## A STUDY OF THE ACCURACY OF PHOTOCRAMREIRIC METHODS

IN RIGHT OF -NAY DETERMINATION

TO: K. B. Woods, Director Joint Highway Research Project

FROM: H. L. Michael, Associate Director Joint Highway Research Project

September 18, 1964
FIle Mo: I-4-20
Project No: C-36-32T

A final report entitled "A Study of the Accuracy of Photograrmetric Methods in Right-oi-Way Determination" by Lee A. Cobourn, Graduate Assistant on our staff, is attached. The report was ritilized as an MSCE thesis by Kr. Coboum. The research was conducted under the direction of Professors Sandor Veres and R. D. Miles.

This report discusses methods and equipment used in determining highway right-of-way by photogrammetry. The equipment consisted primarily of a Kelsh Plotter with an attached coordinatograph. The methods and equipment were evaluated by comparing photogrametric results with field results.

It was concluded in the report that the Kelsh Plotter with an attached coordinatograph provides an accuracy which is adequate for right-oirvay determination.

The report is presented to the Board for the record.
Respectfully submitted,

## 21.2. mild

H. L. Michael Associate Director

Attachments
Copy:

| F. L. Ashbaucher | J. F. McLaughlin |
| :--- | :--- |
| J. R. Cooper | R. D. Miles |
| W. L. DoIch | R. E. Mills |
| H. B. Goetz | M. B. Scott |
| F. F. Have | J. V. Smythe |
| F. S. Hill | E.J. Yoder |
| G. A. Leonard |  |

## Final Report

A STUDY OF THE ACCURACX OF PHOTOGRARMETRIC METHODS
IN RIGHT-OTMWAY DEIERMITAITON
by

Lee A. Cobourn<br>Graduate Assistant

Joint Highway Research Project
F17e: 1-4-20
Project: Ca36-32T

Purdue Uaiversity Iafayette, Indiana September 18, 1964

## ACKNOWLEEDKHENTS

The author wishos to express his most sincere appreciation to the following people and groups whose help meant so mach in the preparation of this theais.

The Joint Highway Research Project, K. B. Woods, Director, for providing the funds and equipment necessary to carry out this project.

Professor Sandor A. Veres, Purdue University, for his continuous support and advice. Professor Veres, as the author's major professor, never ceased to lend encouragement and help throughout the project.

Professor Robert D. Miles, Purdue University, for his advice throughout the project and for his editing of the final report.

Mr. B. J. Cole of the National Surveying Instruments, Incorporated of Chicago for the use of a Geodimater $4 D$.

Patrick G. Harvey, Gordon J. Arnott, and Captain Leo F. Ratter, Jr., classmates of the author, for their help during various stages of the projeot.

## TABTE OF CONTEATS

Page
LIST OF TABLES ..... 7
LIST OF IJILUSTRATIONS ..... V1
ABSTRACT ..... vii
INTRODUCTION ..... 1
Previous Investigations ..... 1
Purpose and Scope ..... 4
Slte Selection ..... 4
PIOTO CONTROL ..... 10
Elements of Photo Control ..... 10
Selection of Control Points ..... 11
Equipment ..... 12
Field Procedures ..... 24
Computation ..... 16
Vertical Control ..... 16
FTEDD CHECK SURVEY ..... 17
PHOTOGRAPHY ..... 20
KELSH PLOTTER AND ATTACHMENTS ..... 23
Kelsh Plotter ..... 23
Coordinatograph ..... 25
Ifght ..... 26
Base Measurement ..... 26
MEASURERTENTS FROM KELSH ..... 30
Calibration ..... 30
Principal Distance Detemination ..... 30
Measurements ..... 33
CALCULATIONS PERFOFMED ON KELSH DATA ..... 35
Correction Formulas ..... 35
Computations ..... 40

Digitized by the Internet Archive in 2011 with funding from
LYRASIS members and Sloan Foundation; Indiana Department of Transportation

## TABIE OF CONTEJTS (contimued)

Pege
RESULTS AND ANALYSIS OF DATA ..... 42
Test Area One ..... 42
Test Area Tho ..... 56
Test Area Threo ..... 64
CONCLUSIONS AND RECOMMENDATIONS ..... 70
Conclusions ..... 70
Recommendations ..... 71
BIBLIOGRAPHY ..... 74
APPENDIX A ..... 76
Transformation Formulas ..... 76
APPENDIX B ..... 78
Sample Computation ..... 78
APPENDIX C ..... 84
IFighway Right-of-Way Area Determination ..... 84
APPENDIX D ..... 93
Glossary of Terms ..... 93

## ABSTRACT

Cobourn, Lee Albert, MSCE, Purdue University, August, 1964. A Stady of the Accuracy of Photogrammetric Mothods In Bight-o1-Way Determination. Major Professor: Sandor A. Veres.

Photogrametry is being used more and more for cadastral surveys. Some work is now being done in determining right-of-way areas for proposed highways from aerial photographs.

This research was conducted to determine if the accuracy required for right-of-way determinations could be accormplished using adapted second order stereoplotting instruments. A Kelsh Plotter with an attached coordinatograph was used in this project.

Three areas on the Purdue University Campus were selected as test sites. In each test area distances and areas were determined from coordinates obtained using the Kelsh Plotter and attached coordinatograph. The values obtained in this manner were compared to resilts obtained by fleld survey methods.

Methods are given for relating individual lots to the center-line of a proposed highway and for detemining the areas taken by the right-of-way. This is done for both urban and rural areas.

The results of this research indicate that large scale photography (1 inch represents 250 feet) can be used for determining right-of-ways for proposed highways. When points are well defined in urban areas, distances can be measured to an expected accuracy of $\pm 0.25$ feet. For rural areas,

## LIST OF TABLES

Table Page

1. Comparison of Distances From Test Area One ..... 45
2. Comparison of Areas From Test Area One ..... 51
3. Comparison of Distances From Test Area Two ..... 57
4. Comparison of Areas From Test Area Two ..... 60
5. Comparison of Distances From Test Area Three ..... 65
6. Comparison of Areas From Test Area Three ..... 68

## LIST OF IILDUSTRATIONS

Figure Page

1. Married Student Courts - Test Area One ..... 6
2. University Park - Test Area Two ..... 7
3. Dairy Farm - Test Area Three ..... 8
4. Radial Distortion Curves ..... 21
5. Kelsh Plotter and Attachments ..... 24
6. Detemnination of Principal Distance ..... 32
7. Correction of $Y$-Model Coordinate ..... 38
8. Coordinate Determination for Pight-of-Way ..... 87
when fence posts are used, the expected accuracy of distance measurement is $\pm 0.50$ feet. The accuracy of areas 18 dependent upon the size of the area. Errors in both distance and areas are caused by the errors in coordinates obtained. Therefore the longer the distance, or larger the area, the maller the residual error will be.

## DATRODUCTION

## Previous Investigations

Photogrametry is prosently being used for many highway purposes ranging from route location through and including cross aections for desizn purposes. There have been some states using photogranmetrically compiled maps on which to baso right-of-way determination. Among the leaders of these are California and Texas.

The California Division of Fighways uses a zoiss C8 Stereoplandgraph to determine measurement data for metes and bounds descriptions of right-of-way which are relinquished to local authorities (I)*. Included in this are frontage roads near a freeway. Their method includes the use of prosignalized targets** and Geodimeter measured fleld control. Using their first order Zeiss C8 Stereoplanigraph, they detemene the state plane coordinates of fence posts located along freeways. These fances are the protective type chain-link fences normally placed near the right-of-way Ine. The fence posts are then used as reference for writing a metes and bounds description of the right-of-way which is to be turned over to local authority.

On a test project of State Sign Route 17 north of Santa Cruz, differ ences between fleld distances and distances computed from the C8 coordinates

[^0]ranged from +0.5 feet to -0.3 feet (1). According to this report, cost savings ranging from 50 percent on rural projects to 70 percent on urban projects were reallzed.

In 1958, the Texas State HHghway Department experimented with photogrametrically compiled maps to show property comers for use in right-ofway determination (2). In this project private photogramentic companies were contracted to prepare map mamscripts of proposed Dallas Freeway areas. They were to show both planimetric and topographic details. The requirements were for the map scale to be one-inch represents 20 feet, with horizontal errors less than 0.5 feet and vertical errors of less than 0.3 feet.

On the planimetric base map the highway personnel plotted the centerline and Hight-of-way lines of the new freeway, the block lines, street right-of-way lines, and the individual property lines as taken from deed abstracts. This map was then used to scale the distances needed to prepare metes and bounds descriptions of the remaining property. At the time of the report, 280 out of 300 parcels had been obtained and no major problems had developed (2). Further testing, using the same mothods, resulted in over 600 parcels of land being obtained without serious problems (3).

Other experiments have taken place in which photogramnety has been applied to cadastral surveys. In 1961 Philip F. Scudierl of Purdue University wrote a thesis comparing photogrametrically compiled right-of-way data to conventional survey data (4). He plotted property comers with a Kelsh Plotter and then scaled distances off the map manuscript. This method proved promising in rural areas but not accurate enough for urban areas.

James M. Anderson of Cornell Oniversity recently completed a study in which he used a hild A-7 Autograph to determine coordinate position of
test points (5). ikis results indicated a standard pooition ermor of 0.32 feet in $X$ and 0.23 feet in $Y$. The standard errors in distances ranged from 0.13 feet to 0.18 feet.

The Ohio State Highway Department found, in an investigation using a Wild A-7 Autograph, that average pobition error was 0.43 foet (6). The average error of distances was found to be 0.40 feet. The scale of this photograph however was 1 inch represents 1000 feet while that used by Anderson was 1 inch represents 250 feet.
S. J. G. Bind in a thesis submitted at the University of Toronto in 1963 compared photogrametric and ground methods for a legal surveg of Vineland, Ontario (7). Canada has laws which provide for resurvey and regiatration of large areas where errors have been perpetuated over a period of years. In this comparison, all reestablished property corners were presignalized. The measurement of the presignalized properts comers was completed using 1 inch represents 400 feet photograph on a W1ld A-7 Autograph. Results indicated that the standard deviation error in position was 0.17 feet when four control points per model were used.

One thing in common with all of the above experiments, except for Scudieri's, was the use of flrst order stereoplotting instruments. Sereral of the authors indicated that to obtain their achieved accuracy only first order instrments could be used. The following described research was undertaken with the thought that an adapted second order stereoplotting instrument could achieve very nearly the same results.

## Purpose and Scope

The purpose of the research was to determine if the accaracy required for right-of-way mapping could be accomplished by ueing a second order storeoplotting instrument. A Kalsh Flotter with an attached coordinatograph was used to detemine state plane coordinates of specifled points which could be used in determining property comers and center-line control points of a proposed new highway. The coordinates were read directiy from the coordinatograph, thus eliminating plotting and scaling errors. A mathematical adjuatment of the machine coordinates was made in an attempt to eliminate a portion of the residual photogrametric errors. The results of the photogrametric data were compared to data obtained by precise fleld surveying methods; thus, allowing a diroct comparison of the two methods. The photogrammetric data and methods were then applied to rlght-of-way determination in an effort to make this research applicable for use by the Indiana State Highway Commission.

## Site Selection

One of the very first considerations of this research was to select adequate test sites. The following criteria were used in determining the test sites:

1. The test site should be as near as possible to actual conditions.
2. The test site should be one which was reasonably accessible to the personnel conducting the research.
3. The test site should be one in which an accurate ground survey could be made in order to insure that data comparisons were meaningful.
4. The test site should be one in which good photography was readily obtainable.

Purtue Univerelty has aerial photographs of the campus taken every four years. The photographs were most recently taken in April, 1963 by Chicago Aerial Survegs. They used a Zeiss Aerotopograph camera having a six-inch focal length. The photography will be diecussed in detail later In this paper. However, it should be mentioned here that the photograpko was excellent and very much usable in the Kelah Plotter.

With the above criteria in mand, three test areas were selected on and near the Purdue University Campus in West Lalayette, Indlana. The test sites were not areas of actual proposed highways; however, they did satisfy the final three criteria. In addition, the test areas could be construed as being the same as an actual area through which a proposed highway would pass; therefore satisfying criteria mumber one.

Test area one is a portion of the married students housing on the Purdue Campus (Figure 1). In this area there are abundant sidewalks intersecting each other. The comers of these sidewalk intersections were used as presignalized points on which a determination of the accuracy of the method could be analyzed. The overlay on Figure 1 shows the location of the points, selected boundary lines and the aress deterinned.

Test area two is located north of State Street and west of the main portion of the Purdue Campus in a section sabdivided and known as Univeredty Park (Figure 2). This area is composed of normal city blocks containing from four to ten lote per block. In this area block boundaries were determined. A method is given for determining the distances and areas of each lot. and for referring the lots to the proposed highway.

Test area three is located northwest of the Purdue Campus on farmland which comprises a part of the Purdue Dairy Farm (Figure 3). In this area





the fence comers were taken as property comers. In addition to the area and distance comparison, a hypothetical highoy was passed through the area and an example computed showing the method of determining areas and descriptions of condemed property. Flgure 3 shows the fence comers used, and the boundary lines and areas enclosed by the comers. In addition, the figure shows the hypothotical highway as it crosses the test area and gives the points of intersection of the highway with the topothetical property lines.

PHOTO CONTROL

## Elements of Photo Control

The accuracy obtained using photogrametric methods $c a n$ be no better than the field control which is necessary to control the photos. This must be kept in mind when planning photogrametric control surveys.

Control surveys are necessary in order to make the photogrametric model a scale representation of the ground. Scaled distances between two points on the model must agree with the horizontal distance measured on the ground. It requires that the relative difference in elevations between points on the model must agree with the ground difference. The control surveys are therefore divided into two parts; the horizontal control and the vertical control. Both are equally important.

