil A, specimens for the PSD test were trimmed from oven-dried compacted samples. The dimensions of individual PSD specimens were approximately 12 x 8 x 8 mm. After trimming, the PSD specimens were placed in an oven for ten minutes in order to get rid of moisture which may have been absorbed by the specimens during trimming. The PSD specimens were then kept in glass bottles and placed in the desiccator.

For Soils B and C, after permeability testing, the PSD specimens were carefully trimmed with a razor blade into cubes approximately 8 mm on a side. The method for drying Soils B and C in this study was the freeze drying technique. The method consists of quickly freezing a small soil specimen to cryogenic temperatures, followed by vacuum drying the frozen sample to remove water by sublimation. This process eliminates the air-water menisci shrinkage effect, which may alter the soil structure, by replacing it with an air-solid system. The small shrinkage that does occur (less than 5% as determined by testing a few specimens initially) after freeze-drying indicates that minimal sample disturbance probably occurred.

The freeze-drying apparatus and procedures used in this study followed Garcia-Bengochea (1978). Appendix A-1 provides details of this apparatus and procedures.

After the specimens were dried, the PSD tests were conducted. The apparatus and procedures used in this study was similar to those used by Garcia-Bengochea (1978) and White (1980). Appendix A-2 provides details of PSD testing equipment and step by step procedures. However, it is worthwhile to point out here that both penetrometers (with intrusion capacity of 0.2 cc and 1.2 cc respectively) were used in
the present study. This is because the small penetrometer (with capacity 0.2 cc), as used by Garcia-Bengochea (1978) and White (1980), cannot properly measure the PSD of the high permeability soils A and B. Also note that the large penetrometer (with capacity 1.2 cc) cannot be placed in the Amico porosimeter; therefore it can only go through low pressure intrusion which measures the distribution of the large size pores (20–500 µm). For these reasons, a combination of data measured by both penetrometers is used to show the distribution of the entire range of pore sizes.
CHAPTER IV PRESENTATION AND DISCUSSION

4.1 COMPACTION

4.1.1 Sample Nomenclature

Each sample tested in this study was assigned a code name based on the soil type, compactive effort, compaction method, and relative water content with respect to its optimum. This code designation is listed in Table 4-1.

4.1.2 Dry Density versus Water Content Relationships for Soils B and C.

The dry density versus water content relationships utilized in this study were determined from the standard and modified Proctor compaction tests. The specifications for these impact compaction tests are listed in Table 4-2. The Proctor test results are listed in Table 4-3. The statistical relationships between dry density and water content obtained from a least squares analysis of these data are shown in Figure 4-1. Satisfactory fit to the data is indicated by the high coefficients of multiple determination listed in Table 4-3.

A California type kneading compactor was used to compact most of the samples tested (i.e., Soils B and C) in this study. The kneading compaction dry density and water content points used in the testing program are shown in Figure 4-1. The gauge pressures utilized for each of these points are listed in Table 4-4. For Soil C, both results of high and medium compactive efforts are presented. However,
TABLE 4-1 Sample Code Designation

Soil Type

0 — Soil A (Ottawa sand only)
SC1 — Soil B (90% Ottawa sand + 10% Kaolin)
SC3 — Soil C (70% Ottawa sand + 30% Kaolin)

Compactive Effort

For Soil A:
L — Relatively low effort
H — Relatively high effort

For Soils B and C:
M — Medium effort (specified in ASTM D698)
H — High effort (specified in ASTM D1557)

Water Content

0 — Optimum water content
0-D — Very slightly dry of optimum
D — Dry of optimum
W — Wet of optimum

Compaction Method

nil — Kneading compaction
P — Pluvial compaction

Example:

SC1MO: Soil B (90% Ottawa sand + 10% Kaolin), compacted by medium effort at optimum water content by kneading compaction.
<table>
<thead>
<tr>
<th>Test Series</th>
<th>Volume of Mold</th>
<th>Height of Hammer Drop</th>
<th>Number of Layers</th>
<th>Weight of Hammer</th>
<th>No. of Blows</th>
<th>Gross Energy KJ/m³ (ft⁻¹-lb/ft³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified Proctor</td>
<td>944 (1/30)</td>
<td>457 (18)</td>
<td>5</td>
<td>44.5 (10)</td>
<td>25</td>
<td>2693 (56,250)</td>
</tr>
<tr>
<td>Standard Proctor</td>
<td>944 (1/30)</td>
<td>305 (12)</td>
<td>3</td>
<td>24.5 (5.5)</td>
<td>25</td>
<td>593 (12,375)</td>
</tr>
</tbody>
</table>

Ref: ASTM - D698 and D1557
<table>
<thead>
<tr>
<th>Data Point</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil Compaction</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type Effort</td>
<td>Medium</td>
<td>Medium</td>
<td>High</td>
<td>Medium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>w/c</td>
<td>6.6</td>
<td>7.0</td>
<td>4.0</td>
<td>6.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_d$ w/c</td>
<td>8.1</td>
<td>10.4</td>
<td>6.3</td>
<td>8.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_d$ w/c</td>
<td>1.72</td>
<td>1.95</td>
<td>1.83</td>
<td>1.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi_d$ w/c</td>
<td>1.79</td>
<td>2.01</td>
<td>1.88</td>
<td>1.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_d$ w/c</td>
<td>1.88</td>
<td>2.01</td>
<td>1.89</td>
<td>1.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_d$ w/c</td>
<td>1.96</td>
<td>1.97</td>
<td>1.93</td>
<td>1.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_d$ w/c</td>
<td>1.96</td>
<td>1.97</td>
<td>1.93</td>
<td>1.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_d$ w/c</td>
<td>12.3</td>
<td>12.6</td>
<td>14.3</td>
<td>12.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_d$ w/c</td>
<td>14.2</td>
<td>14.2</td>
<td>15.2</td>
<td>18.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Remark: The units for water content (w/c) and dry density ($\gamma_d$) are percent and Mg/m$^3$ respectively.
Figure 4-1  Dry Density versus Water Content Curves for Soils B and C

