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A Thermodynamic Basis for Predicting Falling-Film Mode Transitions

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ABSTRACT

Horizontal-tube, falling-film heat exchangers are used in many air-conditioning and refrigeration systems. Depending on the geometry, flow rate and fluid properties, when a liquid film falls over a series of horizontal tubes several distinct flow patterns can be manifested: the droplet mode, the jet mode, and the sheet mode. A thermodynamic analysis is undertaken to predict the transitions between these modes. By seeking thermodynamic equilibrium between two neighboring modes, a scaling relation is developed for the transitional Reynolds number. This theoretical framework is successful in explaining the well-established empirical correlation of transitional Reynolds number with modified Galileo number. The scaling relation also suggests a tube-spacing effect on the mode transitions. Using limited data and prior results from the literature it is found that this effect is likely to exist. The implications and limitations of this thermodynamic approach to predicting two-phase flow patterns are discussed, as is its incompleteness.

1. INTRODUCTION

A falling liquid film is used for heat and or mass transfer in a number of important technical applications, such as sea water desalination (Fletcher, 1975), condensers and spray evaporators (Honda et al., 1987; Moeykens et al., 1996), and absorption systems (Jeong and Garimella, 2002; Perez-Blanco, 1988). Often, the arrangement is such that a liquid film falls from one horizontal tube to another below it, as a heating or cooling fluid flows inside the horizontal tubes. Recent thorough reviews of falling-film heat exchangers have been provided in the literature (Ribatski and Jacobi, 2005; Thome, 1999). It is well known that when the liquid film falls from tube to tube, the flow can manifest several different flow patterns; these so-called falling-film modes are depicted in Figure 1.

Three distinct falling-film modes have been observed: the droplet mode, jet (or column) mode, and sheet mode, as well as mixed modes (Mitrovic, 1986). Early observations of the falling-film mode transitions on smooth tubes in quiescent surroundings disclosed that the liquid flow rate per unit length of tube, \( \Gamma \), was important to determining which mode would prevail (Dhir and Taghavi-Tafreshi, 1981). Further experimental work disclosed that the thermophysical properties of the fluid were important, usually represented by a modified Galileo number† (Honda et

† The naming of this parameter has been debated, with some suggesting Kapitsa number is more appropriate; however, the use of modified Galileo number is widely adopted in the germane literature.
A number of strictly empirical relationships have been presented in the literature to relate the Reynolds number at a mode transition to fluid properties (Ga or an equivalent) (see Mitrovic, 1986; Honda et al., 1987; Armbuster and Mitrovic, 1994; Hu and Jacobi, 1996, Roques et al., 2002). Hu and Jacobi (1996) provided a study of falling-film mode transitions, which included transitional modes, hysteresis in the transitions, and limited findings for non-quiescent surroundings for a wide range of $Ga^{1/4}$ (see also Ruan et al., 2009). Recently, Mitrovic (2005) reviewed the extant transition criteria and demonstrated that they were all essentially in agreement.

Further experimental work has been conducted to characterize the falling-film modes and the transitional Reynolds numbers for cases other than smooth tubes in a quiescent gas. The departure site spacing for the droplet and jet modes, $\lambda$, was studied experimentally, and found to be closely related to wavelength associated with the Taylor instability (Hu and Jacobi, 1998). Experimental studies of local film thickness have shown that the film thickness generally follows Nusselt theory on the upper part of the tube, with departures from Nusselt theory near the bottom of the tube (Gstoehl et al., 2004). The falling-film mode transitions for low-finned tubes have been studied in detail, and the experimental results show that a functional dependence of $Re$ on $Ga$ can fit the data well (Honda et al., 1987; Roques et al., 2002; Roques and Thome, 2003). A rather awkward classification of falling-film modes and transitions in the presence of a flowing gas was presented by Wei and Jacobi (2002), and later refined and clarified by Ruan et al. (2009). Very recently there has been some work on the falling-film modes of nanofluids (Ruan et al., 2010), and falling-films on flat tubes (Wang et al., 2010). All of this research also shows that a functional dependence of $Re$ on $Ga$ can fit the data well, and it is very common to use $Re = AGa^{1/4}$, where the constant $A$ is experimentally determined for a particular mode transition (alternatively, $Re = AGa^B$, where $B=1/4$).

