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Power Handling of Electrostatic MEMS Evanescent-Mode (EVA) Tunable Bandpass Filters

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Abstract—This paper presents the first theoretical and experimental tunable MEMS cavity filters. The theoretical analysis indicates that the frequency-dependent RF voltage inside a narrow-band filter may play an important role in the generation of electromechanical nonlinearities such as frequency response distortion, frequency shift, and bifurcation instability. This analysis also reveals that the filter’s power handling capability is dependent on several critical factors, including the capacitive gap, stiffness of the diaphragm actuator, and the overall quality factor ($Q$) of the evanescent-mode (EVA) resonators. A nonlinear computer-aided design (CAD) model is proposed as a practical tool for capturing the important tradeoffs in high-power design. An EVA tunable resonator and a two-pole 2% filter are fabricated and measured as vehicles to validate the theory and the CAD model. Specifically, a medium-power filter with a tuning range of 2.35–3.21 GHz (1.37:1) and an extracted unloaded quality factor ($Q_u$) of 356–405 shows measured power levels of 23.4 dBm (0.22 W) before bifurcation instability occurs. The measured IIP3 of this filter are 52.1 dBm. The theory and modeling, backed up by the measurements, provide significant insights into the high-power design of electrostatic tunable cavity filters.

Index Terms—Evanescent-mode cavity filter, intermodulation, microelectromechanical systems (MEMS), nonlinearity, quality factor ($Q$), tunable filter, self-actuation.

I. INTRODUCTION

RECENTLY, microelectromechanical systems (MEMS) evanescent-mode (EVA) tunable cavity filters for RF/microwave frequencies have received considerable research attention for their merits of wide tuning range, high unloaded quality factor ($Q_u$), reduced size/weight, and a large spurious-free region [1]–[4]. Furthermore, the electostatic MEMS tuners require almost zero dc power, making such filters great candidate components for a wide range of applications. Examples of such applications include automatic test instrumentation, wireless communication, and sensing systems. These applications have varying power handling requirements, ranging from milliwatts to tens of watts. Therefore, it is important to understand the power-handling capabilities of such MEMS EVA tunable filters.

The power-handling capabilities of RF/microwave filters are limited by several factors including dielectric breakdown, gas discharge, thermal breakdown, and device nonlinearities [5]. The critical high-power phenomena for MEMS tunable filters include solid dielectric breakdown, gas discharge, and electromechanical nonlinearities of the MEMS tuning elements. In this paper, we focus on the last one and, in particular, the effects of “self-actuation” and intermodulation distortion (IMD) on the power handling of EVA tunable resonators and filters. Self-actuation refers to the actuation of the movable MEMS micro-structure caused by the electrostatic attractive force stemming from the RF signal power [6]. IMD refers to the generation of unwanted amplitude modulation of signals due to device nonlinearities. From a system point of view, IMD limits the maximum power that a MEMS tunable filter can handle without introducing excessive in-channel and cross-channel interferences.

There have been numerous studies on the power handling of RF MEMS devices, including MEMS varactors [7]–[10], capacitive switches [8], [11]–[13] and metal-contact switches [14]. In [8], theoretical analysis and computer-aided design (CAD) modeling were used to predict the power handling of MEMS varactors and switches. Girbau et al. presented extended analysis by taking into account the large displacement and impedance change during the actuation of the MEMS varactors [10]. A frequency-domain analysis technique was proposed by Innocent et al. to analyze the weak nonlinearities of MEMS varactors and switches [9].

However, the above-mentioned modeling efforts are primarily based on stand-alone MEMS devices, such as a single MEMS varactor or switch. In [8], the nonlinearities of MEMS tunable filters were studied, but the resonant characteristics of the filter were simply modeled as a voltage amplification for the MEMS devices. This is a valid approximation for filters of relatively large fractional bandwidth. However, it does not take into account the frequency dependence of the RF voltage in a resonator. In [15], the authors of this paper demonstrated the modeling and measurement of such nonlinearities in high-$Q_u$ EVA tunable cavity resonators (but not filters).

Compared with our previous work [2], [4], which focused on the design and fabrication technology of tunable EVA resonators and bandpass filters, this paper presents, for the first
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Fig. 1. (a) Concept drawing of MEMS EVA tunable resonator. (b) Spring-mass model of the MEMS diaphragm actuator. (c) Equivalent circuit of the MEMS EVA tunable resonator.

time, a complete validated framework on the power-handling capability of MEMS tunable EVA filters. Building upon our previous work [15], we start by developing for the first time analytical solutions for the nonlinearities of MEMS EVA tunable resonators (Section II). It is shown that the frequency dependence of the RF voltage plays an important role in the modeling of the nonlinearities of EVA resonators. Section III provides a practical circuit CAD model for capturing such nonlinearities in a system-level environment. The theoretical and numerical models are validated in Section IV by measurements on a high-$Q$ MEMS EVA tunable resonator. Power measurements on a two-pole MEMS EVA tunable filter are also presented for the first time with a very good agreement with simulation.