*Curves are somewhat displaced and different in shape from what would have been expected, but are based upon real data.
<table>
<thead>
<tr>
<th>Sample Code</th>
<th>Sample Point</th>
<th>Water Content (%)</th>
<th>Gauge Pressure kPa (psi)</th>
<th>Achieved Dry Density (Mg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC3MD</td>
<td>1</td>
<td>9.1</td>
<td>55.2 (8)</td>
<td>1.87</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>9.2</td>
<td>55.2</td>
<td>1.87</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.4</td>
<td>55.2</td>
<td>1.81</td>
</tr>
<tr>
<td>SC3MO</td>
<td>1</td>
<td>11.0</td>
<td>55.2</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10.9</td>
<td>55.2</td>
<td>1.96</td>
</tr>
<tr>
<td>SC3MW</td>
<td>1</td>
<td>12.5</td>
<td>55.2</td>
<td>1.90</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>12.5</td>
<td>55.2</td>
<td>1.91</td>
</tr>
<tr>
<td>SC3HD</td>
<td>1</td>
<td>6.5</td>
<td>193.2 (28)</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6.6</td>
<td>193.2</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6.2</td>
<td>193.2</td>
<td>1.94</td>
</tr>
<tr>
<td>SC3HO-D</td>
<td>1</td>
<td>8.0</td>
<td>207.0 (30)</td>
<td>1.99</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7.9</td>
<td>207.0</td>
<td>1.99</td>
</tr>
<tr>
<td>SC1MD</td>
<td>1</td>
<td>6.2</td>
<td>34.5 (5)</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6.1</td>
<td>34.5</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.9</td>
<td>34.5</td>
<td>1.87</td>
</tr>
<tr>
<td>SC1MO</td>
<td>1</td>
<td>8.0</td>
<td>41.4 (6)</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8.1</td>
<td>41.4</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.0</td>
<td>41.4</td>
<td>1.87</td>
</tr>
<tr>
<td>SC1MW</td>
<td>1</td>
<td>11.0</td>
<td>34.5</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11.1</td>
<td>34.5</td>
<td>1.88</td>
</tr>
</tbody>
</table>
for Soil B, only the result of medium compactive effect of this soil was presented because 1) the compaction result was little affected by the magnitude of the impact compaction effort; and 2) the appropriate gauge pressures necessary to fit the results of kneading compaction to the desired density as defined by the modified Proctor compaction curve were not able to be found in the present study.

4.1.3 Pluvial Compaction Results for Soil A

The results of Soil A obtained from the pluvial compaction are listed in Table 4-5. The results indicate that the desired density can be replicated by controlling the diameter of hole and the height of drop.

4.2 PERMEABILITY

The permeability was measured by a falling head apparatus under a back pressure of 345 kPa (50 psi). The hydraulic gradient during the tests ranged between 2 to 5. The permeabilities were calculated from the usual falling head equation (Taylor, 1948)

\[
k = \frac{aL}{\Delta tA} \ln \frac{h_o}{h_t}
\]  

(4-1)

where \(k\) = permeability

\(a\) = cross-sectional area of the standpipes

\(A\) = cross-sectional area of sample

\(\Delta t\) = time increment

\(h_o\) = total head at zero time

\(h_t\) = total head after time \(\Delta t\).

Figure 4-2 shows a schematic diagram of the falling head test.
<table>
<thead>
<tr>
<th>Sample Code</th>
<th>Compacted Sample Point</th>
<th>Diameter of Hole (mm)</th>
<th>Height of Drop (mm)</th>
<th>Number of Layers</th>
<th>Achieved Dry Density (Mg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>6.8</td>
<td>108</td>
<td>5</td>
<td>1.62</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>6.8</td>
<td>108</td>
<td>5</td>
<td>1.62</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6.8</td>
<td>108</td>
<td>5</td>
<td>1.62</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>6.8</td>
<td>108</td>
<td>5</td>
<td>1.62</td>
</tr>
<tr>
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<td>1.62</td>
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<tr>
<td>OPH</td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3.8</td>
<td>108</td>
<td>5</td>
<td>1.72</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3.8</td>
<td>108</td>
<td>5</td>
<td>1.72</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3.8</td>
<td>108</td>
<td>5</td>
<td>1.72</td>
</tr>
<tr>
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<td>4</td>
<td>3.8</td>
<td>108</td>
<td>5</td>
<td>1.72</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>3.8</td>
<td>108</td>
<td>5</td>
<td>1.72</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>3.8</td>
<td>108</td>
<td>5</td>
<td>1.72</td>
</tr>
</tbody>
</table>
Figure 4-2  Schematic Diagram of Falling Head Permeability Test (After Garcia-Bengochea, 1978)
It is evident from the figure that

\[ h_0 = (h_{Ao} + u_o) - (h_{Bo} + u_o) \]
\[ = h_{Ao} - h_{Bo} \]  
\[ (4-2) \]

where \( u_o \) = back pressure on the system

and

\[ h_t = h_{At} - h_{Bt} \]  
\[ (4-3) \]

The permeability is then calculated from Equation (4-1).

The variation of the room temperature during permeation of most of the permeability tests was negligible (e.g., \( \pm 0.4^\circ C \)), although in a few tests the variation was up to \( \pm 0^\circ C \) or so. For each test, an average temperature was taken and the following equation suggested by Taylor (1948) was used to calculate and to report the permeability:

\[ k_{20^\circ C} = k_T \cdot \frac{\mu_T}{\mu_{20^\circ C}} \]  
\[ (4-4) \]

where \( k_{20^\circ C} \) = permeability at temperature \( 20^\circ C \)

\( k_T \) = permeability at temperature \( T \) of test

\( \mu_T \) = viscosity of water at temperature \( T \)

\( \mu_{20^\circ C} \) = viscosity of water at temperature \( 20^\circ C \).