There must be at least one known distance and three known elevations in order to control a stereoscopic model. The distance and elevations must be determined on the ground using points which are identiflable on the photograph. It is desirable to have four or flve known elevations and two know distances. This provides a check and also yields a better fitting model.

The location of the control points on a model is important. Distances are preferably expressed by coordinates. This allows for position control which defines the distance. Ideally, two of the horizontal control points would be in opposite diagonal corners of a model. If more than two are present, then the third should be located in another comer of the model.

For the vertical control it must be remembered that three points define a plane in space. Therefore, the three vertical control points must be located in the form of a triangle in which the apexes are located near different edges of the model. Normally, for large ecale mapping, vertical control is provided in all four corners and in the aiddle of the model.

## Selection of Control Points

The selection of control points greatly influences horizontal scaling of a model. The ideal situation is to have presignalized points of a size designed for the particular flight altitude and to have the point a color combination which allows for positive identification and yet does not have a great deal of contrast. As this research project was based on photography already taken, the design of signals was not a consideration.

There were however, on two of the test areas, presignalized control points. These points were established prior to photograpky under the direction of Professor Sandor A. Veres. The targets consisted of four one-foot by three-foot arms placed in the form of a cross with the center open. The targets were painted gray on roadway and sidewalk surfaces. They proved satisfactory. However, they possibly could have been smaller in size.

The existing presignalized control points were situated in such a fashion that only one was usable on each of test areas one and two. Test area three, at the Dairy Farm, contained no presignalized control points. State plane coordinates were established on the control points by Professor Veres for the use of any subsequent users of the photograpty.

In both test areas one and two, it was deemed advisable to locate the remaining control points by polar survey methods from the existing preelgnalized control points. This method eliminates much time spent in pield work. It does not however offer any check on the results, so extreme care must be taken. Although no presignalized control points were in test area three, there were some nearby which were used to establish the horizontal control.

In test area one, four horizontal control points were established in addition to the presignalized point. Three of these points were comers of sidewalk intersections and the Pourth was a fire kydrant (see FIgure 1). Test area two contained one presignalized horizontal control point and three points located at sidewalk intersections (Figure 2). Test area three had only two horizontal control points, both of which were fence posts (Flgure 3). The analysis of the quality of the control points will be considered in the discussion of each test area later in the report.

## Equipment

The horizontal control was established using a wild T-2 Theodolite for direction measurement and a Geodimeter ld for distance measurement. The WIld T-2 Theodolite reads directiy to one second of are and can be estimated to one tenth of a second of arc. It features an optical ploneret for accurate set-ups.

The Geodimeter 4 is an electronic distance measuring device which uses light rays as its instrument of measurement (8). The 4 D model is accurate to within $\pm 0.04$ feet for distances as short as 50 feet or as long as 20 miles.

The Geodimeter measures distances indirectiJ by measuring the time nequired for a light beam to pass from tho Ceodimeter, loceted at one end of the line to be measured, to a reflector placed at the other end, and back to the Geodimeter. The time measurement is made by counting the number of light pulses and fractions thereof that occur within this distance. The fractions are determined by introducing a known variable electric delay in the instrument until a certain phase relationship is accomplished between returning light pulses and one intemal reference signal. This condition is displayed as a "zero reading" on the instrument indicator. The whole number of light pulses is computed by ropeating the measurement with two other modulation frequencies. If the Ceodmeter position is fixed, these "zero readings" are obtained at regular intervals, as determined by the modulation frequencies.

The Ceodimeter principle may be described as follows. A stable rule is placed with one end at the reflector end of the line. The rule is divided into intervals corresponding to the distance between "zero readings", which is appraximately 8 feet, or 2.5 meters, for the model 40. The last interval, within which the Ceodimeter is situated, provides a fine graduation enabling its position to be determined to within 0.04 feet.

If the frequency is changed, the interval between the nzero readings" will change. This new frequency measures the distance a second time. A third frequency measures the distance the third time. The three readings, which comprise one set of readings, must agree within very close tolerances or the distance is remeasured. The distance is therefore actually measured three times in a very short period of time and with a very high degree of accuracy.

There are three types of targets which can be used to reflect the Iight back to the Geodimetor. The one used in this project was a systom of tetrahedron prisms mounted in a metal housing which is attached to a tripod. The $90^{\circ}$ comer in the back of the tetrahedron is ground th a tolerance of $\pm 1$ second of angle. The target can therefore be mis-directod up to $20^{\circ}$ from the light source and stinl return the light satisfactorily.

The Geodimeter and accessories used in this project were very generously offered free of charge by National Surveying Instrunents Incorporated of Chicago, a Geodimeter representative.

## Fleld Procedures

As previously stated, polar survey methods were used in establishing the horizontal control in all test areas. The procedures used were the same in each area. The Wild T-2 Theodolite was set up on the existing presignalized control point and sighted on a nearby control point on which state plane coordinates had also been established. This line served as a reference for azimuth from which the bearings of the remaining lines were calculated. After sighting and recording the direction on the azimuth station, the remaining control stations were observed and readings recoried in a clockwise direction. A horizon closure was obtained on the infitial sighting with a rejection linit of three seconds. The telescope was then inverted and the process repeated. This comprised one set of readings. Three sets of readings were obtained yielding six total observations for each direction. The initial setting on the azimuth station was varled on each reading by approximately $1 / 6$ of $360^{\circ}$ or $60^{\circ}$ to insure that errors in the circle were eliminated. Ifkewise the minute and second settings were varied. The angles between the control points were detemined by subtracting
the initial direction from the following direction. The six angles those detemined between ang two control points were averaged with the average value being used in subsequent computations. The criteria was established prior to observations that the maximum allowable difference between mean value and an observation would be four seconds or the observation would not be counted. All observations were within this lindt. The standard error of each angle was computed and found to be less than one second in all cases.

For distance measurement, the Ceodineter was set up on the existing presignalized control point. The targets were placed on the remaining control stations. The distances were then deteruined using the standard Geodimeter operating procedures. Some trouble was encountered with the light source and later on with the sensitivity of a recording meter. Both troubles were corrected by the author under the direction of Professor Veres. The Geodimeter used was a demonstration model which had not been used for any actual measurements for some time prior to the author's use. This was in all probability the cause of the trouble. The results, although difflcult to obtoin, were believed to be very good. The basis of this belief is that three measurements are actually made during one set of readings. If close agreement is obtained on these three readings, them the instrument, according to manufactarer's reports, must be functioning properly and the distance must be good. There was however some question raised at a later date about the quality of one distance made in test area one. This will be discussed in the section analsing the results of test area one.

## Computation

After the fleld work was comploted, the next step was to comprite the state plane coordinates of all control points. The bearings of all lines radiating from the presignalized control point were determined from the azimuth line. By knowing the bearing and distance to each control point, it was very easy to determine the difference in $X$ and $Y$ coordinates by simple trigonometry. The differences in $X$ and $I$ were then added or subtracted remembering that northerly and eastorly differences from the indtial station are positive.

## Vertical Control

The vertical control for the photographs was obtained using a $2 e 1 s s$ self-leveling level. This level is equipped with a compensating mechanism that automatically levels the telescope when a bulls-eye bubble level is centered. In the determination of the levels for test area one and two, a single line of differential levels was rum. It involved 32 instrument setups with an average distance of 400 feet between setups and took only $21 / 2$ bours. This is less than five minutes per setup. The closure error was 0.01 feet. Test area three, boing located within a region of plowed fields and pastures, consumed more time. A closure error of 0.03 feet was obtained.

## FIFID CHECK SURVEY

A' complete and accurate ground survery was completed in order to have information with which to compare the photogrametric data. It was desirod to have the survey as accurate as possible and jet not be too tire consumag.

For this reason it was decided to use a Wild T-la Theodolite for angle measurenent and an engineerg' 200-foot steel tape for distance measurement. The wild T-la is read directly to one minute with estimations of one tenth of a minute or six seconds being possible. An extra foot at the beginning of the steel tape and the last foot were both divided into tenths and hundredths of a foot. The tape was compared to a Geodimeter measured distance both before and after the fleld work in order to know the exact length of the tape.

As mentioned previously, the points selected for test purposes are shown in Figures 1 to 3 for the respective test areas. The points were selected by inspection of the aerial photographs. The purpose was to deIne logical land areas with the selected points.

Test areas one and two consisted entirely of sidewalk intersections. Nails were placed in the designated corner of the sidewalk intersections in order to have a definite point from which to measure. The distance between the points was determined using the above mentioned steel tape. The following procedures were strictly adhered to in all distance measurement:

1. Line was always defined by sidewaik edge or by transit.
2. Tape was always held flat to sidewalk with slope corrections applied where applicable.
3. Temperature was recorded and the correction applied.
4. Incorrect length of tape correction was applied.
5. Measuring marise were carefully scribed in oldewalk.
6. Tension on the tape was kept as constant as possible on all measurements although a tension gauge was not used.
7. Same personnal did the taping on all lines including the comparison with the Geodimeter distance.

The author feels that, although the above procedures are definftely not flirst order work, they are of a quallty which gives a good check on the distances obtained photogrametrically.

The angles required in the area detemination were obtained using a T-la Theodolite. The angles were initialed with a zero reading on the circle, turned, and doubled to provide a check on the work. If one half the doubled angle differed from the flrst angle by more than 0.15 manute (9 seconds) then the angle was repeated until the agreement was satisfactory. A total of 33 areas were involved in the first two test areas, and only five areas had an interior angle closure error of more than one minute. The maximum closure error was 108 seconds in an area where bushes were on line for two out of the four angles.

Each individual area was treated as an ordinary traverse. The area was determined by double parallel distance after being adjusted by the compass rule. The worst closure ratio was $1 / 6,500$ for a 17 sided Agure. The best closure ratio was 1/147,000. Only three areas of the 33 had closure ratios less than $1 / 10,000$. The average ratio was $1 / 40,500$.

In test area three, the polats consisted entirely of feace posts. This required the use of offset atakes in order to determine the area. A traverse similar to that of tast areas one and two was established around the perimeter using the offset stakes. At each offest stake the angle from the previous point was turned to the fence post in question. The distance between the fence post and offset stake was measured. The inftial traverse was closed and adjusted the same as above. Arbitrary coordinates were assigned to the first point and coordinates of the other offset stakes were determined. It was then a simple matter to determine the coordinates of the center of each fence post. By knowing the fence post coordinates It was possible to determine the distance between posts and, by using area by coordinates method (9), to determine the area enclosed by the posts. This total area was divided into three smaller areas for analysis parposes.

It should be mentioned that there was no check on the angles and distances from offset stakes to fence posts, other than by repetition. It was also difficult to detemine the center of the fence post. The base was used and the center determined as close as possible by measurements, but the posta were of different size and shape and were often irregular in shape.

## PHOTOGRAPHY

The aerial photography used in the atudy was obtained in eariy April, 1963 by Chicago Aerial Survegs Company. They flew at 1,500 feet above average terrain. The nominal focal length of the camera was six inches. This provided a photograph scale of $1 / 3,000$ or 1 inch represents 250 feet. The photographs were taken with a standard panchromatic f17m. The diapositives were printed amision down on $91 / 2 \times 91 / 2 \times 1 / 4$ inch plates. The photographs and diapositives were of excellent quality.

The camera used was a Zeiss Aerotopograph Precision Camera, type RKM 15/23. The camera was equipped with a Carl Zeiss Pleogon Lens. The equivalent focal length of the camera is 153.07 millimeters and the calibrated focal length is 153.10 millimeters. The radial distortion characteristics of both equivalent and calibrated focal lengths are illustrated in Figure 4.

Note:
The values given in Flgure 4 were taken from a copy of the report sent to Chicago Aerial Surveys by the United States Bureau of Standards in Washington, D. C. The values given in the report are believed to be in error by one decimal place. The report gave the madmum distortion for equivalent focal length as 0.05 millimeters ( 50 microns). This is more than the worst of the modern aerial camera lenses. The maximure distortion for the Fleogon lens has been determined to te 0.005 millimeters ( 5 microns) (10). If the decimal point was moved one place to the left in all values given by the Bureau of Standards report, then the results would be reasonable. It is believed that there was a typographical error in the Bureau of Standards Feport. This belief is based on Schwidefsky's determination of the five micron maximum distortion for the Pleogon lens (10) and an

DISTORTION REFERRED TO EOUIVALENT FOCAL LENGTH $\begin{array}{llllll}7.5^{\circ} \\ 000 & \frac{15^{\circ}}{0.00} & \frac{22.5^{\circ}}{0.01} & \frac{30^{\circ}}{0.02} & \frac{37.5^{\circ}}{0.04} & \frac{45^{\circ}}{0.05}\end{array}$


DISTORTION REFERRED TO CALIBRATED FOCAL LENGTH $\begin{array}{llllll}\frac{7.5^{\circ}}{0.00} & \frac{15^{\circ}}{0.00} & \frac{22.5^{\circ}}{0.00} & \frac{300}{0.00} & \frac{37.5^{\circ}}{0.02} & \frac{45^{\circ}}{0.02}\end{array}$


ANGLE FROM OPTICAL AXIS (DEGREES)
FIGURE 4
RADIAL DISTORTION CURVES
advertisement from the Zeiss Compang which states that the naximon distortion for the Pleogon lens is four microns (11). If the maxdmum distortion is taken as five microns, then the distortion is considered negligible and the lens is considered a distortion free lens.

## KELSH PLOTTER AND ATTACIARNTS

The Kelsh Plotter was the basic instrument used in this research. It was, however, altered somewhat to make for a mathematical solution rather than a graphical solution. The major addition was that of a Coradi $32 \times 32$ inch manual coordinatograph. Anotber addition was that of a scale, vernier, and index marks for the purpose of measuring the instrunent base. A third addition was that of a mall light which was used in reading the coordinatograph in order to avoid tuming on overhead lights and disrupting the readings. The Kelsh Plotter and attachments are shown in Figure 5.

