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QUASI ONE-DIMENSIONAL STEADY-STATE MODELS FOR GAS LEAKAGE - PART I: COMPARISON AND VALIDATION

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ABSTRACT

Most theoretical work done on gas leakage in compressors is concerned with quasi one-dimensional steady-state models. Four of these, each quite different in mathematical formulation and solution strategy, have been implemented and tested. Two experiments reported in literature were used to validate the models. The results show that choking is likely to occur. Unfortunately none of the models predicts the mass flow rate with sufficient accuracy and reliability, mainly due to poor modeling of viscous effects.

INTRODUCTION

The investigation presented here is part of a larger project which seeks to improve the modeling of leakage flows in twin screw compressors, but the same problem is relevant to other types of compressors. Fleming & Tang [3] estimate the efficiency losses due to leakage well over 10%. A thorough knowledge of the process might help to decrease these losses. Models are valuable for gaining insight and as design tools. Many models of various types have been reported in literature and in this paper four of these are compared and validated with experiments. The selection confines to quasi one-dimensional models for steady-state flows of pure gas. The relative movement of the walls is not taken into account.

This paper will start off with a discussion of the physics of the leakage flows, followed by a brief presentation of the models. Two experiments were selected from literature for validation. These will be explained, together with the results in the sections before the conclusions.

PHYSICS OF THE FLOW

The main leakage paths in twin screw compressors have a converging-diverging geometry. This means that supersonic flow might occur for sufficiently high pressure ratio's. The transition back to subsonic takes place in one or more shock waves. In slender channels the shock is likely to be normal, i.e. perpendicular to the direction of the flow. This type of shock always has supersonic flow upstream and subsonic downstream.

For subsonic flow the boundary conditions are the thermodynamic state at the inlet and the pressure at the outlet (fig.1, case A). In the supersonic flow there is no information traveling upstream, thus the outlet pressure cannot be felt at the inlet. In this case an extra boundary condition is needed for the subsonic flow at the inlet: the sonic point must be as far downstream as possible. This is equivalent with the lowest inlet velocity that results in supersonic flow (cases B..E), a phenomenon known as choking. The outlet pressure is still relevant since it
determines the position of the shock. When the pressure ratio increases the shock moves downstream (cases C & D). When the shock stands outside the computational domain, there will be no solution to this position and the flow remains supersonic up to the outlet (case E).

The quasi one-dimensional equations incorporate the continuity, momentum and energy conservation laws for the main flow direction. These models are not quite one-dimensional since they incorporate the changes in flow area and wetted perimeter perpendicular to this direction. The basic equations in differential form are:

\[
\begin{align*}
\frac{d(\rho u A)}{dx} &= 0 \quad \text{(continuity)} \\
\frac{d(\rho u^2 A) + A dp + \tau P}{dx} &= 0 \quad \text{(momentum)} \\
\frac{d(p\rho^{-\gamma})}{dx} &= 0 \quad \text{(energy)}
\end{align*}
\]

The energy equation has been simplified by assuming isentropic flow. This is not correct but sufficiently accurate. Only the shock is strongly anisentropic and requires a more complex equation. Two models use the analytical solution of the shock (see Shapiro [13]) derived from the three conservation laws.

In addition to the basic equations a model of the fluidum is required. In all models the perfect gas law is employed as the equation of state: \( p = R\rho T \). The gas is also assumed caloric perfect, i.e. with constant specific heats and thus a constant specific heat ratio. The dynamic viscosity is modeled by fitting a straight line with the temperature as parameter: \( \eta = c \cdot T + d \). Fig. 2 contains the fluidum properties (Van Hiele [5]).

\begin{table}[h]
\begin{tabular}{llll}
Freon 22 & dry air \\
\hline
\( R \) & 96.14 & 287.1 & J/kg·K \\
\( \gamma \) & 1.177 & 1.40 & \\
\( c \) & 48.12·10^{-9} & 56.67·10^{-9} & Pa·s/bar \\
\( d \) & 5.637·10^{-6} & -3.349·10^{-6} & Pa·s \\
\end{tabular}
\caption{Fluidum properties used for validation.}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Fig1.pdf}
\caption{Typical pressure profiles for isentropic, inviscid nozzle flow (analytical solution).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Fig2.pdf}
\caption{Fluidum properties used for validation.}
\end{figure}

**PRESENTATION OF THE MODELS**

The four models presented here are all taken from literature, though they have been adapted to the specific needs in this investigation. The models differ in both the mathematical formulation and the solution strategy. The numerical models will be referred to by the name of the first author of the original paper. Each model will be briefly discussed in the following enumeration, for details is referred to literature:

**Analytic:** When the viscous term in the momentum equation is neglected, an analytic solution to the three basic equations can be found. A wide range of literature is available and here the books by Shapiro [13] and Anderson Jr. [1] were used.