A summary of the permeability test results of the compacted samples listed in Table 4-4 and Table 4-5 is given in Table 4-6. In this table the permeability is normalized to an ASTM standard temperature of \( 20^\circ C \).

For Soil C, the dry-side samples had permeabilities one to two orders of magnitude greater than the optimum and wet-side samples.
### TABLE 4-6 Permeabilities of Samples Tested

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Sample Code</th>
<th>Point No.</th>
<th>( k_{20°C} ) (cm/sec)</th>
<th>Average ( k_{20°C} ) (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>4.6 x 10^-5</td>
<td>4.5 x 10^-5</td>
</tr>
<tr>
<td></td>
<td>SC3MD</td>
<td>2</td>
<td>4.6 x 10^-5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>4.2 x 10^-5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SC3MO</td>
<td>1</td>
<td>7.1 x 10^-7</td>
<td>7.3 x 10^-7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>7.5 x 10^-7</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>SC3MW</td>
<td>1</td>
<td>6.1 x 10^-7</td>
<td>6.0 x 10^-7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>5.8 x 10^-7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SC3HD</td>
<td>1</td>
<td>1.8 x 10^-6</td>
<td>1.9 x 10^-6</td>
</tr>
<tr>
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<td>3.0 x 10^-3</td>
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<tr>
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<td></td>
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<tr>
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<td></td>
<td></td>
<td>6</td>
<td>1.3 x 10^{-2}</td>
<td></td>
</tr>
</tbody>
</table>
There was little difference between the permeabilities of the optimum and wet-side samples. These trends were found by the previous researchers, e.g., Lambe (1954), Mitchell, Hooper and Campanella (1965), and Garcia-Bengochea (1978). However, for Soil B which contains only 10% clay by weight, the above trend is not very clear; i.e., the difference between the dry-side and optimum samples was small. Two possible reasons are 1) water content is not a sensitive variable in the compaction of this soil, as indicated in Figure 4-1, and 2) the dry-side samples tested are not sufficiently dry of optimum (see Figure 4-1). Strictly speaking, there is no clear optimum water content on the standard Proctor compaction curve of this soil. This may be the nature of this poorly graded sandy soil.

4.3 **PORE SIZE DISTRIBUTION**

4.3.1 Data Reduction

As mentioned in Section 2.1.1, the absolute applied pressures can be converted to apparent intruded pore diameters based on Equation (2-1). This step requires that the surface tension and contact angle be known. The surface tension used in this study is assumed to be 484 dynes/cm for all soils used (Kemball, 1946 and Bhasin, 1975). The contact angle of mercury on Kaolin is $147^\circ$ (Diamond, 1970) and that of mercury on quartz (sand) is also $147^\circ$ (Ellison, Klemm and Schwartz, 1967). Based on these reported values (both at 25°C), the contact angle of mercury on Soils B and C is assumed to be $147^\circ$. For Soil A, the contact angle of mercury on phenolic resin was determined to be $154^\circ$ following the method suggested by Winslow (1969, 1980). Since the surface tension of phenolic resin is much lower than that of sand, therefore the con-
tact angle of mercury on Soil A was assumed to be 154°.

For each PSD test, the applied pressures and intrusion data were recorded. The pressures recorded during the filling operation were absolute pressures, while those recorded during the porosimeter operation (called high pressure intrusion) were gage pressures. As noted by Bhasin (1975), a net correction of 76 kPa (11 psi) accounting for atmospheric pressure and negative head of mercury is added to the gauge pressures to obtain the absolute pressures. The absolute pressures are next converted to apparent intruded pore diameters by Equation (2-1). The next step in the calculation is to apply the corrections to the intrusion data. The corrections required during low pressure intrusion are to account for the fact that part of the intrusion recorded is actually the compression of the air that was trapped in the penetrometer during the mercury-filling period (refer to Appendix A-2). The high pressure correction to the intrusion data is to account for the compressibility of the sample, penetrometer and mercury, as well as the temperature changes that occur as a result of pressurization. A "blank test" for this correction had been outlined in a proposed ASTM standard, subcommittee D18.14. In this study, a correction of 0.001 ml per ml of mercury in the penetrometer at 103,500 kPa (15,000 psi), as noted by Bhasin (1975), was applied.

A computer program developed by Reed (1977) was utilized to reduce the PSD data. This program consisted of the above calculations and corrections. A significant modification of this program has been made, especially for the graphical presentations of the PSD data and the finite difference approximation (see Section 4.3.2). A listing of this modified computer program and sample output are given
4.3.2 Data Presentation

The PSD data are normally expressed in terms of the cumulative volume of pore space intruded as a function of pore diameter. Since a wide range (e.g., several orders of magnitude) of pore sizes is measured, it is often convenient to present pore diameter on a logarithmic scale. A graphical cumulative distribution with the intruded volumes per gram on the ordinate to an arithmetic scale and the limiting pore diameters (called apparent pore diameters) on the abscissa to a logarithmic scale is a common way to present the PSD test results, e.g., Diamond (1970), Ahmed (1971), Bhasin (1975), Cebeci, et al. (1978), and the proposed ASTM standard, subcommittee D18.14. From Equation (2-5a) it is evident that the ordinate of the density function plot is \(-d(V_t - V)/dx\). Multiplying this derivative quantity by \(x/2.303\) and rearranging gives \(-d(V_t - V)/d(\log x)\). Since the cumulative distribution is expressed as \((V_t - V)\) versus \(\log x\), it is preferred to plot the quantity \(-d(V_t - V)/d(\log x)\) instead of \(-d(V_t - V)/dx\) versus \(\log x\).