## Kel.sh Plotter

The Kelsh Plotter is probably the most used stereoplotting instrument in operation in the United States today. The viewing system is based on the anaglyph principle (12). This means that a narrow beam of red monochromatic light is projected through a glass diapositive and projector lens to the platen of the tracing table. At the same time a blue monochronatic Light is projected through another diapositive and projector lens to the platen of the same tracing table. The diapositives overlap by 60 percent. It is this overlap portion that is used and referred to as a stereoscopic model. The diapositives and their supports must be placed in the same relative position to each other as the camera was when the photographs were taken. When this is achieved, the red and blue images can be focused by

raising or lowering the platen. By dewing the images through spectacles having fllters of the corresponding red and blue colors, a three-dimensional model is perceived. The movable tracing table has a floating mark which is used in the measurements. The working area of the model is enlarged five times that of the diapositives by the projection lenses. For a complete description and discussion of the Kolsh Plotter, the reader is referred to reference (12), a manual on the Kelsh Plotter.

## Coordinatograph

The coordinatograph is a drafting type instrument consisting of $X$ and $Y$ carriages which are at right angles to each other. The X-carriage moves on the Y-carriage. A provision is made for reading the distance moved by the X-carriage. The X-carriage has a movable assembly on which a pencil or pen could be attached for drafting purposes. This assembly also has a provision for reading the distance moved.

The coordinatograph was placed on the working surface of the Kelsh Plotter with the Y-carriage nearly parallel with the Y-ards of the Kelsh instrument (Figure 5). The tracing table of the Kelsh was attached to the movable assembly on the X-carriage by a specially machined piece of metal in a fashion so that the tracing table and the movable asserbly moved as a unit.

The usable dimensions of the coordinatograph were $32 \times 32$ inches. A full model is normally about $30 \times 45$ inches. The coordinatograph width of 32 inches is enough in the $X$-direction to make the complete width of a model usable. In the Y-direction, however, 32 inches is not enough to permit full usage of the model. Only about two thirds of the model can
be used in the I-direction. It is reconmended that anyone plaming on placing a coordinatograph on a Kelsh Plotter should obtain at least a $42 \times 42$ inch size.

Distances are measured on the coordinatograph by notine the difference in $X$ and $Y$ coordinates between two points and computing the distance. The Coradi Coordinatograph has 200 divisions por inch, which means the least count of the reading is 0.005 inchez. This is read by use of an attached magnifying glass which allows for estimations to one half of the least count. Different scales can be introduced into the Coradi Coordinatograph. The one used in this project was the same as the model scale or 1 inch represents 50 feet.

## Light

A small light was placed on the Kelsh so that the readings could be read from the coordinatograph without lighting up the whole room. This reduced the reading time considerably as there was no need to wait for the pupils of the eyes to adjust to the dark again after making a coordinate reading. The light consisted of a small six volt bulb with reflector attached to a movable cord. The light was controlled by a simple toggle switch located on the Kelsh assembly near the Y-carriage of the coordnatograph (Figure 5). It was wired through the stepdown transformer from which the tracing table operates.

## Base Measurement

For the numerical corrections which were applied to the machine coordinates, it was necessary to know the instrument base, or distance between projector lenses. The Kelsh Plotter is not equipped with such a
measuring device so it was necessary to build one. An index mark, which moves with the projector was placed on the right projector assembly oi the Kelsh*. A corresponding index mark was placed on the rectangular I-frame bar. This made it possible to move the projector to the index mark on the X-frame bar whenever desired.

A vernier constructed to match a normal 1:20 engineers' scale was attached to the projector assembly of the left projector. A sir inch 1:20 engineers' scale was then attached to the X-frame bar in a position so that the verufer and the scale matched. Both the scale and the index mark. were attached to the $X$-frame bar with glue. The vernier and index attached to the projector assemblies were dons so on a strip of metal. The metal was shaped so that it could be fastened to the top of the outside frame of the projector assenbly and have the vernier and index match their counterparts on the X-frame bar. The metal strips are held in place under the piston assembly which controls the Y-tilt motion. The vernier and index marks are made from white plastic with the marks scribed and filled with black ink. The scale was an ordinary six inch engineers' scale using the 1:20 ratio.

The scale and the index mark were fastened to the X-frame bar after a model had been oriented. This gave the proper positioning of the tro fixed units, as the base does not vary much on six inch, wide angle, large scale photography. It was then necessary to determine the distance between optical centers of the projectors when the index marks of the right projector were coincident and the zero of the vernier was coincident with the zero of the scale on the left projector (referred to as base constant in future discussions). This was accomplished after the Kelsh had been

[^1]completely calibrated and the coordinatograph was functioning. The callbration of the Kelsh will be discussed in the next section.

The author used the following procedures in determining the base constant:

1. Very carefully placed grid plates in their plate holders and placed them on their projectors.
2. Placed the swing motion in the center of its motion and turned the projector until approjimately parallel with X-frame bas.
3. Matched index marks and the vernier and scale marke.
4. Removed $b x$ motion to insure no accidental movement.
5. Removed color filters from lamp asemblles.
6. Leveled X-frame assembly using 25 second level.
7. Leveled individual projectors using 25 second level.
8. Turned on the light of one lamp.
9. Positioned tracing table under center grid plate intersection so that the floating mark matched the intersection.
10. Locked elevation movement to insure no error (ahould be none if tracing table is in proper adjustment).
11. Read $X$ and $Y$ coordinates from coordinatograph and recorded them.
12. Moved tracing table from intersection and replaced it three more times making four readings.
13. Turned swing motion to one end of its motion and made three more readings. This was to insure that the optical center was being used.
14. Turned swing motion to other end of its motion and made three more readings.
15. Averaged the 10 readings for $X$ and $Y$ coordinates and obtained the mean values for optical center of the first projector.
16. Moved the tracing table to the second projector, repeated the readings, and obtained the mean values.
17. Determined the difference between $X$-readings and $Y$-readings.
18. Used the Pythagorean Theorem to solve for distance (X-bar of coordinatograph was not quite parallel to $X$-axis of Kelsh).

This procedure gave the base constant when the vernier and marks coincided. The instrument base for any model was determined by subtracting the reading on the scale from the base constant.

The 1:20 scale was selected for two reasons. It was large enough so that a vernier could be constructed by hand, and jet small enough so that the minimu desired reading of 0.01 inches could be easily obtained. A large amount of the credit for the design and construction of the base measuring apparatus belongs to Professor Veres.

## MEASURGMENTS FROM KEUSH

## Calibration

Before any measurements could be made from the Kelah, a complete calibration had to be performed. A discussion of the procedures involved will nat be given. However, if the reader is interested, he is referred to reference (12), a manual on the Kelsh Flotter. The calibration included the checking and adjustment, where necessery, of the following itens:

1. Slate levelness
2. Perpendicularity of diapositive plane with optical axis
3. Fiducial marker tabs and principal point
4. Principal distance
5. Agate foot pads of tracing table

The manual on the Kelsh Plotter gives steps to follow and the allowable tolerances for each of the above calibration items.

Distortion free projection lenses were placed in the Kelsh prior to calibration. The camera also had a distortion free lens and the diapositives were printed emulsion down. This then is a distortion free system and therefore the distortion correction cams on the Kelsh were disconnected.

## Principal Distance Determination

The depth rod method of setting the principal distance, as given in the Kelsh manual, was not used. It was felt that the uncertrinty of the distance from the nodal point to the top of the lens was too much and
introduced too much error for a numerical analysis. The method uzed will therefore be described.

Figure 6a shows a sketch of an end Niew of the Kelsh Plotter showing the slate surface and one projector along with the light rege from pointe "a" and "b" on the diupositive. The image "a" on the diapositive passes through the nodal point of the projection lens "o" and continues in a straight line and intersects the slate surface at $A_{1}$. The image ray strikes the lowest position of the tracing table platen at $A_{2}$ and likewise strikes the platen at its highest position at $A_{3}$. When the image ray strikes on the platen at $A_{2}$ and $A_{3}$ the points are projected vertically to points $A_{2}^{\prime}$ and $A_{3}^{\prime}$ respectively on the plotting surface. The distance $A_{2}^{\prime} A_{3}^{\prime}$ is also equal to $A_{2} L$. The Triangle $A_{3} L A_{2}$ is similar to triangle oca. In a like fashion triangle $B_{3} M B_{2}$ is similar to triangle ocb. But $B_{3} M=A_{3} L$ as they are both the difference betwoon the high and low positions of the tracing table platen, and oc is common to both triangle oca and ocb. Therefore, triangle abo is similar to a triangle formed by placing triangles $\mathrm{B}_{2} \mathrm{~B}_{3} \mathrm{M}$ and $A_{2} A_{3} I$ back to back as shown in FIgure $6 b$. Therefore co: $\left(B_{3} M=A_{3} I\right)$ as $a b:\left(B_{2} M+L A_{2}\right)$. By knowing $a b,\left(B_{3} M=A_{3} L\right), B_{2} M$, and $L A_{2}$ it is possible to determine co or the principal distance.

The distance $a b$ can be predetermined on a grid plate. $\left(B_{3} M=A_{3} L\right)$ is the vertical movement of the tracing table platen from its low to high position and can be detemined from the elevation counter. $B_{2} M$ and $A_{2} I$ can be measured on the plotting surface from the pertical projections of $A_{2}, A_{3}, B_{2}$ and $B_{3}$.

This method of determining the principal distance is no better than the depth rod method unless the measurements are performed very accurately.

a. SIDE VIEW SKETCH OF KELSH PLOTTER

b. SIMILAR TRIANGLES

FIGURE 6

For this research project $B_{2} M$ and $A_{2} L$ were measured on the coordinatograph. $\left(B_{3} M=A_{3} L\right)$ was measured from the elevation counter with the gear ratio such that the last number read 0.10 feet on a 1 inch represents 50 soot scale. A negative grid plate was used on which the relative positions of the grid intersections were determined by a Nistri Stereocomparator to an accuracy of $\pm 4$ microns. Therefore ab was accurately known.

The procedure is to determine the above mentionod distances and compute $c o=$ principal distance. By comparing the actual principal distance with the focal length of the camera it is possible to determine the amount of error. The principal distance is then changed to agree with what it should be, and a check is made to be sure it is correct. Most likely two or three checks will be made before reaching the correct value.

## Measurenents

Work on the test models was begun after the calibration was completed. All three test models were graphically plotted prior to the numerical solution. These formed the base maps for Figures 1-3.

In the numerical solution $9 l l$ models were interior and relatively oriented as described in the Kelsh manual (12). The leveling of the model was also as described in the mamal. As the primary interest of the project was distances and areas based on coordinates, it was deemed advisable to use numerical absolute orientation in regards to scaling. Coordinates of each control point were detemined and the distances between tine points computed. If the computed distance did not match the ground distance the model was enlarged or reduced until an exact flt was established.

When the orientation procedures and momerical scaling were completed to satisfaction, the roadings on the test points were begun. The readings consisted of two using normal vision and two using pseudo vision. The pseudo vision readings were necessary in order to use the correction formulas. These will be discussed in the next section. Each reading in normal and pseudo vision consisted of detormining the $X$ and $I$ coordinate of each test point.

A repeatability check was made 38 an additional check on the photography, Kelsh system, coordinatograph, and operator. It was found that on a presignalized control point the $X$ coordinate could be determined with a standard error of 0.057 feet. The standand error in the I direction was 0.071 feet. For a sidewalk intersection the standard error in $X$ was 0.155 feet and in $Y$ was 0.075 feet.

## CALCULATIONS PERFORMED ON MELSH DATA

## Corroction Formulas

There were two sets of calculations performed on the Kelsh data. The first set was with corrections and the second set was without corrections. The corrections consisted of mathematical formulas which were applied to both $X$ and $Y$ coordinates in an attempt to correct various inherent photogrammetric errors. The errors referred to could be causod by incorrect orientation, atmospheric refraction, air turbulence, or nany other possible sources. Nobody knows for sure 271 the causes of the inherent error for ang one photogrammetric model. Several noted photogramatrists have derived correction formulas with the assumption that the inherent errors are due to relative orientation. Professor Sandor A. Veres has derived formulas which attempt to correct the coordinates regardless of the cause of the error. The derivation of his formulas was presented in Photogrametric Engineering in January 1964 (13).

In Professor Veres' derivation, he indicates that the actual coordinate position on a model differs from the true position. The differance is caused by the various inherent errors as mentioned above. If a model would be oriented using both normal and pseudo vision, the results would differ. If, however, a model's orientation was the average of that using normal vision and that using pseudo vision, then the actual location of a point would be much closer to its true position. In first order instruments
the time involved for the second orientation makes the process uneconomicel. In second onder instruments, euch as a Keleh Plotter, there are no profisions for reading the orientation elements and, therefore, no way to dotermine an average orlentation. For these reasons Professor Veres derived his formulas for use with normal vision orientation only, whlle atilizing both normal vision and pseudo vision Kelsh readings. He derived his formulas with the understanding that the peeudo fision position of a point is in error by considerably more than is the normal fision position of a point. Because the pseudo vision position of a point is in error by considerably more than the normal vision position of a point the normal Fision coordinate readings and pseudo vision coordinate readings camot be averaged when only normal vision orientation has taken place.

Professor Veres' formulas were derived for and tested on a Whld A-7 Autograph. The formulas contain certain approximations. However, tests indicated the formulas can help the position of a point up to 40 percent. His X-coordinate correction formula is

$$
X_{p}=0.5\left(X_{p}^{\prime}+X_{p}^{\prime \prime}\right)+0.5\left[\frac{X_{p}^{\prime}}{b}\left(\frac{X_{p}^{\prime \prime}-X_{p}^{\prime}}{2}\right)+\frac{X_{p}^{\prime \prime}}{b}\left(\frac{X_{p}^{\prime \prime}-X_{p}^{\prime}}{2}\right)+\frac{X_{p}^{\prime \prime}-X_{p}^{\prime}}{2}\right] \ldots \text { (a) }
$$

where $\quad X_{p}=$ corrected $X$-coordinate
$X_{p}^{\prime}=X$-coordinate from normal vision
$X_{p}^{\prime \prime}=X$-coordinate from pseudo Vision
$b=$ instrument or airbase (dependent upon how coordinates are being read; for examle, ground scale or instrument distance)

The wild A-7 has a provision for switching the optical train so that an observer sees a $90^{\circ}$ rotation in the position of the model. This means that what was X-motion is now Y-motion. Therefore, it is possible to read
the Y-coordinate of a point by making the $90^{\circ}$ rotation and reading the X-coordinate from the instrument. Because of this feature the Y-coordinate correction formula was identical to the $X$-coordinate correction formala.