**Ishii:** By assuming incompressible flow and ignoring the inertia terms in the momentum equation, the energy equation becomes superfluous. Since the density is constant the continuity equation can be employed as an explicit analytical equation for the velocity, leaving only one differential equation\(^1\). This approach was adopted by Ishii et al. [6]. The equation can be integrated by Simpson's scheme. The iterative shooting method has been employed to obtain the velocity such that the pressure drop coincides with the boundary conditions (see Press et al. [10] for the numerical methods). Since the model is incompressible, it cannot incorporate shock waves.

**Xiuling:** A model that directly solves the equations presented above was published by Xiuling et al. [15] and was later used by Zhen & Zhiming [16]. For this investigation the model was extended with normal shocks. They

\(^1\)When Ishii's model is derived directly from the momentum equation as stated in this paper, the viscous terms are four times as strong as in the equation presented by Ishii. Up to now this factor has not been accounted for. In effect he uses \( a = 0.35/4 = 0.09 \) in the computation of the friction coefficient.
employ a shooting method to convert the boundary value problem into an initial value problem: the inlet velocity is determined such that the outlet condition is satisfied. The equations are written as a set of explicit ordinary differential equations which are solved with Euler integration (see Press et al. [10]).

**Anderson:** Anderson Jr. [1] discretized the basic equations by a finite differencing method. For this the equations are written in conservation form: the momentum, energy and mass flow rate are solved instead of the velocity, pressure and density. The main advantage is that these properties are conserved through shocks, avoiding strong gradients. The momentum equation has been extended with a viscous term for this investigation. The equations are used in transient form. The resulting set of non-linear algebraic equations is then solved with MacCormack's scheme (see MacCormack & Paullay [7]). This algorithm solves the equations by time integration starting from an estimated initial state until the solution stabilizes. It is capable of solving shocks from the basic equations, though the shock is smeared out in space. The shock capturing forbids the use of the isentropic flow assumption for the energy equation. It is replaced by the 'full' equation containing thermal, kinetic and potential (pressure) energy terms.

The viscous models employ the semi-empirical relation of Blasius to obtain the wall shear stress through the flow coefficient: \( \xi = a \cdot Re^b \) with \( a = 0.3164 \) and \( b = -1/4 \) (VDI-Wärmeatlas [14]). Only Ishii chooses a slightly different value: \( a = 0.35 \). The Reynolds number is based on the hydraulic diameter according to \( D = 4A/P \).

**COMPARISON WITH REPORTED EXPERIMENTS**

Though quite a number of leakage experiments are reported in literature, only two were found that were described with sufficient detail to be used for validation. The results of both of them have been reproduced based on the mass flow rates predicted by the models.

**Experiments by Ishii**

A vessel was incorporated in a refrigeration system running with Freon 22. After heating up the vessel was disconnected from the rest and depressurized through a nozzle into the atmosphere (fig. 3). The pressure in the vessel was recorded as a function of the time. By assuming adiabatic expansion of a caloric perfect gas the mass flow rate can be related to this pressure. The experiment is presented in the same paper as the model.

![Figure 3: Simplified experimental setup by Ishii. The detail on the right shows the nozzle geometry. The pressure just before the nozzle was recorded as a function of the time. The clearance height is 10±4 μm.](image)

The validation started with determining the temperature inside the vessel by assuming isentropic expansion starting at 14.7 bar and 55 °C. The results were used as input to the four models for pressures ranging from 2 to 15 bar. The predicted mass flow rates could be nicely fitted to straight lines (fig. 4). These fits were then used to simulate the depressurization of the vessel, again under the assumption of isentropic expansion of the gas in the vessel.

The predicted pressure curves are shown in figure 6. The simulations were run for the intended height of 10 μm, plus the extreme values of the inaccuracy. The analytical model and the model by Ishii look quite acceptable,
both enclosing the measured data between the lines for 6 and 10 \( \mu \text{m} \), the latter being slightly better. The two compressible viscous models clearly predict the mass flow rate too low.