The density function value can be obtained by (1) directly taking the derivative of \(F_{vx}(x)\) which is a curve-fitted function of the above cumulative pore size distribution curve, or (2) using a finite difference approximation. The first method is obvious and its accuracy depends on goodness of curve-fitting. In this study, the pore size density function of soils is obtained in this manner (see Section 4.3.3). The second method was first used by García-Bengochea (1978) for the graphical presentation of the PSD test data. However, since pore size is a continuous random variable, it is appropriate to present a pore
size density function instead of a frequency polygon, which was used by Garcia-Bengochea. To approximate the density function a frequency histogram is plotted (i.e., \( \Delta(V_t - V) \) versus \( \log x \), see Figure 4-3a). For a good approximation, the class width, \( \Delta(\log x) \), has to be an appropriate constant.

Assume a constant \( c \) is chosen, then

\[
\log x_{i-1} - \log x_i = c \quad (4-5)
\]

or

\[
\frac{x_{i-1}}{x_i} = 10^c \quad (4-6)
\]

From Equation (2-1), it is evident that the applied pressure is inversely proportional to the pore diameter. Therefore this approximation method requires that the applied pressure, \( P \) be raised as follows:

\[
\frac{P_i}{P_{i-1}} = 10^c \quad (4-7)
\]

Note that \( \Delta(V_t - V) \) represents change in intruded pore volume, which is the difference between the two successive intrusions, and it can be calculated directly from intrusion record. The pore size density function plot is approximated by connecting the points of \( [\log d_i, f(\log d_i)] \) as shown in Figure 4-3b, where \( \log d_i \), the midpoint of each class, is calculated as follows:

\[
\log d_i = \frac{1}{2} (\log x_{i-1} + \log x_i) \quad (4-8)
\]

or

\[
d_i = \sqrt{x_{i-1} \cdot x_i} \quad (4-9)
\]

and \( f(\log d_i) \) is the probability density at \( \log d_i \), which is calculated as follows:
Figure 4-3(a) Frequency Histogram

Figure 4-3(b) Finite Difference Approximation to PSD Density Function
A few specimens were tested and presented according to this finite difference approximation method. Figure 4-4 shows a comparison of the above two approaches. It is evident that these two methods match up very well if an appropriate constant \( c \) is used. The value of \( c \) should be as small as possible. However, the appropriate value of \( c \) depends on the pore size characteristics of soils. The wider the pore size range is the larger the value of \( c \) should be. In this study, the value of \( c \) varied from 0.1 to 0.2.

In addition, some investigators (e.g., Lohnes, et al., 1976 and Diamond, et al., 1972) prefer to express pore size distribution as a dimensionless term (e.g., percentage) for the convenience of the mathematical treatment. Equations (2-6) and (2-7) provide the theoretical basis for this approach. If the pore size distribution data are to be correlated with soil properties, it is desirable to express the pore size distribution in mathematical forms. Hence it is convenient to express pore size distribution as a percentage. In Equation (2-7), the term \( V_t \) represents the total void volume that can be intruded by mercury. According to this definition, the cumulative distribution for pores with size greater than a lower limit pore diameter (which depends on the capacity of porosimeter) should be 100% no matter how much unintruded pore space is left. On the other hand, if the term \( V_t \)
Figure 4-4 Comparison of Finite Difference Approximation Method and Direct Differentiation for Samples SC3MD (Six PSD Data Sets)

*The unit of density function is indeed (cc/g/chosen diameter interval), but the chosen diameter interval is dimensionless (see Eq. 4-11).*
represents total void volume that exists in the sample, the cumulative distribution for pores with size greater than a lower limit pore diameter may not be 100%, but depends on how much pore space is unintruded.

4.3.3 Pore Size Distribution Curves and Fittings of the Curves

Following the preceding discussion of data presentation, the pore size distribution curves directly generated from the PSD data are shown in Appendix A-4. In addition to the cumulative distribution curves, some figures also show the density function curves whenever the finite difference approximation method is applicable.

At least three PSD specimens were tested for each compacted sample and two compacted samples were tested for each sample code. In general, the replication of the PSD tests was satisfactory. However some scatter is present in these data points. The reasons for scatter are:

1. Some of the specimens might have suffered more disturbance than others during trimming and transportation of the specimens. This is especially a problem for Soil B in which insufficient cohesion is created by the small amount of clay (i.e., 10%).

2. The compacted samples themselves might not be completely homogeneous.

3. Measurement errors may also account for scatter in replicate pore size distribution curves, especially when a trapped air pocket on the surface of the specimen happens at the filling stage (see Appendix A-2) or when the pore volume of a specimen exceeds the intrusion capacity of the porosimeter.

4. The variability of replicate compacted samples may also account for the scatter shown in these figures.
A piecewise cubic function based on the least squares method was selected to fit the PSD curve. A computer program called "SPLINE" in the Purdue University CDC 6500-6600 computer library was used to do the least squares analysis. A description of "SPLINE" and some sample output are listed in Appendix A-5. The coefficients of the piecewise cubic functions for all samples are also listed in this Appendix.

The next step is to calculate the functional values of the pore size distribution and its probability density and then to present the graphical results of the PSD tests. Figure 4-5 shows a flow chart of the procedures. A computer program for this calculation is listed in Appendix A-6. Figure 4-6 through Figure 4-15 show these graphical results. Each of the figures consists of a cumulative distribution and its density function with the volume for gram on the ordinate to an arithmetic scale and the limiting pore diameter (or called apparent pore diameter) on the abscissa to a logarithmic scale.

The density function provides a useful complement to the cumulative distribution curve for representing pore size characteristics. The pore size density function curves displayed bimodal characteristics for clay-sand mixtures (i.e., Soils B and C). This was the case found by Garcia-Bengochea (1978) for clay-silt mixtures. However, a single mode characteristic was found for Soil A. This is logical since Soil A contains only sand grains.

