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PASSIVE CONTROL OF TURBOMACHINE NOISE
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ABSTRACT

Discrete-frequency tones generated by unsteady blade row interactions are of particular concern in turbomachinery design. In the annular inlet and exit ducting, rotor-stator interactions generate acoustic waves at the multiples of rotor blade pass frequency. This rotor-stator generated discrete-frequency noise is characterized as a summation of propagating acoustic waves over the multiples of the rotor blade pass frequency. Aerodynamic detuning is accomplished by the replacement of alternate stator vanes with short chord splitter vanes. The detuned stator vane row influences the unsteady aerodynamics and acoustic response of the rotor-stator interaction. The unsteady aerodynamics and acoustic response of detuned vane row are modeled analytically as a two-dimensional flat plate cascade operating in inviscid compressible subsonic flow with small unsteady perturbations. The linearized continuity and momentum equations are solved using wave theory. The model is applied to the interaction of a 16 bladed rotor and a 36 vaned stator with a reduced frequency of 8.0. The detuned stator vane row incorporates 36 half-chord splitters with 36 full chord airfoils. The optimum configuration was determined for the detuned stator row: offset 0.3 chord, spacing ratio 0.3, detuned pitch spacing 1.7 and reduced frequency 5.2. The tuned and detuned stator vane rows were modeled over a range of operating conditions corresponding to a range of Mach numbers from 0.09 to 0.4. Maximum reductions of 8 dB were realized, and aerodynamic detuning was effective over nearly the entire range of operating conditions.

INTRODUCTION

Aeroacoustics is an increasingly important issue in the design of advanced turbomachinery. For a rotor and stator in a duct, Figure 1, the noise signature includes a broadband noise level with large spikes or tones at multiples of the blade passing frequency. The discrete frequency tones may not contribute significantly to the overall noise level but are the main source of irritating screech noise.

Discrete-frequency tones are generated by periodic unsteady aerodynamic interactions between adjacent blade rows. Namely, turbomachine blade rows are subject to spatially nonuniform inlet flow fields resulting from either potential or viscous wake interactions. Potential flow interactions result from variations in the pressure field associated with the blades of a given row and their effect on the blades of a neighboring row moving relative to it. These interactions are of concern when the axial spacing between neighboring blade rows is small or the flow Mach number is high. Wake interactions result from the impingement of wakes shed by one or more upstream rows upon the flow through a downstream blade row. This type of excitation can persist over considerable axial distances. Both of these interactions result in the generation of acoustic waves which may propagate unattenuated and interact with other airfoil rows.

Progress in noise reduction, specifically far-field discrete-frequency noise, is dependent on innovative passive noise control techniques. Aerodynamic

Figure 1. Discrete tone generation and noise spectrum
detuning is a relatively new concept for passive noise control. It is defined as designed airfoil-to-airfoil differences of an airfoil row. Thus, aerodynamic detuning influences the airfoil-to-airfoil unsteady aerodynamics of the airfoil row. These differences affect the fundamental driving force of discrete-frequency noise generation, the unsteady stator vane surface pressure. Due to aerodynamic detuning, the airfoils do not respond in a classical traveling wave mode typical of a conventional uniformly spaced tuned stator vane row.

This paper is directed at the investigation of aerodynamic detuning for passive control of turbomachine frequency tones. An analytical model is developed to predict the unsteady aerodynamics and subsequent acoustic response of an aerodynamically detuned stator vane row. Aerodynamic detuning is accomplished by the replacement of alternate stator vanes with short chord splitter vanes. The detuned stator vane row influences the unsteady aerodynamics and acoustic response of the rotor-stator interaction. The unsteady aerodynamics and acoustic response of the detuned vane row are modeled analytically as a two-dimensional flat plate cascade operating in inviscid compressible subsonic flow with small unsteady perturbations. The linearized continuity and momentum equations are solved using wave theory. The model is then applied to the interaction of a 16 bladed rotor and a 36 vaned stator with a reduced frequency of 8.0 and the optimum configuration determined.

