Non-monotonic pressure dependence of resonant frequencies of microelectromechanical systems supported on squeeze films

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Non-monotonic pressure dependence of resonant frequencies of microelectromechanical systems supported on squeeze films

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Abstract

The resonant frequencies of released microcantilevers, microbeams, and microplates are among the most important response characteristics for microelectromechanical systems such as resonators, sensors, and radio frequency (RF) switches. It is generally believed that the resonance frequencies of such structures decrease monotonically as the surrounding gas pressure is increased from vacuum conditions. However, we find that for microbeams supported on gas films the natural frequencies of the device can first increase and then decrease with increasing gas pressure from vacuum, with the extent of non-monotonicity depending on device geometry. This anomalous property of a wide class of microelectromechanical systems is explained in terms of the competing inertial and compressive effects of the supporting squeeze film.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

A wide class of microelectromechanical systems (MEMS), which include resonators, accelerometers, inertial sensors, comb drives, mirrors, and pressure sensors, consists of suspended structures separated from a substrate by a thin gas film. The damping and natural frequency of such MEMS structures are among their most important performance metrics—they dictate the safe bandwidth for operation for non-resonant devices, and for resonant devices they dictate the sensitivity and speed of the device. From the early days of MEMS modeling it has been recognized that the coupling of the vibrating structure with the surrounding gas film modifies significantly its dynamic response [1–7], especially its effective damping [8–11]; however, the change of natural frequencies due to gas films has been much less investigated [11–14].

2. Background

It is generally thought that the natural frequencies of MEMS decrease monotonically as the surrounding gas pressure is increased from vacuum, in line with the common observation for freely vibrating structures that the fluid added mass effect at increased pressures leads to a decreased resonance frequency. In this work we show that in the presence of a squeeze gas film, the resonance frequencies can first increase and then decrease as the pressure is increased from vacuum conditions. This non-monotonic pressure dependence of natural frequencies can be a subtle or a strong effect depending on the geometrical and structural properties of the device. This interesting observation is explained in terms of the competing inertial and compressive effects in commonly used models of gas films in MEMS [8]. Experimental data are presented on freely vibrating...
Table 1. Nominal dimensions of test devices: freely vibrating cantilever (MikroMasch CSC/12 tipless cantilever), cantilever on a squeeze film (SF) (CADP), doubly clamped beam on a squeeze film (PRISM device). $L$ is the length of the beam, $b$ is the width of the beam, $t_c$ is the thickness of the beam, $g$ is the gap height, $\rho$ is the density of the beam, and $E$ is the Young modulus of the beam.

<table>
<thead>
<tr>
<th>Device</th>
<th>$L$ (µm)</th>
<th>$b$ (µm)</th>
<th>$t_c$ (µm)</th>
<th>$g$ (µm)</th>
<th>$\rho$ (kg m$^{-3}$)</th>
<th>$E$ (GPa)</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free cantilever</td>
<td>130</td>
<td>35</td>
<td>1</td>
<td>$\infty$</td>
<td>2330</td>
<td>169</td>
<td>Single crystal Si</td>
</tr>
<tr>
<td>Cantilever on SF</td>
<td>200</td>
<td>18</td>
<td>2.25</td>
<td>2.25</td>
<td>2330</td>
<td>160</td>
<td>Poly Si</td>
</tr>
<tr>
<td>Doubly clamped on SF</td>
<td>400</td>
<td>120</td>
<td>3</td>
<td>3</td>
<td>8192</td>
<td>200</td>
<td>Nickel</td>
</tr>
</tbody>
</table>

microcantilevers, microcantilevers supported on a gas film, and radio frequency (RF) switches to describe this effect.

