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NUMERICAL SIMULATION OF THE PISTON SECONDARY MOTION IN A RECIPROCATING COMPRESSOR

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ABSTRACT

Piston dynamics play a fundamental role in two critical processes related to reciprocating compressors performance. The first is the refrigerant leakage through the radial clearance, which may cause considerable loss in the compressor volumetric efficiency. The second one is the viscous friction associated with the lubricant film; certainly a significant factor in the compressor energy consumption. A detailed analysis of the piston oscillatory motion inside the clearance is also vital for understanding processes related to wear and noise generation. This study presents a mathematical model for ringless piston lubrication. The corresponding computer program can be used to evaluate the piston radial trajectory and the hydrodynamic and viscous forces as a function of the crank angle, given the geometry, the mechanism kinematics, the fluid properties and the force related to the gas compression. The main goal is developing a simulation program to be used as a reliable tool for reciprocating compressors piston design. The results explore the effects of some design parameters and operating conditions on the stability of the piston.

INTRODUCTION

The piston assembly in reciprocating compressors is recognized as a major source of mechanical friction and reduction in volumetric efficiency, due to gas leakage. The piston lubrication problem solution becomes significant considering its effects on performance and reliability as well as the noise generation through several correlated phenomena. A compromise has to be achieved between a radial clearance small enough to prevent gas leakage and large enough to minimize the friction loss. To fully benefit from this balance, any contact between piston and cylinder must be avoided, in order to prevent penalties in terms of wear and noise.

Several works published lately have dealt with lubrication characteristics between piston and cylinder in reciprocating motion. Li et al. (1983) have shown through an analytical model that piston skirt friction significantly increases if the wrist-pin is located in an unfavorable position. The use of analytical models in the solution of this kind of problem has been common, in spite of some cases its validity is somewhat questionable. A significant development of numerical analysis for piston assemblies lubrication has been presented by Zhu et al. (1982, 1983). Those works numerically investigated an automotive piston motion, lubrication and friction in mixed lubrication, considering piston and cylinder surface profiles, waviness, roughness and both thermal and elastic deformations. Results obtained using the simulation program showed that good hydrodynamic lubrication minimizes the possibility of piston impact against the cylinder and reduces the frictional loss. Another important contribution is the conclusion that elastohydrodynamic may be important depending on the forces involved. Gommed and Etsion (1993, 1994) and Etsion and Gommed (1995) presented a mathematical model for analyzing gas lubricated ringless pistons. A number of piston shapes were explored and an improved design was obtained with noncylindrical profiles. Hu et al. (1994) showed that the assumption of axisymmetry may be too idealistic for a real situation, so unless one is sure about the perfect cylindricity of piston and cylinder, even a well developed simulation program may lead to unrealistic results. Nakai et al. (1996) studied the piston-ring assemblies for refrigerating compressors, considering the effect of surface roughness. One important conclusion of this work is that the effect of oil supply is more important than surface condition.
None of the works cited dealt with ringless piston lubrication in reciprocating compressors. In this regard, it is performed a dynamic analysis for the thin lubricant film between piston and cylinder in presence of oscillatory secondary motion. The main goal was to develop a computational tool to optimize the compressor project relative to performance (power dissipated versus leakage), using reliability and noise criteria as restrictions. The trajectory is evaluated by solving the equations of the mechanism dynamics. The program allows to simulate compressor starting and variable speed, since the formulation considers transient operation. Pressure fields into the piston/cylinder clearance are evaluated solving the Reynolds equation by means of the Finite Volume Method in each step of the time evolution. The modeling considers all dimensional characteristics and allows the consideration of a number of geometrical errors that commonly appears in the reciprocating compressor mechanism. Lubricant is considered cavitation-prone. The main results are the piston path itself, the oil film thickness, leakage, power consumption, forces and pressures acting on the piston.

**PROBLEM FORMULATION**

A typical piston-cylinder assembly encountered in small reciprocating compressors is depicted in figure (1).

![Figure 1](image)

*Figure (1) - Compressor mechanism sketch*

The complete problem formulation has been published elsewhere (Prata et al., 1998) and only the main picture will be described here.