Unfortunately, the above is not true for the Kalsh Plotter. It was therefore necesaary to derive a correction formula for the Y-coordinate. The following derivation is for the $Y$-coordinate correction for use on the Kelsh Plotter. It was adapted by Professor Veres from his previous work.

From Flgure 7, $P$ represents the true location of a point, $P^{\prime}$ represents the normal Vision location, and $P^{\prime \prime}$ represents the pseudo Vision location of the point. $X_{p}^{\prime}$ and $Y_{p}^{\prime}$ represent the nornal vision model coordinates and $X_{p}^{\prime \prime}$ and $Y_{p}^{\prime \prime}$ represent the pseudo Vision model coordinates. $X_{p}$ and $Y_{p}$ represent the true coordinates of the point $P$.

It can be seen from Flgure 7 that

$$
\begin{equation*}
Y_{p}=0.5\left(Y_{p}^{\prime}+Y_{p}^{\prime \prime}\right)+\Delta Y \tag{b}
\end{equation*}
$$

From similar triangles

$$
\begin{equation*}
\Delta Y=\Delta X\left(\frac{Y_{p}^{\prime}-Y_{p}^{\prime \prime}}{X_{p}^{\prime \prime}-X_{p}^{\prime}}\right) \tag{c}
\end{equation*}
$$

From X-coordinate correction equation (a) it can be seen that

$$
\Delta X=0.5\left[\frac{X_{p}^{\prime}}{b}\left(\frac{X_{p}^{\prime \prime}-X_{p}^{\prime}}{2}\right)+\frac{X_{p}^{\prime \prime}}{b}\left(\frac{X_{p}^{\prime \prime}-X_{p}^{\prime}}{2}\right)+\frac{X_{p}^{\prime \prime}-X_{p}^{\prime}}{2}\right]
$$

If it is assumed that

$$
\frac{x_{p}^{\prime}}{b}\left(\frac{x_{p}^{\prime \prime}-x_{p}^{\prime}}{2}\right)=\frac{x_{p}^{\prime \prime}}{b}\left(\frac{x_{p}^{\prime \prime}-x_{p}^{\prime}}{2}\right)
$$



FIGURE 7

CORRECTION OF Y-MODEL COORDINATE
then $\Delta x=0.5\left[\frac{x_{p}^{\prime}}{b}\left(x_{p}^{\prime \prime}-x_{p}^{\prime}\right)+\frac{x_{p}^{\prime \prime}-x_{p}^{\prime}}{2}\right]$
From use, the above assumption has proven to be very nearly true. Substituting (d) in (c)

$$
\Delta Y=0.5\left[\frac{X_{p}^{\prime}}{b}\left(X_{p}^{\prime \prime}-X_{p}^{\prime}\right)+\frac{X_{p}^{\prime \prime}-X_{p}^{\prime}}{2}\right] \frac{Y_{p}^{\prime}-Y_{p}^{\prime \prime}}{X_{p}^{\prime \prime}-X_{p}^{\prime}}
$$

## Simplifying

$$
\begin{equation*}
\Delta Y=0.5\left[\frac{X_{p}^{\prime}}{b}\left(Y_{p}^{\prime}-Y_{p}^{\prime \prime}\right)+\frac{Y_{p}^{\prime}-Y_{p}^{\prime \prime}}{2}\right] \tag{e}
\end{equation*}
$$

Substituting (e) in (b) Jields the flnal result

$$
\begin{equation*}
Y_{p}=0.5\left(Y_{p}^{\prime}+Y_{p}^{\prime \prime}\right)+0.5\left[\frac{X_{p}^{\prime}}{b}\left(Y_{p}^{\prime}-Y_{p}^{\prime \prime}\right)+\frac{Y_{p}^{\prime}-Y_{p}^{\prime \prime}}{2}\right] \tag{f}
\end{equation*}
$$

Equations (a) and ( $f$ ) were the ones used in the computations.
As was previously mentioned, the correction formulas were derlved to correct for a portion of the errors which show up on a model as parallax. The results of this research showed that for a photography scaie of 1 inch represents 250 feet there is a possibility of overcorrection by using the correction formulas. On two of the test reodels, only a very small amount of residual parallax was present. The correction formilas overcorrected slightly for these models. The third test model contained significant residual parallaxes. In this model the correction formulas improved the results by about 35 percent. It can therefore be concluded that the correction formulas should be used when significant residual parallaxes are detected by the steroplotter operator and should not be used when significant residual parallaxes are not detected.

Significant residual parallaxos are not readily defina ble on the Telsh Plotter. By significant, the authr refers to when residual parallaxes are evident to the stereoplottor operator. Residual parallaxes show up as a blurrod plotting mark when the glasses are worn or as a slight separation of the colored images in the I-direction when glasses are not worn. A difference in elevation near the point being measured also causes parallax although in some instances it is not readily detected by the stereoplotter operator.

The use of the correction formulas is therefore dependent on the judgment of the stereoplotter operator. The overcorrection is small compared to the possible improvement, however, and the correction formulas should therefore be used if it cannot be clearly decided whether or not the corrections are needed.

Computations

The computations using corrections consisted of the following steps:

1. Make corrections to the model coordinates using the above correction formulas.
2. Using the corrected coordinates, transform the machine coordinates to ground coordinates using the following transformation formulas (14).

$$
\begin{aligned}
& X=A_{X}^{\prime}+B_{Y}^{\prime}+C^{\prime} \\
& Y=A_{Y}^{\prime}-B_{X}^{\prime}+C^{\prime \prime}
\end{aligned}
$$

where $X$ and $Y=$ ground coordinates of a point
$X^{\prime}$ and $Y^{\prime}=$ machine coordinates of the same point
$C^{\prime} \quad=$ translation constant in X-diroction
$C^{\prime \prime} \quad=$ translation constant in Y-direction
$A$ and $B=$ transformation constants representing a rotation and a chango in scale.

See Appendix A for an explanation of tho above formulas and tomes.
3. Compute distances from the transformed coordinates using the Pythagorean Theorem.
4. Compute areas from transformed coordinates using area by coordinates method (9).

The computations without corrections were basically the same as those with corrections only the corrections were not made. The nodel coordinates were transformed to ground coordinates and the distances and areas computed in the same manner as before.

A sample computation showing all steps from the Kelsh data to the computed distances and areas is given in Appendix. B. The example is computed using the correction formulas. If correction formulas are not used, only the normal vision readings should be used.

## RESULTS AND ANALYSIS OF DATA

## Test Area One

This test area consists primarily of city and urban lot sizo parcels. The smallest area determined was 7,800 square feet. The average parcel size was 25,000 square feet. This is slightly larger than the ordinary urban lot, but should give an idea of the expected results. The test area consisted of 27 individual areas and 96 distances. The points used were, as a whole, very sharp and well defined.

There were originally three control points located in the test area. They were (Figure 1): 33, a white cross; 33A, a sidewalk intersection; and 30A, a fire hydrant. When performing the numerical absolute orientation, It was discovered that the control was erroneous by about 0.50 feat. This error had not shown up in the graphical plotting of the model. After much computational checking, with little success, it was decided that such a sanall error could be made negligable by using the coordinate transformation (14), if sufficient control points were used in the transformation. The coordinate determinations were therefore made with the existing scale error. Two additional definable points were designated as control points and their ground coordinate positions were determined. The T-2 Theodolite was used for angle measurement but the Geodimeter was not available. It was necessary therefore to use the 100-foot engineers' tape for distance measure. The lines were measured three times each, using the same criteria as was
listed for the check surveys. The additional points were both oidenalk intersections and are given as 47 and 54 in Figure 1.

One of the characteristics of the coordinate transformation is that the control points used in the determination of the constants should, when detemining the transformed machine to ground coordinates, agree with the actual ground coordinates. If they do, then it can be considered that the determination of the ground coordinates and the machine coordinates are free of blunder. This is not referring to the scale error, as the transformation eliminates most of it.

Since the control was uncertain in this model, it was decided to try various combinations of three and four control points to see which combination gaw the best fitting solution. Point 33, the white cross, was used In all the combinations as it was the point from which the control was established, and also was the best defined point on the model. It was found that use of 33,41 , and 54 gave, by far, the best fltting solution. It could, therefore, be concluded that somehow there were errors, either ground or model, in points 33 A and 30A. A fleld check indicated that the ground position of 33 A was good but showed a discrepancy of 0.46 feet in the distance used to determine the position of 30A. This result agreod with the transfomation results so the error in this point was considerod found. The discrepancy in point 33A was attributed to a possible spread of the images of the sidewalk in the emulsion of the diapositive. This is feasible as the sideralk was very light in color and the surrounding grass area was very dark. Inage spread often occurs in these circumstances. The transformation results agreed with this conclusion.

Four readings were made on each test point, two with normal vision and two with pseudo vision. This is twice what would be used in practice. If corrections were used, one reading each of normal and pseudo Fision would be satisfactory. If corrections were not to be used, then two nomal readings are sufficient.

The distances computed from the test point coordinates are given in Table 1. Distances were computed both with corrections and without corroctions. These distances were compared to fleld distances and the residual errors are given. The standard residual error using corrections was $\pm 0.265$ feet. Without corrections it was $\pm 0.250$ feet. The distances without corrections were 9 percent better than the distances with corrections. It is to be noted that 75 percent of the distances without corrections had residual errors of less than 0.30 feet. The maximurn residual error without corrections was 0.62 feet and with corrections was 0.64 feet. Without corrections, 57 percent of the distances had resicual errors of less than 0.20 feet and 35 percent of less than 0.10 feet.

It should be pointed out here that the error distribution has nothing whatsoever to do with the length of the line. For example, distance 54-53 is $4 山 4$ feet long and has a residual error of -0.26 feet without correction and -0.22 feet with correction. Distance 5-4 is 46 feet long and has a residual error of +0.20 feet without correction and +0.33 feet with correction.

Table 2 shows the areas as computed from coordinates and from fleld data. A comparison is made botween fleld areas and the two coorvinate areas. The residual error is expressed in square feet and 2150 as a percentage of the fleld area. The standard residual percent error without

TABLEE I
Comparison of Distances From Test Area One

| Points | Distances (feet) |  |  | Residual Error (feet) <br> (Fleld-Computed) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Compated |  |  |  |
|  | Fleld | With Correction | Without Correction | $\begin{aligned} & \text { With } \\ & \text { Correction } \end{aligned}$ | Without Correction |
| 1-6 | 92.71 | 92.47 | 92.48 | +0.30 | +0.23 |
| 6-5 | 134.80 | 134.91 | 134.88 | -0.11 | -0.08 |
| 5-4 | 46.69 | 46.36 | 16.49 | +0.33 | $+0.20$ |
| 4-3 | 49.42 | 49.26 | 49.25 | +0.16 | +0.17 |
| 3-2 | 45.62 | 46.10 | 45.64 | -0.48 | -0.02 |
| 2-1 | 83.45 | 83.53 | 83.54 | -0.08 | -0.09 |
| 6-7 | 126.08 | 126.64 | 126.34 | -0.56 | -0.26 |
| 7-8 | 137.41 | 137.33 | 137.32 | +0.08 | +0.09 |
| 8-5 | 126.29 | 126.08 | 126.38 | +0.21 | -0.09 |
| 7-12 | 92.37 | 91.95 | 92.27 | +0.42 | +0.10 |
| 12-11 | 90.03 | 89.97 | 90.39 | +0.06 | -0.36 |
| 11-10 | 45.60 | 45.36 | 45.77 | +0.24 | -0.17 |
| 10-9 | 49.17 | 49.05 | 49.07 | +0.12 | +0.10 |
| 9-8 | 46.80 | 46.61 | 46.67 | +0.19 | +0.13 |
| 12-16 | 64.10 | 64.27 | 64.15 | -0.17 | -0.05 |
| 16-15 | 76.38 | 76.64 | 76.66 | -0.26 | -0.28 |
| 15-14 | 47.17 | 46.89 | 46.62 | +0.42 | $+0.49$ |
| 14-13 | 59.97 | 59.84 | 59.98 | +0.13 | -0.01 |
| 13-11 | 32.57 | 32.73 | 32.25 | -0.16 | +0.32 |