3. Experiment

Three prototypical experimental test devices are examined; they are as follows: a freely vibrating slender single crystal cantilever with no squeeze film layer (MikroMasch CSC/12 tipless), a slender polysilicon cantilever on a squeeze film (Center for Integrated Nanotechnologies Cantilever Array Discovery Platform), and a doubly clamped wide nickel beam on a squeeze film with gold electrodes coated with a silicon nitride dielectric layer (Center for the Prediction, Reliability, Integrity, and Survivability of MEMS RF switch). The nominal dimensions and physical properties of these devices can be found in table 1. Experiments were conducted using a Polytec MSA-400 laser Doppler vibrometer and a Suss vacuum probe station in which the pressure is decreased from atmospheric to 1 mTorr in controlled steps. A thermocouple monitors the local temperature near the device inside the chamber.

Two different mechanisms of excitation were used to excite the test structures. The cantilever test structures (freely vibrating and squeeze film cantilevers) were affixed to a piezoelectric shaker. The cantilever devices were then excited at their first transverse bending mode to an amplitude of less than 1 nm (well within the linear regime); the excitation was then terminated and the subsequent free response, or ring down, was measured by the laser Doppler vibrometer at the free end of the beam. The doubly clamped beam was tested in a similar manner, however an electrostatic excitation was used, rather than inertial, and the response was measured at the midpoint of the beam.

Using a Hilbert transform the envelope of the signal amplitude, $|X(t)|$, and the phase of the signal, $\phi(t)$, are calculated and fit to the solution of a single degree of a freedom harmonic oscillator to extract the damping ratio, $\zeta$, and natural frequency, $\omega_n$ [13]. The internal structural damping was determined by extrapolating the total damping measured at extremely low pressures to zero pressure [13] and was subsequently subtracted from the previous data, such that the data reflects only gas damping. Five freely vibrating cantilever structures, 4 squeeze film cantilever structures, and 14 doubly clamped squeeze film structures were tested. For each device tested multiple replicates of the experiment were performed at each pressure.

4. Results

For the freely vibrating cantilever 15 replicate measurements were made at each pressure, and for the cantilever on a squeeze film and the doubly clamped beam 5 replicate measurements were made at each pressure. Figure 1 shows representative data for one of each of the three types of devices tested. While the measured gas damping versus pressure plots show the expected trend of monotonic increase with increasing pressure, the natural frequency versus pressure plots show a significant difference between the data sets. The freely vibrating cantilever exhibits a natural frequency shift that is intuitive; the natural frequency of the device decreases as the pressure in the chamber is increased. The cantilever on a squeeze film exhibits an increase in natural frequency as the pressure is increased from vacuum, followed by a decrease in natural frequency as the pressure is further decreased. Note that figure 1 for the cantilever on a squeeze film only shows data below 100 Torr, this is to focus on the increase in the natural frequency of the device as pressure is increased. As the pressure is further increased the natural frequency of the device continues to decrease. Finally, the doubly clamped beam on a squeeze film shows a strong non-monotonic dependence on the natural frequency and pressure change. As the pressure in the chamber is increased the natural frequency increases, until at a critical pressure the natural frequency begins to decrease. To summarize, as the surrounding gas pressure is increased from vacuum, (a) the natural frequencies of the freely vibrating cantilever decrease monotonically, (b) the natural frequencies of the slender cantilever on a squeeze film increase slightly and then decrease in a weakly non-monotonic manner, and (c) the natural frequencies of the wide doubly clamped beam increase and then decrease in a strongly non-monotonic manner. In all cases the gas damping increases monotonically with increasing pressure as expected.

5. Discussion

To help rule out other possible sources of the non-monotonicity observed we note that the observed variation of temperature in the chamber was less than 0.5% (absolute temperature scale) for all experiments performed; the temperature on average was 22 °C. The minimum temperature recorded was 21.6 °C, and the maximum temperature recorded was 23.3 °C. This measurement helps to eliminate additional factors such as a temperature-dependent elastic modulus, or thermal expansion effects that could be caused by local heating of the device due to the excitation used. Additionally all experiments were performed in dry nitrogen, to mitigate any humidity effects.
During sample loading the chamber was trickle purged with nitrogen to ensure that adsorbed water and other gas species were fully evacuated.