II. THEORETICAL ANALYSIS

A. Review of MEMS EVA Tunable Cavity Resonators/Filter

Fig. 1 shows a concept drawing of the EVA tunable resonator proposed in [2]. The tunable resonator consists of an evanescent-mode resonant cavity, a thin metallic diaphragm tuner, and a bias electrode placed above the diaphragm tuner. The resonant frequency and $Q_u$ of the cavity resonator are found to be dependent on the cavity size, post size, and the gap $g$ between the post top and the top wall of the cavity. The resonant frequency is very sensitive to $g$ when $g$ is small. When a bias voltage is applied to the bias electrode, the thin diaphragm is pulled away from the post, changing $g$ and, thus, the resonant frequency. The $Q_u$ of this tunable resonator is inherently high due to the distributed nature of the cavity resonator. A MEMS EVA tunable resonator with a tuning ratio as high as 2.6:1 and $Q_u$ of 650 at 5 GHz has been demonstrated in [4]. The same technology was also used to make a two-pole 0.7% bandwidth filter with a tuning range of 3.0–4.7 GHz and insertion loss of 3.55–2.88 dB [4].

The EVA tunable resonator is a distributed implementation of a lumped-element resonator [2]. The electric field is predominantly concentrated in the gap region between the capacitive post and the diaphragm, which represents an effective capacitor; the sidewalls of the cavity and the capacitive post constitute a shorted coax line, which is effectively an inductor. Therefore, the EVA resonator can be modeled as an $LC$ tank shown in Fig. 1(c), where $C_r$ and $L_r$ are the equivalent capacitor and inductor, respectively, and $R_u$ accounts for losses in the resonator. In the equivalent circuit of Fig. 1(c), the input and output coupling to the resonator is modeled by ideal transformers.

B. Self-Actuation in MEMS EVA Tunable Resonators

MEMS EVA tunable resonators are essential building blocks of EVA tunable filters. In order to understand the power-handling capability of the EVA tunable filters, it is critical to first understand the power-handling capability of the EVA tunable resonators. Here, we focus on the analysis of the RF self-actuation in EVA tunable resonators.

The mechanical behavior of the thin diaphragm actuator can be modeled by a simple 1-D spring-mass model shown in Fig. 1(b). The diaphragm actuator is subject to three primary forces, given here.

1) The electrostatic force $F_{DC}$ from the bias electrode.

Assuming that electric field only exists in the overlapping area between the bias electrode and the diaphragm actuator, $F_{DC}$ can be approximated by

$$F_{DC} = \frac{\varepsilon_0 W^2 V_{DC}^2}{2\left(d_0 + x\right)^2}$$

where $W$ is the width of the bias electrode, $V_{DC}$ is the bias voltage, $d_0$ is the initial gap between the post and the diaphragm, and $x$ is the deflection of the diaphragm. Equation (1) neglects the effect of the fringing field, which can be taken into account by the nonlinear circuit model explained in Section III.

2) The electrostatic force $F_{RF}$ from the RF signal power [6]. Using parallel-plate capacitance for $C_r$, $F_{RF}$ is given by

$$F_{RF} = \frac{\varepsilon_0 \pi a^2 |V_{RF}|^2}{4\left(d_0 - x\right)^2}$$

where $a$ is the post radius and $V_{RF}$ is the peak–peak RF voltage between the post and the diaphragm. Again, the fringing-field contribution to $F_{RF}$ is taken into account by CAD modeling in Section III.

3) Mechanical restoring force $F_k$.

Assuming linear deflection, $F_k$ is given by

$$F_k = k x$$

(3)
where \( k \) is the spring constant of the diaphragm.

In the analog tuning range mode, electromechanical equilibrium at a particular gap requires these forces to balance at that gap as

\[
F_{DC} + F_{RF} - F_k = 0. 
\]  
(4)

At low input RF power, the deflection of the diaphragm actuator is dominated by the electrostatic force between the diaphragm and the dc biasing electrode. When the input power is increased, the RF-induced electrostatic force \( F_{RF} \) starts to affect the deflection of the diaphragm. Specifically, it starts pulling the diaphragm towards the capacitive post causing nonlinear responses. In a narrowband resonator/filter, this nonlinear response is further pronounced by the input and output transformers.

Inserting (1)–(3) into (4), we have

\[
\frac{r_o W^2 V_{DC}^2}{2(d_0 + x)^2} - \frac{\varepsilon_0 \pi a^2 |V_{RF}|^2}{4(g_0 - x)^2} + kx = 0. 
\]  
(5)

Note that the dc bias increases the capacitive gap \( g \) and therefore reduces \( F_{RF} \). In other words, when a dc bias is applied to tune the resonant frequency higher, the power-handling capability will also increase. Therefore, the worst case scenario is seen when no dc bias is applied. In the following analysis, we assume no dc bias and look at the nonlinear response of the EVA tunable resonators solely due to RF power. As will be shown later, omitting the dc bias signal effect does not undermine the generality of the conclusions drawn from the analysis presented in this section.