For the bimodal characteristics of Soils B and C, the small pore mode occurred consistently around 0.1 μm and the large pore mode was between 10 and 100 μm. For the single modal characteristic of Soil A, the large pore mode occurred around 200 μm. It is obvious that the mode and its probability density may serve as useful qualitative
Figure 4-5  Flow Chart of Procedures for the PSD Curves
Figure 4-6  PSD Curves of SCIMW
Figure 4-7 PSD Curves of SCIMO
Figure 4-8 PSD Curves of SCIMD
Figure 4-9  PSD Curves of SC3MW
Figure 4-10  PSD Curves of SC3MO
Figure 4-11  PSD Curves of SC3MD
Figure 4-12  PSD Curves of SC3HO-D
Figure 4-13 PSD Curves of SC3HD
Figure 4-14  PSD Curves of OPL
Figure 4-15 PSD Curves of OPH
descriptors for the soil fabric.

For the purpose of correlating the theoretical permeability model with the pore size distribution data, it is preferable to express the cumulative distribution and density function of pore size as a percentage. It should be noted that at the maximum pressure exerted, about 10 to 15 percent of the pore space was not intruded for the soils tested. The reasons for this have not been established with certainty. Some of the pores may have entry ways too narrow to penetrate with mercury at 103,500 kPa (15,000 psi), and most likely some of them may be in a network of pores not continuous with the boundaries of the specimen. Moreover, a minor shrinkage of the specimens during sample drying could account for part of the discrepancy.

A significant unintruded pore space is not unusual for pore size distributions determined by the mercury intrusion technique. About 25% of the pore space was not intruded by mercury in the study by Diamond, et al. (1972) for hydrated cement paste. A value of 20% unintruded pore space was reported by Lohnes, et al. (1976) for selected Hawaiian soils (e.g., Manana soil, consisting of 70% sand, 16% silt and 14% clay). A 45% unintruded pore space was reported by White (1980) for a medium plastic clay, St. Croix clay. However, only about 5% unintruded pore space was reported by Garcia-Bengochea (1978) and Reed (1977) for clay-silt mixtures. Note that the above values are all under the maximum intruded pressure of 103,500 kPa (15,000 psi) except the one used by Diamond, et al. (1972), in which a maximum pressure of 517,500 kPa (50,000 psi) had been applied. With these facts in mind, the discontinuity of pores may be the main reason of the existence of the unintruded pore space
if a proper specimen drying method is used. However, Reed, et al. (1980) have expressed the view that nearly all pore space in soil systems is connected.

Fortunately, this unintruded pore space effect is of minor importance in this study which deals with flow through porous materials, because the pore channels through which water passes are probably larger than those penetrated by mercury at pressure of 15,000 psi.

An attempt at a correction for this unintruded pore space effect was made by Meyer (1953). A modified form of his work is given in Appendix A-7.

Because of this unintruded pore space, the graphical results of the pore size distribution and its density function are presented in both manners discussed in Section 4.3.2. An example is shown in Figures 4-16 and 4-17. The rest of them are shown in Appendix A-8.

4.3.4 Comparison of Pore Size Curves

Figures 4-18, 4-19, and 4-20 show the effect of water content on pore size distribution. For SC3M samples (see Figure 4-18), the pore size density function shows a bimodal characteristic with differences in the large pore mode, and almost no changes in the small pore mode. The large pore mode decreases as water content increases from dry of optimum to optimum water content. There is little difference in the large pore mode of the optimum and the wet side samples. In general, the trend as indicated above is similar to that found by Garcia-Bengochea (1978).

Comparing the small pore modes displayed in Figures 4-18 and 4-19, it is evident that the small pore mode is almost the same in spite of
Figure 4-16  PSD Curves of SC3MD (Percent of Total Pore Volume Existing)
Figure 4-17  PSD Curves of SC3MD (Percent of Total Pore Volume Intruded)
Figure 4-18 Comparison of PSD Curves for Samples Compacted at Different Water Contents (SC3M series)
Figure 4-19 Comparison of PSD Curves for Samples Compacted at Different Water Contents (SC3H series)
Figure 4-20 Comparison of PSD Curves for Samples Compacted at Different Water Contents (SCIM series)
an increase of the compactive effort. This phenomenon will be discussed later. Unlike the trend in Figure 4-18, the large pore modes of the dry side and the optimum samples are quite similar in Figure 4-19, although they do show a more significant difference in the cumulative distribution curves. This may be due to inaccurate location of points of the optimum samples (see Figure 4-1). The optimum samples were indeed slightly dry of optimum water content, thus the samples were coded as SC3H0-D. Comparing results of permeability tests (see Table 4-6), the difference between permeability of Sample SC3HD and Sample SC3H0-D is much smaller than that of Sample SC3MD and Sample SC3MO. The similar trend when comparing pore size characteristics and those of permeability implies that pore size distribution could be a sensitive indicator of permeability.

Figure 4-20 shows that the difference between pore size distribution of the dry side and optimum or wet side samples is small. The reasons noted in Section 4-2 for comparison of permeability measurements on the same soil compacted at different water contents may also contribute to the small differences between these pore size characteristics.

As noted in Section 2.1.3, seepage occurs only through the pores, and what is directly related to the permeability is the characteristics of the pores themselves. Thus the permeability of each of the samples increases with increasing large pore mode. Insignificant difference in the large pore mode is reflected by insignificant difference in permeability.

Figure 4-21 and Figure 4-22 show the effect of compactive effort on pore size characteristics. Note that these comparisons are only semi-
Figure 4-21 Comparison of PSD Curves for Samples Compacted at Different Compactive Efforts (Soil C)
Figure 4-22 Comparison of PSD Curves for Samples Compacted at Different Compactive Efforts (Soil A)
qualitative because the water content was not controlled at the same value. Figure 4-21 shows that as the compactive effort was increased, the large pore mode decreased, while the small pore mode remains unchanged. Figure 4-22 also shows the same trend except that there is no small pore mode. Recalling the permeability of these soils, again it decreases with decreasing large pore mode. This trend was also found by Garcia-Bengochea (1978).