<table>
<thead>
<tr>
<th>model</th>
<th>clearance</th>
<th>( c )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytic</td>
<td>6</td>
<td>31.65</td>
<td>17.79</td>
</tr>
<tr>
<td>-inviscid</td>
<td>10</td>
<td>52.78</td>
<td>29.12</td>
</tr>
<tr>
<td>-compressible</td>
<td>14</td>
<td>73.83</td>
<td>41.26</td>
</tr>
<tr>
<td>Ishii</td>
<td>6</td>
<td>46.24</td>
<td>38.49</td>
</tr>
<tr>
<td>-viscous</td>
<td>10</td>
<td>94.73</td>
<td>78.36</td>
</tr>
<tr>
<td>-incompressible</td>
<td>14</td>
<td>152.3</td>
<td>126.6</td>
</tr>
<tr>
<td>Xiuling</td>
<td>6</td>
<td>18.02</td>
<td>-3.662</td>
</tr>
<tr>
<td>-viscous</td>
<td>10</td>
<td>21.11</td>
<td>-6.256</td>
</tr>
<tr>
<td>-compressible</td>
<td>14</td>
<td>32.34</td>
<td>-8.207</td>
</tr>
<tr>
<td>Anderson</td>
<td>6</td>
<td>model fails</td>
<td></td>
</tr>
<tr>
<td>-viscous</td>
<td>10</td>
<td>model fails</td>
<td></td>
</tr>
<tr>
<td>-compressible</td>
<td>14</td>
<td>38.92</td>
<td>-33.73</td>
</tr>
</tbody>
</table>

Figure 4: Predicted mass flow rates for the experiment by Ishii. The coefficients of the curve-fits \((m = c \cdot p + d)\) are tabulated on the left. The graph shows the predicted values (marks) and the fits (lines) for a clearance of 14 \( \mu \text{m} \).

Experiments by Peveling

Peveling [9] used standard compressed air unit as high pressure source. After a reduction valve, which provided control over the pressure, the air was discharged into the atmosphere through a nozzle (fig. 5). The mass flow rate was recorded through the pressure drop over a standard orifice. Peveling expresses his results as flow coefficients\(^2\).

The models of Ishii and Xiuling have been used to compute the flow coefficients. The analytical model hardly has any relevance, while Anderson’s model is ignored because of its poor numerical behavior. The results are presented in figure 7. Xiuling’s model predicts the correct form of the curves, but it overestimates the viscous effects, resulting in too low values of the flow coefficients. Ishii’s model does not give any sensible results.

\(^2\) The flow coefficient is defined as the effective mass flow rate over the theoretical. The latter is usually taken from analytical solution with simplified boundary conditions: the inlet velocity is assumed zero and the pressure in the throat is assumed to coincide with the outlet pressure. Especially the latter is a rather crude assumption.
Figure 6: Simulation of the experiment by Ishii based on the mass flow rates predicted by the four models (clockwise from the top-left): Analytic, Ishii, Anderson, Xiuling. The numbers in the legend's give the minimal clearance height in micro meters. The model of Anderson fails for small clearances.

Figure 7: Computation of the flow coefficients with two of the models. On the left the predictions by Xiuling's model, on the right by Ishii's model. The legend's give the clearance in mm.
POSSIBILITY OF EXPERIMENTAL CONFIRMATION OF CHOKING

On first sight there seems to be a contradiction between the occurrence of choking and the linear relationship between the mass flow rate and the pressure ratio. It must be explained by the way the experiments were performed. In both cases the density at the inlet varied along with the pressure. As a result the mass flow rate still increases with the pressure ratio when the flow chokes. Apparently the linear relationship still holds for choking. By keeping the inlet condition constant while the outlet pressure is variable this effect cannot occur and the mass flow rate should stabilize above the critical pressure ratio.

CONCLUSIONS

None of the models can be regarded as generally applicable. The two compressible viscous models clearly overestimate the viscous effects, resulting in too low predictions of the mass flow rate. But the model of Xiuling seems to show the right kind of behavior for both experiments. Worse is the model of Anderson, which has severe numerical problems when the viscous effects are large. The model by Ishii performs well for his own experiments, but has no relevance to those by Peveling. Since it disregards all non-viscous effects, it won't work in wider channels. The results suggest that the mass flow rate is linear proportional to the pressure ratio when the density is varied along with the pressure. Therefore choking cannot be shown by the experimental setup of Ishii or Peveling.

As a logical follow up the viscous model has been optimized. This is the subject of a parallel paper [11].

References

[1] J.D. Anderson Jr.; Computational fluid dynamics - The basics with applications; McGraw-Hill, Singapore, 1995; Ch. 7
[8] G. Neumann (thesis); Berechnete, prozeßbeeinflussende Strömungsvorgänge in Schraubenverdichtern; Ruhr-Universität, Bochum, 1986
[12] B. Sängfors; Computer simulation of the oil injected twin screw compressor; Proceedings of the 1974 International Compressor Engineering Conference at Purdue, Purdue University, West-Lafayette, Indiana, 1974; pp.517-525
[14] VDI-Wärmeatlas; VDI-Verlag, Düsseldorf, 1988; Sec. La