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SURGING IN COIL SPRINGS

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ABSTRACT

One potentially important consideration in compressor noise control is the effect of surging in coil springs. While studies on spring surging have been pursued in the past [3] this study employs the receptance technique to analyze a mechanical system characterized by spring surging.

1. INTRODUCTION

The objective of analyzing coil spring surging is in part to understand its contribution to the response of a larger system. The usual but sometimes not justifiable hypothesis is that springs do not contribute resonances of their own to a composite system. The system was considerably simplified to capture the essence of the spring surge problem. The results illustrate phenomena which are observed by practical engineers when they try to achieve noise control by changing spring rates.

For a real compressor, say a refrigeration compressor supported inside a hermetic shell by three mounting springs which may deflect each in three directions, the receptance formulation is only a little more complicated.

The elements of the approach include determining the equation of motion for a coil spring as a continuous system, the receptances for such a spring, and the total system receptance expression, for three sub-systems B, C and D, C being the surging spring. Once the foundation formulas are obtained, a numerical analysis of a typical system will be performed. This isolation involves the parameters of internal spring damping rate and spring stiffness. The final section will be devoted to the application of the receptance method to an idealized compressor shell.

2. SYSTEM ACCOUNTING FOR SPRING SURGING EFFECTS

In addition to the coil spring of mass M, stiffness K, and internal damping rate C, designated sub-system C, the system is divided into two other sub-systems, B and D as shown in Figure 1. B is an extremely simplified model of the compressor body and D is an extremely simplified model of one mode of a compressor shell. Damping of sub-system C occurs in the spring itself; in other words, there is no ‘external’ damper- the damping of the coil spring occurs via material damping or by means of a plastic sleeve stretched around the spring. This will allow for a continuous damping effect across the length L, rather than a net damping effect at the two end points of the spring.

2.1 The Sub-System B and D Receptances

In general, the receptance of a system is simply the ratio of harmonic displacement at one point to a harmonic force at another point. See references [1,2] for receptance definitions. The receptances are, for system B and D,

\[ \beta_{11} = \beta_{22} = \beta_{12} = \frac{1}{(k_1 - m_1 \omega^2) + j\omega B}, \; \delta_{33} = \delta_{44} = \delta_{43} = \frac{1}{(k_2 - m_2 \omega^2) + j\omega D} \]  

2.2. Derivation of Sub-System C Receptance

A coil spring has mass M, length L, stiffness K, and an internal damping coefficient C. The displacement along the x direction is u(x,t). An infinitesimal element dx from the spring length is shown in Figure 2. Thus, the force F created in the spring is related to deflection by
\[ F = KL \frac{\partial u}{\partial x}. \]  

(2)

Determining a constant mass per unit length \( m' = \frac{dm}{dx} = \frac{M}{L} \), the element \( dx \) is acted on by forces shown in Figure 3. A force balance and Newton’s 2nd law gives

\[ \frac{\partial^2 u}{\partial t^2} + \frac{C}{m'} \frac{\partial u}{\partial t} = \frac{KL}{m'} \frac{\partial^2 u}{\partial x^2}. \]  

(3)

The displacement solution is of the general form

\[ u(x,t) = (A_1 e^{-j\kappa x} + B_1 e^{j\kappa x}) e^{j\Omega t}. \]  

(4)

Applying this to the equation of motion yields

\[ \kappa_1 = \sqrt{\frac{m'\omega^2 - Cj\omega}{c^2 m'}}, \quad c^2 = \frac{KL}{m'}. \]  

(5)

Using this general expression for the equation of motion of the coil spring, the two boundary condition of Figure 4 are introduced and the unknown constants \( A_1 \) and \( B_1 \) are solved for. When \( A_1 \) and \( B_1 \) are known, the receptances at the left hand endpoint \( \gamma_{22} \) and \( \gamma_{32} \) are evaluated:

\[ \gamma_{22} = \frac{u(0,t)}{F_2 e^{j\Omega t}} = \frac{e^{j\kappa_1 L} + e^{-j\kappa_1 L}}{j\kappa_1 KL(e^{j\kappa_1 L} - e^{-j\kappa_1 L})}, \quad \gamma_{32} = \frac{u(L,t)}{F_2 e^{j\Omega t}} = \frac{2}{j\kappa_1 KL(e^{j\kappa_1 L} - e^{-j\kappa_1 L})}. \]  

(6, 7)

Similarly, receptances \( \gamma_{23} \) and \( \gamma_{33} \) are evaluated. The boundary conditions, however, are reversed and are shown in Figure 5.

\[ \gamma_{23} = \frac{u(0,t)}{F_3 e^{j\Omega t}} = \frac{2}{j\kappa_1 KL(e^{j\kappa_1 L} - e^{-j\kappa_1 L})}, \quad \gamma_{33} = \frac{u(L,t)}{F_3 e^{j\Omega t}} = \frac{e^{j\kappa_1 L} + e^{-j\kappa_1 L}}{j\kappa_1 KL(e^{j\kappa_1 L} - e^{-j\kappa_1 L})}. \]  

(8, 9)

Note the symmetry of the sub-system C receptances: \( \gamma_{22} = \gamma_{33} \), \( \gamma_{32} = \gamma_{23} \).