Figures 4-23, 4-24 and 4-25 compare the pore size characteristics of Soils B and C compacted by medium effort dry of optimum, optimum and wet of optimum water contents, respectively. The large pore mode of the soils decreases with increasing clay content, while the small pore mode remains constant, but its probability density increases with increasing clay content.

Figure 4-26 compares the pore size characteristics of Soils A, B, and C with different compaction variables. The small pore mode remains the same in spite of differences in compactive effort and amount of clay, while its probability density decreases with decreasing clay content. Note that there is no small pore mode in soil A, which follows from the fact that Soil A has no clay content. The large pore mode of the soils increases with increasing sand content as mentioned above. Comparing the permeability values and pore size characteristics of these soils (see Table 4-6 and Figure 4-26) shows that the permeability decreases with a decrease in the large pore mode.

For the soils tested, the large pore mode and its probability density appears to be the best indicator of changes in soil fabric caused by varying water content, compactive effort, and soil type. This was also found by Garcia-Bengochea (1978).
Figure 4-23  Comparison of PSD Curves for Different Soils (B and C) Compacted by Medium Effort at Dry of Optimum Water Content
Figure 4-24  Comparison of PSD Curves for Different Soils (Band C) Compacted by Medium Effort at Optimum Water Content
Figure 4-25  Comparison of PSD Curves for Different Soils (B and C) Compacted by Medium Effort at Wet of Optimum Water Content
Figure 4-26 Comparison of PSD Curves for Different Soils (A, B and C)
The hypothesis, proposed by Garcia-Bengochea (1978), that the shape of the differential pore size distribution (called pore size density function in present study) in the small pore region (referred to as small pore mode) may serve as a "signature" of a given clay type, is supported by the data in this study. Again, the small pore mode is related to the size range of clay particles present, and the probability density of the small pore mode is in proportion to the amount of clay minerals in the soil.

4.4 **PREDICTION OF PERMEABILITY BY PROPOSED MODEL**

The objective of this section is to correlate the permeability and the PSD of the selected soils by applying the theoretical model proposed in Section 2.2.2.

Before going into a detailed discussion of the proposed model, it is worthwhile to examine the following functional relationships between void ratio, e and permeability, k as presented by Taylor (1948) and Lambe (1951):

(a) $k$ vs. $e^3/(1 + e)$
(b) $k$ vs. $e^2/(1 + e)$
(c) $k$ vs. $e^2$
(d) $\log k$ vs. $e$

Since the permeabilities of the soils studied vary over five orders of magnitude, it is very unlikely that linear relationships for these relationships exist. Therefore the logarithm of $k$, instead of $k$, was used for the regression analyses. The statistical programs of the Statistical Package for the Social Sciences (SPSS) in the Purdue University CDC 6500-6600 computer library were used to do least squares analysis and regressions in this study. The coefficient of determination
(R^2) of the regressions of k with respect to the functional parameters of e mentioned above are shown in Table 4-7. Although R^2 was around 0.6, the differences between the measured and the calculated permeabilities are generally greater than one order of magnitude and, in some samples are greater than two orders of magnitude. The results noted above imply that gross volumetric parameters (e.g. void ratio or porosity and their functions) cannot appropriately account for changes in permeability of compacted soils.

Previous discussions in Section 2.2 provided the theoretical background for predicting the permeability from the PSD of soils. From the qualitative observations discussed in Section 4.3.4, it is evident that the permeability is related to the distribution of pore sizes about the large pore mode. Moreover, the proposed model is very sensitive to the larger pore diameters (see Equation 2-19). Therefore the pores of diameter less than 1um were considered to be negligible in the calculation of permeability. A physical explanation of choosing 1um as the lower limit of pore sizes which is accessible to flow is the concept of "threshold diameter" given by Murota and Sato (1969). In their study, a threshold pore diameter of 1.8um was chosen for silty clay. On the other hand, the upper limit of pore sizes in this study was bounded by the pressure capacity of the mercury porosimetry. Hence Equation (2-19) can be rewritten as:

\[
k = \frac{\gamma n}{32\mu} \int_1^{x_{\text{max}}} \int_1^{x_{\text{max}}} \bar{x}^2 G(x_j) \cdot f(x_i) \cdot f(x_j) \, dx_i \, dx_j \quad (4-12)
\]

Numerical integration of Equation (4-12) was accomplished by a computer program listed in Appendix A-9. Note that the equivalent diameter, \(\bar{x}\) was assumed to be the smaller of each pair of connected
<table>
<thead>
<tr>
<th>Sample code</th>
<th>Measured Permeability k(cm/sec)</th>
<th>Calculated Permeability (cm/sec) — Based on Regression Equation $k = 10^a + bx$</th>
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The term \( G(x_j) \) defined in Equation (2-18) contains an undetermined function, \( V \). Thus in order to predict permeability from Equation (4-12), \( V \) must be assumed. In this study, \( V \) was assumed to be a linear function of pore diameter (i.e. \( V(x_j) = v \cdot x_j \)) in order to simplify the calculations. The pore size density function \( f \) was determined previously (see Section 4.3.3 and Appendix A-5) and was expressed as a dimensionless term. In other words, \( f \) was a piecewise cubic function which fitted its corresponding PSD curve (i.e., Figures A-8-1 through A-8-20).

The computer program listed in Appendix A-9 provides an iterative method to seek a suitable value of \( v \) at which a good prediction of permeability is obtained. In this computer program the criterion of a good prediction of permeability is determined by the user. In the present study, a criterion of a 15% error with respect to the measured permeability was arbitrarily chosen. Two other criteria were number of iterations and the relative change of the calculated permeability between the two successive values of \( v \). For each iterative run, the main feature of the computer program listed is the numerical integration of the double integral in Equation (4-12). A subroutine called "CADRE" in the Purdue University CDC 6500-6600 computer library was utilized to carry out this numerical integration.

