1996

Mathematical Model of Reciprocating Compressor With One or Several Stages for the Real Gases

B. S. Chrustalev  
*State Technical University*

V. B. Zdalinsky  
*State Technical University*

V. P. A. Bulanov  
*GRAZ Plant*

Follow this and additional works at: [http://docs.lib.purdue.edu/icec](http://docs.lib.purdue.edu/icec)

[http://docs.lib.purdue.edu/icec/1108](http://docs.lib.purdue.edu/icec/1108)

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information. Complete proceedings may be acquired in print and on CD-ROM directly from the Ray W. Herrick Laboratories at [https://engineering.purdue.edu/Herrick/Events/orderlit.html](https://engineering.purdue.edu/Herrick/Events/orderlit.html)
MATHEMATICAL MODEL OF RECIPROCATING COMPRESSOR
WITH ONE OR SEVERAL STAGES FOR THE REAL GASES.

B.S. Christalev, V.B. Zdalinsky
(Compressor Dept., State Technical University St. Petersburg, Russia)
V.A. Bulanov (GRAZ plant, Penza, Russia)

Abstract

Hereby a method of thermodynamic processes investigation within a stage of a reciprocating compressor is suggested by means of the mathematical modelling. It is capable to evaluate the effect of different factors on the compressor efficiency and to obtain the information concerning the quality and performance of a machine existing or being designed.

A thorough analysis of different compressor construction schemes approved the fact that the whole range of the compressor design variety can be described by the following four schemes: (Fig.1). By means of this schematization, a universal mathematical model was developed with the following simplifications:

1. In the case of the interstage communication volumes being twice or more times greater than the cylinder volumes of the previous stages a multistage compressor is considered as an assembly of different separate stages with a constant values of the interstage gas pressure [1]. Suction and discharge gas parameters are defined on the following way:
   - Suction side: pressure ($p_1$) and temperature ($T_1$) are equal to the gas parameters of the media or those of the suction cavity; for the higher stage after the interstage cooler it is considered to
   - Discharge side: pressure ($p_5$) is equal to the gas pressure of the foregoing volume; the temperature is defined according to the suggested empirical formula:

\[ T_5 = T_{5\text{ad}} - 0.2(T_{5\text{ad}} - T_1) \]

$T_{5\text{ad}}$ is the temperature defined by the suction temperature and pressure and pressure drop based on adiabatic process;

2. The volumes of the suction and discharge chambers should be not less than a quarter of the cylinder working volume; this is satisfied in the most compressor design examples;

3. The cross section of the pipes within the compressor should be large enough to have the average gas velocity defined from the piston velocity not exceed the sonic one (in practice not exceed 30% of the latter);

4. Parameters of the suction and discharge system should be defined so to avoid the resonance effect of the gas pulsations on the lower frequencies.

5. Flow path of a compressor consists of the following elements: cavities of constant or variable volume and heat exchange areas, as well as the hydraulic resistance elements of constant (suction and discharge systems, piston and valve seals) and variable (self-acting valves) cross-sections;
6. Because of the negligible heat exchange effect, a simplified approach is suggested: average temperatures and heat exchange coefficients are introduced based on the designer experience.

7. The compression and expansion processes in the neighbouring cavities 6 and 7 (Fig. 2) are considered to be adiabatic, suction and discharge processes - isobaric.

8. Compressing media - the perfect (or real) gas.

As the main thermodynamic variables in the computer model specific internal energy and density are used. All the rest (gas pressure temperature, enthalpy) can be determined from the basic two ones.

The mathematical model of the stage has a block structure and is capable to assume the reality of the compressing media. To do it, a polynomial dependencies $z = f(T, p)$ and $T = f(p, u)$ should be constructed, covering the working range of the process.

9. The valve dynamic model is described as a single degree of freedom, but because of the block structure it could be easily changed to a more complicated one [2, 3].

Fig. 2. represents the block structure of the two-stage compressor based on the generalized stage model.

