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PULSATIONS IN LIQUID-GAS MIXTURES

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ABSTRACT

Pulsations in liquid-gas mixtures are investigated using a modal series approach. The examples which demonstrate the new approach are taken from the hydraulic industry, specially dealing with a water-air mixture in a pipe and its transient response to valve closing. This was done in order to compare the new solution with existing d'Alembert wave solutions. The approach can be directly translated to two phase refrigerant in condensers and evaporators. It is the foundation for possible future research in gas pulsation modeling of compressors taking evaporator and condenser dynamics into account.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>Dynamic pressure in a pipeline</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>Change in static pressure at location x</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Equilibrium static pressure in fluid</td>
</tr>
<tr>
<td>$c_e$</td>
<td>Effective speed of sound in two phase mixture</td>
</tr>
<tr>
<td>$K_e$</td>
<td>Effective bulk modulus of fluid in two phase mixture</td>
</tr>
<tr>
<td>$K_G$</td>
<td>Bulk modulus of gas</td>
</tr>
<tr>
<td>$K_L$</td>
<td>Bulk modulus of liquid</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>Effective density in two phase mixture</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>Density of fluid</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>Density of liquid</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Density of gas</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass flow rate</td>
</tr>
<tr>
<td>$t_v$</td>
<td>Valve closing time</td>
</tr>
<tr>
<td>$v_0$</td>
<td>Initial velocity</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>Natural frequency</td>
</tr>
<tr>
<td>$\zeta_e$</td>
<td>Damping coefficient</td>
</tr>
<tr>
<td>$\omega_d$</td>
<td>Damped frequency</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Horizontal displacement</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Displacement fluctuation</td>
</tr>
<tr>
<td>$\Lambda_n$</td>
<td>Mode shape</td>
</tr>
<tr>
<td>$T_n$</td>
<td>Modal participation coefficient</td>
</tr>
</tbody>
</table>

INTRODUCTION

Currently, condenser and evaporator lines are modeled as anechoic termination when modeling gas pulsation in compressors [1]. The inclusion of the pulsations in condenser and evaporator lines in gas pulsation models for compressor simulation is one of the next necessary steps in the theoretical investigation of noise sources in the refrigeration system. It requires mathematical models which match the mathematical models of the current compressor gas pulsation simulation. While the wave action of liquid-gas mixtures has been of interest to the hydraulic industry for many years, the modeling has been based on d'Alembert type wave construction solutions and is not compatible.

In this paper, a modal expansion type approach for liquid-gas systems is presented which is compatible with the current gas pulsation modeling in compressors. However, in order to verify the solution approach, the examples presented in this papers are taken from the hydraulic industry. The approach can be directly applied to the needs of the refrigeration and air conditioning industry.

THEORY

The wave equation for a liquid can be derived along the lines found in the reference [1]:

\[ \Delta P - \rho_0 \frac{\partial^2 \xi}{\partial t^2} = 0 \]
where the speed of sound is corrected for pipe elasticity [2] and gas bubbles in the liquid,

\[ c_e = c_o \sqrt{1 + \frac{\Delta K}{\rho_e h E}} \quad \text{and} \quad c_v = \sqrt{\frac{K_e}{\rho_e}} \left[ K_e + K_v (1-\alpha) (\rho_e (1-\alpha) + \rho_v \alpha) \right]. \]  

Note that the gas bubble correction is different from reference [3], where the density of the gas, \( \rho_v \), had been neglected as small. The notation is that of reference [1]; \( \alpha \) is the void ratio. See Figure 1.

Figure 1. Effective speed of sound in two phase flow with respect to the void fraction.

SOLUTIONS BY MODAL ANALYSIS

The following solution approach is the main contribution of this paper. It departs from the typically used d'Alembert (wave travel) solution for two phase flow used by the hydraulic industry and uses modal series expansion instead. The basic one-dimensional undamped wave equation is defined as [2]:

\[ \frac{\partial^2 \xi}{\partial t^2} = c_v^2 \frac{\partial^2 \xi}{\partial x^2}. \]  

Using the superposition principle, the complete solution of equation (5) is the sum of all the individual modes:

\[ \xi(x,t) = \sum_{n} \left( A_n \cos \omega_n t + B_n \sin \omega_n t \right) \times (C_n \cos k_n x + D_n \sin k_n x). \]  

Case 1: Suddenly Closing Valve in Hydraulic System. This case will probably not be of interest to the refrigeration industry, but it is solved in order to verify the modal against existing d'Alembert solutions in the hydraulic industry. Deceleration caused by sudden valve closure of a liquid filled pipeline generates a pressure surge.