The work described above was experimental, and almost all of the research on this topic has relied heavily on experiments. There have been two theoretical or semi-theoretical approaches to predicting mode transitions.

In one semi-theoretical approach, a single droplet departure site was studied, and using empiricism to estimate the droplet size, the Reynolds number at which the rate of droplet production would necessarily exceed the frequency associated with capillary oscillations was asserted to represent a droplet-to-jet transition (Yung et al., 1980). While this approach provides a basis for predicting that transition, it cannot be adopted for any of the other transitions. It is a highly restricted theoretical framework confined to address one and only one of the transitions in one direction. However, with rearrangement (see Hu and Jacobi, 1996) it has the result of suggesting from theory that $Re \sim Ga^{1/4}$.

The second approach to theoretical prediction of the mode transitions has been to use linear stability analysis (e.g., Joo et al., 1991; Grant and Middleman, 1966). This approach assumes a base state (sheet or jet) and makes a determination as to when that state will be dynamically unstable. However, such a stability analysis only allows for transitions in the direction of decreasing Reynolds number, because such an analysis cannot start with a droplet-mode base state and consider a transition to the jet mode. Nor can such an analysis start in a jet-mode base state and
consider transitions to the sheet mode. In this theoretical framework, the transition from jet to droplet is fundamentally different from the transition from droplet to jet; likewise for the sheet-to-jet and jet-to-sheet transitions. However, the experimental evidence shows that hysteresis in the transitions is often very small; hysteresis is often neglected. A theoretical framework that is applicable to transitions in one direction—which essentially ignores transitions in the other direction—when transitions in both directions occur at essentially the same Reynolds numbers appears to be missing an essential element of the physics.

There is currently no successful theoretical framework for predicting the falling-film mode transitions in any unified sense, and thus even the functional forms of empirically based transition criteria are not understood. Furthermore, the lack of a general theoretical basis restricts the generality of transition criteria to their empirical range, and leaves further experimental studies of falling-film flows without guidance as to what parameters might be important.

2. A NEW THEORETICAL FRAMEWORK

Zivi (1964) was the first to use thermodynamic arguments to characterize two-phase flow morphology. In developing a void fraction estimate for annular flow, a model was developed that ostensibly sought the void fraction corresponding to minimum entropy production, invoking the ideas of Prigogine (1961). In the model development, a “finite length conduit” was identified as the system, and the inlet flow was specified only as “saturated”. The flow was stated to be either adiabatic or diabatic, with pressure and vapor quality assumed to change with position along the conduit (but not in time). The simplest form of the model was then developed by minimizing the kinetic energy flux exiting the conduit. Although not stated, the model appears to have assumed the exiting flow was at the dead-state temperature, pressure, and elevation, and all external entropy generation was due to kinetic energy. Even if that were true, since the quality was explicitly assumed to change from inlet to outlet there must have been an entropy change from inlet to outlet. If the flow was adiabatic, then the entropy change must have been due to internal entropy generation. If the flow was diabatic there must have been entropy production associated with heat transfer.

The analysis was extended to consider wall friction effects and then liquid entrainment effects on exiting kinetic energy. Wall friction is not the only source of internal entropy generation for the flow; for example, liquid-vapor shear, of the order of wall shear, was ignored. At no point in the analysis, was the state of the inlet flow, quality changes or heat transfer considered in evaluating entropy flow. The work of Zivi (1964), while seminal, does not ensure a flow morphology corresponding to minimum entropy production.