TABLE 1 (continued)

| Points | Distances (feet) |  |  | Residual Error (feet) <br> (Fleld-Computed) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Computed |  |  |  |
|  | Fleld | With Correction | Without Correction | With Correction | Without Correction |
| 16-17 | 149.27 | 148.95 | 149.16 | +0.32 | +0.11 |
| 17-18 | 79.21 | 79.25 | 79.29 | -0.04 | -0.08 |
| 18-19 | 95.75 | 95.16 | 95.44 | +0.59 | +0.31 |
| 19-15 | 51.14 | 50.85 | 50.79 | +0.29 | +0.35 |
| 19-21 | 233.18 | 232.79 | 232.89 | +0.39 | +0.29. |
| 21-23 | 51.05 | 50.95 | 51.10 | +0.11 | -0.05 |
| 23-22 | 47.79 | 47.47 | 47.50 | +0.32 | +0.29 |
| 22-26 | 60.21 | 60.10 | 60.10 | +0.11 | +0.11 |
| 26-13 | 134.07 | 134.13 | 134.25 | -0.06 | -0.08 |
| 18-20 | 233.63 | 233.25 | 233.26 | +0.38 | +0.37 |
| 20-21 | 203.54 | 103.45 | 103.45 | +0.09 | +0.09 |
| 36-35 | 60.04 | 59.88 | 60.02 | +0.16 | +0.02 |
| 35-34 | 60.25 | 60.20 | 60.07 | +0.05 | +0.18 |
| 34-31 | 126.80 | 116.90 | 116.83 | -0.10 | -0.03 |
| 31-37 | 124.01 | 124.15 | 124.00 | -0.14 | $+0.01$ |
| 37-36 | 116.87 | 116.88 | 216.85 | -0.01 | +0.02 |
| 34-33 | 40.70 | 40.13 | 40.65 | $+0.57$ | +0.05 |
| 33-32 | 47.13 | 47.05 | 47.00 | +0.08 | +0.13 |
| 32-30 | 109.19 | 109.05 | 109.14 | $+0.14$ | +0.05 |
| 30-29 | 47.37 | 47.47 | 46.90 | -0.04 | +0.47 |
| 29-27 | 40.92 | 40.28 | 40.37 | $+0.64$ | +0.55 |

TABLE 1 (continued)

| Points | Distances (feet) |  |  | Residual Error (foet) <br> (Field-Computed) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Computed |  |  |  |
|  | Fleld | With Correction | Without Correction | With Correction | Without Correction |
| 27-28 | 216.84 | 116.94 | 216.91 | -0.10 | -0.07 |
| 28-31 | 195.23 | 195.15 | 195.16 | +0.08 | +0.07 |
| 27-26 | 64.05 | 64.55 | 64.46 | -0.50 | -0.4] |
| 23-24 | 69.04 | 69.18 | 69.18 | -0.14 | -0.14 |
| 24-28 | 120.07 | 120.00 | 119.80 | +0.07 | +0.27 |
| 20-25 | 69.07 | 69.13 | 69.14 | -0.06 | -0.07 |
| 25-24 | 156.90 | 156.63 | 256.95 | +0.27 | -0.05 |
| 35-2 | 166.60 | 166.51 | 166.47 | +0.09 | +0.13 |
| 38-39 | 216.36 | 216.17 | 116.33 | +0.19 | +0.03 |
| 39-43 | 234. 35 | 234.23 | 234.20 | +0.12 | +0.15 |
| 43-44 | 116.60 | 136.32 | 216.25 | +0.28 | +0.35 |
| 4-38 | 234.65 | 234.47 | 234.44 | $+0.18$ | +0.21 |
| 39-40 | 133.81 | 133.63 | 134.13 | +0.18 | -0.22 |
| 40-42 | 234.47 | 234.55 | 234.58 | -0.08 | -0.11 |
| 42-43 | 120.55 | 120.44 | 120.51 | +0.11 | $+0.04$ |
| 42-46 | 82.20 | 82.10 | 82.12 | +0.10 | +0.08 |
| 46-45 | 236.89 | 236.83 | 237.47 | +0.06 | -0.52 |
| 45-44 | 82.02 | 82.36 | 82.35 | -0.34 | -0.33 |
| 46-47 | 107.29 | 106.96 | 106.93 | +0.33 | +0.36 |
| 47-48 | 90.18 | 90.51 | 90.43 | -0.33 | -0.25 |
| 48-49 | 178.11 | 177.79 | 178.05 | +0.32 | +0.06 |

TABLE 1 (continued)

| Points | Distances (feet) |  |  | Fesicual Error (feet) <br> (Fleld Computed) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Computed |  |  |  |
|  | Fleld | $\begin{gathered} \text { With } \\ \text { correction } \end{gathered}$ | Without Correction | With Correction | Wh thout Correction |
| 49-45 | 173.74 | 173.90 | 173.85 | -0.16 | -0.11 |
| 40-54 | 314.32 | 314.18 | 313.70 | +0.14 | +0.62 |
| $54-53$ | 444.52 | Wh4. 74 | W44. 78 | -0.22 | -0.26 |
| 53-52 | 209.75 | 209.69 | 220.09 | +0.06 | -0.34 |
| 52-47 | 118.15 | 118.38 | 118.18 | +0.23 | -0.03 |
| 48-51 | 74.22 | 74.25 | 74.14 | +0.07 | +0.08 |
| 51-50 | 173.78 | 173.82 | 174.10 | -0.04 | -0.32 |
| 50-49 | 74.10 | 74.04 | 74.03 | $+0.06$ | +0.07 |
| 52-51 | 217.14 | 217.01 | 226.60 | +0.13 | $+0.54$ |
| 53-55 | 112.92 | 113.30 | 113.29 | -0.38 | -0.37 |
| 55-56 | 159.01 | 158.81 | 158.75 | $+0.20$ | +0.26 |
| 56-57 | 51.07 | 51.29 | 51.68 | -0.22 | -0.61 |
| 57-52 | 106.58 | 106.51 | 106.49 | +0.07 | +0.09 |
| 56-59 | 213.75 | 213.32 | 213.52 | $+0.43$ | +0.23 |
| 59-58 | 51.09 | 51.36 | 51.38 | -0.27 | -0.29 |
| 58-57 | 223.17 | 223.72 | 223.73 | -0.55 | -0.56 |
| 55-60 | 214.00 | 213.80 | 213.82 | +0.20 | +0.18 |
| 60-59 | 158.91 | 158.77 | 158.87 | +0.14 | $+0.04$ |
| 62-61 | 69.42 | 69.12 | 69.31 | +0.30 | +0.11 |
| 61-64 | 108.17 | 108.60 | 108.55 | -0.43 | -0.38 |
| 64-63 | 77.72 | 77.33 | 77.59 | +0.39 | +0.13 |

TABILE 1 (continued)

| Points | Distances (feet) |  |  | Residual Error (feet) <br> (Fleld Computed) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Computed |  |  |  |
|  | FYeld | With Correction | Wi thout Correction | With Correction | Whthout Correction |
| 63-62 | 107.07 | 107.51 | 107.37 | -0.44 | -0.30 |
| 67-68 | 311.34 | 310.99 | 311.20 | +0.35 | +0.14 |
| 68-65 | 130.45 | 130.49 | 130.56 | -0.04 | -0.11 |
| 65-66 | 315.55 | 315.47 | 325.59 | +0.14 | -0.04 |
| 66-67 | 130.29 | 230.54 | 130.55 | -0.25 | -0.26 |
| 70-69 | 295.47 | 295.24 | 295.47 | +0.17 | 0.00 |
| 69-68 | 60.00 | 59.84 | 59.78 | +0.16 | +0.22 |
| 67-70 | 55.72 | 55.54 | 55.54 | $+0.18$ | +0.18 |
| 71-72 | 294.52 | 293.97 | 294.31 | +0.55 | +0.21 |
| 72-69 | 123.02 | 122.93 | 123.05 | +0.09 | -0.03 |
| 70-71 | 122.90 | 123.00 | 123.01 | -0.10 | -0.11 |
| 74-73 | 309.30 | 309.35 | 309.71 | -0.05 | -0.47 |
| 73-72 | 66.04 | 66.47 | 66.46 | -0.43 | -0.42 |
| 71-74 | 61.25 | 61.25 | 61.25 | 0.00 | 0.00 |

96 distances

## $\underset{\text { Error }}{\underset{\text { Rean }}{\text { Mesidual }}}=\frac{\Sigma \mid \underline{L I}}{n}$

$\underset{\text { Error }}{\substack{\text { Standard } \\ \text { Residual } \\ \text { Ero }}}=\sqrt{\frac{\sum E^{2}}{n}}$

With
Corrections
$\pm 0.215$ foet
$\pm 0.265$ feet

Without Corrections
$\pm 0.196$ feet
$\pm 0.250$ feet

TABLE 1 (continued)

|  | W1th Corrections | Without Corrections |
| :---: | :---: | :---: |
| Distribution of Rosidual Errors (feet) | $\begin{aligned} & 1 \text { at } 0.64 \\ & 6 \text { from } 0.50-0.59 \\ & 7 \text { from } 0.40-0.49 \\ & 14 \text { from } 0.30-0.39 \\ & 13 \text { from } 0.20-0.29 \\ & 30 \text { from } 0.10-0.19 \\ & \frac{25}{96} \text { from } 0.00-0.09 \end{aligned}$ | ```I each at 0.62 and 0.61 4 from 0.50-0.59 5 from 0.40-0.49 13 from 0.30-0.39 17 1rom 0.20-0.29 21. from 0.10-0.19 34 from 0.00-0.09 96``` |
| Percent of Distance With a Residual Error of Less Than | With Correction | Without Corrections |
| 0.60 feet | 99\% | 98\% |
| 0.50 feet | 93\% | 94\% |
| 0.40 feet | 85\% | 88, |
| 0.30 feet | 71\% | 75\% |
| 0.20 feet | 57\% | 57\% |
| 0.10 feet | 26\% | 35\% |

TABLE 2

| Area Number | Area (square feet) |  |  | Residual Error (square feet) (Field-Computed) |  | ```Residual Error Expressed As Percent of Total Area = Residual Error ``` |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Field | Computed |  |  |  |  |  |
|  |  | With Corrections | Without Corrections | With Corrections | Without Corrections | WIth Corrections | Without Corrections |
| A | 10,128.47 | 10,124.97 | 10,108.63 | +13.50 | +19.84 | 0.13 | 0.20 |
| B | 17,172.18 | 17,198.42 | 17,195.15 | $-26.24$ | -22.97 | 0.15 | 0.13 |
| C | 10,532.56 | 10,485.75 | 10,536. 46 | $+46.80$ | +3.91 | 0.44 | 0.04 |
| D | 7,814.61 | 7,843.15 | 7,820.11 | -28.54 | $-5.49$ | 0.37 | 0.07 |
| E | 11,518.91 | 11,499.54 | 11,515.68 | +29.37 | + 3.23 | 0.26 | 0.03 |
| F | 47,762.58 | 47,766.63 | 47,742.46 | - 4.05 | +20.12 | 0.01 | 0.04 |
| G | 20,102.53 | 20,038.44 | 20,064.34 | +64.09 | +38.19 | 0.32 | 0.19 |
| H | 23,251.63 | 23,134.22 | 23,172.19 | $+117.47$ | +79.44 | 0.50 | 0.34 |
| I | 14,496.80 | $14,494.56$ | 14,489.59 | $+2.24$ | + 7.21 | 0.01 | 0.05 |
| J | 17,387.02 | 17,355.36 | 17,368.59 | +32.65 | $+18.43$ | 0.18 | 0.11 |

TABLE 2 (continued)

| Area Number | Area (square feet) |  |  | Residual Error (square feet) (Fleld-Computed) |  | Residual Error Expressed As Percent of Total Area $=$ $\frac{\text { Residual Error }}{\text { Field Area }} \times 100$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Fiela | Computed |  |  |  |  |  |
|  |  | $\begin{gathered} \text { With } \\ \text { Corrections } \end{gathered}$ | Without Corrections | With Corrections | Without Corrections | With Corrections | Without Corrections |
| K | 14,509.51 | 14,520.36 | 24,508.54 | -10.86 | +0.96 | 0.07 | 0.01 |
| L | 10,758.86 | 10,752.40 | 10,768.60 | +6.46 | - 9.74 | 0.06 | 0.09 |
| M | 27,314.68 | 27,247.70 | 27,198.84 | +72.98 | +115.84 | 0.27 | 0.42 |
| N | 29,788.66 | 29,751.97 | 29,818.59 | +36.68 | -29.93 | 0.12 | 0.10 |
| 0 | 19,472.70 | 19,492.67 | 19,519.74 | -19.97 | -47.04 | 0.10 | 0.24 |
| P | 39,396.07 | 39,365.94 | 39,415.66 | +30.13 | -29.59 | 0.08 | 0.05 |
| Q | 142,692.56 | 142,740.48 | 142,670.64 | -47.92 | +21.91 | 0.03 | 0.02 |
| R | 13,037.83 | 13,016.47 | 13,035.54 | +21.36 | + 2.29 | 0.16 | 0.02 |
| s | 10,971.19 | 12,025.51 | 21,008.73 | -54. 32 | -37.54 | 0.50 | 0.34 |
| T | 23,511.16 | 23,542.05 | 23,581.23 | -30.89 | -70.07 | 0.13 | 0.30 |

TABLE 2 (continued)

| Area Number | Area (square feet) |  |  | Residual Error (square feet) (Fleld-Computed) |  | Residual Error Bepressed As Percent of Total Area $=$ $\frac{\text { Residual Error }}{\text { Field Area }} \times 100$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Fleld | Computed |  |  |  |  |  |
|  |  | With Corrections | Without Corrections | With Corrections | Without Corrections | With Corrections | Without Corrections |
| U | 7,907.44 | 7,901.93 | 7,919.37 | + 5.51 | -11.93 | 0.07 | 0.15 |
| $\nabla$ | 11,106.38 | 11,161.60 | 11,206.49 | -55.23 | -100.11 | 0.50 | 0.90 |
| W | 33,997.47 | 33,928.09 | 33,932.30 | +69.38 | +65.16 | 0.20 | 0.19 |
| X | 40,856.61 | 40,869.94 | 40,907.91 | -13.33 | -51.30 | 0.33 | 0.13 |
| $Y$ | 17,213.61 | 17,153.39 | 17,160.90 | +80.22 | +52.71 | 0.47 | 0.31 |
| 2 | 36,264.98 | 36.220 .84 | 36,271.74 | +24.14 | - 6.76 | 0.07 | 0.02 |
| AA | 18,986.61 | 19,025.05 | 19,043.31 | -38.44 | -56.70 | 0.20 | 0.30 |

$\begin{array}{cc}\begin{array}{c}\text { With } \\ \text { Corrections }\end{array} & \begin{array}{c}\text { Without } \\ \text { Corrections }\end{array} \\ \pm 0.212 \% & \pm 0.177 \% \\ \pm 0.265 \% & \pm 0.256 \%\end{array}$
TABIE 2 (continued)

|  | W1th <br> Corrections | Without <br> Corrections |
| :--- | :--- | :--- |
| Distribution of <br> Residual Percent Ermor | 3 at 0.50 <br> 2 from 0.40-0.49 | 1 at 0.42 |

corrections is $\pm 0.256$ percent and with corrections is $\pm 0.265$ percent. The standard residual percent error without corrections is deceptively high as the presence of one area with a percent error of 0.90 percent is almost three times that of the next highest percent error. Pertaps a comparison of the mean reaidual percent error gives a bettor picture. The mean residual percent error without corrections is $\pm 0.177$ percent and with corrections is $\pm 0.212$ percent. The areas without corrections were 20 percent better than those with corrections.

