An Analytical Model for the Discharge Process in Scroll Compressors

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AN ANALYTICAL MODEL FOR THE DISCHARGE PROCESS
IN SCROLL COMPRESSORS

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ABSTRACT

This paper describes an analytical model of the porting process in a scroll compressor. The model combines fundamental differential equations describing flow dynamics and valve motion. Scroll geometry routines are incorporated which determines the pocket volumes and discharge flow passages. An acoustic discharge plenum model is also included to simulate the discharge pulses.

The output from the model includes instantaneous pressure and flow information along with the valve motion. This information is directly related to power efficiency and noise ratings. Therefore, this model is used to assist in optimizing the performance of the scroll compressor.

INTRODUCTION

The discharge porting process can have a drastic influence on the performance and noise characteristics of a scroll compressor. Since a primary source for noise and efficiency losses is directly attributed to the flow dynamics during the discharge porting process, it is imperative that this process be fully understood. In addition, some compressors designed for the refrigeration market, are being fitted with a discharge valve. The dynamic response of the valve will also greatly affect the discharge pulse and noise characteristics of the compressor.

MODEL OVERVIEW

Analytical models of the discharge porting process have been documented in the technical literature[2]. The principles presented in those models have been expanded to include a more comprehensive representation of a scroll compressor. The underlying concepts used in this model are explained below and shown in Figure 1.

- The model includes four control volumes (central scroll pocket, the twin adjacent pockets and the hub).
- The analysis begins at the instant of the port exposure to one of the adjacent pockets and continues for one cycle (360° of crank rotation).
- The volume and flow path areas between the control volumes are calculated as a function of crank angle.
- When the three distinct pockets are no longer present, three pseudo-pockets are defined, and the four control volume approach is continued.
- A one-dimensional, compressible flow equation is used to calculate gas flow to/from the other control volumes or out of the compressor.
- Resulting pressure of the control volumes is computed according to the mass flow and volume reduction.
- An optional valve can seal the four control volumes from the plenum, if the plenum pressure exceeds the pressure of the hub.
An acoustic model is used to predict dynamic pressure fluctuations which result from a finite plenum volume. The associated back pressure influences valve response and must be included in the analysis.

![Diagram of Plenum and Optional Valve](image1)

**FIGURE 1**

To relate the framework of Figure 1 to the actual compressor, a sketch of two mating scroll profiles and the corresponding pocket definitions are given in Figure 2.

![Diagram of Scroll Profiles](image2)

**FIGURE 2**

**MODELING THE GEOMETRY OF THE SCROLL PROFILE**

The shape of the majority of the scroll vane profiles is involute. Equations for the coordinates of an involute curve have been well documented. However, due to the strength requirements, using an involute profile for the innermost portion of the vane is not feasible. This innermost profile continues to be a major area of development and is far from standardized.

However, a routine was formulated that generates the coordinates of this profile from manufacturing information. Thus, x and y coordinates of the entire scroll vane profile can be determined and used for pocket volume and flow area calculations.
MODELING THE SCROLL POCKETS

Determination of the volume of a pocket formed by involute has been well documented. One form of the equation for a pocket, defined by the wrap angle, $\varphi$, to its leading seal point is given as:

$$V(\varphi) = 2\pi H \left( r_1 + r_2 (\varphi + \pi) \right)$$  \hspace{1cm} (1)

Thus, the volume of the crescent shaped pockets, formed by involute vanes, can be readily computed through Equation 1. However, once the leading flank seal point has progressed beyond the true involute portion of the vane, the Equation 1 is not valid. Therefore an alternative method of determining the volume of these critical inner pockets is needed.

With sufficient information to uniquely define the vane profile, as discussed in the previous section, the volume at any configuration can be closely approximated. At any crank angle, the profile which enclose distinct volumes, can be determined. A typical enclosed volume is illustrated in Figure 3.

![FIGURE 3](image)

To calculate the volume of the non-involute pocket, the projected area must be determined. The area within any 2-D curve, enclosed within n-line segments, can be calculated [3] as:

$$A = \frac{1}{2} \left[ (x_1y_2 + x_2y_3 + \ldots + x_{n-1}y_n + x_ny_1) - (x_2y_1 + x_3y_2 + \ldots + x_{n}y_{n-1} + x_1y_n) \right]$$  \hspace{1cm} (2)

Equation 2 can be used to approximate the projected area of the non-involute pockets. Finally, the volume of these innermost pockets can be computed as

$$V_{pocket} = (A)(H)$$  \hspace{1cm} (3)

ALLOWABLE FLOW AREAS

Similar to previous models[2], several flow paths have been identified, which connect the four control volumes. These areas all vary with the crank angle and are illustrated in Figure 4 and defined below.

Flank Separation ($A_{d1}$) : Connects the adjacent pockets to the central pocket.

Outer Port ($A_{d2}$) : This area is the portion of exposed port which is located in the adjacent pocket and connects that adjacent pocket to the hub.

Dimple Area ($A_{d3}$) : Some compressors have a dimple machined in the base of the orbiting scroll to release gas trapped in the twin adjacent pocket. This area connects the twin adjacent pocket with the central pocket.

Inner Port ($A_{d4}$) : This area is the portion of exposed port which is located in the central pocket and connects the central pocket to the hub.
Discharge Area ($A_{ds}$): This area connects the hub with the plenum. When a valve is present, this area is the outer perimeter of the valve multiplied by the lift of the valve.