With no dc electrostatic force, (5) is simplified as

\[
\varepsilon_0 \pi a^2 |V_{RF}|^2 = 4kx(g_0 - x)^2 
\]  
(6)

where \( V_{RF} \) can be calculated by linear circuit analysis of Fig. 1 as

\[
V_{RF} = \frac{2}{(j\omega C_r + \frac{1}{j\omega L_r} + \frac{1}{R_o})} \sqrt{\frac{2P}{n^2 Z_0}} 
\]  
(7)

where \( n \) is the transformation ratio, \( Z_0 \) is the port impedance, and \( P \) is the RF power from the input port.

Inserting (7) into (6) and rearranging both sides of the equation, we obtain

\[
\frac{2\varepsilon_0 \pi a^2 P}{n^2 Z_0} = kx(g_0 - x)^2 \left( j\omega C_r + \frac{1}{j\omega L_r} + \frac{1}{R_o} \right) + \frac{2}{n^2 Z_0} x^2. 
\]  
(8)

Note that \( C_r \) is directly related to the deflection of the MEMS actuator. We use the parallel-plate model for the capacitance calculation. The neglected fringing-field term is taken into account in the circuit models developed in Section III and shown here as

\[
C_r = \frac{\varepsilon_0 \pi a^2}{g_0 - x}. 
\]  
(9)

Putting (9) into (8) and rearranging both sides, we obtain

\[
\frac{2\varepsilon_0 \pi a^2 P(\omega L_r)}{n^2 Z_0} = kx \left\{ \frac{\varepsilon_0 \pi a^2 \omega^2 I_r}{(g_0 - x)^2} + \left( \frac{1}{R_o} + \frac{2}{n^2 Z_0} \right) \right\}. 
\]  
(10)

Equation (10) can be further simplified by making a few more substitutions to yield

\[
\frac{x}{g_0} \left\{ \left( \frac{1}{\omega \omega_0} \right)^2 - \frac{g_0 - x}{g_0} \right\}^2 + \left( \frac{g_0 - x}{g_0} \right) \left( \frac{\omega}{\omega_0} \right)^2 / Q^2 \right\} = \left( \frac{\omega}{\omega_0} \right)^2 F 
\]  
(11)

where

\[
\omega_0 = \frac{1}{L_r C_r} = \frac{g_0}{\varepsilon_0 \pi a^2 L_r} 
\]  

is the small-signal resonant frequency of the resonator, the doubly loaded quality factor of the resonator is given by

\[
Q = \frac{1}{\omega_0 L_r \left( \frac{1}{R_o} + \frac{2}{n^2 Z_0} \right)} 
\]

and

\[
F' = \frac{2\varepsilon_0 \pi a^2 P\omega_0 L_r}{k\pi^2 Z_0 g_0^2} - \frac{2PL_r}{k\pi^2 Z_0 g_0^2}. 
\]

We now define a normalized varactor gap

\[
\tilde{g} = \frac{g_0 - x}{g_0} 
\]

and normalized frequency

\[
\tilde{\omega} = \left( \frac{\omega}{\omega_0} \right)^2. 
\]

Equation (11) can then be further simplified to

\[
(1 - \tilde{g}) (\tilde{\omega} - \tilde{g})^2 + \tilde{g} \tilde{\omega} \tilde{g} Q^2 = \tilde{\omega} F'. 
\]  
(12)

Equation (12) is the nonlinear equation describing the relationship between the normalized deflection of the diaphragm and the RF power. It is a third-order equation in terms of \( \tilde{g} \) and has three solutions in the complex domain. Among the three solutions, the ones in the real domain give the amplitude of the normalized diaphragm deflection under certain external RF power.

For small input power, i.e., small \( F \), only one solution is in the real domain. This corresponds to the case of small signal input [Fig. 2(a)]. In the limiting case of \( F \to 0 \), the frequency response of the resonator is symmetrical around the resonant frequency.

As \( F \) increases, the resonant frequency becomes lower and the frequency response starts to “bend” towards it. This asym-
metrical distortion in the frequency response can be intuitively understood if we consider the establishment of the frequency response in an iterative manner.

Fig. 3(a)-(d) shows the case when a moderately high-power input RF signal is applied at a frequency higher than the resonant frequency. $F_{HF}$ pulls the diaphragm actuator closer to the capacitive post, thus lowering the resonant frequency. This, in turn, lowers $V_{RF}$ and $F_{RF}$, causing a negative feedback effect. Due to the mechanical restoring force, the diaphragm actuator will retract away from the post until an equilibrium is achieved.