Although the relatively small number of samples does not allow for a true picture of the error distribution, it should be noted that 81 percent of the areas without corrections and 70 percent of the areas with corrections had a residual percent error of less than 0.30 percent. If a 10,000 square foot city lot has a value without buildings of $\$ 5000$, than the error being referred to amounts to only $\$ 60$.

As might be axpected, the areas having the larger residual percent errors also are bounded by the distances having the larger residual errors. The error in position of the coordinate points is the factor which deter mines the distance and area error. It is therefore important to have the property corners presignallzed or othervise well defined.

The results of this test area show that distances can be determined to an expected accuracy of $\pm 0.250$ feet when the points are well defined. Lot size areas can be determined to an expected accuracy of $\pm 0.256$ percent. The error in the original photo control had no apparent effects on the results. The correction formulas did not help the results of this test model. The correction formolas are derived to help when errors show up as residual parallax. This model contained very little rosidual parallax and, therefore, the correction formulas were of no value.

Test area two consists of normal city blocks with from four to ten lots per block. The coordinates of the block comers were detornined and, from the coordinates, distances and areas of the blocks computed. The distances were all close to 300 feet and the areas near two acres in size. A method is given which relates street intersections or block comers to the individual lots. It will thers be shown how lots can be detorined from block determinations.

Four control points were located in this model, one of which could not be used because of the limitations of the coordinatograph. The control points used are 36 , white cross; 36 A and 35 A , sidewalk inter3ections. The numerical absolute orientation fit very well 80 it was assumed that no blunders were present in the fleld work.

The relative orientation of the model was excellent. The test points used were all sidewalk intersections at the block corners. In several cases there were tree lings or other shadows which made it difflcult to read the points. The results showed that the shadows did not effect the results. Again, as in the first test area, four readings were taken from the Kelsh, although only two would be necessary in practice.

The distances computed from the coordinates are shown in Table 3. The conputed distances, both with and without corrections, are given and compared to the field distances. The standard residual error using corrections was $\pm 0.268$ feet and without corroctions was $\pm 0.244$ feet. This means the distances without corrections were 10 percent better than those with corrections. The mean residual error, however, indicates that the corrections improved the results by 2 percent. It seems that the corrections

TABLE 3
COMPARISON OF DISTAYCES FPOM TEST AREA TWO

| Points | Distances (feet) |  |  | Rosidual Error (feet) <br> (Fleld-Computed) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Computed |  |  |  |
|  | Field | With Correction | Without Correction | $\begin{aligned} & \text { With } \\ & \text { Correction } \end{aligned}$ | Without Correction |
| $A-B$ | 332.64 | 332.31 | 332.24 | +0.33 | +0.40 |
| B-D | 283.99 | 284.05 | 283.83 | -0.06 | +0.16 |
| D-C | 328.76 | 328.75 | 328.66 | +0.01 | +0.10 |
| C-A | 284.13 | 284.47 | 284.32 | -0.34 | -0.19 |
| $\mathrm{E}-\mathrm{F}$ | 325.20 | 325.14 | 324.96 | +0.06 | +0.24 |
| F-H | 289.65 | 289.72 | 289.39 | -0.07 | +0.26 |
| H-G | 333.98 | 333.79 | 333.63 | +0.19 | +0.35 |
| G-E | 283.99 | 283.64 | 283.63 | +0.35 | +0.36 |
| I-J | 339.60 | 339.57 | 339.16 | +0.03 | +0.4 4 |
| J-L | 284.01 | 283.73 | 284.16 | +0.28 | -0.15 |
| L-K | 347.70 | 347.21 | 341.34 | +0.49 | +0.36 |
| K-I | 288.96 | 288.58 | 288.73 | +0.38 | +0.23 |
| $\mathrm{M}-\mathrm{N}$ | 323.82 | 323.68 | 323.98 | +0.14 | -0.16 |
| $\mathrm{N}-\mathrm{O}$ | 265.94 | 265.40 | 265.98 | $+0.54$ | -0.04 |
| O-P | 325.00 | 324.43 | 324.65 | $+0.57$ | +0.35 |
| P-M | 266.35 | 266.38 | 266.25 | -0.03 | +0.10 |
| Q-R | 321.16 | 321.13 | 327.29 | +0.03 | -0.13 |
| R-T | 266.11 | 265.99 | 265.98 | +0.12 | $+0.13$ |
| T-S | 321.23 | 320.92 | 320.83 | +0.31 | +0. 20 |
| S-Q | 266.27 | 266.07 | 266.15 | $+0.20$ | +0.12 |

TABLE 3 (contimued)

made very little change in the results of this model. Wthout correctione, 71 percent of the distances had a residual error of less than 0.30 leet and 58 percent had a residual error of less than 0.20 feet. The same flgures with corrections are 67 percent and 54 percent respectively. With corrections 38 percent of the distances had a resdual error of less than 0.10 feet while without corrections the figure was 13 percent.

It should be pointed out here that the magnitude of the residual distance error is about the same as that for test area one but the distances as a whole are much longer. The relative accuracy is therefore much better when longer distances are used.

Table 4 gives the areas of the blocks and the comparisons of the methods used. The standard residual percent error is $\pm 0.110$ percent without corrections and $\pm 0.134$ percent with corrections. Although the sampling is much too small to gain ano infomation from emor distribution, It should be noted that the madmum residual error was 0.20 percent without corrections and 0.22 percent with corrections. This is much better than the results of test area one. The reason is that the areas are much larger but the residual emor in the coordinates is the same. The conclusion can therefore be drawn that block size areas have a relatively small error in both distance and area.

It should also be noted that the correction formulas again did not help the results. As in test area one, the residual parallax was very slight. The general conclusion can be drawn from these two test areas that the correction formias should not be used with 1 inch represents 250 feet photography when the residual parallax is swall.
TABLE 4

| Area Number | Area (square feet) |  |  | Residual Error (square feet) (Fleld-Computed) |  | Residual Error Expressed As Percent of Total Area $=$ Residual Error $\times 100$ Fleld Area |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Field | Computed |  |  |  |  |  |
|  |  | With Corrections | W1 thout Corrections | With Corrections | Without Corrections | With Corrections | Without Corrections |
| 1 | 93,851.28 | 93,872.22 | 93,787.24 | -20.94 | +64.04 | 0.02 | 0.07 |
| 2 | 94,443.86 | 94,362.02 | 94,259.29 | +81.84 | $+184.57$ | 0.09 | 0.20 |
| 3 | 97,510.48 | 97,321.54 | 97,380.73 | +188.93 | +129.75 | 0.19 | 0.13 |
| 4 | 86,291.88 | 86,214. 23 | 86,256.08 | $+177.65$ | +35.80 | 0.21 | 0.04 |
| 5 | 85,454. 30 | 85,363.12 | 85,381.23 | +91.18 | +73.07 | 0.11 | 0.09 |
| 6 | 85,994.75 | 86,065.10 | 86,012.45 | -70.35 | -17.70 | 0.08 | 0.02 |
| 6 Areas |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Mean Residual <br> Percent Error = $\Sigma$ ! $\% \mathrm{El}$ |  |  |  | $\pm 0.116$ | $\pm 0.092$ |  |  |
| Standard Residual Percent Error$\square$ |  |  |  | $\pm 0.134$ | $\pm 0.110$ |  |  |

TABLE 4 (contimued)

|  | With Corrections | Without Corrections |
| :---: | :---: | :---: |
| Distribution of Residual Percent Error | $\begin{aligned} & 1 \text { at } 0.21 \\ & 2 \text { from } 0.10-0.19 \\ & \frac{3}{6} \end{aligned}$ | $I$ at 0.20 $I$ at 0.13 $\frac{4}{6}$ |
| Fercont of Areas With Residual Percent Error of Less Than | With Corrections | Without Corrections |
| $\begin{aligned} & 0.20 \\ & 0.10 \end{aligned}$ | $\begin{aligned} & 83 \% \\ & 50 \% \end{aligned}$ | $\begin{aligned} & 83 \% \\ & 67 \% \end{aligned}$ |

The next question is, how can individual lots be determined from block determinations? If the center-lines of all streets are located and presignalized prior to photography, then the Kelsh operator can detemine the coordinates of the strect intersections as well as the proposed highway. This gives a tie between the proposed highray control and the existing blocks and lots adjacent to the new highway. By knowing the coordinates of the street intersections it is possible to graphically and mathematicaliy compute the location of all lots within the block. This is done by use of the plats and deeds of the property in the block. By knowing the record distance between street intersections and comparing to the distance obtained from Kelsh determined coordinates, it is knom how well the block actualis fits record measurements. If the distances match within the expected tolerances then it is assumed the block is good and can be subdivided up into individual lots. The block is subdivided using the Kelsh detemined distance as the actual distance and proportioning the lot dimensions. This gives the lot comer coordinates on the same system as the proposed highway control.

A more advantageous situation arises if, in a certain area, it is known that the sidewalks were set a certain distance from the property lines. This eliminates the need for determining the center-lines of streets and presignalizing the points. The coordinates of the sidewalk comers can be determined and the same analysis computed as above.

If the Kelsh distance and record distance between street intersections do not flt within the expected tolerances, then it must be determined whether the Kelsh distance is incorrect or whether the block is not what the record indicates. This can probably be detemined from a comparison of the other distances in the immediate area.

The expected error in any one lot will depend upon at least two factnre. First, are all lots within the block as they are on the original plat? This is something which can be partially detemined by plotting the lots on the Kolsh-made base map. If fence lines, hodgos, drivowxys, and buildings agree within reason, then the error mado by using this method will be emall.

The second consideration is whether there are an original or otherwise controlling monuments within the block. If there are, do they fit the rest of the block as it exists?

If both of the above situations are favorable, then the error per lot would be in direct proportion to the error in the block. As the expected error in distance is 0.25 feet regardless of distance then we can expect this error in our total block length. Therefore if we have five lots in the block, each lot has an error of 0.05 feet. The error in area of a single lot is likewise proportional to the error in the block. It can be seen from the results that the expected error in block size areas is about 100 square feet. If there are ten total lots in the block, then there is an area error of ten square feet per lot. A ten square foot error in a 10,000 square foot lot amounts to only one-tenth of one percent of the total area.

The accuracy of the above method is acceptable in both distance and area for highway condomation purposes. The largest disadvantages to this method are the ever present possibilities of the block not being on the ground as it is in the record, and the prosence of controlling monuments which do not agree. It should be pointed out, however, that if the subdivision is on the ground as it is on the record, then all the monuments should likewise be in agrecment, and thus no significant errors will be caused by the proposed method.

## Test Area Three

This test area was conducted in rural land with all test points being fence poste. The fence poste used defined one large area which was difided into three smaller areas for comparison. A total of eleven fence posts were used as test points with thirteen distances being computed from the coordinates. It proved very difficult to deternine the axact position of the base of the fence poste. This was caused by overhanging brush, nearioy trees, and leaning posts.

The control points were also fence posts. Only two posts were used as control points because of the small number of posts. The two used were 7 A and 7 B as shown in Figure 3. The only trouble with the absolute orientation was the inconsistency in reading the posts. This was overcome by taking several readings and averaging them together.

The relative orientation was not very good. This condition was due primarily to local parallaxes caused by tufts of grass and weeds and small holes caused by cattle hoofs. Some of the test points were located in areas of local parallax and thus are affected. In areas where parallax exists, it is a good idea to take more readings in order to average out some of the error. Four readings were taken on each point; two normal vision and two pseudo vision.

The results of the distance comparisons are given in Table 5. The standard residual error without corrections was $\pm 0.692$ feet and with corrections was $\pm 0.469$ feet. The corrections therefore inproved the results of this area by about 32 percent. Seventy-seven percent of the distances with corrections had a residual error of less than 0.50 feet while the same figure for the distances without corrections was only 38 percent.