It includes all stage variants, reviewed on Fig. 1. Every stage can be derived from the general model by excluding some elements. The structure of the stage model consists of the following elements: cavities of the constant volume (2 and 4) or variable volume (6, 3 and 7), cavities with unlimited volume (1 and 5), discrete hydraulic resistances, which are placed on the exact points of the gas flow path. It is considered that the cavities are connected to each other by gas flows through valves and clearances. Mass flows between any volumes $m_{ij}$ are defined by pressure drop between volumes $i$ and $j$. The indexes $ij$ correspond to the direction of the flow: if $p_i > p_j$, then $m_{ij}$ means the flow streaming from the cavity $i$ to the $j$ one.

The values, corresponding to the gas parameters within the cavities are displayed inside the blocks on Fig. 2. For the compression cavity, as well as for the suction and discharge ones they are the cavity volume ($V$), internal energy ($U$), gas mass ($M$). For the cavities 6 and 7 they are the volume, pressure ($p$) and temperature ($T$).

The gas state variables within the cavity are defined according to the first law of the thermodynamics (in differential form), mass flow equations, as well as the gas state and caloric equations. According to the experience of the authors, that the most effective way is to choose the cavity volume, gas internal energy and gas mass as the main variables for the volumes 2, 3 and 4. The system of governing equations is the following:

$$\frac{dU}{dt} = \alpha \cdot F_w \cdot (T_w - T) - p \cdot \frac{dV}{dt} + \sum_{j1} m_{j1} \cdot \frac{dM}{dt} - \sum_{j2} m_{j2} \cdot \frac{dM}{dt} ;$$

$$\frac{dM}{dt} = \sum_{j1} m_{j1} - \sum_{j2} m_{j2} ;$$

$$p = \frac{M}{V}; \; u = \frac{U}{M}; \; T = f_1 (u, \rho); \; z = f_2 (T, \rho); \; p = z \rho R; \; T_i = u + p / \rho;$$
where U - gas internal energy within the cavity; M - gas mass within the cavity; t - time; V - cavity volume; \( \rho \) - gas density, \( R \) - gas constant; \( p \) - pressure; \( i \) - specific enthalpy; \( \alpha \) - heat exchange coefficient between the gas and the cavity walls; \( z \) - compression coefficient; \( F \) - heat interchange area; T - gas temperature, Tw - temperature of the walls which can be defined approximately by the empiric formulas:

\[
Tw_1 = (1.1...1.25) \, T_1, \quad Tw_2 = (T_1+T_5)/2, \quad Tw_3 = (0.8...0.9) \, T_5;
\]

\( m_{j1} \) - incoming gas mass flow through the \( j_1 \) port; \( m_{j2} \) - outgoing gas mass flow through the \( j_2 \) port, \( u \) - specific internal energy.

By means of the abovementioned equations, a special system of equations can be assembled for each of the cavities.

The steady working condition of a compressor is those when the processes in each of the stages are periodical. Two conditions should be satisfied in this case:

1. Gas mass variation within a compression cycle should be equal to zero. Because, according to the compressor scheme, interstage communication has only one entry, (cross-section 4-5 from the first stage) and only one exit (cross-section 1-2 of the second stage), then this statement can be changed to the following one:

\[
M_{II} = M_{12II} - M_{21II}; \quad M_{I} = M_{45I} - M_{54I}; \quad M_{I} = M_{II}.
\]

Where \( M_{I} \) - gas mass flow through section 1-2 (first stage) in both directions; \( M_{II} \) - gas mass flow through section 4-5 (second stage) in both directions;

2. The internal energy variation after a revolution should be equal zero.

In general, the processes within the interstage communication are quite complicated, especially in the heat exchanger, with the effects of unsteady flow and heat exchange. But in the case of properly working heat exchanger the suction temperature of the second stage can be determined as the temperature of the coolant (\( T_c \)) with some additional value. The pressure remains constant in the communication.

So, both of the conditions correlates the suction gas values of the second stage to the discharge values of the first one. Finally, they can be the following:

\[
p_{5I} = p_{1II} = p_{\text{int}}, \quad T_{1II} = T_c + \Delta T.
\]

The formulated conditions are capable of linking of the system of equations for each of the stages together, compressing the real gas. It is important that they are independent from the gas type.