UNSTEADY AERODYNAMIC MODEL

To analyze the unsteady aerodynamics of an aerodynamically detuned cascade, it is necessary to develop an understanding of the fundamentals of two-dimensional subsonic compressible inviscid flow as applied to a uniformly spaced or tuned cascade [1]. This tuned cascade analysis is then extended to the detuned cascade.

An aerodynamically detuned cascade is depicted in Fig 2. It is the combination of two uniformly spaced tuned cascades, denoted as Cascade A and Cascade B for convenience and identified by the indices A and B. These tuned cascades have the same stagger angle $\psi$ and circumferential spacing $S$. The coordinate system for Cascade A is $(x, y, z)$, with the Cascade B coordinate system $(x', y', z')$. Note that $z$ and $z'$, the chordwise coordinates, are not orthogonal to the axial and tangential coordinates $(x, y)$ or $(x', y')$. The origins for the Cascade A and B coordinate systems are at the leading edge of the zeroth airfoils of each cascade. The airfoils of Cascade A may have different chord lengths than those of Cascade B. The airfoil chords are denoted by $C_A$ and $C_B$, with the Cascade A to B airfoil chord ratio defined as $C_r = C_A/C_B$. Additional parameters to define the detuned cascade geometry include the following. The tangential distance between adjacent Cascade A and B airfoils in the detuned cascade is $S_r$. The Cascade A to Cascade B airfoil spacing ratio is $S_r = S_r/S$. The chordwise offset $o_s$ is the distance in the $z$ direction from the stagger line of Cascade A to the stagger line of Cascade B. A tuned cascade thus corresponds to a detuned cascade when $\alpha_{\text{detuned}} = 2\alpha_{\text{tuned}}$, $S_{\text{detuned}} = 2S_{\text{tuned}}$, $S_r = 0.5$, $C_r = 1.0$, and $o_s = 0.0$.

PLANE WAVE SOLUTIONS OF THE LINEARIZED EULER EQUATIONS

The two-dimensional inviscid compressible flow continuity and momentum equations linearized about a uniform mean flow are given in Equation 1 where $U$ and $V$ are the steady freestream velocities in the axial and tangential $(x, y)$ directions, $u$ and $v$ are the corresponding unsteady perturbation velocity components, $\rho_o$ and $\rho$ are the freestream and perturbation densities and the perturbation pressure is $p$.

The velocity and pressure perturbations are assumed to be harmonic in time and space, Equation 2, where $\bar{u}$, $\bar{v}$ and $\bar{p}$ are complex constants specifying the magnitude of the perturbation velocities and pressure, $\alpha$ and $\beta$ are the axial and tangential wave numbers and $\omega$ is the frequency.

The pressure and density perturbations are related through the isentropic relation $\frac{\partial p}{\partial \rho} = a^2$ where $a$ is the speed of sound.

Hence, $\bar{p} = a^2 \bar{\rho}$.

Reduction of the linearized continuity and momentum equations leads to a system of homogeneous algebraic equations.

\[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} + \rho_o \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0
\]

\[
\frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} + \rho_o \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0
\]

\[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} + \frac{1}{\rho_o} \frac{\partial p}{\partial x} = 0
\]

\[
\frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} + \frac{1}{\rho_o} \frac{\partial p}{\partial y} = 0
\]
For a nontrivial solution, the determinant of the coefficients must be zero. Expansion of the determinant leads to the characteristic equation 
\[
[(\omega + \alpha U + \beta V)^2 - a^2(\alpha^2 + \beta^2)][\omega + \alpha U + \beta V] = 0
\]
that has two families of solutions.

When \([\omega + \alpha U + \beta V] = 0\), the solution corresponds to vorticity waves that are simply convected with the mean flow, with no associated pressure perturbations. The axial wave number for this vorticity wave solution is \(\alpha = -\frac{\omega + \beta V}{U}\).

The solution family for the case when \([(\omega + \alpha U + \beta V)^2 - a^2(\alpha^2 + \beta^2)] = 0\) corresponds to a pair of irrotational pressure or acoustic waves, with one propagating upstream and the other downstream. The axial wave numbers for these pressure waves are \(\alpha = \frac{U(\omega + \beta V) \pm a\sqrt{(\omega + \beta V)^2 - (\alpha^2 - U^2)\beta^2}}{a^2 - U^2}\).