TABLE 5
COMPARISONS OF DISTANCES FROM TEST AREA THREE


13 Distances
$\begin{aligned} & \text { Mean } \\ & \text { Residual } \\ & \text { Error }\end{aligned}=\frac{\Sigma \mid E l}{n}$
$\begin{aligned} & \text { Standard } \\ & \text { Residual } \\ & \text { Error }\end{aligned}=\sqrt{\frac{\sum\left(E^{2}\right)}{n}}$

Without Corrections
$\pm 0.378$ feet $\pm 0.585$ feet
$\pm 0.469$ feet $\pm 0.692$ feet

```
TABLE 5 (continued)
```

|  | With Corrections | Without Corrections |
| :---: | :---: | :---: |
| Distribution of. Residual Error (feet) | $\begin{aligned} & 1 \text { at } 0.97 \\ & 1 \text { at } 0.78 \\ & 1 \text { at } 0.70 \\ & 0 \text { from } 0.60-0.69 \\ & 0 \text { from } 0.50-0.59 \\ & 2 \text { from } 0.40-0.49 \\ & 2 \text { from } 0.30-0.39 \\ & 3 \text { from } 0.20-0.29 \\ & 1 \text { from } 0.10-0.19 \\ & 2 \text { from } 0.00-0.09 \\ & \hline 13 \end{aligned}$ | 1 at 1.36 1 at 1.11 1 at 0.99 1 at 0.79 1 from $0.60-0.69$ 3 from $0.50-0.59$ 0 from $0.40-0.49$ 2 from $0.30-0.39$ 1 from $0.20-0.29$ 1 from $0.10-0.19$ $I$ from $0.00-0.09$ $I 3$ |
| Percent of Distances With a Residual Error of Less Than | With Corrections | Without Corrections |
| 1.00 feet <br> 0.90 feet <br> 0.80 feet <br> 0.70 feet <br> 0.60 feet <br> 0.50 feet <br> 0.40 feet <br> 0.30 feet <br> 0.20 feet <br> 0.10 feet | 100\% <br> 92\% <br> $92 \%$ <br> 77\% <br> $77 \%$ <br> $77 \%$ <br> 62\% <br> 46\% <br> $23 \%$ <br> $15 \%$ | $\begin{aligned} & 85 \% \\ & 77 \% \\ & 77 \% \\ & 6 \% \\ & 62 \% \\ & 38 \% \\ & 38 \% \\ & 23 \% \\ & 15 \% \\ & 8 \% \end{aligned}$ |

Table 6 show the comparison of the aroas. The standard resicual percent error was $\pm 0.133$ percent with corrections and $\pm 0.224$ percent without corrections. The corrections improved the area determanation by Lil percent. The greatest residual percent error with corrections was only 0.18 percent. This shows again that the larger the area the smaller the area error, even though the residual distance error may be larger in magnitude.

Two factors should be pointed out about this test area. Flrst, the correction formulas belped considerably where in the previous test areas the corrections hurt the results somerhat. In this model, residual parallaxes were present in much greater amounts than in the previous models. It can therefore be concluded that when residual parallaxes are present in photograply of 1 inch represents 250 feet scale, that the correction formulas help the results and should therefore be used.

A second factor about test area three concerns the identification problem of using fence posts. It often happens in rural Indiana lands that the property comers are actually marked by fence comers. It was therefore very feasible for them to be used as property comers in this project. The author feels, however, that the simple addition of a cardboard target nafled to the fence post property comer would decrease the observation error and therefore increase the accuracy considerably. This would also help the Kelsh operator in picking out the actual property corner from other fence intersections.

An example showing how Kalsh data can be used to determine areas taken by proposed highways is given in Appendix C. The limits of the topothetical highway are shown in Figure 3.
TABLE 6

| Area Number | Area (square Feet) |  |  | Residual Error (square feet) (Fleld-Computed) |  | ```Resfdual Error Expressed As Percent of Totel Area = Residual Error ``` |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Fleld | Computed |  |  |  |  |  |
|  |  | $\begin{gathered} \text { With } \\ \text { Corrections } \end{gathered}$ | Wi thout Corrections | With Corrections | Without Corrections | $\begin{aligned} & \text { With } \\ & \text { Corrections } \end{aligned}$ | Kỉthout Corrections |
| 1 | 134,586.21 | 134,699.61 | 134,889.98 | -113.40 | -303.77 | 0.08 | 0.23 |
| 2 | 285,241.44 | 284,718.75 | 284, 382.55 | +522.69 | +858.89 | 0.18 | 0.30 |
| 3 | 142,088.56 | 147,884.06 | 141,813.50 | +204. 50 | +275.06 | 0.14 | 0.19 |
| 4 | 561,916.21 | 561,302.42 | 561,086.03 | +613.79 | +830.18 | 0.11 | 0.15 |

4 Areas
W1th
Corrections
$\pm 0.128$
$\pm 0.133$
W1thout
Corrections
$\pm 0.218$
$\pm 0.224$
COIPARISON OF AREAS FROM TEST AREA THREE
TABLE 6 (continued)

|  | With Corrections | Without Corrections |
| :---: | :---: | :---: |
| Distribution of Residual Percent Error | $\begin{aligned} & 3 \text { from } 0.10-0.19 \\ & 1 \text { from } 0.00-0.10 \\ & \overline{4} \end{aligned}$ | 1 at 0.30 1 at 0.23 2 from $0.10-0.19$ 4 |
| Percent of Areas With Residual Percent Error of Less Than | With Corrections | Whthout Corrections |
| 0.30 | 100\% | 75\% |
| 0.20 | 100\% | 50\% |
| 0.10 | 25\% | 0 |

## CONCLUSIONS AND RECOMMENDATIONS

## Conclusions

A number of conclusions may be reached from this research project which might alter the existing approach to right-oi-way determination and make it faster and more accurate than it is today.

1. Photographo at a scale of 1 inch represents 250 feet is adequate for rlght-of-way determination using the methods described in this report.
2. The coordinatograph should be large enough 80 that the whole plotting surface of the Kelsh-type instrument cen be used.
3. The least count of 0.005 inches on the coordinatograph is acceptable for detemining machine coordinates needed to calculate distances and areas.
4. The least count of 0.01 inches on the instrument base measuring device is acceptable for determining the instrument base needed if correction formulas are applied to the machine coordinates.
5. Sidewalk intersections which can be used in some instances for block comer deteminations are adequate as area points.
6. Fence posts which could be used as property corners in ruml areas are of a questionable nature as area points because of the inability of the plotter operator to locate the center of the base of the post.
7. Correction formulas improve the results when the stereoplotter operator detects signiflicant residual parallaxes (see page Lo). Overcorrection may occur if the correction formulas are used (with photography scale of 1 inch represents 250 feet) when local parallaxes are not detected.
8. Residual errors in calculated areas and distances are caused by the errors in the machine coordinates of the points used.
9. Residual orrors in distance of $\pm 0.25$ leet can be expectod regardless of distance, in urbañ areas whem points are well deflned.
10. Residual errors in distance of $\pm 0.50$ feet can be axpected, regardless of distance, in rumi areas ir fence posts are used.
11. Lot size areas can be determined to residual percent errors of approximately 0.20 percent when comers are well doflned.
12. Block size areas can be determined to residual percent errors of approximately 0.10 percent when corners are well defined.
13. Fural areas can be determined to residual percent errors of approximately 0.14 percent when fence posts are used.
14. A second order stereoplotting instrument, such as a Kelsh Plotter, with an attached coordinatograph provides an accuracy which is adequate for right-of-way determination.
15. The numerical analysis using the described equipment and procedures approaches the accuracy which first order stereoplotting instruments have obtained in similar tests.

From the above conclusions the author reaches the general conclusion that a Kelsh-type second order stereoplotting instrument with an attached coordinatograph is applicable to right-of-way determination both in rural and urban areas. Although economics was not a prime consideration of this project, time records kept by the author indicated that the photogrametric approach was much faster than fleld methods and probably therefore more economical.

## Recomendations

It is reconmended that the Indiana State Highway Commission develop a project and use the methods herein described to determine the full economic advantages of photogrametry in right-of-way studies.

It is the author'e opinion that the following methods should be ueed on the project:

1. Presignalized points should be used as photo control.
2. Signals for presignalized points should be dealgned for the particular photography which is being taken.
3. The slgnal should be a square containing a circular central portion which has a diameter of one-third the side of the square.
4. The diameter of the circle should be such that at the model scal. the plotting mark of the plotting instrument just covers the circle.
5. Signals for presignalized points should be light colored in the cixcular portion and darker in the square portion.
6. High contrast colors should be avoided as these yield blurred images on the photograph. Circles of yellow and squares of blue are good color combinations.
7. Control surveys should be completed using accurate equipment and methods.
8. The photography should be taken with a good quality camera having radial lens distortion of less than seven microns as otherwise the resultant errors are too large.
9. The Kelsh-type instrument should be equipped with a distor tion free lens assembly on which the focal length has been determined by the National Bureau of Standards.
10. The projector lens pair should be matched for lens character istics as closely as possible in order to have symetrical effects on the model surface.
11. The Kelsb-type instrument should be completely and accurately calibrated.
12. The principal distance calibration should be corpleted using the described method.
13. Numerical absolute orientation should be used for scaling purposes.
14. Points used to define distances and areas should be well defined.
15. Slgnals attached to the top of fence posts shonid eliminate a large portion of the observation error caused by using fence posts.
16. The determination of urban 20 ts should be rade from the streot intercoctions or block comers.
17. Coordinate points should be raad twice to elimanite blunders.
18. Machine coordinates should be transformed to ground coominates to eliminate small scale and rotation ermes.
19. The coordinatograph used should be large enough to accormodate the whole model surface and should have an automatic read-out.
20. A high speed computer should be used for the calculations.

BIBIIOGRAPHY

## BIBIIOGRAPHY

1. Katibah, G. P., "The Appilcation of Precise Fhotogrametric Methods To Right-of-Way Relinquishment Surveys," Paper presented to HHghray Research Board annual meeting, Washington D. C., Jamary, 1964.
2. Henry, Hubert A., "Development of Photogrametric Methods for Paght-of-Way Operations in Texas, "Photogranmetry: Developments and Applications 1960, Highway Research Board Bulletin \%o. 283, Washangton D. C., January, 1961.
3. Clarke, L. E., "Photogrenmetry in Preparing Right-of-Way Maps and Deeds," Texas Hyphways, Vol. VII, No. 8, August, 1960.
4. Scudieri, Philip F., "An Analysis of Photogranmetry Appiled to Pight-of-Way Surveys," Thesis, Purdue University, August, 1961.
5. Anderson, James M., "Accuracy of Planimetric Positions Detormined from Large-Scale Photography," Photogrametric Engineerine, Vol. XXIX, No. 5, September, 1963.
6. Veres, Sandor A., "Photogranmetry - Its Relation to Reght-of-Way," Speech delivered before Anmal Regional Educational Seninar of American Right-of-Way Association. Columbus, Ohio, November 1, \& 2, 1962.
7. BLird, S. J. G., "Comparison of Photogranmetric and Ground Survey Methods for Legal Survey At Vineland, "Thesis, University of Toronto, 1963.
8. AGA Company, The Geodimeter Models 2, 3, and 4, Stockhols, Sweden, 1960.
9. Brinker, Fussell C. and Tayior, Warren C., Eementary Surveying, Scranton, Pennsylvania, International Textbook Compary, 1958.
10. Schridefsky, K., Translation by Fosberry, John, An Outiine of Photogrammetry, Iondon, Sir Isaac P1tman and Sons, Itc., 1959.
11. Zeiss Aerotopograph Munchen, Aerfal Survey Cameras, Germany.
12. The Kelsh Instrument Company, The Kelsh Flotter, Baltimore, Maryland.
13. Veres, Sandor A., "The Effect of Flxation Disparity on Photogrametric Processes," Photogrammetric Engineering, Dol. Ex, No. I, Jenuary, 1964.

I4. Doyle, Frederick J., Coordinate Transiormations, Mapping and Charting Research Laboratory Technical Namorandum Nunber 1, Ohdo State Undrersity, Columbus, Ohio.
15. Kells, Iyman M. and Stotz, Ferman C., Analytic Coonetry, Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1949.
16. Moffltt, Francis M., Photogrammetyy, Scranton, Pennsylvanda, Inter national Textbook Comprny, 1959.
17. American Socioty of Civil Engineers, Definitions of Surveying, Mapping, and Related Terms, New Tork, 1954.
18. G. Coradi Itd., Instruction for Coradi's Mamal Coordinatograph, Zuerich, Switzerland.

## APPENDIX A

TRANSFORMATION FORMULAS

## APPENJDIX A

## TRANSFORMATION FORMULAS

The transformation formulas are (14)

$$
X=A X^{\prime}+B Y^{\prime}+C^{\prime} \quad Y=A Y^{\prime}-B X^{\prime}+C^{\prime \prime}
$$

where $\quad X$ and $Y$ ground coordinates of a point
$X^{\prime}$ and $Y^{\prime}=$ machine coordinates of the same point
$A$ and $B=$ transformation constants representing rotation and scale changes
$C^{\prime}$ and $C^{\prime \prime}=$ translation constants for and $X$ and $I$ respectively
$A=\frac{\Sigma\left(X_{0}^{\prime} X_{0}+Y_{0}^{\prime} Y_{0}\right)}{\sum\left(X_{0}^{\prime 2}+Y_{0}^{\prime 2}\right)}$
$B=\frac{\Sigma\left(Y_{0}^{\prime} X_{0}-X_{0}^{\prime} Y_{0}\right)}{\Sigma\left(X_{0}^{\prime 2}+Y_{0}^{\prime 2}\right)}$
where $X_{0}=X-X_{g}$ and $Y_{0}=Y-Y_{g}$ (determined for each of the control points)
where $X_{g}=\frac{\sum(X \text { used in transformation) }}{n}$
$Y_{g}=\frac{\sum(Y \text { used in transformation) }}{n}$
$n=$ number of points used in transformation
$X_{0}^{\prime}=X^{\prime}-X_{g}^{\prime} ; Y_{0}^{\prime}=Y^{\prime}-Y_{g}^{\prime}$ (determined for each of the control points)
where $X_{g}^{\prime}=\frac{\sum\left(X^{\prime} \text { used in transformation) }\right.}{n}$

$$
\begin{aligned}
& Y_{g}^{\prime}=\frac{\sum\left(Y^{\prime} \text { used in transformation }\right)}{n} \\
& C^{\prime}=X_{g}-A X_{g}^{\prime}-B Y_{g}^{\prime} \quad C^{\prime \prime}=Y_{g}-A Y_{g}^{\prime}+B X_{g}^{\prime}
\end{aligned}
$$

In order to $u s e$ the transformation formulas, ilrat find $X_{g}, Y_{q}, X_{g}^{\prime}$, and $I_{g}^{\prime}$. If three control points are used, then the three gound coordinates and the three machine coordinates of the same points are used to dotermine the above. $X_{0}^{\prime}, Y_{0}^{\prime}, X_{0}$, and $Y_{0}$ are determined neoxt and then $A$ and $E$ are computed. When this is completed $C^{\prime}$ and $C^{\prime \prime}$ can computed. $A, B, C^{\prime}$, and $C$ " are constants which will be used for each of the subsequent machine coordinates to be transformed. $X$ and $Y$ are computed after the constants are determined.

