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Gas pulsations in thin, curved or flat cavities due to multiple mass flow sources

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ABSTRACT

If compressor manifolds are thin compared to the shortest wavelength of interest, the general procedure developed by the authors [1, 2] for the analysis of curved, thin gas cavities can be applied to the analysis of gas pulsations in multi-cylinder compressor head cavities. The procedure for single input single output systems is extended to systems of multi-inputs. Four pole parameters and transfer functions for each input/output set are obtained separately. The pressure response at the output port excited by each input is calculated by multiplying the input volume velocity from each compressor valve with the corresponding transfer functions. The total pressure response is obtained by superposition of the pressure response excited by each input. A rectangular box type compressor head cavity with refrigerant discharged from a two-cylinder compressor is analyzed as an application example.

INTRODUCTION

Four pole parameters are very useful for the analysis of composite acoustic systems and have been widely used in gas pulsations in cavities, sound propagation in porous media, and sound transmission in stiff panels and resistive screens. Basic discussions of the concept and derivation of four poles of some acoustic elements are found in references [3, 4, 5, 6]. A general formulation of four pole parameters for gas pulsations in thin, flat shell or plate like muffler elements, which can utilize limited narrow spaces, has been developed by the authors [1, 2]. In this study, this formulation is applied to cases of multiple inputs following the superposition schemes reported by Singh [7] and Kim [8].

The above procedure can be utilized in the analysis of gas pulsations in compressor or engine manifolds with thin head geometries. Specifically, a flat rectangular compressor head cavity with refrigerant discharged from a two-cylinder compressor is analyzed as an application example. The ability to analyze gas pulsations in compressor head cavities and manifolds is important since pulsations excite structural vibrations and also travel to the condenser where they become a major noise source.

Soedel et al. [9] presented a simulation model for the discharge cavity and plenum chamber pressures of a two cylinder compressor. The discharge system was described as a multi-degree-of-freedom Helmholtz resonator. It was later extended to multi-cylinder compressor manifolds by Soedel and Baum [10, 11], and a four cylinder compressor was analyzed as an application example.

Singh and Soedel [7, 12] derived the four pole parameters of a two-cylinder compressor discharge system by the lumped model approach and investigate the fluid interaction of each discharge plenum. The plenums were connected to each other by a common gas collection cavity. Kim and Soedel [8, 13] extended this work to the analysis of multi-cylinder compressors using a distributed modeling approach, since the lumped model is only valid if the largest gas cavity dimension is small enough compared to the shortest wavelength of interest.

The multi-cylinder compressors investigated in the past generally consist of a discharge plenum attached to each cylinder and a common gas collection cavity connecting all the discharge plenums and discharging the gas through the exhaust pipe to the condenser. Sometimes, this design is unnecessarily complicated. A typical design by one compressor company uses only one flat, thin common gas collection cavity attached to both cylinders. The refrigerant is discharged directly into the common gas cavity from both cylinder valves without passing through any discharge restrictions or plenums. The advantage of this design is easy manufacturing and efficient utilization of narrow spaces. However, at least to the knowledge of the authors, a theoretical two dimensional analysis of these types of compressor manifolds has not been done. The three dimensional analysis by Kim and Soedel [14] can in principle handle these cases also, but while the three dimensional analysis has the advantage that it does not rely on a "thin" wave guide assumption, it is by nature more cumbersome.
ANALYSIS PROCEDURE FOR A MULTI-INPUT GAS CAVITY

The procedure for the analysis of a multi-input gas cavity is introduced in this section. A typical thin multi-input gas cavity is shown in Figure 1.

Four Pole Parameters for Thin Acoustic Cavity Elements

The acoustical system characteristics are described in the distributed parameter format, which allows two dimensional gas pulsations in a curved thin waveguide. The two dimensional analysis is applicable when the thickness of the gas cavity is sufficiently small compared to the shortest wavelength of interest [1, 2]. The volume flow rate \( Q_i \) and acoustic pressure \( P_i \) at each input and \( Q_o, P_o \) at the output in frequency domain can be related as

\[
\begin{bmatrix}
Q_i \\
P_i
\end{bmatrix} =
\begin{bmatrix}
A_i & B_i \\
C_i & D_i
\end{bmatrix}
\begin{bmatrix}
Q_o \\
P_o
\end{bmatrix},
\]  

(1)

where \( A_i, B_i, C_i, D_i \) are the four pole parameters and \( i = 1, 2, 3, \ldots, m \). If the pressure responses at the input points and the output point are available, the four pole parameters can be obtained [1, 8]:

\[
A_i = \frac{f_o(\bar{r}_i, \omega)}{f_i(\bar{r}_o, \omega)},
\]  

