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USING SEGMENTED COMPRESSION EXPONENT TO CALCULATE THE EXHAUST TEMPERATURE OF THE SINGLE STAGE HIGH PRESSURE RATIO AIR COMPRESSOR

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ABSTRACT

Today, to calculate the terminal temperature of compression of the single stage high pressure ratio compressor, the traditional method is using the constant process exponent. This calculation method creates a large error in the result compared with the measurement. To eliminate the error the author brings out a calculation method which separates the high compression process into server segments. The new method segments the whole compression process of the cylinder into entropy increment process, isentropic process and entropy decrement process which are all have a low pressure ratio and it is suitable for engineering uses. By using different exponent of process, the calculation result is more closely to the real process. Based on this method, the paper leads out the equations of the compression terminal temperature and the exhaust temperature of the high pressure ratio compressor with considerations of suction, discharge pressure loss, suction temperature rise and discharge temperature drop.

Key Words: Air compressor; Engineering thermodynamics; Thermodynamic process

The single stage high rotating mini air compressor with high pressure ratio cooling system (simply called as “high pressure ratio compressor”) is far behind from the multistage air compressor on the same level in performance. But users prefer the former one much better for its simple structure, flexibility and low price. Because the pressure ratio of the cylinder in this type of compressor is high, when we use the traditional constant exponent of compression (expansion) to do the thermodynamic calculations, the error will be large. This is especially shown on the compression terminal temperature. To minimize the error, designers have to revise the process exponent, but this method will create large errors on items such as the indicated work and etc. In this paper, the author brings out a calculation method which separates the high compression process into server segments. We can think that the high compression process in the cylinder is composed by three processes with low pressure ratio: the entropy increment segment, entropy decrement and isentropic segment. The three working cycles with low pressure ratio have different polytropic index of process and efficiency, so make calculations of them separately will
make the result more closely to the real process than the traditional method. This paper based on the method above and discussed the calculation of the compression terminal temperature in cylinder, and took the suction, discharge loss of pressure, suction temperature rise and discharge temperature drop into account to lead out the discharge temperature equation.

THE REASONS OF WHY SEGMENTED THE CALCULATION OF HIGH COMPRESSION PROCESS

Figure 1 is the pressure volume chart (P—V curve) of the high pressure ratio compressor. In this figure ab——the suction process; bc——the compression process; cd——the discharge process; da——expansion process. To simplify the calculation, the process bc and da can be thought as two polytropic processes who’s exponent of process separately equals to a constant \( m = \text{constant}, \ n = \text{constant} \).

\[
PV^m = \text{Constant} \tag{1}
\]
\[
PV^n = \text{Constant} \tag{2}
\]

In the real process \( m \) and \( n \) are all variables, they are determined by the quantity of heat \( q \) exchanged with environment by the working substance during the process. They are also the functions of crankshaft’s angle \( \varphi \).

\[
m = m(\varphi) = m'(q) \tag{3}
\]
\[
n = n(\varphi) = n'(q) \tag{4}
\]

Because the compression ratio \( (V_0 + V_h)/(V_0 + V_d) \) of the high pressure ratio compressor is large, then the \( X_{bc} \gg X_{ad} \) in the piston stroke \( S_{bc} \). We can say that the exchange of heat between working substance and environment is mainly occurred in \( X_{bc} \). For the value of \( X_{bc}/S_{bc} \) is much larger than the low pressure ratio compressor’s, the changes of transferred heat’s quantity \( \Delta q \) is also much larger than the low pressure ratio compressor. So the index of compression process of the high pressure ratio compressor \( m \) changes much larger than low pressure ratio compressor. Similarly, the expansion ratio \( (V_0 + V_a)/V_0 \) of the high pressure ratio compressor is large, then the \( X_{da} > X_{ab} \) in the piston stroke \( S_{da} \). That is the exchange of heat between working substance and environment is mainly occurred in \( X_{da} \). For the value of \( X_{da}/S_{da} \) is much larger than the low pressure compressor, the changes of transferred heat quantity \( \Delta q \) is also much larger than the low pressure compressor. So the index of expansion process of the high pressure ratio compressor \( n \) changes much larger than low pressure compressor. If we consider the values of \( m \) and \( n \) of the high pressure ratio compressor as constant, that must lead to a large calculation error. If we use the \( m(\varphi), \ n(\varphi) \) as compression and expansion polytropic index of process separately, we can get eq(5) and (6).