The approach put forward now is also an attempt to provide a thermodynamic basis for determining something about two-phase flow patterns. In the current work, the conditions at falling-film mode transition are sought. Consider a falling-film flow as a thermodynamic system and assume its thermodynamic properties can be determined, and that they depend upon the falling-film mode. Assume that if two modes are in thermodynamic equilibrium, then either mode is available, and at that condition a transition between the modes can occur. An alternative but equivalent view is that if two falling-film modes (each not at equilibrium with the dead state) are at steady state and thermodynamically equivalent to each other, then they both must be producing entropy at the same minimum rate, and therefore either mode is allowable under Prigogine’s principal.

2.1 Sheet/Jet mode transitions

Consider two thermodynamic systems, the falling-film sheet mode, and jet mode, as shown in Figure 2, with both systems isothermal and adiabatic at the temperature and pressure, \( T_0 \) and \( P_0 \), respectively. These two systems have the same material flowing through them at the same rate, and are in thermodynamic equilibrium—thermodynamically equivalent—when their Helmholtz potentials are equal. Thus, recognizing that there are no work or heat interactions, the arguments above suggest that mode transition is possible when

\[
(E - T_0 S)_S = (E - T_0 S)_f
\]

or

\[
M_S (e - T_0 s)_S = M_f (e - T_0 s)_f
\]

The transitional Reynolds number is sought between the sheet and jet modes, and at transition both of the flow patterns have that Reynolds number. Thus, the mass flow rates into and out of each of the control volumes are equal.
In each case the liquid on the tube takes the form of a thin film, and at transition the rate at which liquid free-falls between the tubes is the same. Therefore, it is assumed that \( M_S \approx M_J \), thus

\[
\left( e - T_0 s \right)_S \approx \left( e - T_0 s \right)_J
\]  

(3)

Furthermore, both systems are comprised of the same substance at the same temperature and pressure. Therefore the transition criterion becomes simply

\[
e_S \approx e_J
\]  

(4)

For the sheet and the jet mode, the specific energy within the control volume is

\[
e = \frac{1}{M} \int \rho \left( u + \frac{V^2}{2} + g(z - z_0) \right) d\mathcal{V} + \sigma
\]  

(5)

In Eq. (5), the first term in the integrand is internal energy, the second is kinetic energy and the third is potential energy; the final term in the equation is the interfacial energy. As argued above, because the systems both have the same substance at \( T_0 \) and \( P_0 \), both have the same internal energy (and \( \sigma \)). Both systems are assumed to have a thin film on the tube and a free-fall to the next tube, and assuming viscous effects are small in the freefall, the distribution of velocity and therefore mass in the vertical direction in the inter-tube space is the same. For this reason the kinetic and potential energy is approximately the same for each mode. Thus, the transition criterion simplifies to the condition of equal liquid-vapor interfacial area:

\[
A_{v,S} \approx A_{v,J}
\]  

(6)

For a length of tube, \( L \), with film of uniform thickness \( \delta \), the interfacial area in the sheet mode is (refer to Fig. 2)

\[
A_{v,S} \approx 2Ls + \pi(d + 2\delta)L
\]  

(7)

As a first approximation for the interfacial area in the jet mode, assume there are \( N_J \) jets of uniform diameter, \( d_J \), then the area for the jet mode is

\[
A_{v,J} \approx N_J \pi d_J s + \pi(d + 2\delta)L
\]  

(8)

The number of jets is taken to be \( N_J = L/\lambda \), and the spacing \( \lambda \) is assumed to be the fastest-growing Taylor wavelength (see Hu and Jacobi, 1998),

\[
\lambda = 2\pi \sqrt{3\sigma / g \rho}
\]  

(9)

so

\[
N_J = \frac{L \sqrt{\rho g}}{2\pi \sqrt{3\sigma}}
\]  

(10)

The diameter of jets follows from the conservation of mass, with total mass flow rate of the liquid \( \Gamma \), and the average downward liquid velocity in the jet \( V_J \).

\[
\Gamma L = N_J \rho V_J \pi d_J^2 / 4
\]  