When the input signal is applied at a lower frequency, as shown in Fig. 3(e)-(h), the scenario can be quite different. The $V_{RF}$ lowers the resonant frequency in a similar fashion as in the previous case. However, as the resonant frequency moves closer to the input signal, the induced $V_{RF}$ increases, creating a positive feedback process. Due to this increased $V_{RF}$, the resonant frequency will become still lower until equilibrium is achieved.

From this conceptual experiment, it is obvious that the EVA resonator reacts differently to input RF signals below and above its resonant frequency. This behavior leads to the asymmetrical response shown in Fig. 2. It is worth noting that the frequency response curves in Fig. 3 are all drawn in linear scale for easier illustration.

When the input power becomes even larger, the situation becomes more complex as $F$ reaches a critical value $F_c$. It is noted that there is still a one-to-one correspondence between $\hat{g}$ and $\hat{\omega}$ for $F < F_c$. For $F > F_c$, however, all three solutions to (12) can be real. In this case, there are three possible $\hat{g}$ values for a certain range of frequencies $\tilde{\omega}_1 < \hat{\omega} < \tilde{\omega}_2$ [Fig. 2(d)]. Such a phenomena is often referred to as "bifurcation" [20].

C. Critical RF Power

In order to predict the critical power-handling capability of MEMS EVA tunable resonators and filters, it is important to calculate the value of the critical input RF power $P_c$, which presents itself in (12) as $F_c$. We first observe that the condition $\partial \hat{g}/\partial \hat{\omega} = 0$ holds at $\tilde{\omega}_1$ and $\tilde{\omega}_2$ [which correspond to points C and D in Fig. 2(d)]. Differentiating (12) with respect to $\hat{\omega}$ yields

$$
\frac{1}{(1 - \hat{g})} \left[ 2(\hat{\omega} - \hat{g}) \left( 1 - \frac{\partial \hat{g}}{\partial \hat{\omega}} \right) + \frac{2\hat{g} \hat{\omega}}{Q^2} \frac{\partial \hat{g}}{\partial \hat{\omega}} + \hat{g}^2 \frac{\partial^2 \hat{\omega}}{\partial \hat{\omega}^2} \right]
$$
In order to satisfy the condition $\frac{d\varphi}{d\omega} \to \infty$, we set the coefficient of the $d\varphi/d\omega$ term in (13) to zero to yield

$$2(1 - \hat{g}) \frac{\hat{\varphi}}{Q^2} - 2(1 - \hat{g})(\hat{\omega} - \hat{g}) - (\hat{\omega}^2 - \hat{g}^2\hat{\omega}) = 0$$

which can be rearranged as a quadratic equation in terms of $\hat{\omega}$ as

$$\hat{\omega}^2 - \left( \frac{2\hat{g}}{Q^2} - \frac{3\hat{g}^2}{Q^2} + 4\hat{g} - 2 \right) \hat{\omega} + (3\hat{g}^2 - 2\hat{g}) = 0.$$  (14)

$\hat{\omega}_1$ and $\hat{\omega}_2$ can then be found by simultaneously solving (14) and (12).

However, the calculation of $F_c$ does not require the solution for $\hat{\omega}_1$ and $\hat{\omega}_2$. We observe that points $C$ and $D$ reduce to a single point when $F = F_c$ [Fig. 2(c)]. In other words, the two solutions to (14) coincide with each other. Setting the discriminant of (14) to zero, we obtain

$$\left( \frac{2\hat{g}}{Q^2} - \frac{3\hat{g}^2}{Q^2} + 4\hat{g} - 2 \right)^2 - 4(3\hat{g}^2 - 2\hat{g}) = 0.$$  (15)

Equation (15) is a fourth-order equation in terms of $\hat{g}$ and has four solutions in the complex domain. Of the four solutions, only one is physically meaningful ($0 < \hat{g} < 1$). It gives the normalized gap value $\hat{g}_c$ that corresponds to point $C(D)$ in Fig. 2(c). Its closed-form formula is rather involved but can be analytically found by using the root-finding formula or, more conveniently, a symbolic mathematics software package such as Mathematica [19].

With the help of Mathematica, we can use the power series expansion to get a more practical and simplified formula for $\hat{g}_c$. In the limit of $Q \gg 1$, we have

$$\hat{g}_c = 1 - \frac{1}{Q} + \frac{3}{Q^2} - O(Q^3).$$  (16)

Putting (16) into (14), we can find the normalized frequency $\hat{\omega}_c$ at which bifurcation occurs

$$\hat{\omega}_c = 1 - \frac{2}{Q} + \frac{11}{2Q^2} - O(Q^3).$$  (17)

Putting (16) and (17) into (12), we can solve for $F_c$ as follows:

$$F_c = \frac{2}{Q^2} + \frac{11}{Q^3} - O\left( \frac{1}{Q^4} \right).$$  (18)

Therefore

$$P_c = \frac{kZ_0g_0^2}{2L} \left[ \frac{2}{Q^2} + \frac{11}{Q^3} - O\left( \frac{1}{Q^4} \right) \right].$$  (19)