\[
PV^{m(\varphi)} = \text{Constant} \tag{5}
\]
\[
PV^{n(\varphi)} = \text{Constant} \tag{6}
\]

Using the two equations above in the thermodynamics calculation of high pressure ratio compressor, we can get the accurate result, but they are not suitable for the
engineering uses. To solve this problem, this paper brings out a segment calculation method, that is along with the three directions of the entropy's change \( ds \) of the working substance \((ds > 0; ds = 0; ds < 0)\), segment the high compression process into entropy increment process, isentropic and entropy decrement process. The three processes are all have low pressure ratio, so the original thermodynamics calculation is changed as the calculations of 3 low pressure ratio processes with different \( m \) and \( n \). This method is also helpful in using lots of experience coefficients of the low pressure ratio compressor.

**THE THREE STAGES OF THE HIGH PRESSURE RATIO PROCESS**

From eq(3) we know that \( m \) is determined by \( q \). In the compression process the value of \( q \) is changeable, \( \Delta q \) should be measured by the entropy changes \( \Delta s \) of the working substance.

\[
\Delta q = T \Delta s \tag{7}
\]

While \( \Delta s > 0 \), entropy is increased and the working substance absorbed heat; when \( \Delta s = 0 \), entropy keeps constant and working substance is in adiabatic state; when \( \Delta s < 0 \), entropy is decreased and working substance discharged heat. From eq(2) and (7) we know that:

\[
m = m^* (T, \Delta s) \tag{8}
\]

This equations indicates that \( m \) is determined by the working substance's \( T \rightarrow s \) relation during the process. Figure 2 is the \( T \rightarrow s \) curve of the high pressure ratio compressor. From the compression line \( bc \) we can analyze the directions of entropy changes during the compression process. That is at the beginning \( \Delta s > 0 \), at the middle \( \Delta s > 0 \) and at the end \( \Delta s > 0 \). Form the thermodynamics we also know that the polytropic indexes of the three processes are \( m_1 > K \), \( m_2 = K \) and \( 1 < m_3 < K \) separately. So based on theories above this paper segments the high pressure ratio process into 3 stages. The first stage is the compression process with entropy increasing, \( m_1 > K \); The second stage is the isentropic compression, \( m_2 = K \); the third stage is the decreasing entropy compression, \( 1 < m_3 < K \). If \( \varepsilon_1 \), \( \varepsilon_2 \) and \( \varepsilon_3 \) are the pressure ratio of the three stages separately, we can get:

\[
\varepsilon_1 \cdot \varepsilon_2 \cdot \varepsilon_3 = \varepsilon \tag{9}
\]

The pressure ratio relations \( \varepsilon_1 : \varepsilon_2 : \varepsilon_3 \) of the three stages depends on the structure of the compressor, the mean piston speed, clearance volume and the transfer heat square etc. To avoid complexity of the problem, just think them as equal pressure ratio.

\[
\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon^{\frac{1}{3}} \tag{10}
\]

Figure 3 is such a \( T \rightarrow s \) curve of the high compression process composed by three stages which obeying the equal pressure ratio relations. In the figure the \( bc \) process with pressure ratio \( \varepsilon \) is divided into \( bc_1 \)—— the entropy increment compression process, \( c_1c_2 \) ——— the isentropic compression process; \( c_2c \) ——— the entropy decrement compression process. \( Tc_1 \), \( Tc_2 \) and \( Tc \) are the compression terminal temperature of the three processes.
Pe is the compression terminal temperature of whole process bc, it can be worked out from Te and Tc2.