(11)
At the bottom of a tube, the downward liquid velocity is \( V_J \approx 0 \). The liquid departs the tube and falls freely over a vertical distance \( s \). At the top of the next tube, the impinging liquid has a downward velocity, \( V_J = \sqrt{2gs} \). The average liquid velocity in the jets is taken to be the arithmetic mean
\[
V_J \approx \sqrt{gs/2}
\]  
(12)

and using Eq. (12) and Eq. (10) in Eq. (11) yields the average jet diameter
\[
d_J \approx (2^{7/4})(3^{1/4}) \sqrt{\frac{\Gamma \sigma^{1/2}}{g \rho^{3/2} s^{1/2}}}
\]  
(13)

Substituting Eqs. (13) and (10) into Eq. (7) gives the jet-mode interfacial area
\[
A_{v,J} \approx (24^{1/4})L \sqrt{\frac{\Gamma s^{3/2}}{\rho^{1/2} \sigma^{1/2}}} + \pi(d + 2\delta)L
\]  
(14)

Finally, substituting Eqs. (7) and Eq. (14) into Eq. (6) and rearranging to yield a scaling relationships for the sheet/jet transition criterion
\[
\Gamma \sim \sqrt{\sigma \rho s}
\]  
(15)

Using the definitions of \( Re \) and \( Ga \), and introducing the capillary length \( \xi = \frac{\sigma}{\rho g} \) the transition criterion is
\[
Re \sim Ga^{1/4} \frac{s}{\xi}
\]  
(16)

This relationship, which is based on thermodynamic arguments, provides a basis for expecting transition criteria of the form \( Re = A Ga^{1/4} \) for the sheet/jet falling-film mode transitions. However, it further suggests that the dimensionless tube spacing plays a role—that role will be explored in more detail later.

### 2.1 Jet/Droplet mode transitions

Applying the same thermodynamic analysis with the same simplifying assumptions, the transition criterion for jet/droplet mode transitions is found to be
\[
A_{v,J} = A_{v,D}
\]  
(17)

Assuming the number of droplet-producing sites equal to the number of jets, \( N_J \), the droplet-mode interfacial area is
\[
A_{v,D} \approx N_J N_D \pi d_D^2 + \pi(d + 2\delta)L
\]  
(18)

where \( N_D \) is the number of droplets in a single droplet-producing site falling between the departure site on the bottom of a tube and the impingement site on the top of the tube, and \( d_D \) is the diameter of the falling droplets.

Yung and co-workers (1980) found experimentally that \( d_D \approx \xi \), and in particular they recommended
\[
d_D = 3\xi
\]  
(19)

The conservation of mass requires
\[
\Gamma \lambda = \frac{\rho \pi d_D^3}{6 \tau_D}
\]  
(20)

where \( \tau_D \) is the time between successive droplet departures.

The time required for a droplet to depart the bottom of one tube and impinge on the top of the tube below is
\[
\tau_F = \sqrt{\frac{2s}{g}}
\]  
(21)

Taking the number of drops in freefall as \( \tau_F / \tau_D \), with Eq. (9), Eqs. (20) and (21) give
\[
N_D = \frac{12 \Gamma \sqrt{6}}{g d_D^3} \sqrt{\frac{s \sigma}{\rho}}
\]  
(22)
Substituting Eqs. (19) and (22) into Eq. (18) and using the result along with Eq. (14) in Eq. (17) the scaling relationship for the jet/droplet mode transition is

\[ \Gamma \sim \sqrt{\rho \sigma \zeta} \]  

(23)

Finally, as before, using the definitions of \( Re \), \( Ga \), and using \( \xi \), the jet-droplet transition criterion also can be expected to also follow

\[ Re \sim Ga^{1/4} \sqrt{s/\xi} \]  

(24)

