1996

Investigation of a Multistage Scroll Compressor With Oil-injection

Z. Qu  
Xi'an Jiaotong University

A. B. Tramschek  
University of Strathclyde

Follow this and additional works at: http://docs.lib.purdue.edu/icec

http://docs.lib.purdue.edu/icec/1165
INVESTIGATION OF A MULTISTAGE SCROLL COMPRESSOR WITH OIL-INJECTION

Qu Zong chang
Associate Professor
Compressor Engineering Research Centre
Xi'an Jiaotong University, China

A.B. Tramschek
Senior Lecturer
Department of Mechanical Engineering
University of Strathclyde, Glasgow, UK

ABSTRACT

The working process of a multistage scroll compressor with oil injection is analysed and discussed. A mathematical model of the working process and an optimisation procedure are established. Experimental results obtained from a 2 stage oil injected scroll compressor are used to demonstrate the correctness of the model.

INTRODUCTION

New types of scroll compressor have been studied in the compressor field by many research workers. Because they offer many advantages over other types of compressor, scroll compressors occupy a position of major importance in the freezer and air conditioning world. Their use in pneumatic handling systems is however limited. The main reason against their use is that at the higher pressure ratios required, the internal leakage increases considerably and causes a marked decrease in volumetric efficiency. This is because the compression process relies on a good seal between the fixed and moving scrolls in the compressor. Where specific manufacturing tolerances are required a gap must exist. Additionally heat transfer within the scroll compressor is poor and the higher working temperature causes the lubricating oil to deteriorate and leads to a decrease in the working life of the moving parts.

For the above reasons, more efficient cooling measures have to be used with high pressure scroll compressors. Multistage stage compression and oil injection are thus used. If this procedure is adopted then a key question becomes, what are the optimum stage pressure ratios? This question poses a key problem in the design of multistage scroll compressor systems.

The paper presents a mathematical model and an optimisation technique to determine the optimum internal pressure ratios for a multistage oil injected scroll compressor.

MATHEMATICAL MODEL

1. General considerations

The optimisation of the internal pressure ratio for a multistage scroll compressor is usually constrained by the fact that the free air delivery is required to be constant. In a multistage
machine this means that optimisation of the internal pressure ratios leads to the need to establish the moment of discharge when considering the relative movement of the fixed and moving scrolls. The paper chooses the specific work (i.e. the work per unit of volume displacement) as an objective function.

Consider the \( i \)th stage of a multistage compressor with oil injection. The work input for the stage can be calculated from

\[
W_i = W_i^\prime + W_i^\prime \prime
\]

where  
- \( W_i \) = the total work input to the stage  
- \( W_i^\prime \) = the indicated work for the stage  
- \( W_i^\prime \prime \) = the work associated with all the energy losses

\[
W_i = \frac{m_i}{m_i - 1} P_i V_i (\gamma_i \frac{m_i}{m_i - 1} - 1) + W_i^\prime \quad (1)
\]

\[
\gamma_i = \frac{P_i}{P_1} = \text{the stage pressure ratio}
\]

The above assumes that the suction temperature is the same for all stages (i.e. perfect intercooling).

If the polytropic index \( \gamma_i \) is assumed to be same of each stage of a compressor,

\[
m_i = m_2 = \ldots m_i = m_{01}
\]

If \( W_i \) includes all the energy loses, it can be calculated respectively for the following two cases:

1. The internal pressure ratio equal to the external pressure ratio

\[
W_i = P_i V_i (\frac{k}{k - 1} \frac{m_{in}}{m_{in} - 1} (\gamma_i^{\frac{m_i}{m_i - 1}} - 1)) \quad (2)
\]

2. The internal pressure ratio not equal to the external pressure ratio

\[
W_i = P_i V_i (\frac{k}{k - 1} \frac{m_{in}}{m_{in} - 1} (\gamma_i^{\frac{m_i}{m_i - 1}} - 1) + W_i^\prime \prime) \quad (3)
\]

Where \( W_i^\prime \prime \) is an additional work input attributable to the effects of over or under compression.