THE CALCULATION OF THE COMPRESSION TERMINAL TEMPERATURE
Te WITH HIGH PRESSURE RATIO

The Calculation Of Tc1

Tc1 is the terminal temperature of entropy increment compression process bc1. It can be worked out from the terminal temperature Te of the isentropic process be (Figure 3).

\[ T_{e}^{1} = T_{e}^{1} \frac{K-1}{K} = T_{e}^{1} \frac{K-1}{3K} \]  

(11)

Because the increment of entropy of the working substance, temperature will rise \((T_{c1} - T_{e})\). The indicated work consumed by the compression in bc1 line is larger than in be line, usually we use the adiabatic efficiency \(\eta_{ad-i}\) to compare them.

\[ \eta_{ad-i} = \frac{L_{ad}}{L_{i}} \]  

(12)

In this equation, \(L_{ad}\) — the indicated work of the isentropic compression process be, \(L_{i}\) — the indicated work of the entropy increment process. If we assume the isobarics specific heat as \(C_{p}\), then we can get eq(13) and (14)

\[ L_{ad} = C_{p}(T_{e} - T_{b}) \]  

(13)

\[ L_{i} = C_{p}(T_{c1} - T_{b}) \]  

(14)

Substituting equations (13) and (14) into (12), and considering eq(10), then \(T_{c1}\) is worked out:

\[ T_{c1} = T_{b} (1 + \frac{\frac{K-1}{3K} - 1}{\eta_{ad-i}}) \]  

(15)

The Calculation Of Tc2

Because \(Tc_{j}\) is the initial temperature of the second stage and \(m_{2} = K\), then \(Tc_{2}\) is worked out.

\[ T_{c2} = T_{c1}^{m_{2}} = T_{c1}^{m_{2}} \frac{K-1}{3K} (1 + \frac{\frac{K-1}{3K} - 1}{\eta_{ad-i}}) \]  

(16)

The Calculation Of Te

\(T_{e}\) is the terminal temperature of the entropy decrement compression process \(c_{2}c_{4}\). We can get it from the terminal temperature \(T_{e4}\) \((T_{e4} = T_{c3})\) of the isothermal process \(c_{2}c_{4}\) (Figure 3). The indicated work \(L_{a}\) of the isothermal compression process \(c_{2}c_{4}\) is minimum. While in the \(c_{2}c\) compression the working substance has to discharge heat to environment, so \(L_{a}\) is larger than \(L_{a}\). We often use the isothermal efficiency \(\eta_{is-i}\) to show the relations of them.
\[
\eta_{l_{a-i}} = \frac{L_{i_s}}{L_i}
\]  
(17)

\[L_{i_s} \text{ and } L_i \text{ used in the above equation are defined as:}
\]
\[
L_{i_s} = RT_c \ln \varepsilon_3 = \frac{1}{3} RT_c \ln \varepsilon
\]  
(18)
\[
L_i = C_p (T_c - T_{c1})
\]  
(19)

Substituting equations (18) and (19) into (17), and considering the relations of
\[C_p = \frac{KR}{K-1}\]  
and (16), we can get \(T_c\)
\[
T_c = T_{c2} \frac{K-1}{K} \left(1 + \frac{K-1}{3K} \cdot \ln \varepsilon \right) \left(1 + \frac{\varepsilon^{\frac{3K}{K-1}} - 1}{\eta_{a-d}} \right)
\]  
(20)