![Diagram](image)

**FLOW MODEL**

The discharge model assumes polytropic compression, which is commonly defined by Equation 4.

$$p_{pocket} = p_{suction} \left( \frac{\rho_{pocket}}{\rho_{suction}} \right)^n$$  \hspace{1cm} (4)

Flow between control volumes can be described using the steady, one dimensional, compressible flow Equation 5.

$$\frac{dm_{pocket}}{dt} = c_d A_d \sqrt{\frac{2}{\gamma}} \rho_{pocket} \Delta p_{pocket}$$  \hspace{1cm} (5)

The pressure difference, $\Delta p_{pocket}$, in Equation 5 is between the two control volumes for which the mass flow is calculated. The gas density, $\rho_{pocket}$, is that of the higher pressure control volume, which will be the gas that flows. The flow area path areas, $A_d$, that connect the control volumes are obtained through the process described above. The flow coefficient, $c_d$, must be appropriately determined from either experimental methods [4] or computational fluid dynamic modeling.
Combining Equations 4 and 5 and solving for the pressure differential term gives the governing equation for pressure change in a scroll pocket. [1]

\[
\frac{dp}{d\phi} = - \left( \frac{n_{pocket}}{V_{pocket}} \right) \left( \frac{dV_{pocket}}{d\phi} \right) + \left( \frac{c_d A_d}{\omega} \sqrt{\frac{2 \Delta p_{pocket}}{\rho_{pocket}}} \right)
\]

Since the discharge model includes four control volumes, Equation 6 is used to describe the pressure change differential in all four pockets. The actual pressures, as a function of crank angle can be calculated through a Runge-Kutta numerical routine for solving first order differential equations.

The pocket volumes and flow areas are numerically determined, using the approach outlined above. The volume differential in Equation 6 can be computed from a finite difference procedure using the numerical volume data as calculated in Equations 1 or 3. An alternative approach is to curve fit the numerical data, and use the differential of that curve fit equation.

**VALVE MODEL**

When a valve is present, additional dynamic equations are required. The valve can be approximated as a single degree of freedom, spring-mass system. This approximates the first-mode vibration of a reed valve. While a reed valve is a multiple degree of freedom system, with a large number of natural frequencies, this simplified model is sufficient because the discharge pressure harmonics do not strongly excite higher modes. Therefore, Equation 7 can be used to govern the motion of scroll compressor valves.

\[
\frac{d^2x_v}{d\phi^2} + 2\xi \omega \left( \frac{dx_v}{d\phi} \right) + (\omega^2) x_v = \left( \frac{A_f}{m_v \omega^2} \right) \left( p_{plenum} - p_{hub} \right)
\]

Notice that the forcing function on the spring-mass system is the pressure difference across the valve multiplied by the force area subjected to this difference. The force area has been previously derived [5].

Equation 7 represents a second order differential equation that can be used to determine the valve displacement, velocity and acceleration. In practice, Equation 7 can be reduced to two, simultaneous, first order differential equations. Therefore, displacement, velocity and acceleration of the valve, as a function of crank angle can be calculated through a Runge-Kutta numerical routine for solving first order differential equations. This is exactly the same numerical procedure used to determine the pressures in the control volumes.

**ACOUSTIC MODEL**

A finite plenum volume causes dynamic pressure fluctuations which will create back pressure on a valve. The plenum back pressure will have a significant effect on the valve response, as seen in Equations 7 and 9, and dictates the need to model the interaction of the discharge flow and the plenum pressure pulsations. The most common procedure to model the plenum is with a transfer matrix method that is accomplished in the frequency domain [2].

A four-pole transfer function is used to relate the pressure in the port to mass flow through the port. The pressure Fourier coefficients, P, are obtained from this transfer function method. Using a common inverse Fast Fourier Transform routine, the time domain dynamic pressure, at the inlet to the plenum, is determined. These pressure values are represented as \( p_{hub} \) in Equation 7 and are used to recalculate the valve response. A calculation procedure iterates on this entire process until the pressures at the inlet of the plenum have converged.
RESULTS

The results of this model include plots of the volume and pressure for the four pockets. Of course, this information is directly related to compression work. The influence of design parameters, such as valve shape and stiffness, inner vane geometry, port location shapes and sizes, on power consumption can be readily determined.

The instantaneous flow between the pockets and plenum is another desirable output from the model. Although not a clear association, this information is related to flow induced noise. Finally, the valve displacement, velocity and acceleration can be obtained from the model. This data can be used to assess the structural reliability of the valve, along with providing insight to any valve impact noise.

CONCLUSION

Analytical models of the discharge process have been extremely useful in optimizing the performance of a scroll compressor. The model described in this paper has been built on previous versions [2], and could be viewed as a second generation. The results from this model have been instrumental in improving the capability of scroll compressors, especially in the refrigeration market. Still, the model contains some notable assumptions. It should be expected that such models will be further developed to provide even greater insight into the discharge porting process. This promises to be an area of dialog for the next several years.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>involute wrap angle</td>
</tr>
<tr>
<td>( r_i )</td>
<td>arbitrary swing radius from which wrap angles are measured</td>
</tr>
<tr>
<td>( r_p )</td>
<td>involute generating circle radius</td>
</tr>
<tr>
<td>( H )</td>
<td>height of scroll vane</td>
</tr>
<tr>
<td>( r_{or} )</td>
<td>orbiting radius</td>
</tr>
<tr>
<td>( \omega )</td>
<td>fundamental frequency, or speed, of compressor (rad/sec)</td>
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<tr>
<td>( A )</td>
<td>projected area of pocket</td>
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<tr>
<td>( A_{df} )</td>
<td>flow area</td>
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<tr>
<td>( n )</td>
<td>polytropic exponent</td>
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<td>( \rho_i )</td>
<td>density of gas at state i</td>
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<td>( p_i )</td>
<td>pressure of gas at state i</td>
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<td>volume of a general scroll pocket</td>
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<td>orifice flow coefficient</td>
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<td>( A_f )</td>
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REFERENCES