The traditional approach is close to the compressor actual work: The computer model starts at the initial conditions which correspond to the real ones in most cases. After some relaxation process the solution would become periodical within the computational error. According to the experience, the conversion of this method is not very fast and depends greatly on the geometrical data of the compressor. For the particular compressor several thousand iterations are required.

To accelerate the computation, the following scheme was applied, based on the described conditions: While the suction and discharge pressure should be equal, the mass flow should be equal as well for both of the stages. The procedure includes the creation of the dependencies and their comparison.
As an illustration of the approach, a methane compressor is presented with a high variation of the suction pressure of the working process. The machine is used for the displacement of methane from a vessel with the suction pressure varying from 20 to 150 bar and discharge pressure of 250 bar. The compressor operates in the continental Russia so the winter and summer suction gas temperature variation should be taken into consideration.

On Fig.3 a graphic solution of the interstage gas pressure is presented. Thorough analysis of the performance at different working conditions provided the evaluation of the effect of suction gas parameters on the compressor characteristics (Fig.4) as well as the definition of the appropriate valve parameters dynamics of which depend greatly on the working mode.

On Fig. 4 a non-linear characteristics can be seen because of the gas reality. In the case of the suction pressure exceeding 150 Bar a mode of 'blowing through' takes place, when both suction and discharge valves of the second stage are being opened (Fig.5). In this mode the power consumption decreases substantially.

Acknowledgement.

The work described above is based on more than 30 year experience of the experimental and theoretical studies provided under the supervision of Dr. Igor Pirumov, Professor of the Compressor Dept., State Technical University of St.Petersburg, Russia, in the field of gas parameter and valve dynamic simulation. The authors greatly acknowledge the great contribution of knowledge they accumulate during many years of working together.

References.


2. Isakov V.P., Khrustalev B.S., Self-acting valves of the reciprocating compressors for various usage purposes.- Chemical and Oil Machine-Building,11, pp.67-70- in Russian.


Fig. 1

Stage N I

1. Cylinder upper
   V6(T), U6, M6

2. Suction cavity
   V2, U2, M2

3. Compressor cylinder
   V3(T), U3, M3

4. Discharge cavity
   V4, U4, M4

5. Cylinder under
   V7(T), U7, M7

Interstage volume:
   p_{int} = p_{51};
   p_{1II} = p_{int};
   T_{1II} = T_e + \Delta T;

SUCTION

DISCHARGE

Fig. 2

Stage N II

1. Cylinder upper
   V6(T), U6, M6

2. Suction cavity
   V2, U2, M2

3. Compressor cylinder
   V3(T), U3, M3

4. Discharge cavity
   V4, U4, M4

5. Cylinder under
   V7(T), U7, M7

SUCTION

DISCHARGE

p_{5II}, T_{5II}
Fig. 3

\[ M_1, M_\text{II} (\text{Nm}^3/\text{h}) \]

Fig. 4

\[ p_{\text{int}} (\text{MPa}), \quad N_i (\text{kW}) \]

Fig. 5

\[ N_\text{I} (2.0 - 25.0 \text{ MPa}) \]

\begin{align*}
\text{St. I} & : \quad P_1 = 2.00 \\
& : \quad P_5 = 5.65 \\
& : \quad n_c = 1.306 \\
& : \quad n_e = 1.306
\end{align*}

\begin{align*}
\text{St. II} & : \quad P_1 = 5.65 \\
& : \quad P_5 = 25.00 \\
& : \quad n_c = 1.479 \\
& : \quad n_e = 1.483
\end{align*}

\[ N_\text{II} (14.5 - 25.0 \text{ MPa}) \]

\begin{align*}
\text{St. I} & : \quad P_1 = 14.50 \\
& : \quad P_5 = 25.00 \\
& : \quad n_c = 1.960 \\
& : \quad n_e = 1.948
\end{align*}

\begin{align*}
\text{St. II} & : \quad P_1 = 25.00 \\
& : \quad P_5 = 25.00 \\
& : \quad n_c = 2.621 \\
& : \quad n_e = 2.605
\end{align*}

\[ \phi \]