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Minimizing Thermally Induced Interfacial Shearing Stress in a Thermoelectric Module

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Abstract
The problem of minimizing the level of the thermally induced interfacial shearing stress in a Thermo-Electric Module (TEM) is addressed using analytical and finite-element-analysis (FEA) based modeling. The maximum stress is calculated for different leg sizes. Good agreement between the analytical and FEA predictions has been found. It is concluded that the shearing stress can be effectively minimized by using thinner legs with compliant interfaces.

I. Introduction

Thermal (“internal”) loading, and the resulting stresses and deformations can be defined as those that are associated with the change in temperature, and/or as those, which depend on thermomechanical properties of the employed materials. Thermally induced stresses and strains can be due to dissimilar materials that expand/contract at different rates during temperature excursions, and/or to the nonuniform distribution of temperature (e.g., temperature gradient). Examples of the devices that are prone to be inversely affected by the thermal stresses during their operation are Thermo-Electric Modules (TEMs). TEMs are receiving increasing attention with the development of energy technology and thermoelectric materials [1,2]. Although most of the research focused on improving thermoelectric material properties [3], few researchers pay attention to the mechanical properties of these materials as well as mechanical stability of module under operational condition [4, 5].

In this work, an analytical model is developed to evaluate the level of thermal stresses in a simplified two-leg TEM design. The goal of this analysis is to develop and employ the simplest and most physically meaningful model for the assessment of the role of the finite size (length) of the bonded areas (legs) on the maximum stress in the assemblies of the TEM type. Finite Element Modeling (FEM) software, ANSYS 14 [6], is used to confirm the analytical model presented in this work. The numerical examples are expected to guide the design of a mechanically robust TEM structure.

The rest of this paper is organized as follows. In section II the analytical model is described. In section III case studies are presented and the results for these case studies are discussed.

II. Analytical modeling

A. Interfacial compliance

Analytical modeling has been based on the approach using the interfacial compliance concept [7, 8]. In accordance with this concept, the longitudinal interfacial compliance of a strip subjected to shear loading applied to its long edge (Fig. 1) was found from the following approximate formula for the displacements of this edge:

\[
 u_0 = -\frac{1-v^2}{2Eh} \int_0^x Q(\xi) d\xi + \kappa q(x) \quad (1)
\]

Here $E$ and $v$ are the modulus of elasticity and Poisson’s ratio for the strip material, $\kappa$ is the interfacial compliance, $h$ is the thickness of the strip, $b$ is its width, $q(x)$ is the shear force per unit strip length, and $Q(\xi)$ is the force at the $x$ cross section. The interfacial compliance $\kappa$ can be calculated as

\[
 \kappa = \frac{\sum_k \gamma_k M(u_k) \sin \alpha_k x}{\frac{2}{E} \sum_k \gamma_k \sin \alpha_k x} \quad (2)
\]

where

\[
 M(u_k) = \left(1 + \frac{v}{2}\right) \left(3 - v - (1+v)u_k \coth u_k \coth u_k + (1+v)u_k - \frac{2(1-v)}{u_k}\right)
\]

&

\[
 G = \frac{E}{2(1+v)}, \quad u_k = \alpha_k h = \frac{kn h}{2},
\]

&

\[
 \gamma_k = \frac{1}{2} \int_0^1 \tau_0(x) \sin \alpha_k x dx
\]

Fig. 1. Elongated strip subjected to shear loading.

The paper concludes in section IV with a summary and possible future plans.
We have shown (Fig.2) that the $\kappa$ value is dependent on the geometry and the material properties of the structural element in question and is only slightly dependent on the applied shearing load. The general expression (2) can be approximated by simplified relationships [7, 8]

$$\kappa = \begin{cases} \frac{h}{2\pi b} & \frac{h}{l} < 0.5 \\ \frac{h}{2\pi b} & \frac{h}{l} > 2 \end{cases}$$  (3)

Accordingly, in our further analysis we use the formulas (3) when the $h/l$ ratio is below 0.5 or above 2, and the general formula (2), when the $h/l$ ratio is between 0.5 and 2.0. The $\kappa$ values computed for different materials employed in TEM are shown in Fig. 3, assuming that the shear loading is uniformly distributed along the interface.

B. Shear Stress in a Thermo-Electric Module

For the TEM schematically shown in Fig.2, the longitudinal interfacial displacements can be predicted using the following approximate equations

$$u_1(x) = -a\Delta t_1 x + \lambda_1 \int_0^x T(\xi) d\xi - \kappa_1 \tau(x)$$

$$u_2(x) = -a\Delta t_2 x - \lambda_1 \int_0^x T(\xi) d\xi + \kappa_1 \tau(x)$$  (4)

which are similar to those used in [7], where, however, dissimilar adherend materials were considered. In the equations (4), $\alpha$ is the coefficient of thermal expansion (CTE) of the components’ (adherends’) material, $\Delta t_1$ and $\Delta t_2$ are the change in temperature of the components (from the manufacturing temperature to the operation temperature), $\lambda_1$ is the axial compliance of one of the bonded components, $E_1$ and $v_1$ are the elastic (Young’s) modulus and Poisson’s ratio of the component’s material, $h_1$ is the component thickness, $T(x)$ is the axial thermally induced force acting in the cross-sections of the components, $\tau(x)$ is the interfacial shearing stress, and $\kappa_1$ is the longitudinal interfacial compliance. The latter is due to both the material of the components and the material(s) of the bond.

