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Optimization of Cylindrical Halbach Permanent Magnet Array Dimensions for Magnetic Refrigeration

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ABSTRACT

Halbach cylindrical permanent magnet arrays (HCPMA) are promising for magnetic refrigeration applications since they have strong magnetic field inside the cylinder and weaker magnetic field outside. To investigate the effect of dimensional ratios of a HCPMA on the cooling performance of the magnetic refrigerator, FEMM (Finite Element Method Magnetics) 4.0 software was used. For different external radii, average magnetic fields inside the cylinder were calculated. Magnetization curves of Gd (Gadolinium) element were used to calculate magnetization entropy changes and cooling energies for obtained average magnetic field values. Cooling energy to magnet volume ratio was calculated to determine dimensions of the Halbach array which allows most efficient magnet usage. In addition to magnet dimensions, effect of using soft magnetic materials inside and outside the permanent magnet cylinder on magnetic field and cooling energy was also investigated.

1. INTRODUCTION

Magnetic refrigeration is an environment friendly and potentially energy efficient alternative refrigeration technology, which employs magnetocaloric effect (MCE). MCE is the temperature difference created by applying magnetic field on magnetocaloric material (MCM). External magnetic field causes alignment of electronic spins of MCM and decrease in magnetic component of total entropy ($S_M$). Since the total entropy of the system must remain constant, by 2nd law of Thermodynamics, the lattice entropy of the material increases resulting in a decrease in temperature (Pecharsky and Gschneidner, 2006).

The main components of magnetic refrigeration technology are MCM, magnetic field source and the heat transfer. There are research groups focusing on improving MCE of Gadolinium and its alloys are well known MCM materials. In the literature there examples of magnetic refrigerators with high cooling power in the range between 100 W and 600W, which were obtained by magnetic fields in of 4-5 T. (Zimm et al., 1998; Hirano et al., 2002).
However, this magnetic refrigerators use electromagnets or superconducting magnets as the magnetic field source. Since the electromagnets and superconducting magnets need complex cooling systems, which increase the electricity consumption dramatically and they increase the dimensions of the refrigerator they are not suitable for home usage. On the other hand, permanent magnets have no need for electricity to work or any cooling mechanisms. Besides to these advantages of permanent magnets, it is difficult to produce magnetic field over 1 T by using these magnets. So, it is important to have specific permanent magnet designs which can produce higher magnetic field.

Halbach array permanent magnet arrays are special magnet designs which can produce strong magnetic field in one side of the array while a weaker magnetic field in the other region. Halbach cylindrical permanent magnet arrays (HCPMA) are known to be appropriate for magnetic refrigeration applications since they produce strong magnetic field (up to 2.6 T (Lee and Jiles, 2000)) inside the cylinder while the magnetic field outside the cylinder is much weaker (Figure 1) (Allab et al., 2005; Bjork et al., 2008; Lee and Jiles, 2000; Nielsen et al., 2008; Vasile and Muller, 2006). It is also known that using soft magnetic material in the magnetic field source enhances the magnetic field in desired regions. There is research showing that adding soft magnetic material inside or outside the HCPMA can increase the magnetic field inside the bore of the cylinder (Allab et al., 2005, Lee and Jiles, 2000; Lee et al., 2002).

In the present study, we used 2D finite element software, FEMM (Finite Element Method Magnetics) 4.0, to analyze average magnetic field, $B_{ave}$, in two types of HCPMA with varying dimensions. The entropy change $\Delta S$ obtained by the generated magnetic fields and related cooling energy, $Q_c$, of the system was calculated by using relevant theory. The effect of outer radius to inner radius ratios on $Q_c$ and maximum cooling energy for a given volume of magnet ($Q_c/V_{mag}$) were investigated. Also, effect of using soft magnetic materials inside and outside of the HCPMA on $B_{ave}$ and $Q_c$ was investigated.

