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OPTIMIZATION OF DIMENSIONAL PARAMETERS
OF SCROLL COMPRESSOR
GEOMETRIC MODEL WITH ARBITRARY REAL NUMBER OF TURNS

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ABSTRACT

This paper focuses on the geometric theory of scroll compressor, which includes the determination of the discharge angle, the calculation of the volume in different compression stages, the axial gas force under the condition that the number of scroll turns equals an arbitrary real number.

NOMENCLATURE

φ : expanding angle of the involute (rad)
α : initial angle of the involute (rad)
θ : orbiting angle (rad)
a : radius of basic circle of involute (m)
P : pressure of gas (pa)
m : adiabatic exponent
V : volume (m³)
h : height of scroll (m)
Ror : eccentricity of the crank (m)
Pp : pitch of scroll (m)
F : gas force (N)
A : area (m²)
M : scroll turns
X, Y : coordinate (m, m)

INTRODUCTION

The scroll compressor is a new type of positive displacement compressor. It has
the advantages of high efficiency, light vibration and low noise, small size and light weight. It has the great potentialities and prospect of a wide application. Geometric theory is the basis on which the mathematical model for optimization of dimensional parameters of the scroll compressor is founded. The theory of the number of scroll turns equaling an integer are presented in many references [1], [2], but in practice, the number of the scroll turns often does not equal an integer. In this paper the geometric model for the scroll of any real number of turns is studied.

GEOMETRIC THEORY

1. Calculation of Discharge Angle

Discharge angle is the rotational angle of the main shaft where the compression pocket begins to connect with the discharge pocket. It affects not only the volume of the discharge chamber but also the present number of compression chambers. Fig. 1.a and Fig. 1-b show the cutter circle’s interference with the orbiting scroll and the fixed scroll respectively, and cutter circle is represented by dashed lines. As shown in Fig. 1.a, Q is the intersection of the cutter circle and the external profile of the orbiting scroll. At this time it is the mating point of orbiting scroll and the fixed scroll and the rotational angle of the main shaft is defined as \( \theta_Q \). If the orbiting angle \( \theta > \theta_Q \), the mating scrolls will separate from each other at the innermost mating point and thus the compressed gas will push into the discharge chamber because the involute before point Q is cut away by the cutter. The expanding angle of this interference point \( \phi_Q \) must satisfy the following equation:

\[
\phi_Q^2 + 2\phi_Q \sin(\phi_Q - \alpha) + 2\cos(\phi_Q - \alpha) = (\pi - \alpha)^2 - 2
\]

which could be solved with the digital method. Therefore, \( \theta_Q \) can be obtained by

\[
\theta_Q = \frac{3}{2} - \phi_Q + \alpha
\]

Fig. 1.b shows the intersection P of the internal profile of the fixed scroll and the cutter circle. Similarly, \( \theta_P \) is defined as in previous analysis. If \( \theta > \theta_P \), the mating scrolls also may separate from each other at the innermost mating point because of the interaction between the involute and the cutter circle. The expanding angle of the mating point P, \( \phi_P \), is restricted by the equation:

\[
\phi_P^2 + 2\phi_P \sin(\phi_P + \alpha) + 2\cos(\phi_P + \alpha) = (\pi - \alpha)^2 - 2
\]

which can be solved with the same method used above, and also \( \theta_P \) is obtained by

\[
\theta_P = \frac{5}{2} - \phi_P - \alpha
\]

The discharge angle \( \theta_d \) should be the minimum of \( \theta_Q \) and \( \theta_P \), that is:

\[
\theta_d = \min(\theta_Q, \theta_P)
\]

2. Calculation of The Volumes of Different Chambers:
The working process of the compression pocket can be divided into three stages: the suction stage, compression stage and discharge stage. The analysis of the volume of the different chambers, which change with the rotational angle $\theta$, is the basis of the dynamic computation. These volumes are determined by the dimensional parameters such as the number of scroll turns, scroll height, scroll thickness, and so on.

i) Compression Volume.