The boundary conditions (Figure 2) are

\[ P(0,t) = -K_v \frac{\partial \xi}{\partial x}(0,t) = 0, \quad \text{and} \quad \xi(L,t) = 0. \]  

The natural frequencies for the normal vibration modes are

\[ \omega_n = k_n c_v = \frac{(2n-1) \pi c_v}{2L}. \]  

Figure 2. Valve in pipe system.
where $n=1,2,3,...$ The natural undamped modes are
\[ A_n(x) = \cos k_n x. \]  

The general solution of the damped wave equation is the summation of the individual undamped eigenfunctions:
\[ \xi(x,t) = \sum_{n=1}^{\infty} T_n(t) \cos k_n x, \]  

where the $T_n(t)$ are the modal participation coefficients. The general solution (11) satisfies all boundary conditions, because each natural mode satisfies the boundary conditions. Substituting equation (11) into the damped wave equation (2) gives, after some manipulation which evokes the orthogonality of natural modes,
\[ \xi(x,t) = \sum_{n=1}^{\infty} \frac{2 v_n \sin k_n L}{\omega k_n L} e^{-\sigma_n t} \sin \omega_n t \cos k_n x \]  

The pressure wave is
\[ P(x,t) = \sum_{n=1}^{\infty} K_n \frac{2 v_n \sin k_n L}{\omega k_n L} e^{-\sigma_n t} \sin \omega_n t \sin k_n x \]  

Results. In equation (13), the pressure wave is expressed as the summation of individual eigenfunctions. The convergence of the mode summation techniques is illustrated in Figure 3. In equation (13), the answer requires that $n \to \infty$. However, good approximations can be expected for sufficiently large $n$. It was found that 1000 terms were more than enough to make the solution of the modal analysis converge smoothly. The results agree with theoretical and experimental data presented in reference [2].

Figure 3. Influence of suddenly closing valve at valve location when $n$ term are added from 10, 100, and 1000 terms.

Case 2: Gradually Closing Valve. The modal analysis can also be applied to a slowly closing valve in a pipeline. The boundary conditions are
\[ \frac{\partial \xi}{\partial x}(0,t) = 0, \text{ and } \xi(L,t) = \frac{1}{\rho A} \int M dt, \]  

where $M$ is mass flow rate [Kg/s] through the valve as function of time. The basic equation of motion for a liquid in a uniform pipeline is given by equation (2). Here, the displacement is rederived as a combination of displacement at the exit of the pipeline and the displacement fluctuation along the pipe:
\[ \xi(x,t) = \tilde{\xi}(L,t) + \eta(x,t) \]  

where $\eta$ is a function of the displacement fluctuation. Substituting this into equation (2) gives
\[ \frac{\partial^3 \eta}{\partial t^3}(x,t) - C_i \frac{\partial^2 \eta}{\partial t^2}(x,t) - C_i \frac{\partial^2 \eta}{\partial x^2}(x,t) = -\frac{\partial^2 \xi}{\partial x^2}(L,t) + C_i \frac{\partial \xi}{\partial t}(L,t). \]  

In equation (16), the last term $C_i \frac{\partial \xi}{\partial t}(L,t)$ represents damping at the valve and is assumed to be insignificant. It is equivalent to saying that the pipe at the valve location does not provide damping. This is an assumption which has been proven to be correct in the gas pulsation literature. Equation (16) can be simplified as
\[
\frac{\partial^2 \eta}{\partial t^2}(x,t) - C_x \frac{\partial \eta}{\partial t}(x,t) - c_x \frac{\partial^2 \eta}{\partial x^2}(x,t) = - \frac{\partial^2 \xi}{\partial t^2}(L,t)
\]  
(18)

Modified boundary conditions are obtained by substituting equation (16) into equations (14) and (15):
\[
\frac{\partial \eta}{\partial x}(0,t) = 0, \quad \text{and} \quad \eta(L,t) = 0.
\]  
(19, 20)

Solving for the natural frequencies gives, as before,
\[
\omega_n = \frac{2n - 1}{2L} c_x,
\]  
(21)

where \(n=1,2,3,\ldots\) The natural modes are
\[\Lambda_n(x) = \cos k_n x.\]  
(22)

It is again assumed that the solution to equation (18) is expressible as a modal series,
\[
\eta(x,t) = \sum_{n=1}^{\infty} T_n(t) \Lambda_n(x),
\]  
(23)

where the \(T_n(t)\) are the modal participation coefficients. Substituting equation (23) into equation (18) gives, after some manipulation and evoking orthogonality of natural modes,
\[
\eta(x,t) = \sum_{n=1}^{\infty} \frac{2 \cos k_n x \sin k_n L}{k_n L \rho \omega_n} \left( \int_{0}^{1} \left( \frac{d M}{dt} \right)_{\infty} e^{-\omega_n \tau} \sin \omega_n (t - \tau) \ d\tau \right). \]  
(24)

The pressure equation becomes
\[
P(x,t) = \sum_{n=1}^{\infty} \frac{2 c_x \sin k_n x \sin k_n L}{\rho L} \left( \int_{0}^{1} \left( \frac{d M}{dt} \right)_{\infty} e^{-\omega_n \tau} \sin \omega_n (t - \tau) \ d\tau \right). \]  
(25)