At the onset of bifurcation, the critical deflection $x_c$ and frequency $f_c$ are, respectively, given by

$$x_c = g_0 \left[ \frac{1}{Q} - \frac{3}{Q^2} + O(Q^3) \right]$$  (20)

$$f_c = \frac{\omega_0}{2\pi} \left[ \frac{2}{Q} + \frac{9}{4Q^2} + O(Q^3) \right].$$  (21)

It is also interesting to note that, in the limit of $Q \to 0$, the solution to (14) can be expanded (using Mathematica) as

$$\hat{g}_c = \frac{2}{3} \frac{1}{Q^2} + \frac{21}{3} \frac{1}{Q^3} - O(Q^4)$$  (22)

and

$$\hat{\omega}_c = \sqrt{\frac{2}{3} Q - \frac{21}{3} Q^2} + O(Q^4).$$  (23)

This is intuitively understood because, as $Q \to 0$, the resonator is heavily loaded and approaches a transmission structure instead of a resonant structure. The bifurcation instability occurs at $\omega_c = 0$ at a normalized gap of $g_c \to (2)/(3)$, which is simply the instability point of an electrostatically actuated parallel-plate actuator [6]. Therefore, the dc instability can be regarded as a special case of the analysis developed in this section.

Equation (19) gives the critical input power level at the onset of bifurcation. Note that dc bias is assumed to be zero in the above analysis. Therefore, (19) gives the minimum upper limit of power-handling capability of a MEMS EVA tunable resonator.

$P_c$ in (19) is shown to be dependent on a few factors, including the stiffness $k$ of the diaphragm actuator, initial gap $g_0$, and the overall quality factor $Q$. Whereas $Q$ is often determined by system-level requirements, appropriate $k$ and $g_0$ can be chosen to improve the power-handling capabilities of MEMS EVA tunable resonators/filters. However, in most applications, other specifications, such as actuation voltage and tuning, often need to be taken into account as well. For example, improving power-handling capability by increasing $k$ and $g_0$ comes at the cost of increased actuation voltage or reduced tuning range. These interdependencies are examined quantitatively in Section III.

It is important to mention that the analysis given in this section is based on a general nonlinear varactor model and the general conclusions from the above analysis hold true for any tunable resonator using parallel-plate electrostatic MEMS switches/varactors as the tuning elements.

D. MEMS EVA Tunable Filters

The self-actuation behavior of a MEMS EVA tunable filter can be analyzed following a similar approach as the EVA tunable resonator. Fig. 4 shows a schematic of a general coupled-resonator bandpass filter. $M_{ij}$ are the elements of the coupling matrix and denote the direct and cross coupling between the resonators [17].

The loop equations for each of the resonators in the filter can be written in a matrix form given in (24), shown at the bottom of the following page.

With knowledge of the coupling matrix, the current in each resonator can be solved. The voltage on the $j$th MEMS varactor is then given by

$$V_{RFj} = \frac{i_j}{j\omega C_j}.$$  (25)

Inserting (25) and (9) into (5), one can obtain the nonlinear equation describing the self-actuation behavior of the EVA tunable filters. However, this equation can become very compli-
cated for higher order filters. A simpler and more practical way of solving the nonlinear equation is through the use of a numerical CAD model, which is the subject of Section III.

### III. CAD MODELING

In Section II, theoretical analysis on nonlinearity of the MEMS EVA tunable resonator is presented. It is important to develop a more practical design tool in order to take into account second-order effects such as fringing field capacitance and model more complicated structures such as higher order filters. Here, we present the modeling of nonlinearities of EVA tunable resonators through a nonlinear CAD model.

#### A. Nonlinear CAD Model

The analysis of Section II-B shows that the nonlinearity of the EVA tunable resonator is primarily caused by the nonlinearity of the equivalent varactor $C_r$. The electromechanical characteristics of $C_r$ are governed by $V_{DC}$ in (1), $V_{RF}$ in (2) and $F_k$ in (3). These equations are coupled with each other, and analytical solutions are difficult to obtain. However, an iterative approach can be utilized to numerically solve these equations.

Fig. 5 outlines this process. First, the RF voltage $V_{RF}$ across the varactor $C_r$ is found from (7) by setting $C_r$ to its initial value, i.e., the mechanical deflection of the diaphragm is only determined by the dc bias voltage $V_{DC}$. Then, $V_{RF}$ is used to calculate the RF force $F_{RF}$ exerted on the diaphragm according to (2). $F_{RF}$ is in turn used to calculate the deflection $x$ of the diaphragm, which gives an updated value to $C_r$. This process is repeated until the solution converges. A failed convergence indicates that the RF power is large enough to cause self-pulling of the diaphragm.