As shown in Fig. 2, B and C are two engagement points on the internal profile of the fixed scroll, while B' and C' are on the external profile of the free scroll with their involute angles noted as $\phi_B$, $\phi_C$, $\phi_{B'}$ and $\phi_{C'}$ respectively. $S_1$ is the area that is formed by the tangent TC when it sweeps from C to B (see Fig. 3) and can be obtained by

$$S_1 = \frac{1}{2} \int_{\phi_B}^{\phi_C} a^2 \phi^2 d\phi = \frac{1}{6} a^2 (\phi_B^3 - \phi_C^3)$$

Similarly, the area that is formed by the tangent $T'C'$ when it sweeps from $C'$ to $B'$ (see Fig. 3) and can be obtained by

$$S_2 = \frac{1}{2} \int_{\phi_B}^{\phi_C} a^2 \phi^2 d\phi = \frac{1}{6} a^2 (\phi_{B'}^3 - \phi_{C'}^3)$$

Therefore, the volume of the compression chamber is given by

$$V_N = 2h(S_1 - S_2)$$

which can be expanded as

$$V_N = \pi P_\epsilon (P_e - 2t)(2N - 1 - \theta_\pi/h)h \quad \text{when} \quad 0 < \theta < \theta_d$$

$$V_N = \pi P_\epsilon (P_e - 2t)(2N - \theta_\pi/h)h \quad \text{when} \quad \theta_d < \theta < 2\pi$$

where $N$ stands for the Nth compression chamber counting from the discharge chamber.

ii) Discharge Volume.

As shown in Fig. 4, the shaded area is the discharge pocket, which can be calculated by

$$V_d(\theta) = \frac{1}{3} a^2 h[(\frac{5}{2} \pi - \alpha - \theta)^3 - (\frac{3}{2} \pi - \alpha - \theta)^3] - 2a^2 \alpha (\frac{3}{2} \pi - \theta)^2 - \frac{2}{3} a^2 h \alpha^3 - sh$$

$0 < \theta < \theta_d$

$$V_d(\theta) = \frac{1}{3} a^2 h[(\frac{9}{2} \pi - \alpha - \theta)^3 - (\frac{7}{2} \pi - \alpha - \theta)^3] - 2a^2 \alpha (\frac{7}{2} \pi - \theta)^2 - \frac{2}{3} a^2 h \alpha^3 - sh$$

$\theta_d < \theta < 2\pi$

where

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iii) Suction Volume.

In the previous geometric analysis of the scroll compressor, attention was primarily directed to the compression volume and discharge volume because the pressure in the suction chamber was considered balanced by the suction pressure. With the development of the study of the scroll compressor, some new technologies have been adopted such as the back-pressure mechanism. The effect of back-pressure is that the concept of the balance is changed. Thus, it is necessary to analyze the volume of the suction chamber.

The whole working volume, including the suction chamber, compression chamber and discharge chamber is constant, while the rotational angle \( \theta \) changes from 0 to \( 2\pi \) as shown in Fig. 5. If the constant volume is denoted as \( V_c \), the suction volume \( V_s \) can be obtained:

\[
V_s = V_c - \sum_{N=1}^{n} V_N - V_d
\]

where \( n \) will be analysed in the following section.

3. Analysis of Axial Gas Force:

The pressure in different chambers can be obtained:

\[
P_s \quad \text{in the suction chamber}
\]

\[
P_n = P_s \left[ \frac{V_h}{V_N(\theta)} \right]^m \quad \text{in the compression chamber}
\]

\[
P_d \quad \text{in the discharge chamber}
\]

where \( V_h \) is the displacement volume and can be obtained by \( V_h = V_n(\theta) \), and \( \theta \) is a constant rotational angle of the scroll compressor and can be evaluated by \( \theta = [M-\text{int}(M)] \times 2\pi \). Therefore, the axial gas force is:

\[
F_a = \frac{1}{h} \left\{ \sum_{N=1}^{n} P_s \left[ \frac{V_h}{V_N(\theta)} \right]^m V_N + P_d V_d + P_s V_s \right\} - F_b
\]

where \( F_b \) is caused by back-pressure and can be calculated according to different back-pressure mechanisms. Otherwise the meaning of \( n \) is the same as that in the previous section and can be obtained by the following equations:
if \( \theta_d > \theta_t \), \[ n = \begin{cases} \int(M - \frac{1}{2}) & \theta_d > \theta > \theta_t \, \quad 2\pi > \theta > \theta_d \\ \int(M - \frac{3}{2}) & \theta_d < \theta < \theta_t \, \quad 0 < \theta < \theta_d \, \quad \theta_t < \theta < 2\pi_d \end{cases} \]

if \( \theta_d < \theta_t \)

CONCLUSION

This paper analyses and gives the formulas of the discharge angle, the volumes of different compression pockets and the axial gas force, which are essential to the geometric theory for the scroll compressor with arbitrary real number turns. It is not inclusive for geometric theory because the analysis of other parts are similar to the scroll compressor with integer turns and are presented in many references.

REFERENCES


Fig. 1 determination of the discharge angle
Fig. 2 compression volume

Fig. 3 area s1 and s2

Fig. 4 discharge volume

Fig. 5 the whole working volume