The analysis shows from thermodynamic considerations that the falling-film mode transitions should follow the commonly adopted dependence on \( Ga \), but with an additional dependence on tube spacing, reflected by \( \sqrt{s/\xi} \). Hu and Jacobi (1996) found transitional Reynolds numbers to have very little tube-spacing dependence, reporting almost no effect for \( Ga^{1/4} \approx 530 \). However, Wei and Jacobi (2002) reported the transitional \( Re \) to depend on \( s/\xi \). They showed that for \( Ga^{1/4} \approx 36 \), \( Re/Ga^{1/4} \) increased with \( s/\xi \), went through a sharp local maximum at \( s/\xi \approx 5 \) then continued a slow increase with \( s/\xi \). A study by Roques et al. (2002) reported similar behavior for \( Ga^{1/4} \approx 90 \), with the transitional Reynolds numbers increasing with tube spacing, going through a sharp local maximum then becoming insensitive to tube spacing. Wei and Jacobi pointed out that the sharp maximum occurred at a tube spacing associated with a change in the shape of the jets (described by Hu and Jacobi, 1998). In the next section data from prior work are used to explore whether or not the scaling suggested by the new theoretical work can improve data correlation over that achieved with the conventional scaling.

3. COMPARISON TO DATA

In earlier work, measurements of the mode transitions were undertaken using the apparatus and methods described elsewhere (see Hu and Jacobi, 1996; Ruan et al., 2009; Wei and Jacobi, 2002; Ruan et al., 2010; Wang et al., 2010). The purpose of that earlier work was not to explore tube-spacing effects on the falling-film mode transitions; however, the data span a range of tube spacing—in a limited way—that allows a preliminary examination of these effects in the context of the new theory. Most of the data allowing this analysis were obtained with ethylene glycol as the liquid (\( 27 < Ga^{1/4} < 35 \); \( 1500 < Ga^{1/4} \sqrt{s/\xi} < 6800 \)). Limited data were also obtained with R-123 (\( Ga^{1/4} \approx 350 \); \( 22(10^3) < Ga^{1/4} \sqrt{s/\xi} < 9(10^3) \)) and water (\( Ga^{1/4} \approx 525 \); \( 15(10^3) < Ga^{1/4} \sqrt{s/\xi} < 9(10^4) \)). Typical measurement uncertainties for \( Ga^{1/4} \), \( 1/4 /Ga \sqrt{s/\xi} \) were \( \pm 1% \); in the range of the data presented, at low and high \( Re \) the typical uncertainties in \( Re \) were less than \( \pm 10% \) and \( \pm 1% \), respectively (see reports cited above for details).

Transitional Reynolds numbers for ethylene glycol are shown in Figures 3-5 for both the conventional and the new scaling. Transitions from the jet mode to the jet-sheet mode (jet/jet-sheet) and from the jet-sheet mode to the jet mode (jet-sheet/jet) are shown. From Figure 3 alone it is not clear which scaling is superior; however, taken in aggregate, Figures 3-5 qualitatively suggest that the \( Ga^{1/4} \sqrt{s/\xi} \) scaling collapses the data as well as the \( Re \sim Ga^{1/4} \) scaling. In order to quantify the goodness of fit for each scaling and each transition, the sum of Chi-square was computed, and the results are provided in Table 1.

\[ \sum \chi^2 = \sum \left( \frac{Re_{meas} - Re_{fit}}{U_{Re}} \right)^2 \]  

(25)

The results given in Table 1 show that \( Re \sim Ga^{1/4} \) provides a slightly better correlation to data for two mode transitions, the jet-sheet/jet and the droplet-jet/droplet transitions. However, using \( Re \sim Ga^{1/4} \sqrt{s/\xi} \) provides better correlation for the other four mode transitions, sometimes with dramatic improvements in data representation (e.g., for the jet-sheet/jet and sheet/jet-sheet transitions). If all of these data are considered together, using \( Re \sim Ga^{1/4} \sqrt{s/\xi} \) results in the sum of Chi-squared being reduced to 20% of its value for \( Re \sim Ga^{1/4} \). These limited data with ethylene glycol as a working fluid support the new theory as superior to the current empirical approach.