The above process can also be implemented using commercially available circuit simulators. Building upon previous work by Lu [18], a nonlinear voltage-controlled capacitor model (Fig. 6) is constructed in Agilent Advanced Design Systems (ADS) using four-port Symbolically Defined Devices (SDD) [21].

The voltages at the four ports of the model are defined as follows.

1) Port 1: Diaphragm deflection $x$.
2) Port 2: Electrostatic force on the diaphragm $F_x = F_{RF} + F_{DC}$.
3) Port 3: RF voltage $V_{RF}$.
4) Port 4: DC bias voltage $V_{DC}$.

An example EVA tunable resonator is simulated with the equivalent nonlinear circuit model. The resonant frequency and $Q_u$ of the resonator are 2.4 GHz and 1000, respectively. The external quality factor $Q_e$ is assumed to be 50. The nominal parameters of the tunable resonator are listed in Table I. Fig. 7 shows the simulated large-signal $S_{21}$ at different input power levels with no dc biasing.

The simulation shows that, for the particular design parameters and for low-input-power signals (<10 dBm), $S_{21}$ remains quite linear. As the input power is increased, nonlinearities start
to appear. For input power in the range of 15–20 dBm, self-biasing causes the diaphragm to deflect towards the capacitive post, leading to a drift in the resonant frequency and distortion to the shape of the resonance peak. At a power of ~30 dBm, the RF-induced attractive force is sufficiently large to pull the diaphragm into the capacitive post. This can be seen in the instability point of the diaphragm deflection plot in Fig. 7, where a sudden jump in the diaphragm deflection is observed. When the diaphragm is pulled into the post, the resonator can no longer be tuned. The diaphragm will restore to its original position when the RF power is turned off. Assuming no dielectric discharge or breakdown, the critical power sets the higher limit to the power-handling capabilities of the MEMS EVA tunable resonators.

### B. High-Power Design Considerations

It is shown in (19) that the critical power $P_c$ is strongly dependent on the overall quality factor $Q$, $Q$ is related to the unloaded quality factor $Q_u$ and external quality factor $Q_e$ by

$$\frac{1}{Q} = \frac{1}{Q_u} + \frac{1}{Q_e}. \tag{26}$$

Whereas $Q_u$ is often determined by the resonator technology, $Q_e$ can vary considerably according to the design’s specifications. Fig. 8 shows the calculated large-signal $S_{21}$ at the onset of instability for resonators with different $Q_e$. Linear responses are included as a comparison. The nominal parameters of the resonators in this calculation are listed in Table I.

It is shown in (19) that the power-handling capability is also dependent on the gap $g$ and spring constant $k$. Fig. 9(a) shows the large-signal $S_{21}$ for resonators with the same input power of 33.8 dBm but varying $g_0$ of 2, 5, 10, and 20 μm. The nominal parameters of the resonators are shown in Table I. The resonant frequencies are kept the same for all resonators by setting the post radii values to 0.316, 0.5, 0.707, and 1 mm, respectively. With 33.8-dBm input power, the frequency response of the resonator with $g_0 = 10$ μm is at the onset of bifurcation. For
Fig. 8. Large-signal simulation of resonators with different external couplings.

Fig. 9. Large-signal simulation of resonators with (a) different capacitive gaps and (b) different spring constant. The resonant frequency and $C_e$ are kept the same by setting appropriate post radius. The nominal parameters of the simulated resonator are listed in Table I.

Fig. 10. Simulated actuation voltage and $P_c$ of the EVA tunable resonator with respect to (a) the spring constant and (b) the initial gap. The nominal parameters of the simulated resonator are listed in Table I.

The penalties paid with increasing $g_0$ and $k$ are reduced tuning range or higher actuation voltage. For example, Fig. 10 shows the calculated actuation voltage of a tunable resonator (parameters listed in Table I) with respect to $k$ and $g_0$ for 1.5:1 tuning ratio. It can be seen that $P_c$ can be increased by 20 dBm by increasing $g_0$ from 2 to 10 $\mu$m. The required actuation voltage needs to be increased by almost 400 V to maintain the same tuning ratio.

C. DC Bias

Section III-B shows that high RF signal power can lead to a shift in the resonant frequency. This shift is always towards the lower frequency due to the attractive electrostatic force. Intuitively, this frequency shift can be compensated by increasing the dc bias voltage, which serves to pull the diaphragm actuator away from the post and increase the resonant frequency.

Fig. 11 shows the simulated linear and nonlinear response of a tunable resonator with 26-dBm input power. The nominal parameters of the resonator are listed in Table I. The dashed curve represents the linear response of the resonator with no dc bias voltage. The dotted curve represents the nonlinear frequency response when an input RF power of 26 dBm is fed through the resonator. A close-up plot of the frequency response at 2.4 GHz is shown in Fig. 11(b). With 26-dBm input power and no dc bias, a frequency shift of 40 MHz can be observed. The solid curve shows that this frequency shift can be compensated by applying 25-V bias voltage. However, the asymmetric distortion can still be observed in the frequency response. Although additional dc biasing can compensate for the frequency shift, it cannot prevent the self-pulling instability in EVA tunable resonators.