Piston location is determined by the top and bottom eccentricities with respect to the cylinder axis; both are function of the time and should be predicted by solving the piston equations of motion. Comparing to the model presented by Prata et al. (1998), another degree of freedom has been added to the piston motion. Besides the motion in a plane parallel to the cylinder axis and perpendicular to the wrist-pin axis, the piston is allowed to move along the wrist-pin axis. This characteristic is important to take into account the possibility of a cylinder axis with a slight angular deviation from the plane of the mechanism (plane $XZ$ in figure 1), which is considered in this work. The piston and connecting rod free body diagrams yelds to the following equations,

$$F_\alpha + F_\beta = mca^2 \left[ \ddot{e}_z - z_{cm}(\ddot{e}_i - \ddot{e}_s)/L \right]$$

(1)
In equations (1) and (2) \( F_h \) and \( M_h \) are the hydrodynamic total force and its momentum, \( F_r \) is the radial force acting on the wrist-pin, \( F_{tr} \) and \( M_{tr} \) are the friction force and its momentum, \( F_{\phi} \) is the force due to the compressed gas, \( m \) is the piston mass and \( J_p \) is the piston momentum of inertia. Quantities \( M_h, M_{tr} \) and \( J_p \) are respective to the wrist-pin axis. For reference to the other variables, see figure (1). Equations (1) and (2) are used to evaluate the piston displacement on the plane \( XZ \). On the plane \( XY \) the piston location is determined assuming a frictionless contact between wrist-pin and connecting rod, thus the integration of the hydrodynamic force projected on the \( Y \) direction forces must vanish. The piston axial movement is determined by the mechanism kinematics. Gas pressures above and below the cylinder can be obtained experimentally or evaluated from a companion program that performs the overall compressor simulation.

Some important assumptions have been made when writing the governing equations. The piston-cylinder clearance is assumed much smaller than the piston radius, thus the pressure radial gradients are very small comparing to the axial ones. The solid parts are stiff enough to assume negligible deformations. Lubricant is considered a newtonian fluid with constant properties; its flow is laminar and with no entrance effects. Considering the above mentioned assumptions, the pressure field can be obtained solving the Reynolds equation,

\[
\frac{\partial}{\partial \theta} \left( h \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial \xi} \left( h \frac{\partial p}{\partial \xi} \right) = -12 \mu R^2 \left( \frac{V_z}{2R} \frac{\partial h}{\partial \xi} - \frac{\partial h}{\partial \theta} \right)
\]

where \( \xi = z/R \), \( \mu \) is the oil viscosity and \( h \) the local oil film thickness. Given the piston top and bottom eccentricities, the piston axial location and the piston-cylinder interface surface profile, \( h \) can be calculated using simple geometric relations. Once the pressure in the oil film is known, one can evaluate all the variables appearing in equations (1) to (4). Basically there are three unknown quantities (piston top and bottom eccentricities and pressure in the oil film) to be evaluated solving equations (1), (2) and (5).

**NUMERICAL METHODOLOGY**

The numerical solution starts prescribing values for the piston eccentricities and radial velocities, which are used to evaluate an initial oil film pressure distribution. Because the periodic characteristic of the problem, the convergence solution should not depend on the initial guess. An implicit formulation is employed here. From the geometrical parameters and the equations for the mechanism kinematics, values of piston location, velocity and acceleration along \( Z \) and connecting rod acceleration are determined for the next time step. In-cylinder gas pressure is then re-evaluated. An iterative process is then needed to determine the piston radial velocity, and it is performed using a Newton-Raphson procedure to find values that satisfies equations (1) and (2). Piston radial position and acceleration are obtained from the radial velocities. The pressure field is determined integrating equation (5) through a finite volume approach (Prata and Ferreira, 1990). Whenever cavitation occur, the oil film pressure is replaced by a gas pressure which is interpolated between the pressures used as boundary conditions, depending on the axial location.

Using a time step corresponding to five degrees of the crankshaft angle, convergence in the Newton-Raphson algorithm is achieved at most in ten iterations. A converged periodic solution for the piston trajectory requires about 300 cycles.

**RESULTS AND DISCUSSION**

The results to be presented were obtained for a typical R134a reciprocating compressor employed in domestic refrigeration. All simulations were performed at ASHRAE LBP check-point conditions.
Figures (2) and (3) depict the piston top and bottom trajectory throughout a cycle for an existing compressor and a modified model with a prototype piston, respectively. Both figures show the results for the piston with and without undercut, which is used to minimize power consumption. It becomes clear that the results with or without undercut are qualitatively the same, in spite of some deviation may lead to different results when optimizing the design. It is also clear that the proposed prototype presents a more stable motion, which also lead to a lower power consumption and oil leakage comparing to the existing model. Table (1) clarifies this comparison.