Limited data for water and R-123 are compared to the two scaling relations in Figure 6 where the sheet/jet-sheet transition for R-123 shows scatter far beyond the experimental uncertainty, with \( Re \) ranging from about 340 to 490
at $Ga^{1/4} \approx 340$. Likewise, the figure shows significant scatter for the jet-sheet/sheet transition of water, with $Re$ ranging from 520 to 610 at $Ga^{1/4} = 530$. In contrast, when plotted in the new scaling, these limited data show much less scatter. The number of measurements available with these fluids is much smaller than with ethylene glycol, and no statistical analysis was undertaken. Nevertheless, the results shown in Figure 6 are supportive of the new scaling.

![Graphs showing transitional $Re$ for ethylene glycol: $\Delta$ jet/jet-sheet transition, and $\nabla$ jet-sheet/jet transition. The lines show a Chi-square linear fit to the data: (a) $Re$ dependence on $Ga^{1/4}$, (b) $Re$ dependence on $Ga^{1/4} \sqrt{s/\xi}$.](image)

**Figure 3:** Transitional $Re$ for ethylene glycol: $\Delta$ jet/jet-sheet transition, and $\nabla$ jet-sheet/jet transition. The lines show a Chi-square linear fit to the data: (a) $Re$ dependence on $Ga^{1/4}$, (b) $Re$ dependence on $Ga^{1/4} \sqrt{s/\xi}$.

![Graphs showing transitional $Re$ for ethylene glycol: $\Delta$ jet-sheet/sheet transition, and $\nabla$ sheet/jet-sheet transition. The lines show a Chi-square linear fit to the data: (a) $Re$ dependence on $Ga^{1/4}$, (b) $Re$ dependence on $Ga^{1/4} \sqrt{s/\xi}$.](image)

**Figure 4:** Transitional $Re$ for ethylene glycol: $\Delta$ jet-sheet/sheet transition, and $\nabla$ sheet/jet-sheet transition. The lines show a Chi-square linear fit to the data: (a) $Re$ dependence on $Ga^{1/4}$, (b) $Re$ dependence on $Ga^{1/4} \sqrt{s/\xi}$.

<table>
<thead>
<tr>
<th>Mode Transitions</th>
<th>jet to jet-sheet</th>
<th>jet to sheet/sheet</th>
<th>sheet to jet-sheet</th>
<th>jet-sheet to jet</th>
<th>drop to drop-jet</th>
<th>drop-jet to drop</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum \chi^2$</td>
<td>$Ga^{1/4}$</td>
<td>994.8</td>
<td>4,795.0</td>
<td>2,919.0</td>
<td>494.4</td>
<td>527.7</td>
<td>3,008.0</td>
</tr>
<tr>
<td>$Ga^{1/4}$</td>
<td>1214</td>
<td>45,317.0</td>
<td>11,634.0</td>
<td>169.9</td>
<td>4,015.0</td>
<td>2,648.0</td>
<td>64,997.9</td>
</tr>
</tbody>
</table>
Figure 5: Transitional $Re$ for ethylene glycol: $\Delta$ droplet/droplet-jet transition, and $\nabla$ droplet-jet/droplet transition. The lines show a Chi-square linear fit to the data: (a) $Re$ dependence on $Gd^{1/4}$, (b) $Re$ dependence on $Gd^{1/4}(s/\xi)^{1/2}$.

Figure 6: Transitional Reynolds numbers: $\Delta$ jet-sheet/sheet for water, and $\nabla$ sheet/jet-sheet R-123. The lines show a Chi-square linear fit to the data: (a) $Re$ dependence on $Gd^{1/4}$, (b) $Re$ dependence on $Gd^{1/4}(s/\xi)^{1/2}$.

4. CONCLUSIONS

Based on a hypothesis that when two flow patterns (falling-film modes) are at thermodynamic equilibrium a transition between them can occur, a highly simplified thermodynamic analysis was undertaken. The analysis indicated that near transition the liquid-vapor interfacial energy is the main contributor to differences in Helmholtz potential of the falling film modes, and through a simplified analysis of interfacial energy a new scaling relationship for the transitional Reynolds numbers for all the falling-film mode transitions was found to be $Re \sim Gd^{1/4}(s/\xi)^{1/2}$. At least two earlier reports suggested a tube-spacing effect on the mode transitions (Wei and Jacobi, 2002; Roques et al., 2002), and using limited data available from earlier work, the new scaling was demonstrated to provide a better correlation to the data than a scaling which neglects tube-spacing effects. The data support the hypothesis; however, clearly this view of the mode transitions is incomplete.