The output of the iterations discussed above for the samples tested is also given in Appendix A-9.

Following the above discussions and the results given in Appendix A-9, the values of \( v \) for all samples studied are listed in Table 4-8. The next step is intended to correlate \( v \) to the pore size characteristics of soils. Table 4-9 lists some of the possible
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<th>Sample Code</th>
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TABLE 4-9  Possible Correlations between PSD Parameters and v

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Remarks:

a) $D_{25}$ means 25 Percentile (a diameter) of the cumulative distribution curve, i.e., the subscripts represent the percentile of the distribution.

b) $LD_{25}$ means the log of diameter $D_{25}$.
correlations between PSD parameters and v-values. Simple linear regression by the method of least squares was used to evaluate these possible correlations, and the following satisfactory linear relation was established with $R^2 = 0.97$ and an adjusted $R^2_a = 0.96$:

$$\ln v = -6.15 - 40.07 \left( \frac{D_{40}}{D_{25}} \right) + 118.79 \left( \frac{D_{40}}{D_{10}} \right) - 78.35 \left( \frac{D_{60}}{D_{10}} \right)$$  \hspace{1cm} (4-13)

With Equation (4-13) and the PSD of soils, the value of $v$ can be determined (see Table 4-8). The permeability can then be calculated by using the computer program described above. Table 4-10 compares the calculated and the measured permeabilities. A linear regression was performed and the result as shown in Figure 4-27 was excellent.

In summary, the proposed permeability model can provide an excellent predictive tool. Thus a functional relationship between the PSD and the permeability of compacted soils is clearly demonstrated.

4.5 APPLICABILITY OF THE PROPOSED PERMEABILITY MODEL

Attempts were made to broaden the range of prediction of permeability using independently determined data. Ten sets of PSD curves along with permeability values were taken from Garcia-Bengochea (1978). The procedures described above were carried out for both the author's and Garcia-Bengochea's data. The following regression equation was established for the v-value ($R^2 = 0.89$):

$$\ln v = -3.6031 - 2.5957 \left( \frac{D_{40}}{D_{25}} \right) + 14.9678 \left( \frac{D_{40}}{D_{10}} \right) - 7.2562 \left( \frac{D_{60}}{D_{10}} \right) + 4.6320 \left( \frac{D_{60}}{D_{25}} \right) - 2.7129 \left( \frac{D_{25}}{D_{10}} \right) - 3.8473 \left( \frac{D_{60}}{D_{40}} \right)$$  \hspace{1cm} (4-14)
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</tr>
<tr>
<td>SC3HD</td>
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<td>$1.9 \times 10^{-6}$</td>
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Regression Equation ($R^2_a = .99$)

\[ Y = .997X - .001 \]

where

- $Y = \log$ of permeability(calc.)
- $X = \log$ of permeability(meas.)

Figure 4-27  Prediction of Permeability by Proposed Model
The values of \( v \) calculated from Equation (4-14) were then used in the calculation of permeability. The information from each data set, including sample code, PSD curve-fitting result, the \( v \)-value, and the calculated and measured permeabilities are given in Appendix A-10. Note that the computer program listed in Appendix A-9 was used to perform the above calculations. In addition, a regression analysis was performed on the permeability data (i.e., the calculated and the measured permeability). The result is shown in Figure 4-28. It is evident that the proposed permeability model provides as excellent theoretical tool for the prediction of permeability for a wide range of soil types.
Regression Equation ($R^2 = .98$)

$$Y = .99X^{-.03}$$

where

$Y$ = log of permeability (calc.)

$X$ = log of permeability (meas.)

Figure 4-28 Prediction of Permeability by Proposed Model (Author's and Garcia-Bengochea's Data)
5.1 SUMMARY

The theoretical background for the determination of the pore size distribution of soils has been critically reviewed. Mathematical interpretations of PSD were given in detail. For the soils studied, both the pore size distributions and pore size density functions were presented. The pore size distributions from original PSD data were expressed as intruded volume per gram of specimen. In addition, they were also expressed as a dimensionless term (e.g., percentage) for convenient mathematical treatment. The pore size density function could be obtained either from direct differentiation of a curve-fitted cumulative distribution function, or by finite difference approximation. Moreover, the results of both methods agree very well.

The soils used in the study were a sand and two sand-clay mixtures. Pluvial compaction was used to prepare compacted samples of sand while kneading compaction was used to prepare samples of sand-clay mixtures. The testing program consisted of the permeability test and the PSD test. The experimental procedures of these tests for sand-clay mixtures (called Soils B and C in this study) generally followed García-Bengochea (1978). For the sand (called Soil A), a small amount, 0.5% by weight, of powdered phenolic resin was mixed with sand before compaction. Oven drying at a temperature of 170°C for 1 h was carried out after compaction in order to create a "cohesion." This technique
enabled PSD specimens to be prepared and PSD tests to be conducted. Thus, the characterization of sand fabrics by their PSD was possible.

Qualitative descriptions of the pore size characteristics of the soils studied were given through their pore size density functions. When displayed on a log diameter scale, the pore size density function of Soil A showed a single modal characteristic while those of soils B and C showed bimodal characteristics. The location of the mode (at which a peak was observed) and its probability density could serve as a useful qualitative descriptor. For the bimodal characteristics of Soils B and C, the small pore mode occurred consistently around 0.1 μm and the large pore mode was between 10 and 100 μm. For the single modal characteristic of Soil A, the large pore mode occurred around 200 μm.

The hypothesis, proposed by Garcia-Bengochea (1978), that the shape of the differential pore size distribution (called pore size density function in the present study) in small pore region (referred to as small pore mode) may serve as a "signature" of a given clay type, is supported by the data obtained in this study. In addition, changes in soil fabric caused by varying water content and compactive effort were generally reflected by changes in large pore mode and its probability density.