2.3. Receptance Model of System

The sub-systems are generalized as block diagrams as shown in Figure 6. To obtain the system A (the total system) receptances it is necessary to break the system between each of the arrows and generalize the force and displacements for each sub-system. This is shown in Figure 7. Setting up displacement expressions and following the procedure outlined in references [1,2] yields

\[ \alpha_{31} = \frac{\beta_{22} \gamma_{23} \delta_{33}}{(\gamma_{22} + \beta_{22})(\delta_{33} + \gamma_{33}) - \gamma_{23}}. \]  

(10)

Because of damping in some or all parts of the system, \( \alpha_{31} \) is a complex number. To account for this, the magnitude and phase of the receptance will be considered when response behavior is analyzed.

3. NUMERICAL RESULTS AND DISCUSSION

3.1. The Effect of Surging on the Response

The first observation to be made is the noticeable difference in a typical system response (receptance \( \alpha_{31} \)) when spring surging is considered, as shown in Figure 8. The natural frequencies of sub-systems B and D were selected to be 6283 and 2000 rad/s, respectively. The spring dimensions and properties were selected to be typical for small heat pump compressors.

Several observations can be made. First, notice that the external system spikes (due to the resonances of systems B and D) at \( \omega = 6283 \) rad/s and \( \omega = 2000 \) rad/s are magnified when surging occurs. Each of the dashed spikes that do not occur at \( \omega = 2000 \) or 6283 rad/s represent the surge frequencies.
In addition to the overall increase at the system resonance, note the near coincidence effect of a spring surge resonance with the sub-system D natural frequency at 2000 rad/s. The spring surge peak is increased as a result of this near coincidence. Clearly, surge natural frequency intervals are a major consideration in the design of such a system. The separation between the coil spring resonance will clearly dictate whether coincidence is likely to occur and thus increase system response. Furthermore, it may be required to operate at a specific frequency or over a range of given frequencies and in these cases, spring surging would reduce the operable ranges. If spring surging can be ignored, then a large range between the external sub-system natural frequencies exists in which to operate. However, if spring surging cannot be ignored, one must then consider how to appropriately space the intervals so as to avoid detrimental amplifications.

A final note about the system is that spring natural frequency harmonics will be present at the higher frequencies as well (i.e. those to the right of 6283 rad/s). On the other hand, when surging is ignored the response behavior naturally decreases at these higher frequencies (a false sense of security is created).

3.2. Use of Coil Spring Damping to Minimize Surge Influence

Of interest here is the extent to which internal spring damping can reduce system response. A plastic sleeve around the coil spring might be one effective way to create internal damping. A typical response for various amounts of damping is shown in Figure 9.

It is noted that damping of the coil spring appears to be a useful tool in reducing the contribution of the coil spring to the overall system receptance. However, while it dampens the coil spring resonances, it does not seem to make a significant contribution to damping the external sub-system controlled natural frequencies.

3.3. Variation of Coil Spring Rate, K

First consider a lower value of K. The classical hypothesis to test here is whether a lower spring rate will always reduce the response of system A. If one ignores spring surging, a case may be made that the most desirable K for response isolation is the lowest one possible. However, in the case of surging, it will be shown that this is not necessarily always true.

Consider the value of a typical spring stiffness. In Figure 10, the solid line demonstrates that the intervals between surge natural frequencies are smaller for K reduced by 17% than for the original larger spring rate values (superimposed as the dashed curve). Notice that the surge resonance which was at approximately 7000 rad/s before has now moved to the left and exhibits a near coincidence behavior of the coil spring natural frequency with the natural frequency of sub-system B at 6283 rad/s. The peak has been split into two, each part of which is higher than the original system A receptance. This is one case which demonstrates that a lower K is not necessarily advantageous.

Next, consider an even much lower spring stiffness (K reduced by 33% from original value). In Figure 11, the interval between spring surges is much smaller than before, and coincidence is a great deal more likely. Note that a second surge peak of the solid line now coincides with the system A resonance at 2000 rad/s. Thus with a much smaller K, coincidence with other sub-system natural frequencies occurs more frequently and refutes the argument that a softer spring rate invariably reduces overall system response. (But there is a trend of a lower average off resonance response with decreasing spring rate).