To explain the natural frequency shift phenomenon based on fluid-structure interaction we resort to a modified Reynolds equation-based approach in the same vein as Fukui and Kaneko [15]. The fact that the beam is long and slender and is oscillating transversely with small amplitude gives rise to the assumption that axial fluid effects are negligible, and thus we analyze the fluid dynamics in a two-dimensional cross-section of the beam, as seen in figure 2. We utilize the theory of Veijola [8], who derived a modified Reynolds equation which accounts for gas inertia and rarefied gas effects through a relative flow rate coefficient, \( Q_{pr} \), and includes the slip boundary condition between the gas and the structure. From this equation a complex-valued squeeze force, \( F_s \), is derived. This theory has been validated in the low pressure regime (high Knudsen numbers) [11, 13].

The complex squeeze force is best studied in the frequency domain [11, 16, 17]. Starting with the Bernoulli–Euler beam theory we write the equation of motion as

\[
\rho_c b t_c \frac{\partial^2 W(x, t)}{\partial t^2} + E I \frac{\partial^4 W(x, t)}{\partial x^4} = f_s(t) + f_d(t),
\]

where \( W(x,t) \) is the transverse displacement of the beam, \( \rho_c \) is the beam density, \( b \) is the beam width, \( t_c \) is the beam thickness, \( E \) is the elastic modulus of the beam, \( I \) is the area moment of inertia, \( f_s \) is the complex squeeze force per unit length, and \( f_d \) is an external excitation force applied per unit length. Next we assume a separable solution in time and space, \( W(x, t) = \phi_n(x) q_n(t) \), where \( \phi_n(x) \) is the \( n \)th normalized eigenmode of the beam [18], and \( q_n(t) \) is the \( n \)th temporal component of the solution. The complex squeeze force in the frequency domain is then given by [8]

\[
F_s = \frac{\phi_n(x) \widetilde{q_n(\omega)} \gamma P_A b}{g} \left[ \frac{2}{\beta} \tanh\left(\frac{\beta}{2}\right) - 1 \right],
\]

where \( \omega \) is the frequency of oscillation of the beam, \( \widetilde{q_n(\omega)} \) is the \( n \)th temporal component of the solution, \( \gamma \) is the polytropic index, in this case equal to unity (isothermal process), \( P_A \) is the ambient pressure, \( b \) is the width of the beam, \( g \) is the gap height between the beam and the underlying substrate, \( \beta \) is a non-dimensional parameter given by \( \beta = \sqrt{i \sigma Q_{pr}} \), and \( v_t \) is the velocity of the beam cross-section. The parameter \( i \) is defined as \( \sqrt{-1} \), \( \sigma \) is the squeeze number, and \( \sigma = \frac{12 \rho_c v_t^2}{\gamma P_A g^2} \eta \), where \( \eta \) is the fluid viscosity. \( Q_{pr} \) is the relative flow rate coefficient given by

\[
Q_{pr} = \frac{12}{i \Re} \left[ 1 - \frac{\frac{2}{\sqrt{\Re} \eta} - \frac{\sqrt{\eta} \sigma \lambda}{\sqrt{\frac{2}{\Re}}} \tanh\left(\frac{\sqrt{\frac{2}{\Re}}}{\sqrt{\eta} \sigma \lambda}\right)}{1 + \frac{\sqrt{\eta} \sigma \lambda}{\sqrt{\frac{2}{\Re}}} \tanh\left(\frac{\sqrt{\frac{2}{\Re}}}{\sqrt{\eta} \sigma \lambda}\right)} \right],
\]

\( \Re \)
is the mean free path of the fluid, \( \sigma_p \) is the momentum accommodation coefficient [8], \( \lambda \) is the mean free path of the fluid, \( \rho \) is the fluid density, and \( Re \) is the unsteady Reynolds number, \( Re = \frac{\rho \omega g}{\eta} \).