The propagation of the unsteady pressure perturbations or acoustic waves, described by the values of \(\alpha\), depend on the values of the arguments under the radical. Three possibilities exist.

* \((\omega + \beta V)^2 - (a^2 - U^2)\beta^2 = 0\), there is one real axial wave number. Only one wave is created that propagates in the tangential direction. This is an acoustic resonance condition where the frequency is the cut-off frequency.

* \((\omega + \beta V)^2 - (a^2 - U^2)\beta^2 > 0\), two waves propagate without decay, one upstream and the other downstream. This behavior is termed superresonant for a subsonic mean flow field. These acoustic waves are cut-on.

* \((\omega + \beta V)^2 - (a^2 - U^2)\beta^2 < 0\), two waves are created. This behavior is termed subresonant and the waves decay exponentially with axial distance. These acoustic waves are cut-off.

The unsteady aerodynamic loading on the blading is modeled by replacing the airfoils with bound vortex sheets. The vorticity distribution is then expanded in a Fourier series in the tangential direction, with the various harmonics specified by the index \(r\). Unsteady cascade periodicity requirements then specify the tangential wave number \(\beta = \frac{\sigma - 2\pi}{S}, \quad r = 0, \pm 1, \pm 2, \cdots\) where \(\sigma = -2\pi n \frac{N_B}{N_V}\) is the interblade phase, \(n\) is the rotor harmonic, \(N_B\) is the number of rotor blades, \(N_V\) is the number of stator vanes, and \(r\) is an arbitrary integer.

### AERODYNAMICALLY DETUNED CASCADE KERNEL FUNCTION

The kernel function \(K\) given in Equation 3 is defined to satisfy the upwash integral equation
\[
w(z) = \int_0^\Gamma(z_0) K(\Delta x, \Delta y) dz_0
\]
where \(\bar{W}_1, \bar{W}_2\) and \(\bar{W}_3\) are upstream and downstream going pressure and vorticity waves [1]. Specifying the upwash \(w(z)\), and calculating the kernel function, the bound vortex strength \(\Gamma(z_0)\) can be determined. The bound vortex strength is proportional to the differential unsteady surface pressure. The solution for the unsteady bound vorticity distribution is found by the method of collocation and matrix inversion subject to the Kutta condition at the trailing edge using standard methods.

The kernel functions derived for the tuned cascade are applied to aerodynamically detuned cascades [2]. This is accomplished through the geometric factors \(\Delta x\) and \(\Delta y\). Namely, the values of \(\Delta x\) and \(\Delta y\) are determined by the spacing ratio, the chord ratio, and the offset, which specify the detuned cascade geometry.

The geometric factors \(\Delta x\) and \(\Delta y\) become more complicated for a detuned
cascade but are found in the same manner as for the tuned cascade. The upwash is specified at location \((x, y)\). For the vortex located at \((x_0, y_0)\) on Cascade A, \(\Delta x\) and \(\Delta y\) take the form \(\Delta x = x - x_0\) and \(\Delta y = y - y_0\) for upwash at \((x, y)\) on Cascade A and \(\Delta x = x' - x_0\) and \(\Delta y = y' - y_0\) for upwash at \((x', y')\) on Cascade B. Similarly for the vortex located at \((x_0', y_0')\) on Cascade B, \(\Delta x\) and \(\Delta y\) take the form \(\Delta x = x' - x_0\) and \(\Delta y = y' - y_0\) for upwash at \((x', y')\) on Cascade B and \(\Delta x = x - [x_0 + \text{os} (\cos \psi)]\) and \(\Delta y = y - [y_0 + \text{os} (\sin \psi)]\) for upwash at \((x, y)\) on Cascade A. These factors must be specified for each case: the upwash specified on Cascade A with a vortex on Cascade A or Cascade B given by kernel functions \(K_{AA}\) and \(K_{BA}\) respectively and the upwash specified on Cascade B with a vortex on Cascade A or Cascade B given by kernel functions \(K_{AB}\) and \(K_{BB}\) respectively.