The condition of the compatibility of the interfacial displacements can be written as

$$u_1(x) = u_2(x) + \kappa_0 \tau(x)$$  (5)

where $\kappa_0$ is the interfacial compliance of the buffering layer, if any. Substituting the displacements (4) into the compatibility condition (5) and solving the obtained equation for the interfacial shearing stress, with appropriate boundary conditions, one can obtain the following expression for the interfacial shearing stress:

$$\tau(x) = k \frac{\Delta t}{2\lambda_1} \left[ \frac{\sinh kx}{\cosh kl} + \frac{\tanh kl}{2k(\frac{kl}{2l})^2} \frac{\sinh 2kl + \cosh 2kl}{\cosh k(l - x)} \right], \frac{l}{2l} \geq 1$$  (6)

Here

$$k = \frac{2\Delta t}{\lambda_1} \sqrt{\kappa}$$  (7)

is the parameter of the interfacial shearing stress.

The maximum interfacial shearing stress takes place at assembly edge:

$$\tau(l) = k \frac{\Delta t}{2\lambda_1} \tanh kl \left[ 1 + \frac{1}{2k(\frac{kl}{2l})^2} \frac{\sinh 2kl + \cosh 2kl}{\cosh k(l - x)} \right], \frac{l}{2l} \geq 1$$  (8)

This relationship indicates that by decreasing the product $kl$ one could reduce the maximum interfacial shear stress at the bonding region.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young Modulus (GPa)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceramic Component</td>
<td>380</td>
<td>0.28</td>
</tr>
<tr>
<td>Copper Stripe (metallization) on Ceramic</td>
<td>115</td>
<td>0.31</td>
</tr>
<tr>
<td>Sn-Sb solder layer</td>
<td>44.5</td>
<td>0.33</td>
</tr>
<tr>
<td>Be₂Te₃ leg</td>
<td>47</td>
<td>0.4</td>
</tr>
</tbody>
</table>
III. Case studies

The analytical solution described in section 2 is applied to the TEM structure Fig. 2. The material properties are given in Table I. Three different assembly sizes of 10, 20 and 40mm (L=5, 10, and 20mm) were chosen. The value of l, the half-length of the bonded regions, has been varied to evaluate its effect on the maximum interfacial shear stress. The temperature difference between the top and the bottom components (ΔT) is 130ºC (160-30). Fig. 4 shows how the maximum interfacial shearing stress is changing versus bonded region length for different assembly sizes based on the analytical model. As it could be seen, as the bonded region length decreases the maximum interfacial shear stress reduces.

Finite Element Modeling (FEM) software, ANSYS, has been used to simulate the same TEM assembly. 8 nodes plane 223 element in plane strain mode were used. The structure is meshed with very fine square elements. Each element is 25x25μm² and there were around 400000 elements in this structure. The boundary conditions of the simulations were set according to the boundary conditions in the analytical model. The material properties are set according to Table I. The coefficient of thermal expansion (CTE) for the ceramic plates is set to 6.5 × 10⁻⁶ (1/ºC).

It is shown that by decreasing the bonded region length the maximum shear stress would drop by a factor of 5. On the other hand, by decreasing the TE leg thickness, the maximum shear stress would increase. Therefore, employment of thinner and longer legs could indeed result in a substantial stress relief, thereby leading to a more mechanically robust TEM. In [5] a similar conclusion was achieved with 3D simulation of 2 leg thermoelectric module.

3D simulation has been done to confirm what is obtained analytically. The meshed structure is shown in Fig. 8. Symmetry is used and a quarter of the model is simulated. In these simulations the assembly length was 10mm and the TE leg thickness was 4mm. By changing the bonded region length from 4mm to 0.2mm (l changed from 2mm to 0.1mm), the maximum shear stress dropped by a factor of 13. A comparison between the results is shown in Fig. 9.
IV. Conclusion

The longitudinal interfacial compliance for the uniform and linearly distributed shear loading along the interface of a long-and-narrow strip has been evaluated in application to assemblies of the TEM type. The evaluated compliances were employed, using analytical modeling, to calculate the maximum shear stress in a TEM design with two legs at the ends. It is shown that the maximum interfacial thermally induced shearing stress occurs at the leg’s corner and employment of thinner and longer legs could indeed result in a substantial stress relief. This leads to a more robust design than in the existing (conventional) TEM systems.

REFERENCES