Next we perform a Fourier transform of (1) and then a Galerkin projection onto the undamped eigenmodes, \( \phi_n(x) \), of the structure, normalized such that in the case of the cantilever beam the tip displacement is unity, and for the doubly clamped beam the midpoint displacement is unity [18]. The transfer function between the motion of the beam and an external forcing, for a given mode number \( n \), is given by

\[
\frac{g_n(\omega)}{F_d(\omega)} = 1 \left\{ \Omega_n^2 \int_0^L \phi_n(x) \, dx \right\} \left[ 1 - \left( \frac{\omega}{\Omega_n} \right)^2 \right] - \text{Re} \left( \frac{1}{\Omega_n^2 \rho_c t_c} \phi_n \left( \frac{2}{\beta} \tanh \left( \frac{\beta}{2} \right) - 1 \right) \right) - i \text{Im} \left( \frac{1}{\Omega_n^2 \rho_c t_c} \phi_n \left( \frac{2}{\beta} \tanh \left( \frac{\beta}{2} \right) - 1 \right) \right) \right\}. \tag{4}
\]

Thus, the natural frequency is given by the transcendental equation (recall that \( \beta \) is a function of \( \omega \))

\[
\left( \frac{\omega}{\Omega_n} \right)^2 + \text{Re} \left( \frac{1}{\Omega_n^2 \rho_c t_c} \phi_n \left( \frac{2}{\beta} \tanh \left( \frac{\beta}{2} \right) - 1 \right) \right) = 1, \tag{5}
\]

where \( \Omega_n \) is the theoretical value in the vacuum frequency of the device, \( \rho_c \) is the mass density of the beam, and \( t_c \) is the thickness of the beam. (5) is solved numerically for \( \omega \) at various ambient pressures, \( P_A \), with nominal physical device dimensions and properties.

Figure 3 shows the comparison between experimental data and theory. The agreement is quite good for the case of the wide doubly clamped beam, for which the nominal physical dimensions were used. Strong non-monotonicity of the natural frequency is clearly observable. In the case of the cantilever beam on a squeeze film, the prediction is in good agreement when a nominal gap of 2.25 \( \mu m \) is used in the low pressure regime; the prediction becomes much poorer as the pressure is increased. This is likely due to the fact that the \( \frac{1}{g} \) ratio of the cantilever on a squeeze film is quite moderate, and for such moderate ratios the assumptions of the model used begin to break down [8]. In particular it is assumed that the gap height is much smaller than the width of the device; when this is not the case the zero amplitude pressure boundary conditions used in the model need to be reconsidered. Additionally at lower \( \frac{g}{b} \) ratios, damping and added mass effects due to the sidewalls and the surface of the device need to be considered [8]. However, even in the light of these discrepancies for the cantilever on a squeeze film, the trend of weak non-monotonicity is still preserved.

The underlying physics of the above result can be explained from the observation that the compressible gas damping model actually contains three distinct effects: the added mass effect (in phase with beam acceleration), as well as stiffness (in phase with beam deflection), and the usual damping component (in phase with beam velocity). The observed changes in frequency can be ascribed to the relative change with pressure of the gas film stiffness and added mass. Figure 4 shows the real (damping) and imaginary (added mass and stiffness) components of force acting on the beam calculated at various ambient pressures for the doubly clamped beam with varying ratio \( \frac{g}{b} \) at the calculated natural frequency.
Figure 4. (a) Added mass and stiffness component (imaginary part) of the force (in newtons) acting on the beam, (b) damping component (real part) of the force acting on the beam (N). Calculations are for the doubly clamped beam at various pressures and \( \beta \) ratios, at the calculated natural frequency, with a prescribed maximum displacement of 1 nm.