Fig. 11 also shows that the frequency distortion is less severe at higher frequencies. This is primarily due to the fact that the diaphragm actuator is farther away from the post and $P_c$ increases proportional to $g_0^2$ as shown in (19).

D. Intermodulation

With the nonlinear model developed in previous sections, the IMD of EVA tunable resonators can be quantitatively investigated.

Fig. 12 shows simulated intermodulation products of an EVA tunable resonator in response to two-tone input signals. The

smaller $g_0$ (5 and 10 $\mu$m), severe bifurcation can be observed; for larger $g_0$ (20 $\mu$m), the frequency response is much less distorted. Similarly, Fig. 9(b) shows the large-signal $S_{21}$ for resonators with varying spring constant. It is seen that the power handling is improved with increasing spring constant.
nominal parameters of the resonator are listed in Table I. The frequency separation of the two-tone input signals is 10 kHz, which is the mechanical resonant frequency of the diaphragm actuator. The third-order intermodulation product (IM3) of the tunable resonator with 0- and 80-V bias are compared in the plot (the resonant frequencies are 2.4 and 2.8 GHz, respectively). Slightly smaller IM3 is observed for the 80-V case due to the higher capacitive gap.

Fig. 13 shows the comparison of the output powers of the fundamental frequency and the IM3 products. With small input power (<20 dBm), both the fundamental output and the IM3 increases linearly with the input power. As the input power increases, compression of both the fundamental output and the IM3 can be observed. This compression is caused by the self-actuation of the diaphragm actuator, leading to a lowering in resonant frequency and thus an increase in return loss for the input signals.

E. Nonlinearities of EVA Tunable Filters

With the nonlinear CAD resonator model, the high-power response of coupled-resonator EVA filters can also be quantitatively investigated. Fig. 14 shows the simulated responses of a two-pole and a four-pole direct-coupled Butterworth filter at 2.4 GHz with 2% fractional bandwidth. The filters are designed with two and four resonators, respectively. The nominal parameters of the resonators are the same as those listed in Table I. The inter-resonator couplings are achieved with J-inverters, which are modeled by T-section inductor networks.

The RF voltages are different on each resonator in a coupled-resonator filter. Therefore, the power handling is determined by the resonator that experiences the highest RF voltage. The RF voltage distribution is, in general, dependent on the internal and external coupling coefficients of the filter. Fig. 15 shows the simulated RF voltages on the resonators in the two-pole and four-pole filters with identical input and output couplings. In general, the second resonator sees the highest voltage at the band edge. In the two-pole filter case, the first resonator sees the highest voltage. The power handling capability of the filter is determined by the $P_c$ of this resonator. The nonlinear model
can also be used when the input and output coupling coefficients are not identical.

IV. EXPERIMENTAL VALIDATION

A. MEMS EVA Tunable Resonator

A MEMS EVA tunable resonator has been fabricated and measured to validate the theoretical and numerical models developed in the previous sections. The design procedure and fabrication techniques are presented in [2]. Fig. 16(a) shows the CAD drawing of the EVA tunable resonator. Fig. 16(b) and (c) shows the fabricated EVA cavity and assembled EVA tunable resonator.

Small-signal S-parameter measurements were taken using an Agilent 8722ES vector network analyzer (VNA). The CPW feedlines of the resonator are shorted by two pieces of copper tapes to achieve weak coupling so that the \( Q_u \) of the resonator can be extracted with higher accuracy. In large-signal measurements, the copper tapes can be removed to achieve stronger coupling (Fig. 16).

The measured resonator has a tuning range of 1.85–2.84 GHz (1.51:1) with less than 140-V actuation voltage. The high actuation voltage required for achieving this tuning range can be supplied by a voltage driver [22], [23]. The critical parameters of the EVA tunable resonator, such as \( Q_u \) and \( g_{pd} \), are extracted from the small-signal measurements. The overall \( Q \) of the resonator is related to \( Q_u \) and \( Q_e \) by (26). In a weakly coupled resonator, the \( Q_e \) is sufficiently large so that the \( 1/Q_e \) term can be neglected and the measured \( Q \) approaches \( Q_u \). The capacitive gap \( g \) is extracted by matching the Ansoft High Frequency...
Structural Simulator (HFSS) model of the resonator to the measured initial resonant frequency [2]. The initial gap of the resonator is 9 μm and the diaphragm deflects 14 μm before pull-in. The actuation gap δ₀ is approximated to be three times the maximum deflection. The spring constant of the diaphragm actuator is calculated by the pull-in voltage

\[ V_{p1} = \sqrt{\frac{8k\delta_0^2}{27\epsilon_0 W^2}}. \]  

Fig. 17 shows the high power measurement setup used to characterize the self-actuation of the EVA tunable resonator. A 43-dB gain power amplifier (Mini-Circuits ZHL-16 W-42+) is used to amplify the frequency sweep signal from the VNA. The output of the amplifier is protected by a circulator with its isolation port terminated with a 50-Ω load. The amplified signal goes through the EVA resonator and is attenuated by a 20-dB attenuator before going back to the VNA. The setup is calibrated with Agilent 85052D 3.5 mm kit to the end of the input and output port cables of the VNA. The insertion loss of the circulator and the attenuator are subtracted from the measured S₂₁.