**SOLUTION METHOD**

The unknown vortex distributions on Cascades A and B are found by solving the upwash integral equation, Equation 4. The integral is evaluated numerically using the trapezoidal rule. A variable transformation is used to resolve the high gradients near the leading edge. This yields a linear system of equations with the upwash specified and the vortex strength unknown. A polynomial curve fit that implicitly satisfies the Kutta condition \(\Gamma(1) = 0\) is determined to approximate the vortex strength. The coefficients of the curve fit \([\delta]\) are determined by solution of the linear system of equations. With the coefficients determined, the vortex distribution is calculated and the unsteady lift and moment coefficients are determined.

The resulting linear system of equations is given in Equation 5 where the first subscript on \(\delta\) denotes the cascade excited by the convected harmonic gust, with the second subscript denoting the cascade the vortex is on. For example, \(\delta_{AB}\) represents the coefficients for a vortex on the reference airfoil of Cascade B due to upwash on the reference airfoil of Cascade A. The unsteady aerodynamics of Cascades A and B are specified by means of an influence coefficient technique.

**CONVETED VORTICAL GUST EXCITATION AND FAR-FIELD ACOUSTIC RESPONSE**

To solve the upwash integral equation, the upwash is specified as an airfoil excited by a convected vortical gust \(w = w_w e^{i(\alpha - k_z)}\) where \(w_w\) is the magnitude of the gust and \(k = \omega c/W\) is the reduced frequency.

The wakes generated by the upstream rotor convect downstream and cause unsteady upwash on the stator. The stator vane unsteady surface pressures satisfy the flow tangency condition on the stator airfoil surface. Ultimately, the source of the discrete-frequency noise is this stator vane unsteady loading. Once the stator airfoil unsteady loading has been determined, the acoustic response can be calculated. The acoustic response is determined by integrating over the blade chord.

\[
\frac{\bar{p}_1}{\rho_0 W w_w} = \frac{1}{(S/C)} v'_{1,2} \left(\frac{k + \alpha_{1,2} \cos \psi + \beta \sin \psi}{\beta} \right) \int_0^l \Gamma_w(z) \exp \left[-i \left(\alpha_{1,2} \cos \psi + \beta \sin \psi \right) z \right] dz
\]

where \(v'_{1,2} = \frac{\beta^2}{2A} \left(1 - \frac{k}{\beta} \sin \psi + \frac{i k \cos \psi}{\sqrt{\beta^2 - M^2 A}} \right)\), \(A = k^2 + \beta^2 + 2k \beta \sin \psi\), \(S\) is the pitch spacing, \(c\) is the stator chord, \(k\) is the reduced frequency, \(\alpha_{1,2}\) is the upstream or downstream axial wave number, \(\psi\) is the stagger angle, \(\beta\) is the tangential wave number, \(\Gamma_w\) is the nondimensional bound vortex strength, \(z\) is the blade-chord coordinate, and \(M\) is the freestream Mach number.
The acoustic response of the detuned cascade is given by superposition.

\[
\frac{\overline{P}}{\rho_c Ww} = \overline{P}_{\text{AA}} + \overline{P}_{\text{BA}} e^{i(\sigma S_r - k \alpha_0)} + \overline{P}_{\text{AB}} e^{-i(\alpha_0 \beta + \beta_0 S_r)} + \overline{P}_{\text{BB}} e^{i(\sigma S_r - k \alpha_0)} e^{-i(\alpha_0 \beta + \beta_0 S_r)}
\]

where the acoustic response is appropriately phase shifted.