**Valve Closing Description.** Assuming as example a closing behavior that produces a linear change of mass flow, the mass flow rate of a pipeline can be expressed as
\[
M = -\alpha_t + v_s \rho \omega_n, \]  
(26)

where \(v_s \rho \omega_n\) is the initial mass flow rate [Kg/s]. This is typical of what occurs in the hydraulic industry and again our model will be verified against existing d'Alembert solutions. When \(t \leq t_i\), the pressure wave equation is
\[
P(x,t) = \sum_{n=1}^{\infty} \frac{2 \rho \omega_n v_s \sin k_n x \sin k_n L}{\rho L} \left( \int_{0}^{1} \left( \frac{d M}{dt} \right)_{\infty} e^{-\omega_n \tau} \sin \omega_n (t - \tau) \ d\tau \right). \]  
(27)

Note that equation (27) is compatible with the gas pulsation models for compressors. The only change needed is to replace \(M\) of equation (26) by \(M = \dot{M} e^{i\alpha}\) and obtain the steady state version of the solution. Also, the open boundary condition of the example will have to be replaced. Also, the changing quality of the liquid gas refrigeration mixtures needs to be considered. When \(t > t_i\), the pressure wave equation is
\[
P(x,t) = \sum_{n=1}^{\infty} \frac{2 \rho \omega_n v_s \sin k_n x \sin k_n L}{\rho L} \times \left( \int_{0}^{1} \left( \frac{d M}{dt} \right)_{\infty} e^{-\omega_n \tau} \sin \omega_n (t - \tau) \ d\tau \right). \]  
(28)

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**Figure 4.** Pressure affected by gradual valve closure \((t_i=0.01, 2, 4, n1=1000 \text{ terms})\)
Results. The objective of this study is to investigate how well modal analysis can be applied to the liquid hammer problems, but it is also of interest to study liquid hammer as a phenomenon. For example, to reduce the pressure increase, the valve closing time can be adjusted to find an optimal valve closing time. The pressure waves at different valve closing times were investigated and the results are shown in Figure 4. The pressure is shown at the valve location. The wave travel time from the valve to the reservoir takes $L/c_v$ and the wave travel time back to the valve requires the same time. Therefore, if the valve closes in less than $2L/c_v$, then the maximum pressure can be observed at the valve location. When the valve closing time is greater than $2L/c_v$, the maximum pressure does not occur at the valve location. Figure 4.c shows that the pressure wave becomes almost zero after the valve closes at $t_i = 4$, therefore the optimal valve closing time, for this particular case, is when $t_i = 4$. It was observed that the pressure wave fluctuates when the valve closing time is greater than $t_i = 4$. However, if the valve closing time is further delayed then the pressure wave is decreased exponentially. The optimal valve closing time could be
predicted by studying the pressure wave patterns while the valve closes. Good agreement is observed between this result and experimental results in reference [4].

Pressure waves affected by the wall elasticity at different valve closing times are plotted in Figure 5. As shown in these figures, the pressure waves affected by the wall elasticity were delayed since the speed of sound with the wall elasticity effect is less than without the wall elasticity effect. The amplitude of the pressure wave is also attenuated since the speed of sound is also directly associated with the pressure wave amplitude. Figure 5. c shows that the pressure waves become close to zero when the valve closes at t, = 4.5.

The pressure waves in the two phase mixture are studied for various void fractions and the results are shown in Figure 6. The volume of gas is assumed to be relatively small compared to the liquid volume. It was found that the pressure wave amplitude decreases rapidly with small void fraction increases. This is largely because the low air bubble elasticity reduce sudden pressure build ups. These results agree with the experimental data in references [5, 6].

Damping of the pressure waves is affected by several factors related to energy losses. One way to obtain the modal damping coefficients is by assuming reasonable values based on experience and experimental work. An assumed effective viscous damping term was therefore introduced to account for the various possible energy dissipation mechanisms. The pressure waves as function of various modal damping coefficients are plotted in Figure 7. Good qualitative agreement is observed between these results and experimentally obtained results in references [2].

CONCLUSIONS

While the examples of pulsations in liquid-gas mixtures, extended in this paper, were taken from hydraulics, the method is directly applicable to refrigeration systems. Equation (27) is directly compatible with current acoustic type gas pulsation modeling of compressors. The mass flow rate M needs to be replaced by a boundary correction describing the condenser at the expansion valve or capillary tube, and the solution has to take into account the non-uniform quality of the refrigerant in the lines. Discontinuous cross-sections of lines can be handled easily by the modal description. The presented approach will be the foundation of possible future research of the influence of evaporator and condenser line gas pulsations on noise in refrigeration and air conditioning systems.

REFERENCES

1. W. Soedel, Mechanics, Simulation and Design of Compressor Valves, Gas Passages and Pulsation Mufflers, Short Course Notes, Purdue University, 1992.