The measured large-signal frequency responses with several input power levels are shown in Fig. 18. With the current setup, reflection coefficients of the filter cannot be measured and therefore are not presented here. With low RF power (<21 dBm), there is little distortion to the frequency response. As the RF power increases (21 and 28.5 dBm), the S₂₁ exhibits distortion as predicted by the theory and CAD modeling. The onset of instability occurs at 32.1 dBm (1.62 W). As the input power is further increased to 33.5 dBm, a discontinuity in S₂₁ can be observed. The measured results agree very well with ADS simulations. The parameters used in the simulation are listed in Table II.

The intermodulation behavior of the EVA tunable resonators is measured by the two-tone setup shown in Fig. 19. Two Agilent 4433B signal generators were used to generate the two-tone signals, which are then amplified and combined to feed the EVA tunable resonator. The signal is then attenuated before going into an Agilent 4448A Spectrum Analyzer (SA). The intermodulation powers are read from the output spectrum and recorded for several input power levels and frequency separations. Fig. 20 shows an example measured spectrum with an input power of
5 dBm and $\Delta f$ of 10 kHz. Whereas the IM3 is clearly visible, the fifth-order intermodulation product (IM5) is too low to be observed.

The measured and simulated IM3 values with respect to input power $P_i$ (10–25 dBm) with a frequency separation of 20 kHz are shown in Fig. 21. The extracted IIP3 for the resonator is 58.2 dBm. It is to be noted that the mechanical frequency of the diaphragm tuner is in the kilohertz range and therefore the deflection of the diaphragm tuner does not respond to the instantaneous change in RF signal power. In other words, the diaphragm responds only to the RF power envelope change in the kilohertz range. For signals whose envelope change much faster, the tunable filter remains quite linear.

B. MEMS EVA Tunable Filter

A 2% two-pole EVA filter has been designed and fabricated for high-power characterization. The design of the filter follows a similar process described in [4]. Since the fractional bandwidth (FBW) of the tested filter is larger than the FBW of the filter in [4], it is expected that the tested filter will exhibit higher power-handling capability. The nominal dimensions of the EVA resonators used to design the filter are same as in Section IV-A. Fig. 22(a) shows an illustration of the designed two-pole filter.

The fabricated tunable filter is shown in Fig. 22(b) and (c). The measured linear responses of the tunable filter under several bias voltages are shown in Fig. 22(d). The filter is continuously tunable from 2.35 to 3.21 GHz with less than 110-V bias voltage. The measured insertion loss is 1.65–1.42 dB (including the loss of the connectors), which translates to a $Q_o$ of 356–405. The extracted initial gaps for the two resonators are 8.2 and 9.7 $\mu$m, respectively. The input and output transformer ratio is extracted to be 8.47 by matching the ADS simulation with small-signal measurement.

Fig. 23 shows the measured self-actuation characteristics of the filter at 2.4 GHz. The 3-dB bandwidth of the filter is 47.4 MHz (2%). The required bias voltages on the two resonators are 23.4 V and 36.7 V respectively. For input power less than 15 dBm, there is no significant distortion in the frequency response. As the input power increases beyond 15 dBm, the $S_{21}$ bends towards the lower frequency as predicted by the CAD models in Section III. The bifurcation instability of one of the resonators occurs at 23.4 dBm. The measured IIP3 of the filter is 52.1 dBm (Fig. 21). There is a very good agreement between the measurement and simulation using the CAD model. These are the worst case results (0 V bias). No attempt to compensate them with higher bias voltages has been made in these measurements.
V. CONCLUSION

This paper presents a validated complete theoretical framework for estimating the power-handling capabilities of EVA RF MEMS tunable resonators and filters. It has been shown that the frequency-dependent RF voltage inside a resonator must be taken into account when analyzing the nonlinear effects. A practical nonlinear circuit model is also employed for analyzing more complex filter structures. The theory and CAD modeling are validated by power measurements on an MEMS EVA tunable resonator and a medium-power two-pole EVA tunable filter. The measured two-pole 2% EVA tunable filter handles 23.4 dBm (0.22 W) RF power before bifurcation instability occurs. The power handling capabilities of the EVA tunable filter can be increased by either increasing the initial gap or the stiffness of the diaphragm actuator at the expense of increased bias voltage or decreased tuning range. Careful consideration of these parameters is necessary to meet the requirements of specific applications.

REFERENCES


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