An easy example of the incompleteness of this theory is derived from examining a mode transition not discussed so far. It is well established that the jet mode can take two forms: in one form jets depart from directly below the jet
impingement site (in-line jet mode), and one in which the jets depart from a location halfway between two impingement sites (staggered jet mode). The transition between in-line jet and staggered-jet mode is repeatable and systematic (see Hu and Jacobi, 1996); however, the current analysis is unable to even distinguish between these two modes. It might be that a more careful thermodynamic analysis, one that accounts for the shape of the film on the tube, could distinguish between the two modes, but knowing the shape of the film on the tube requires an analysis of momentum transfer. Thus, fluid dynamics plays a key role in this transition, and a theory that does include those effects is incomplete. A more complete analysis would couple the conservation equations for mass, momentum and energy with the thermodynamic framework outlined above.

While the limited data generally support the new scaling over one ignoring tube-spacing effects, that does not imply the new scaling is the best form for capturing such effects. For example, the polynomial form adopted by Roques et al. (2002) may fit their data better than the form suggested by the new theory. The simplified analysis presented in this paper assumed the liquid jets were simple, columns with constant cross-section area—they are not (see Hu and Jacobi, 1998). Nevertheless, for the first time a general theory for the mode transitions has been formulated that explains the Ga^{1/4} scaling and a tube-spacing effect that has been reported in the literature. Further refinements in the analysis are possible, but they are not needed for the current purpose of putting forward the general approach and establishing the thermodynamic mechanism for flow pattern transitions.

A more complete treatment of the mode transitions, perhaps with a direct application of Prigogine’s principle, might also yield information on such phenomena as transition hysteresis, perhaps as an analogy to the spinodal limit. This approach might also be applicable to other two-phase flow transitions. The analysis presented in this paper is very highly simplified. However, it represents the first generalized theoretical approach to predicting the falling-film mode transitions and may lead to further and related work.

5. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>area</td>
<td>m²</td>
</tr>
<tr>
<td>E</td>
<td>extensive total energy</td>
<td>J</td>
</tr>
<tr>
<td>D</td>
<td>tube diameter</td>
<td>m</td>
</tr>
<tr>
<td>e</td>
<td>intensive total energy</td>
<td>J/kg</td>
</tr>
<tr>
<td>Ga</td>
<td>Galileo number ( \rho \sigma^3 / \mu^4 g )</td>
<td>(-)</td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration</td>
<td>m/s^2</td>
</tr>
<tr>
<td>L</td>
<td>tube length</td>
<td>m</td>
</tr>
<tr>
<td>M</td>
<td>mass</td>
<td>kg</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number ( 2 \Gamma / \mu )</td>
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<tr>
<td>S</td>
<td>entropy</td>
<td>J/K</td>
</tr>
<tr>
<td>u</td>
<td>intensive internal energy</td>
<td>J/kg</td>
</tr>
<tr>
<td>Z</td>
<td>vertical location</td>
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<tr>
<td>( \nu )</td>
<td>velocity</td>
<td>m/s</td>
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<tr>
<td>( \delta )</td>
<td>film thickness</td>
<td>m</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>Mass flow rate per unit length</td>
<td>kg/m</td>
</tr>
<tr>
<td>( \mu )</td>
<td>dynamic viscosity</td>
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<td>( \sigma )</td>
<td>surface tension</td>
<td>N/m</td>
</tr>
<tr>
<td>( \xi )</td>
<td>capillary length ( (\sigma / \rho g)^{1/2} )</td>
<td>m</td>
</tr>
</tbody>
</table>

Subscripts:
- \( \text{lv} \) liquid/vapor
- \( o \) baseline
- \( J \) jet
- \( S \) sheet

REFERENCES


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