The additional work is associated with the shaded area of figure 1. In either case the method of calculation is the same. The method is illustrated for the case of under compression which is the situation normally found with scroll compressors

\[
W_i^\prime = P_i (V_i - V_{o1}) - \int_{V_o}^{V_i} pdv
\]

\[
= P_i V_i \left( \frac{m_{in}}{m_{in} - 1} \gamma_i^{\frac{m_i}{m_i - 1}} - \frac{1}{m_{in} - 1} \gamma_i^{1} + \gamma_d \gamma_i^{\frac{1}{m_i}} \right) \quad (4)
\]
In the above $\gamma_i, \gamma_d$ are respectively the internal and external pressure ratios whilst $P_i$ and $V_i$ are the suction pressure and suction volume.

In fact, for the $i$th stage of a multistage compressor with oil injection, the polytropic index $m_0$ is less than $m_{01}$ because there is a heat transfer between the air and the oil. The indicated work $W_i$ is given by

$$W_i = \frac{m_0}{m_0 - 1} P_i V_i (\gamma_i - 1)$$

(5)

The polytropic index $m_0$ depends on the amount of oil injected and can be found by experiment.

Changes in $W_i$ may be ignored and $W_0$ is given still by (2) and (3).

The work input for $i$th stage of a multistage scroll compressor can be calculated by substituting equations (2), (3), (4) and (5) in equation (1).

**Case (1). Internal and external pressure ratios equal**

$$W_i = P_i V_i \left[ \frac{m_0}{m_0 - 1} (\gamma_i - 1) + \left( \frac{k - m_{01}}{k - 1} \frac{m_0}{m_0 - 1} (\gamma_i - 1) \right) \right]$$

(6)

**Case (2). Unequal internal and external pressure ratio**

$$W_i = P_i V_i \left[ \frac{m_0}{m_0 - 1} (\gamma_i - 1) + \left( \frac{k - m_{01}}{k - 1} \frac{m_0}{m_0 - 1} (\gamma_i - 1) \right) \right]$$

$$+ \left( \frac{m_0}{m_{01} - 1} \frac{m_0 - 1}{m_0 - 1} \right) \gamma_i^{m_{01} - 1} \gamma_d$$

(7)

The power input to the stage is given by

$$N_i = W_i n$$

(8)

Where $n$ is the number of cycles per second.

Thus total power input to a multistage compressor is given by

$$N = \sum_{i=1}^{n} N_i$$

(9)

An objective function $F(x)$ can be defined as

$$F(x) = \frac{N}{V} = \frac{Power}{Volume \ delivered \ per \ second}$$

(10)

Optimisation seeks to find the values of the parameters $V$ and $N$ which will yield a maximum value of $F(x)$. In order to aid calculation and increase sensitivity the objective function is normalised, i.e.

$$F(x) = \frac{1}{\nu, \eta_{n}} = 1/(\nu, \eta_{n})$$

(11)

In the above $V_0$ and $N_0$ are respectively the ideal theoretical air displacement and power input. For $F(x)$ to be a minimum the product $\eta_{n}\eta_{n}$ must be a maximum.
The choice of independent parameters is crucial when optimising a particular arrangement (design). In this case the paper uses the amount of oil injected and the stage pressure ratio as independent parameters.

Thus \[ X = (x_{oi}, x_p) \] (12)

Where \( x_{oi} \), \( x_p \) are respectively the amount of oil injected and the stage pressure ratio.

It must be noted that in a multistage compressor the individual stage pressure ratios are not independent and must satisfy the following equation

\[ x_p = \frac{x_p}{(x_{p1} \cdot x_{p2} \cdot \ldots \cdot x_{p(i-1)} \cdot x_{p(i+1)})} \]

There \( x_p \) is the stage pressure ratio and \( x_p \) is total pressure ratio.