2. THEORETICAL AND NUMERICAL ANALYSES

2.1 Halbach Cylindrical Permanent Magnet Array (HCPMA)

As mentioned in the previous chapter, Halbach cylindrical permanent magnet arrays are special since they allow creating strong magnetic field inside the cylinder and much weaker magnetic field outside the cylinder. In our calculations we designed a 16-block HCPMA formed by NdFeB permanent magnets. We calculated the average magnetic field $B_{ave}$ for circular and octagonal HCPMA with 160 mm length, 20 mm inside radius $R_{in}$ and varying outside radii $R_{out}$ (Figure 1).

![Figure 1: Circular and Octagonal HCPMA designs](image-url)
2.2 Entropy Change and Cooling Energy Calculations
After calculation of $B_{ave}$ values for different $R_{out}/R_{in}$ ratios, we assumed that the inside volume was fully filled by Gadolinium element (as MCM) and calculated the related entropy changes ($\Delta S$) around room temperature. Experimental Magnetization-Magnetic field (M-H or M-B) curves were used to calculate entropy change for Gd (Figure 2). McMichael presents the formula for the $\Delta S$ as given in Equation (1). In Equation (1), to obtain $\Delta S$ at temperature T, area between two curves in the M-H curve is divided to the temperature interval of $\Delta T$ between the curves.

$$\Delta S_M (T + \Delta T/2, H) \approx -\frac{1}{\Delta T} \times \text{Area}$$  \hspace{1cm} (1)

Entropy change values were used to calculate the cooling energy, $Q_c$, the maximum heat given by MCM as a result of a magnetization/demagnetization cycle. Allab et al. gives the formula for $Q_c$ as follows:

$$Q_c = m_{MCM} \times \Delta S \times T_C$$  \hspace{1cm} (2)

In Equation (2), $m_{MCM}$ is the mass of the magnetocaloric material, $\Delta S$ is the entropy change, and $T_C$ is the cold temperature. In the final step, to investigate the most effective permanent magnet usage, the ratios of cooling power to permanent magnet volume, $Q_c/V_{mag}$, were calculated for different $R_{out}/R_{in}$ ratios.

2.3 Enhancement of Magnetic Field by Soft Magnetic Materials
The effect of soft magnetic material (SM) usage was investigated in two ways: First, SM was placed outside the HCPMA, on the path of the magnetic field inside the cylinder to decrease magnetic field flux leakage. Secondly, SM was added in the bore of HCPMA as a magnetic field concentrator (Figure 3).
For SM analyses the $R_{out}/R_{in}$ ratio was chosen as 2, which gives the highest $Q_c/V_{mag}$ (which is explained given in the next chapter). FeVCo was used as soft magnetic material. $B_{ave}$ values for different $R_{out}/d_{SM}$ ratios were calculated. After obtaining the optimum thickness of the SM shield, $d_{SM}$, $B_{ave}$ and $Q_c$ values were calculated for different concentrator thicknesses, $d_{CON}$.

3. RESULTS

3.1 Effect of $R_{out}/R_{in}$ Ratio on Cooling Power

Cooling energy created by the MCM in the HCPMA changes with varying $R_{out}/R_{in}$ as illustrated in Figure 4. For circular and octagonal HCPMA, the relationship between $Q_c$ and $R_{out}/R_{in}$ has similar behavior.

![Figure 4: Relationship between $Q_c$ and $R_{out}/R_{in}$ for circular (left) and octagonal (right) HCPMAs ($R_{in} = 20$ mm, $L = 160$ mm)]
A peak is observed when the effect of dimensional ratios on the cooling energy per permanent magnet volume, \( Qc/V_{mag} \), is analyzed. This suggests that for a certain \( Rout/R_{in} \) value, the cooling power obtained from a given volume of permanent magnet can be maximized. As illustrated in Figure 5, for \( Rout/R_{in} = 2 \), \( Qc/V_{mag} \) value is maximum for both type of HCPMAs.