(2)

\[
B_i = \frac{1}{f_i(\bar{r}_o, \omega)},
\]  

(3)

\[
C_i = -f_o(\bar{r}_i, \omega) + \frac{f_i(\bar{r}_i, \omega)}{f_i(\bar{r}_o, \omega)} f_o(\bar{r}_o, \omega),
\]  

(4)

\[
D_i = \frac{f_i(\bar{r}_i, \omega)}{f_i(\bar{r}_o, \omega)},
\]  

(5)

where \( \bar{r}_i, \bar{r}_o \) are the input and output positions, respectively.

It is assumed that the output port is connected to an anechoic pipe (in a refrigeration compressor, this is the discharge pipe leading to the condenser) and the system is subjected to a harmonic input point volume flow source [8, 7, 14]. The relation between the output pressure and the volume velocity or, in other words, the impedance at the output port is [4]

\[
Z_{ex} = \frac{P_o}{Q_o} = \frac{\rho c}{S},
\]  

(6)

where \( S \) is the cross section area of the exhaust pipe and \( \rho, c \) the gas density and speed of sound. The assumption is made that \( S \) has to be small compared to the shortest acoustic wavelength of interest.

Therefore, the transfer function or the impedance of the gas cavity subjected to a set of input and output can be formulated from four poles [1, 8]:

\[
Z_i(\omega) = \frac{P_o}{Q_i} = \frac{1}{A_i \frac{S}{\rho c} + B_i}.
\]  

(7)

Analysis Procedure

The properties of the gas discharged into the acoustic cavity have to be identified first. The time series of volume flow rate at each input with the time period \( T_i \) can be either measured from experiment or obtained from computer simulation [7, 15]. Since the volume flow rate is a periodic phenomenon, it can be represented as [16]

\[
q_i(t) = \sum_{n=0}^{N-1} c_n e^{jn\omega_it},
\]  

(8)

where \( N \) is the total number of data sampling points. The Fourier coefficients are

\[
c_n = Q_i(n\omega_i) = \sum_{k=0}^{N-1} q_i(k\Delta t)e^{-j2\pi kn/N},
\]  

(9)
where $\Delta t = \frac{T}{N}$ and $n = 0, 1, 2, \ldots, N - 1$.

Similarly, the pressure response subjected to each input mass pulse can be expressed by a Fourier series in complex form [7]:

$$p_i(t) = P_{nom} + \sum_{n=1}^{N-1} c_n e^{jn\omega_i t}, \quad (10)$$

where

$$P_{nom} = P_{mean} + P_i(0) \quad (11)$$

is the nominal discharge pressure, which is determined by the valve design. Since $P_{mean}$ is the mean pressure which is generally much larger than the acoustic pressure fluctuations and independent of the excitation, it is not important in this investigation. When pressure is mentioned here, it means only the acoustical pressure. $P_i(0)$ is the acoustic pressure at frequency $\omega = 0$.

The pressure response at the output port subjected to each input volume flow rate can be solved by multiplying the volume flow rate by the corresponding impedance:

$$P_o(n\omega_i) = Z_i(n\omega_i)Q_i(n\omega_i). \quad (12)$$

Using the inverse Fourier transformation, the pressure response data in the time domain is obtained in discrete form [16]:

$$p_o(k\Delta t) = \sum_{n=0}^{N-1} P_o(n\omega_i)e^{j2\pi kn/N}, \quad (13)$$

where $k = 0, 1, 2, \ldots, N - 1$.

If the pumping frequency of each source is the same, the pressure response in the frequency domain becomes

$$P_o(n\omega) = [ Z_1(n\omega) \ Z_2(n\omega) \ \ldots \ \ Z_m(n\omega) ] \begin{bmatrix} Q_1(n\omega) \\ Q_2(n\omega) \\ \ldots \\ Q_m(n\omega) \end{bmatrix}. \quad (14)$$

The total pressure response in the time domain can be obtained by an inverse Fourier transformation.

**SIMULATION OF A RECTANGULAR BOX HEAD CAVITY CONNECTED TO A TWO-CYLINDER COMPRESSOR.**

In this section, a two-cylinder compressor with a rectangular box discharge cavity as shown in Figure 2 will be analyzed as an example. For this specific case, the length of the cavity is $a = 0.16m$, width $b = 0.06m$, and uniform thickness $h = 0.015m$. The refrigerant gas properties are $c = 162.9m/s$ and $\rho = 6.04Kg/m^3$. Assume the highest frequency of interest is 2000Hz, the shortest wavelength is $\lambda = c/f = 0.0815m \gg 0.015m$. Therefore, the two dimensional theory developed in references [1, 2] can be utilized in this investigation.