These parameters can vary widely because of the different units brought together to form a multistage compressor. To aid calculation these parameters are again normalised. In this case the normalisation takes the form

\[ x = \frac{x_p}{X_{pio}} \] (13)

In the above \( x_{pio} \) is the original stage pressure ratio. Stage pressure ratios may also be constrained to lie within set limits thus

\[ G_1 = X_p - X_i \geq 0 \]
\[ G_2 = X_s - X_p \geq 0 \] (14)

Where \( x_u, x_l \) are the upper and lower limits.

2. Mathematical model of working processes

The working processes within a stage of a multistage scroll compressor with oil injection can be described in terms of the changes of internal pressure and volume which occur. Many simplifications can be made to the governing equations if the following assumptions are made

(1). Air is an ideal gas and has uniform properties throughout a control volume.
(2). At any moment, the flow through the suction and discharge port and leakage gaps can be treated as isentropic and instantaneously steady. Changes of kinetic and potential energy of the air in a control volume may be ignored.
(3). Oil is incompressible and does not change its phase. The oil/gas mixture within the control volume has uniform pressure and temperature.
(4). The suction temperature is the same for all stages of a multistage compressor.

Application of the First-Law of Thermodynamics and the principle of conservation of mass to a control volume yields

\[ dU(\theta) = \delta Q + dE(\theta + 2\pi) - dE(\theta) + \delta W \] (15)

For the air within a control volume:

\[ dE_{\theta} = dm_{\theta} \cdot h_{\theta} \]
\[ dE_{\theta} = dm_{\theta} \cdot h_{\theta} \] (16)

\[ dU_{\theta} = d[m_{\theta} \cdot u_{\theta}] \]
\[ = m_{\theta} \cdot C_v \cdot dT_{\theta} + C_v \cdot T_{\theta} \cdot dm_{\theta} \] (18)

\[ \delta W_{\theta} = \text{work input to the air} = -Pd[V_{\theta}] \] (19)
\[ \delta Q = \text{heat input to the air} \]
\[ dm_g(\theta) = dm_{go}(\theta + 2\pi) - dm_{go}(\theta) \] \hspace{1cm} (20)

Assuming that there is no heat transfer between the oil and gas and using the ideal gas equation allows equations (16)-(20) to be substituted into equation (15) giving

\[ dT_g(\theta) = \frac{1}{m_g(\theta)} \left[ \left( k \cdot T_g(\theta + 2\pi) - T_g(\theta) \right) \cdot dm_{go}(\theta + 2\pi) - (k - 1)T_g(\theta + 2\pi)dm_{go}(\theta) \right] \]
\[ - (k - 1) \cdot T_g(\theta) \frac{dV_g(\theta)}{V_g(\theta)} \] \hspace{1cm} (21)

The corresponding equation for the oil is

\[ dT_i(\theta) = \frac{1}{m_i(\theta)C_i} \left[ h_i(\theta + 2\pi) \cdot dm_{io}(\theta) - U_i(\theta) \cdot dm_{io}(\theta + 2\pi) \right] \] \hspace{1cm} (22)

Therefore the compression process for an oil injected scroll compressor can be written

\[ \frac{dm_g(\theta)}{d\theta} = \frac{dm_{go}(\theta + 2\pi)}{d\theta} - \frac{dm_{go}(\theta)}{d\theta} \] \hspace{1cm} (23)

\[ \frac{dm_i(\theta)}{d\theta} = \frac{dm_{io}(\theta + 2\pi)}{d\theta} - \frac{dm_{io}(\theta)}{d\theta} \] \hspace{1cm} (24)

\[ \frac{dV_g(\theta)}{d\theta} = \frac{dV_{go}(\theta)}{d\theta} - \frac{1}{\rho_i(\theta)} \frac{dm_i(\theta)}{d\theta} \] \hspace{1cm} (25)