Figure 5. Cross-sections of figure 4 at (a) \( \beta = 8 \) and (b) \( \beta = 40 \). At low \( \beta \) values the added mass and stiffness force is weakly non-monotonic, and the added mass effect is clearly dominant at higher pressures, indicated by negative force values. These negative force values, which are present at larger pressures, will decrease the natural frequency of the device, below the in vacuo value. At large \( \beta \) values the added mass and stiffness force is strongly non-monotonic; even at atmospheric pressures the force values are larger than zero, and thus the natural frequencies will be larger than the in vacuo value, due to the stiffness effect of the squeeze film. In both cases the damping force is monotonic.

with a prescribed maximum displacement of 1 nm. The effect of the ratio of beam width to gap height on the magnitude of the squeeze film forces is substantial. Also, it is quite evident that for large \( \beta \) values there is a clear non-monotonicity in the imaginary portion of the force. At low pressures the added mass and stiffness force is negligible, but as the pressure is increased the compressible gas film acts like a spring and adds stiffness to the structure, thus increasing the natural frequency. As the pressure is further increased the added mass effect of the fluid film becomes more dominant than the stiffness effect, eventually becoming the main contributor of the imaginary portion of the force, which leads to a decrease in natural frequencies. This behavior is in stark contrast to microbeams vibrating far from a substrate, in which the fluid added mass and damping effects are the only fluid forces experienced by the beam as the pressure is increased from vacuum.

It is relevant to ask the question: for which device geometries is this non-monotonic effect significant? (5) is a complex function of several parameters, and \( \beta \) is a function of the squeeze number, \( \sigma \), and \( Q_{pr} \), which in turn are functions of the device geometry, gas properties, and excitation frequency.
However, based on (5) the frequency of driven vibration, \( \omega \), and the ratio of beam width to gap height, \( b/g \), are the main contributing factors of the magnitude of this effect. We can see that for the doubly clamped beam on a squeeze film the width to gap ratio is about an order of magnitude greater than that of the cantilever beam on a squeeze film and its \textit{in vacuo} resonance frequency is also greater. Both of these factors lead to a much more significant frequency shift in the doubly clamped devices.

To illustrate this effect we have plotted the cross-sections of figure 4 at two different \( b/g \) values; this can be seen in figure 5. In figure 5(a) we see that for low \( b/g \) values the added mass and stiffness component of the force is weakly non-monotonic, and at higher pressures the added mass effect is dominant (indicated by the negative value of the force). This force behavior would lead to a slight increase in natural frequency as the pressure is increased from vacuum, followed by a steady decrease in natural frequency. In figure 5(b) we see that the higher values of \( b/g \) lead to the stronger non-monotonicity of the added mass and stiffness force. At higher pressures the added mass effect begins to decrease the force value; however the stiffening effect is still dominant. At atmospheric pressure a device with this \( b/g \) ratio would have natural frequencies higher than the \textit{in vacuo} value, as is seen in the doubly clamped beam in figure 3(a). In both cases of \( b/g \) the damping force is monotonic. In contrast, a beam not close to a substrate (no squeeze film layer) would only experience added mass and damping effects, and thus exhibit a monotonic decrease in natural frequency as pressure is increased from vacuum.

6. Conclusion

We have shown that for microbeams supported on gas films the natural frequency of the device varies non-monotonically as the ambient pressure is increased from vacuum, due to added mass and film stiffness effects. This effect is strongly dependent on the ratio of beam width, \( b \), to gap height, \( g \), as well as the frequency of vibration. For larger \( b/g \) ratios this non-monotonicity is stronger. The non-monotonicity of natural frequency versus pressure is in stark contrast to MEMS operating with no squeeze films, in which fluid film stiffness effects are absent and the natural frequency decreases monotonically as ambient pressure is increased from vacuum conditions.

We anticipate several possible uses for these findings. Resonant pressure transducers can be designed to avoid this effect, passive leak detection systems in packaged MEMS could utilize this effect to quantify a critical pressure level at which too much vacuum has been lost, and novel gap sensors could be designed to correlate this effect with the gap profile beneath the device.

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References