![Figure 5](image-url)

**Figure 5**: Relationship between \( Qc/V_{mag} \) and \( Rout/R_{in} \) for circular (left) and octagonal (right) HCPMAs (\( R_{in} = 20 \) mm, \( L = 160 \) mm)

### 3.2 Effect Using Soft Magnetic Material

It is known that using soft magnetic material (SM) shields on the path of magnetic field inside the cylinder can reduce the magnetic flux leakages and enhance the inside magnetic field (Lee and Jiles, 2000; Lee et al., 2002). When we look at the relationship between increasing SM thicknesses on average magnetic field (\( B_{ave} \)) inside the bore (Figure 6), we can see that increasing the thickness more than 0.1\( Rout \) has no contribution to the inside magnetic field. In other words, the additional soft magnetic material over this thickness will be wasted.

![Figure 6](image-url)

**Figure 6**: The relationship between the \( d_{SM}/R_{out} \) ratio and \( B_{ave} \) (\( R_{out} = 40 \) mm \( R_{in} = 20 \) mm, \( L = 160 \) mm)
The effect of SM inside the cylinder, as a magnetic field concentrator, seems advantageous, since it increases the magnetic field (Lee and Jiles, 2000; Lee et al., 2002). On the other hand, since some space inside the cylinder occupied by SM, the amount of magnetocaloric material used will increase. So it is important to know the effect of adding SM concentrator inside the HCPMA on cooling energy $Q_c$. The relationship between SM concentrator thicknesses ($d_{CON}$) and cooling energy ($Q_c$) was illustrated in Figure 7.

![Figure 7: The effect of $d_{CON}/R_{out}$ ratio on magnetic field inside the cylinder and cooling energy ($R_{out} = 40$ mm $R_{in} = 20$ mm, $L = 160$ mm)](image)

As it can be seen in figure 7, the decrease in amount of MCM dominates the increase in magnetic field as a result of adding SM inside the HCPMA. Although it seems advantageous to increase the magnetic field, there is a loss in cooling power.

4. CONCLUSIONS

By using finite element method for magnetics, the effect of dimensional ratios of HCPMA on cooling performance of a magnetic refrigerator was investigated. The relationship between $R_{out}/R_{in}$ ratio and cooling energy of the system was shown. It is shown that there is a peak value for $Q_c/V_{mag}$, which gives an idea about the effective use of permanent magnet volume in magnetic refrigeration applications of HCPMA. For both the circular and octagonal HCPMAs, maximum cooling capacities per unit volume of magnet ($Q_c/V_{mag}$) were achieved when $R_{out}$ is approximately twice $R_{in}$, which agreed with the literature.

The effect of using soft magnetic material on magnetic field and cooling energy was also investigated. It was shown that using soft magnetic materials as a shield by covering outside of HCPMA increases the inside magnetic field. On the other hand, the magnetic field inside the cylinder does not respond to any increase in soft material thickness over...
0.1\(R_{\text{out}}\). Adding SM inside the cylinder as magnetic field concentrators increases magnetic field but decreases the cooling energy obtained from the MCM placed inside the cylinder.

**NOMENCLATURE**

- \(B_{\text{ave}}\): Average magnetic field flux density (T)
- \(d_{\text{sm}}\): Thickness of soft magnetic material shield (mm)
- \(d_{\text{CON}}\): Thickness of soft magnetic concentrator (mm)
- \(H\): Magnetic field (A/m)
- \(L\): Length of Halbach cylinder (mm)
- \(M\): Magnetization (J/kg.T)
- \(m_{\text{MCE}}\): Magnetocaloric material mass (kg)
- \(R_{\text{in}}\): Inner radius of cylinder of Halbach cylinder (mm)
- \(R_{\text{out}}\): Outer radius of Halbach cylinder (mm)
- \(Q_{\text{c}}\): Intrinsic cooling energy (J)
- \(S\): Entropy (J/kg.K)
- \(T\): Temperature (K)
- \(V_{\text{mag}}\): Permanent magnet volume (m³)

**Subscripts**

- HCPMA: Halbach cylinder permanent magnet array
- MCE: Magnetocaloric effect
- MCM: Magnetocaloric material
- SM: Soft magnetic material

**REFERENCES**


Nielsen, K. K., et al., 2009, Magnetic cooling at Risø DTU, *8th IIR Gustav Lorentzen Conference on Natural Working Fluids, Copenhagen*.