RESULTS

The influence of aerodynamic detuning on the discrete-frequency acoustic response of a rotor-stator interaction is determined by applying the model described herein to a baseline stator with 36 vanes excited by a rotor with 16 blades. At blade pass frequency, this design is “cut-off” where no propagating acoustic waves are generated for subsonic rotor relative Mach numbers. The tuned stator has a pitch spacing of one chord and zero degrees stagger. At two times blade pass frequency, the interblade phase angle \( \sigma \) and the reduced frequency \( k \) are \(-320^\circ\) and 8.0, respectively. Under these conditions, propagating acoustic waves are generated at two times blade pass frequency for freestream Mach numbers over 0.087. Thus, the acoustic response is shown in Fig. 3 for Mach numbers ranging from 0.09 to 0.4 where the transverse velocity convected vortical gust is 5% of the freestream value. The model of tuned stator row requires \( \sigma_{\text{detuned}} = -640^\circ \), \( S_{\text{detuned}} = 2.0 \), \( S_r = 0.5 \), \( C_r = 1.0 \) and \( \alpha_0 = 0.0 \).

To ensure no acoustic waves propagate at blade pass frequency the detuned cascade is composed of 36 half-chord splitter vanes in addition to 36 full-chord vanes. At two times blade pass frequency, the detuned cascade has an interblade phase angle \( \sigma_{\text{detuned}} = -320^\circ \). A linear relationship exists between the pitch spacing and the reduced frequency. If the chordlength of the full-chord vanes remains constant, the spacing of the detuned cascade will be \( S_{\text{detuned}} = 1.0 \) and the reduced frequency will remain \( k = 8.0 \). On the other hand if the spacing remains constant \( S_{\text{detuned}} = 2.0 \), the reduced frequency must be reduced \( k = 4.0 \). The influence of pitch spacing and reduced frequency can be predicted by comparing the acoustic response of the detuned and tuned stators at a 0.2 freestream Mach number, Fig. 4. The optimum spacing and corresponding reduced frequency are \( S_{\text{detuned}} = 1.7 \) and \( k = 5.2 \), respectively.

Using the optimum pitch spacing \( S_{\text{detuned}} = 1.7 \), the influence of spacing ratio can be determined by comparing the acoustic response calculated for the detuned and tuned stator rows for the \( M = 0.2 \) condition, Fig. 5 where the detuned cascade has a chord ratio of 0.5 and chordwise offset of 0.3. The acoustic response is lower for all spacing ratios, and the maximum noise reduction is seen for a spacing ratio of 0.3 where reductions of roughly 4 dB are realized upstream and downstream.
Again using the most beneficial spacing ratio of 0.3, the further influence of chordwise offset can be determined in the same manner for the detuned and tuned stator vane rows, Fig. 6. The maximum reduction is seen for a detuned stator row with offset 0.3 chord. Hence, the maximum reductions remain 3.6 and 4.6 dB upstream and downstream, respectively.

Thus, the optimum configuration \( S_r = 0.3 \) and \( o_s = 0.3 \) has been determined for the detuned stator vane row for the \( M = 0.2 \) operating condition. The relative acoustic response can be determined for Mach numbers ranging from 0.09 (just over the Mach number where the acoustic wave propagates) to 0.4, Fig. 7. For freestream Mach numbers of up to 0.36, the detuned stator row achieves a reduction in the acoustic response. The maximum relative reduction of 8 dB is achieved at a freestream Mach number of 0.09, and the detuned stator row has a 4 dB reduction for a wide range of conditions. Fig. 8 shows the acoustic response of the tuned and detuned stator rows excited by a vortical gust with a transverse velocity 5% of the freestream. As seen in Fig. 8, aerodynamic detuning is effective for all freestream Mach numbers except 0.36.

**SUMMARY**

An analytical model has been developed to determine the unsteady aerodynamics and subsequent acoustic response of aerodynamically detuned vane rows. The model considers compressible flow in two dimensions with small unsteady perturbations in pressure and velocity superimposed. The cascade model considers flat-plate uncambered airfoils at zero incidence with the mean flow.

The overall acoustic response of a tuned blade row was determined over a range of operating conditions and a given vortical gust magnitude. The acoustic response of the tuned stator row was compared to the response of the aerodynamically detuned stator with half chord splitter vanes. The optimum spacing ratio and chordwise offset were determined for a single operating condition. The relative noise reduction was determined for a range of freestream Mach numbers. The detuned blade row had a maximum 8 dB reduction in the upstream and downstream acoustic response, and aerodynamic detuning was effective over a wide range of operating conditions.