**THE CALCULATION OF DISCHARGE TEMPERATURE DROP \(\Delta T_{cd}\)**

The working substance’s temperature is higher than the cylinder wall, it will transfer heat. So there is a temperature drop \(\Delta T_{cd}\) in the discharge process (Figure 2). Using the equation of energy, we can work out \(\Delta T_{cd}\). If the initial energy the working substance has is \(I_c\), the energy of the working substance remaining in the clearance volume at the end of discharge process is \(I_d\), the energy of working substance discharged from the cylinder is \(I_h\) and the total quantity of heat discharged to outside during the whole discharge process is \(Q_w\), then the energy equation is eq(21) (despite the pressure loss).
\[I_d = I_c - I_h - Q_w \]  
(21)

In this equation letter \(I\) means the energy of the fluid working substance, that is the enthalpy.
\[I_d = G_0 C_p T_d \]  
(22)
\[I_c = (G_h + G_0) C_p T_c \]  
(23)
\[I_h = \int_{X_d}^{X_c} C_p T \frac{\pi D^2}{4} \rho dx = \int_{X_d}^{X_c} C_p T \frac{\pi D^2}{4} \frac{P_c}{RT} dx \]  
(24)
\[= C_p \frac{P_c}{RT_c} T_c \frac{\pi D^2}{4} (X_c - X_d) = G_h C_p T_c \]
\[Q_w = h \Delta T \int_{\phi}^{\phi_0} F(\phi) d\phi = \frac{h \Delta T}{6n} \int_{\phi}^{\phi_0} F(\phi) d\phi \]  
(25)
\[F(\phi) = F_0 + \frac{\pi D S}{2} \left[ (1 - \cos \phi) + \frac{\lambda}{4} (1 - \cos 2\phi) \right] \]  
(26)

In these equations, \(G_h, G_0\) —— the discharged mass of working substance, the residual mass in the clearance volume; \(P_c\) —— the beginning pressure of the discharge process; \(T, P, \rho\) —— the temperature, pressure, density of working substance \((P = P_c)\); \(h\) —— the thermal conductivity exponent; \(\Delta T\) —— the mean temperature difference; \(D\) ——cylinder bore; \(S\) —— stroke; \(\lambda\) —— connecting rod-crank radius ratio; \(n\) —— rotational
speed; \( F_0 \), \( F(\phi) \) — the square of the clearance volume, the square of the cylinder wall; 
\( \phi_e (t_c) \), \( \phi_d (t_d) \) — the crank angle (time) when discharge valve is open, the crank angle (time) when discharge valve is close.

Substituting eq(22),(23),(24) and (25) into (21) we can get the result below.

\[
\Delta T_{cd} = T_c - T_d = \frac{h \Delta T}{6nG_0C_p} \int_{\psi_e}^{\psi_d} F(\phi) d\phi
\]  

(27)

THE CALCULATION OF DISCHARGE TEMPERATURE \( T_d \) OF HIGH PRESSURE RATIO COMPRESSOR

From figure 2 we know that the discharge temperature \( T_d \) is \( T_d = T_c - \Delta T_{cd} \).

Substituting eq(20),(27) we can get:

\[
T_d = T_s e^{\frac{K-1}{3K} (1 + \frac{K-1}{3K} \ln e)}(1 + \frac{K-1}{3K} \ln e) - \frac{h \Delta T}{6nG_0C_p} \int_{\psi_e}^{\psi_d} F(\phi) d\phi
\]  

(28)

Because the working substance is heated by the cylinder wall during the suction process, and the pressure loss exist in both suction and discharge processes, these factors should be considered in calculation. If the suction temperature is \( T_s \), the heat exponent is \( \lambda_T \), the relative pressure loss of suction and discharge are \( \delta_s \), \( \delta_d \), and the titular suction, discharge pressure are \( P_s \), \( P_d \) then the \( T_s \) and \( \varepsilon \) in equation (28) will be:

\[
T_b = \frac{T_s}{\lambda_T}
\]

\[
\varepsilon = \frac{P_d (1 + \delta_d)}{P_s (1 - \delta_s)}
\]  

(29)

(30)