Table (1) - Operating characteristics for an existing and a prototype piston models considering the piston with or without undercut.

<table>
<thead>
<tr>
<th></th>
<th>with undercut</th>
<th>without undercut</th>
</tr>
</thead>
<tbody>
<tr>
<td>existing compressor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>minimum oil film thickness (μm)</td>
<td>0.52</td>
<td>1.2</td>
</tr>
<tr>
<td>oil mass flow rate (ml/h)</td>
<td>5.6</td>
<td>4.0</td>
</tr>
<tr>
<td>power consumption (W)</td>
<td>5.8</td>
<td>7.7</td>
</tr>
<tr>
<td>prototype</td>
<td></td>
<td></td>
</tr>
<tr>
<td>minimum oil film thickness (μm)</td>
<td>0.38</td>
<td>0.03</td>
</tr>
<tr>
<td>oil mass flow rate (ml/h)</td>
<td>4.5</td>
<td>4.1</td>
</tr>
<tr>
<td>power consumption (W)</td>
<td>5.1</td>
<td>7.2</td>
</tr>
</tbody>
</table>

Table (2) compares the power consumption, oil flow and minimum film thickness for an existing compressor at various angular speeds. In spite of the rotation has a weak influence on the oil mass flow, the power consumption is strongly affected by this factor. Thus, it becomes clear that for variable speed compressors the optimum piston design depends strongly on the crank rotation. Obviously, one should use the piston that lead to better performance at the rotation the compressor operates in normal conditions.

Table (2) - Operating characteristics as a function of crankshaft angular speed

<table>
<thead>
<tr>
<th>rotation (rpm)</th>
<th>1800</th>
<th>2400</th>
<th>3000</th>
<th>3500</th>
<th>4500</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum oil thickness (μm)</td>
<td>0.03</td>
<td>0.22</td>
<td>0.50</td>
<td>0.52</td>
<td>0.73</td>
</tr>
<tr>
<td>net oil mass flow rate (ml/h)</td>
<td>3.2</td>
<td>2.9</td>
<td>5.8</td>
<td>5.6</td>
<td>5.2</td>
</tr>
<tr>
<td>power consumption (W)</td>
<td>1.8</td>
<td>3.2</td>
<td>4.5</td>
<td>5.6</td>
<td>10.2</td>
</tr>
</tbody>
</table>
Figure (4) show the influence of the piston-cylinder clearance on the maximum value of the piston eccentricities throughout the cycle. Figure (5) illustrates the compromise between viscous friction and leakage.

As seen from the figure, the piston becomes more unstable as the clearance increases, which have also been observed by Li et al. (1983), Zhu et al. (1992) and Gommed and Etsion (1994). Smaller values of radial clearance increase the oil film damping which, in turn, tend to stabilize the piston motion, thus improving piston reliability. However, as shown in figure (5), the use of small clearances may imply severe penalties in terms of overall compressor performance.

Figure (6) compares the piston trajectory for a compressor with the cylinder axis perpendicular to the piston wrist-pin axis to the results obtained when an angle (0.005°) is considered.
It becomes clear that geometric errors lead to very different results comparing to an ideal geometry; the power consumption increases about 40% and the maximum piston eccentricity decreases from 0.5μm to almost zero.

CONCLUSIONS

A comprehensive mathematical model for reciprocating compressors ringless piston lubrication was presented, considering the effects of oil cavitation, piston secondary motion and some deviations from idealistic geometry. It has been shown that the piston oscillation in the skirt-to-bore radial clearance due to unbalanced forces significantly affects both power consumption and oil leakage. The analysis incorporated equations for both piston and connecting rod dynamics, as well as the lubrication equation applied to the variable shape oil film.

A number of factors and parameters have not been explored here. Otherwise, the computer program developed based on the model can be used as helpful toll in assessing the influence of the design parameters and operating conditions on the compressor performance and reliability. Further improvements in the present model should incorporate piston and cylinder deformation and roughness effects, the thermal problem, the gas leakage evaluation, improvements in the piston motion along the wrist-pin model and the prediction of the friction resulting from metallic parts contact.

REFERENCES