Finally consider a much higher sub-system C spring rate: K is increased by a factor of four. This curve is characterized by a much higher mean receptance level, higher peaks (which are receptive features), but much wider surge natural frequency intervals. As a result of this, coincidence is a great deal less likely. The wider spring resonance intervals can be advantageous if the application calls for a system to operate at a specific frequency or in a specific frequency range. One would simply have to consider what design spring rate K would have the lowest mean receptance for the given operation range. The comparison of this larger spring rate (solid line) with the default value (dashed line) is given in Figure 12.

As one can observe, the receptance technique is a very appropriate method for understanding how spring surging contributes to the overall response of a system. An analysis such as this demonstrate its usefulness when a detailed analysis of spring rate design is important. The conclusion drawn is that a lower coil spring rate does create a somewhat lower mean system response, but coincidence is more likely to happen due to the tighter intervals of the spring
resonances. Thus, quantifying spring rate is ultimately an issue of the application and the range of operating frequencies.

4. COMPRESSOR SHELL RECEPTANCE APPLICATION

The preceding system analysis has demonstrated the ease with which a multiple degree of freedom system can be studied. Clearly the example given has been rather basic in nature, yet it has effectively illustrated how the receptance method can be applied to a system which is comprised of several sub-systems of differing design.

Because this study employs the receptance technique, one of its major advantages is that a wide variety of sub-systems can be applied to the general expression for $\alpha_{31}$. For example, one might wish to substitute a more complex sub-system into system A in place of sub-system D. The only requirement beyond what has previously been formulated to make this substitution possible is that the receptance for sub-system D must be formulated or measured. The example that will be investigated here is a compressor shell.

4.1. Compressor Shell Receptance

As mentioned, the system A receptance expression will remain unchanged. In order to substitute a compressor shell into system A, sub-system D must be replaced by a shell fixed to the spring of sub-system C. In this example, a half shell model, simply supported all around, is used.

The numerical values chosen for the system parameters roughly approximate those found in a typical compressor system: length, $L = 0.400$ m; radius, $a = 0.200$ m; thickness, $h = 0.003$ m; and subtended angle, $\alpha' = \pi$ radians (semicircular). The material is steel. Figure 13 illustrates the compressor half-shell model as it joins with sub-system C:

According to reference [1] the receptance for a half-shell at its center is given by the double sum expression

$$\delta_{33} = \frac{4}{phLa\alpha} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ \frac{1}{\omega_{mn}^2} - \frac{\sin^2 \left( \frac{m\pi x^*}{L} \right)}{\sin^2 \left( \frac{n\pi \theta^*}{\alpha} \right)} \right], \quad (11)$$

where, at location 3, $(x^*, \theta^*) = (L/2, \alpha/2) = (0.100, \pi/2)$. The natural frequencies for the $(m,n)$ mode are given by equation (6.12.3), in reference (1).

Notice that the lowest natural frequency occurs in the $m=1, n=4$ mode and has a value of $\omega_{14} = 3939$ rad/s. It is also important to notice that for this particular half shell example, there are 12 natural frequencies below 10000 rad/s. Only these 12 will contribute to the system A response for the frequency range considered in this paper (0 to 10000 rad/s). The $\delta_{33}$ receptance is evaluated using (10) and is plotted for this range in Figure 14.

Next, this modified sub-system D receptance is substituted into the expression for the system A cross receptance $\alpha_{31}$, with all other receptances remaining the same, to yield the composite response plot of Figure 15. The response resonances are labeled C,B, or D if they are primarily due to resonances of sub-systems C,B or D, respectively. The difference here is that instead of having only one system D resonance as before, the shell introduces numerous resonances, each of which can be in coincidence or near coincidence with a surge frequency of the spring. Considered mode by mode, however, the system will behave similarly to the simple cases discussed before.

5. CONCLUSION

The receptance method analysis has demonstrated the following: (1) Spring surging is often significant; it cannot categorically be neglected. (2) Internal damping of the surging coil spring is effective in damping surge resonance. (3) Surge frequencies should be detuned from other system natural frequencies. (4) Lower coil spring rates reduce the average system response, but make coincidence more likely. (5) Higher coil spring rates may become desirable if the driving frequency operation range is between surge peaks, provided the driving frequency is not coincident with other system natural frequencies.
Here, the situation to be analyzed was greatly simplified. However, it gave useful answers which explain what can be seen in engineering practice. In the future, studies involving more than one spring and surge behavior of the coil springs in more than the axial direction should be undertaken.

REFERENCES

Fig. 8. Surging and nonsurging system response.

Fig. 9. Effect of spring damping on surging.

Fig. 10. System A response when K is reduced by 17% (solid line).

Fig. 11. System A response when K is reduced by 33% (solid line).

Fig. 12. System A response when K is increased by a factor of four.
Fig. 13. System D is a shell.

Fig. 14. Receptance $\delta_{33}$ of shell.

Fig. 15. System A response when system D is a shell.