\[ \frac{dT_g(\theta)}{d\theta} = T_g(\theta) \left[ \left( \frac{kT_g(\theta + 2\pi)}{T_g(\theta)} - 1 \right) \frac{dm_{go}(\theta + 2\pi)}{m_g(\theta)d\theta} \right] \]
\[ - (k - 1) \frac{dm_{go}(\theta)}{m_g(\theta)d\theta} - T_g(\theta)(k - 1) \frac{dV_g(\theta)}{V_g(\theta)} \] \hspace{1cm} (26)

\[ \frac{dT_i(\theta)}{d\theta} = T_i(\theta) \left[ \frac{T_i(\theta + 2\pi)}{T_i(\theta)} - 1 \right] \frac{dm_{io}(\theta + 2\pi)}{m_i(\theta)d\theta} \] \hspace{1cm} (27)

There: \( m_{go} \) and \( m_{io} \) are respectively the amount of oil and gas passing through the inter scroll gaps. These quantities are determined from orifice theory. The assumption of equal oil and gas temperatures gives

\[ T_g(\theta) = T_i(\theta) = T(\theta) \] \hspace{1cm} (28)

Solution of the system of equations allows the temperature \( T(\theta) \) to be found and hence the pressure in the working volume can be obtained from the ideal gas equation.
3. Verification of the working process model for a multistage oil injected scroll compressor.

Figures 2 and 3 show a comparison of theoretical and experimental results for a two-stage oil injected scroll compressor. Figure 2 shows the variation of the air delivery for various oil injection rates for constant compressor speed, overall pressure ratio and a fixed interstage pressure. This figure shows that increased oil injection reduces air leakage and leads to a greater air throughput. Figure 3 shows the variation of the quantity of air delivered for a constant oil injection rate, constant compressor speed and varying overall pressure ratio and with the interstage pressure given by $p_i = \sqrt{\frac{p_d}{p_s}}$.

4. The optimisation of internal pressure ratio

The optimisation of the internal pressure ratio was performed using the SWIFT (sequential weighted increasing factor technique) and is outlined in the flow chart shown in Figure 4. Results for a 2 stage machine are presented in Figure 5, where the variation of the resulting theoretically required power input with interstage pressure is shown. This figure is for a constant compressor speed and a constant oil injection rate. For a 2 stage machine the minimum input power was attained when the inter stage pressure (the first stage discharge pressure) is given by

$$p_{d1} = \text{const.} \sqrt{\frac{p_d}{p_s}} \quad (31)$$

Where the constant has a value between 1.2 and 1.25.

For a 3-stage compressor, the respective inter stage pressures were given by

$$P_{d1} = (\text{const.})_1 \sqrt{\frac{P_d}{P_s}} \quad , \quad P_{d2} = (\text{const.})_2 \sqrt{\frac{P_d}{P_s}} \quad (32)$$

Where 1.15 < (const.)_1 < 1.2 and 1.25 < (const.)_2 < 1.3

The quantity of oil-injected amounted to be between 0.7% and 1% of the air displacement.

References:
1. Disheng Wang etc. Mathematical model of the working cycle of oil-injected scroll compressor, proceeding of the 1993 International Compressor Technique Conference, Xi'an China.
2. Qu Zong chang. The optimisation of pressure ratio and valve parameter for multistage compressor. Journal of Xi'an Jiaotong University, 1994, China.
FIG. 1 OVER COMPRESSION AND UNDER COMPRESSION.

FIG. 2 VARIATION OF AIR DISPLACEMENT WITH OIL INJECTED.

FIG. 3 VARIATION OF AIR DISPLACEMENT WITH PRESSURE RATIO

FIG. 4. FLOW CHART FOR OPTIMISATION.

FIG. 5 VARIATION OF TOTAL POWER INPUT WITH INTERSTAGE PRESSURE RATIO

FIG. 6 THEORY VS. EXPERIMENT FOR VARIATION OF AIR DISPLACEMENT WITH PRESSURE RATIO.

FIG. 7 THEORETICAL VS. EXPERIMENTAL DATA FOR VARIATION OF AIR DISPLACEMENT WITH PRESSURE RATIO.