As stated in the previous section, the time series of the volume flow rate discharged from the valve can be measured by experiment or simulated by taking the valve dynamics into account. However, as a practical approximation it is reasonable to simply assume a square wave [15], as shown in Figure 3. The two pistons operate out of phase with respect to each other, which is usually the case for most two-cylinder compressors, at a pumping frequency of 3600rpm or 60Hz. The period of valve opening is approximately $T_d = T/6$, as shown in Figure 3. The stroke in this case is $H = 0.03m$ and the diameter of the cylinder is $D = 0.025m$. The area under the square wave of Figure 3 is approximately the volume discharged per stroke and cylinder. For this example, the maximum amplitude of the square wave is $Q_{max} = V_s/T_d = 0.0053m^3/s$, where $V_s$ is the stroke volume.

The Fourier transformation of the volume flow rate from the time domain to the frequency domain has to be performed first. The sampling frequency (Nyquist rate) has to be at least twice the lowest frequency of interest to prevent signal aliasing [17], i.e., above 4000Hz in this case. A radix-2 fast Fourier transform (FFT) algorithm [18] is adopted for use here. 128 sampling points per cycle are used for this case. The FFT coefficients for both $Q_1(n\omega)$ and $Q_2(n\omega)$ are shown in Figures 4 (a) and (b).

The transfer function or impedance in equation 6 for the rectangular box of uniform thickness case has been solved and reported in reference [1]. It is assumed that the valves can be modeled as point sources. This
is valid if the effective valve port diameters are significantly less than the shortest wavelength of interest. If this is not the case, then the valve flow has to be modeled as a distributed source. For the case of valve locations at \( r_1 = (0.04, 0.03) \) and \( r_2 = (0.12, 0.03) \) and the output point at \( r_o = (0.16, 0.03) \), the transfer functions for \( Z_1(nw) \) and \( Z_2(nw) \) are illustrated in Figures 5 (a) and (b). The cross sectional area of the exhaust pipe is \( S = 0.000113 \text{m}^2 \).

The pressure responses from both inputs are calculated separately by equation 12 and illustrated in Figures 6 (a) and (b), where \( P_{10}(nw) \) is the response excited by input 1 and \( P_{20}(nw) \) is the response to input 2. Using the inverse fast Fourier transform (IFFT), the pressure responses in the frequency domain can be transformed back to the time domain. Figure 7 (a) shows the pressure responses \( p_{10}(t) \) and \( p_{20}(t) \) in the time domain. By simple superposition, the total pressure response is shown in Figure 7 (b). The results are typical for what is measured in two cylinder compressors with this shape of head cavity [19]. A further discussion and experimental verification of this model will be presented in the authors' companion paper [20].

**CONCLUSIONS**

The procedure for the analysis of thin, shell or plate like gas cavities governed by 2-D wave equations [1, 2] was extended to multi-input systems. An example of gas pulsations inside the thin head cavities of a two-cylinder compressor was investigated.

For the analysis of compressor head cavity gas pulsations, each input volume velocity, which can be measured by experiment or simulated by including a valve dynamics model, was transformed from the time domain to the frequency domain by FFT. Four pole parameters and transfer functions for each set of input/output were calculated independently following the procedure in references [1, 2]. The pressure response due to each input excitation was obtained by multiplying the respective transfer functions and the input volume velocities. An IFFT algorithm was used to transform the acoustical pressure from the frequency domain back to the time domain. The total response was obtained by simple superposition of both pressure responses.

**References**


Figure 1: A general thin multi-input gas cavity.


Figure 2: Side view of a two-cylinder compressor.

Figure 3: Time series of the two volume flow rates discharged from the compressor to the head cavity.

Figure 4: FFT coefficients for (a) $Q_1(n\omega)$ and (b) $Q_2(n\omega)$. 
Figure 5: Transfer function for (a) $Z_1(n\omega)$ and (b) $Z_2(n\omega)$.

Figure 6: Pressure responses of (a) $P_{o1}(n\omega)$ excited by input 1 and (b) $P_{o2}(n\omega)$ by input 2.

Figure 7: (a) Pressure responses in time domain, $p_{o1}(t)$; $p_{o2}(t)$. (b) The total acoustical pressure response $p_o(t)$. 805