The relation between the compaction variables and the permeability found in this study generally agree with the trends found by Lambe (1954), Mitchell, et al. (1965) and Garcia-Bengochea (1978). For the prediction of permeability, the relation between permeability and pore geometry was critically reviewed. A new permeability model based on Child and Collis-George's (1950) model was proposed. The
new model had the following characteristics: (1) a probabilistic basis, (2) no arbitrary matching factor required, and (3) consideration of a tortuosity factor. The prediction of permeability by the proposed model was excellent (e.g., the $R^2_a$ of regression of predicted permeability with respect to measured permeability equaled 0.99). Data obtained independently by García-Bengochea (1978) was also used to examine the proposed model. The excellent result [e.g., $R^2_a = 0.98$ for both author's and García-Bengochea's (1978) data] confirmed the applicability of proposed model.

5.2 CONCLUSIONS

1. A technique for the preparation of PSD specimens of cohesionless sands has been developed.

2. Finite difference approximation method is an appropriate approach for determining pore size density functions. The graphical presentation of pore size density function obtained from a finite difference approximation agrees very well with that obtained from direct differentiation of the curve-fitted cumulative distribution function.

3. The curve fitting technique using the computer program "SPLINE" for the PSD data of soils studied was apparently satisfactory.

4. Both pore size distribution and pore size density function are useful for characterizing the pore size characteristics of the soils studied in this research. The mode and its probability density of pore size density function curves served as successful qualitative descriptors, especially in characterizing the changes in soil fabric caused by varying compact-
ion variables. The pore size parameters (e.g., some percentile and their functions) obtained from pore size distributions were successfully used to predict the constant $v$ of the proposed permeability model.

5. The pore size density function of Soil A (Ottawa sand) was found to be singly modal on a log diameter scale. The mode occurred around 200 $\mu$m.

6. The pore size density functions of Soils B and C (sand-clay mixtures) were found to be bimodal on a log diameter scale. A large pore mode occurred between 25 $\mu$m and 100 $\mu$m, and a small pore mode occurred around 0.1 $\mu$m.

The following conclusions (7 through 12) verify conclusions previously reached by Garcia-Bengochea (1978).

7. The large pore mode decreased as water content increased from dry of optimum to optimum water content.

8. The large pore mode decreased as the compactive effort increased.

9. The large pore mode of the soils tested decreased with increasing clay content.

10. The small pore mode remained the same in spite of differences in compactive effort and amount of clay, while its probability density decreased with decreasing clay content.

11. For the soils tested, the large pore mode and its probability density appear to be the best indicators of changes in soil fabric caused by varying water content, compactive effort, and soil type.
The hypothesis, proposed by Garcia-Bengochea (1978), that the shape of the differential pore size distribution (called pore size density function in the present study) in the small pore region (referred to as small pore mode) may serve as a "signature" of a given clay type, is supported by this study. The following conclusions (13 through 18) apply to the proposed permeability model.

13. The coefficient of determination ($R^2$) of the conventional relation between permeability $k$ and void ratio $e$ ($e$ vs log $k$) was 0.54 for the soil samples tested. This result showed that $k$ can not be accurately predicted from total porosity or functions thereof.

14. A model has been proposed which relates PSD to soil permeability which is as follows:

$$k = \frac{\sqrt{\nu}^2}{32\mu} \int_0^{\infty} \int_0^{\infty} \frac{x^2}{G(x_j) \cdot f(x_j) \cdot f(x_j) \cdot dx_i \cdot dx_j}$$

where

$$G(x_j) = e^{-\left(\frac{x_j - x_j}{V}\right)^2}$$

and

$$V = \nu \cdot x_j$$

15. The following regression equation for the constant $\nu$ was obtained for the samples tested in this study ($R^2 = 0.97$ and $R^2_{adj} = 0.96$):

$$\ln \nu = -6.15 - 40.07 \left(\frac{D_{40}}{D_{25}}\right) + 118.79 \left(\frac{D_{40}}{D_{10}}\right) - 78.35 \left(\frac{D_{60}}{D_{10}}\right)$$
16. The excellent predictive result by the proposed model is evident from the following regression equation for the samples tested in this study ($R^2 = 0.99$):

$$Y = 0.997X - 0.010$$

when $Y = \text{the log of the calculated permeability}$

$X = \text{the log of the measured permeability}$

17. Independent data have also been used to examine the proposed model. The following regression equation for the constant $v$ was obtained for the author's and Garcia-Bengochea's (1978) data ($R^2 = 0.89$ and $R^2_a = 0.84$):

$$\ln v = -3.06 - 2.59 \left(\frac{D_{40}}{D_{25}}\right) + 14.97 \left(\frac{D_{40}}{D_{10}}\right) - 7.26 \left(\frac{D_{60}}{D_{10}}\right) + 4.63 \left(\frac{D_{60}}{D_{25}}\right) - 2.71 \left(\frac{D_{25}}{D_{10}}\right) - 3.85 \left(\frac{D_{60}}{D_{40}}\right)$$

18. The applicability of the proposed model is evident from the following regression equation for the author's and Garcia-Bengochea's (1978) data ($R^2 = 0.98$):

$$Y = 0.99X - 0.03$$

5.3 **RECOMMENDATIONS FOR FUTURE RESEARCH**

Future research is recommended in the following areas:

1. Prediction of permeability by the proposed model should
be extended to: (a) other compacted conditions of soils similar to the ones tested here, (b) other laboratory compacted soils and (c) field compacted soils.

2. An investigation of the sand fabric through variation in PSD caused by different compaction methods and/or different sands but at the same total porosity should be carried out. An attempt could then be made to relate PSD of sands to their liquefaction potential.

3. This study established procedures by which PSD can be measured in sand. These procedures can now be used to explain and predict shear and compressibility behavior of sands.

4. Techniques for determining PSD in coarse sands and fine gravels are needed to exploit this parameter in the prediction of behavior of (a) drainage filters and (b) highway subbases.
REFERENCES


## APPENDICES

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