Signs must be very carefully watched throughout the computations. If all the compitations are correct and the coordinates used do not contain blunders, then the computed transformed coordinates of the control points should match the ground coordinates of the points.

## APPENDIX B

## SAMPLE COMPUTATION

## APPENDIX B

## SAMPLE COMPUTATION

The sample computation will begin with the Kelsh determined coordinates for the control points and the pointa defining area B of test area one. The average values for the readings for normal vision ( $X_{p}^{\prime}$ and $Y_{p}^{\prime}$ ) and psoudo vision ( $X_{p}^{\prime \prime}$ and $Y_{p}^{\prime \prime}$ ) are as shown in the following table. Values are in feet throughout the computation.

| Point | $X_{p}^{\prime}$ | $Y_{p}^{\prime}$ | $X_{p}^{\prime \prime}$ | $Y_{p}^{\prime \prime}$ |
| :---: | ---: | :---: | :---: | :---: |
| 33 | 989.975 | 1560.675 | 989.900 | 1560.375 |
| 41 | 395.900 | 1091.900 | 395.925 | 1091.725 |
| 54 | 1003.075 | 1095.175 | 1003.325 | 1095.050 |
| 5 | 561.400 | 1373.400 | 561.875 | 1373.225 |
| 6 | 561.150 | 1508.225 | 561.225 | 1507.875 |
| 7 | 687.400 | 1511.300 | 687.750 | 1511.000 |
| 8 | 687.725 | 1374.050 | 687.975 | 1373.900 |

The machine coordinates refer to an arbitrary coordinate axis and the correction formulas were derlved with the coondinate axds at the center of the left photograph. It was determined from Kelsh readings that the coordinates of this center point were $X_{0}=1349.937$ and $Y_{0}=988.025$.

The direction of increasing coordinates on the abritrary eysten is not necessarily the same as that assumed for the derivation of the correction formulas. In this example the $Y$-coordinates were increasing the same but the $X$-coordinates were not. The following table shows the coordinates referred to the center of the left photo. Signs ere important in this process.

| Point | $X_{0}-X_{p}^{\prime}$ | $Y_{p}^{\prime}-Y_{0}$ | $X_{0}-X_{p}^{\prime \prime}$ | $Y_{p}^{\prime \prime}-Y_{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| 33 | +359.962 | +572.650 | +360.037 | +572.350 |
| 41 | +954.037 | +103.875 | +954.012 | +103.700 |
| 54 | +346.862 | +107.150 | +345.612 | +107.025 |
| 5 | +788.537 | +385.375 | +788.062 | +385.200 |
| 6 | +788.787 | +520.200 | +788.712 | +519.850 |
| 7 | +662.537 | +523.275 | +662.187 | +522.975 |
| 8 | +662.212 | +386.025 | +661.962 | +385.875 |

The next step is to adjust the above coordinates using the correction formulas.

$$
\begin{aligned}
& X_{p}=0.5\left(X_{p}^{\prime}+X_{p}^{\prime \prime}\right)+0.5\left[\frac{X_{p}^{\prime}}{b}\left(\frac{X_{p}^{\prime \prime}-X_{p}^{\prime}}{2}\right)+\frac{X_{p}^{\prime \prime}}{b}\left(\frac{X_{p}^{\prime \prime}-X_{p}^{\prime}}{2}\right)+\frac{X_{p}^{\prime \prime}-X_{p}^{\prime}}{2}\right] \\
& Y_{p}=0.5\left(Y_{p}^{\prime}+Y_{p}^{\prime \prime}\right)+0.5\left[\frac{X_{p}^{\prime}}{b}\left(Y_{p}^{\prime}-Y_{p}^{\prime \prime}\right)+\frac{Y_{p}^{\prime}-Y_{p}^{\prime \prime}}{2}\right]
\end{aligned}
$$

where $X_{p}^{\prime}$ refers to $X_{o}-X_{p}^{\prime}$

$$
\begin{aligned}
& X_{p}^{\prime \prime} \text { refers to } X_{0}-X_{p}^{\prime \prime} \\
& Y_{p}^{\prime} \text { refers to } Y_{p}^{\prime}-Y_{0} \\
& Y_{p}^{\prime \prime} \text { refers to } Y_{p}^{\prime \prime}-Y_{0}
\end{aligned}
$$

b refers to alrbase (airbase used here because all conrdinatas are in terms of ground measure rather than instrument masure)
b was detemined to be 862.385 feet (from base measuring device)
The corrected machine coordinates ( $X_{p}$ and $Y_{p}$ ) are computed eimply by substituting the given values into the equation and solving for $X_{p}$ and $Y_{p}$. Signs of the machine coordinates must be taken into consideration. The corrected machine coordinates are given in the following table.

| Point | $X_{p}$ | $Y_{p}$ |
| :--- | :---: | :---: |
| 33 | +360.034 | +572.637 |
| 41 | +954.004 | +103.928 |
| 54 | +34.6 .624 | +107.144 |
| 5 | +787.964 | +385.411 |
| 6 | +788.696 | +520.272 |
| 7 | +662.140 | +523.315 |
| 8 | +661.928 | +386.045 |

The next step is to transform the corrected machine coordinates to ground coordinates. The formulas used are explained in Appandix A and will not be repeated here. The following table is normally set up to facilitate the computational procedure.

| Point | $X^{\prime}$ | $Y^{\prime}$ | $X$ | $Y$ |
| :--- | :---: | :---: | :---: | :---: |
| 33 | +360.034 | +572.637 | $545,009.18$ | $1,065,014.39$ |
| 41 | +954.004 | +103.928 | $544,547.35$ | $1,064,414.72$ |
| 54 | +346.624 | +107.244 | $544,543.30$ | $1,065,022.33$ |
| 5 | +787.964 | +385.411 |  |  |
| 6 | +788.696 | +520.272 |  |  |
| 7 | +662.140 | +523.315 |  |  |
| 8 | +661.928 | +386.045 |  |  |

The ground coordinates given ( $X, Y$ ) are those determdned on the ground. The following computations are those necessary to compute the transformation constants.

$$
\begin{aligned}
& X_{g}=544,699.943 \\
& Y_{g}=1,064,817.146 \\
& X_{g}^{\prime}=553.554 \\
& Y_{g}^{\prime}=261.236
\end{aligned}
$$

| Point | $X_{0}$ | $Y_{0}$ | $X_{0}$ | $Y_{0}$ |
| :--- | :---: | :---: | :---: | :---: |
| 33 | -193.520 | +317.401 | +309.237 | +197.244 |
| 47 | +400.450 | -157.308 | -152.593 | -402.426 |
| 54 | -206.930 | -154.092 | -156.643 | +205.184 |
|  | 0.000 | +0.001 | +0.001 | +0.002 |

As this is a gravity point method the sums of the above should equal zero. Round-off errors may cause small differences. The constants can now be determined and are:

$$
\begin{aligned}
& A=+0.01184778 \\
& B=+1.00033196 \\
& C^{\prime}=544,432.062 \\
& C^{\prime \prime}=1,065,367.789
\end{aligned}
$$

Once the constants are determined, the machtne coordinates can be transformed to ground coordinates. The control points are transformod as well as other points in order to check for blunders. The transformed control coordinates should very nearly match the ground coordinates. The transformed coordinates are shown in the following table. Normally the transformed coordinates would be written beside the machine coordinates of the previous table but are shown separately to dietinguish between given and computed values.

| Point | $X$ | $I$ | $\Delta X$ | $\Delta Y$ |
| :---: | :---: | :---: | :---: | :---: |
| 33 | $545,009.16$ | $1,065,014.42$ | +0.02 | -0.03 |
| 47 | $544,547.33$ | $1,064,424.70$ | +0.02 | +0.02 |
| 54 | $544,543.35$ | $1,065,022.32$ | -0.05 | +0.01 |
| 5 | $544,826.94$ | $1,064,584.13$ |  |  |
| 6 | $544,961.85$ | $1,064,585.00$ |  |  |
| 7 | $54,963.40$ | $1,064,711.63$ |  |  |
| 8 | $544,826.08$ | $1,064,710.21$ |  |  |

The $\Delta x^{\prime} s$ and $\Delta y^{\prime}$ s are very smail, therefore it is assumed that no blunders exist in the system.

The transformed coordinates of points 5-8 may now be used to determine the distances and area. The distances are computed using the Pythagorean Theorem and the area by coordinates method (9).

$$
\text { Distance 1-2 }-\sqrt{\left(X_{1}-X_{2}\right)^{2}+\left(Y_{1}-Y_{2}\right)^{2}}
$$

The area by coordinates method is a simple method to determine areas when coordinates are given for the points. The area is equal to one-haly the sum of the products obtained by multiplying each $X$-coordinate by the difference between the adjacent Y-coordinates, taken in the same order around the figure.

The results for area B of test area 1 are as follows:

$$
\begin{aligned}
& D_{5-6}=134.91 \text { feet } \\
& D_{6-7}=126.64 \text { feet } \\
& D_{7-8}=137.33 \text { feet } \\
& D_{8-5}=126.08 \text { feet } \\
& \text { Area }=17,198.42 \text { square feet }
\end{aligned}
$$

The above calculations are relatively simple and are quickly performed using a desk calculator, but on an actual job a high speed computer would be a definite advantage.

## APPENDIX C

HIGHWAY RIGHT-OF-WAY AREA DETERMDNATION

## APPENDIX C

## HIOHWAY RIOHTT-OF-WAY AREA DETEFMINATION

This example shows the method of detemining area taken and area remaining for highway right-of-way. The date begins with the corrected transformed coordinates of test area three. Figure 3 shows the location of the bypothetical highway which was passed through the area. Pointe 7 h and 7 B were considered as the center-line control of the highway which would normally be presignalized. The coordinates of these center-line control points were therefore lonown. The proposed hopothetical highway has a 200 foot right-of-way.

The following table lista the corrected transformed coordinates of test area three with 540,000 being subtracted from $X$-coordinates, and 1,070,000 being subtracted from Y-coordinates. The values are easier to work with when this is done.

| Point | $x$ | $x$ |
| :---: | :---: | :---: |
| 7 A | $1,604.33$ | 718.92 |
| 1 | $1,616.55$ | $1,388.73$ |
| 2 | $1,601.07$ | $1,644.90$ |
| $7 B$ | $2,115.49$ | $1,654.74$ |
| 3 | $2,116.25$ | $1,568.47$ |
| 4 | $2,101.85$ | $1,374.74$ |
| 5 | $2,119.83$ | $1,243.80$ |
| 6 | $2,404.54$ | $1,241.62$ |
| 7 | $2,429.31$ | $1,174.08$ |
| 8 | $2,044.52$ | 679.29 |
| 9 | $1,896.25$ | 691.67 |

Using the coordinates of the center-line control points, it is possible to form the equation of the line joining the points. The general equation of a line can be expressed as

$$
\frac{Y-Y_{1}}{X-X_{1}}=\frac{Y_{2}-Y_{1}}{X_{2}-X_{1}}
$$

where $X$ and $Y$ refer to any unknown point on the line
$X_{1}$ and $Y_{I}$ are the coordinates of the flrst point
$X_{2}$ and $Y_{2}$ are the coordinates of the second point (15).
The equation of line $7 \mathrm{~A}-7 \mathrm{~B}$ would be

$$
\frac{Y-718.92}{X-1,604.33}=\frac{1,654.74-718.92}{2,115.49-1,604.33}
$$

which when simplified is

$$
X-0.54622(Y)=1221.644_{1} 28
$$

In like manner, the equation of line I-4 is

$$
X+34.68906(Y)=49,790.29331
$$

A simultaneous solution of the two equations ylelds the coordinates of the center-Iine intersection point "a".

$$
\begin{gathered}
X-0.54622(Y)=1231.64428 \\
\frac{-X-34.68906(Y)=49.490 .29331}{X_{a}=1964.7152} \\
Y_{a}=1378.6934
\end{gathered}
$$

The determination of the intersection coordinates of the right-of-way line and property line is slightly more difficult. The solution for the general case will be derived here. If $A B$ (Figure 8) is the center-line of a highway, then TS is one of the right-of-way lines. CD is a property line which intersects the center-line at "O" and the right-of-way line at "P". $P Q$ is perpendicular to the center-line $A B$ and is equal to one-half of the right-of-way width or 100.00 feet in this example. Ancle $\theta_{I}$ is the angle formed from the horizontal to line $A B$. Angle $\theta_{2}$ is the angle formed from the horizontal to line $C D$.

The slope of a line is equal to the tangent of the angle formed from the horizontal to the line. Therefore, slope of line $A B$ is equal to tangent $\theta_{1}$. Angle $\theta_{I}$ is then equal to arctangent (slope $A B$ ).

But the slope of $A B$ can also be expressed as $\frac{Y_{B}-Y_{A}}{X_{B}-X_{A}}$
Therefore $\theta_{I}=\arctan \frac{Y_{B}-Y_{A}}{X_{B}-X_{A}}$


FIGURE 8
COORDINATE DETERMINATION FOR RIGHT-OF-WAY

In a like fashion $\theta_{2}=\arctan \frac{Y_{D}-Y_{C}}{X_{D}-X_{C}}$
From FIgure 8: $\varphi=\theta_{1}-\theta_{2}$.
Therefore in right triangle OPQ ,

$$
\begin{aligned}
\text { angle } Q & =90^{\circ} \\
\text { angle } 0 & =\varphi=\theta_{1}-\theta_{2} \\
\text { angle } P & =90^{\circ}-\varphi \\
\text { distance } Q P & =100.00 \text { feet }
\end{aligned}
$$

Distance OP therefore $=\frac{100.00}{\sin \varphi}$
From triangle ous

$$
\begin{aligned}
& \Delta X=O P \cos \theta_{2} \\
& \Delta Y=O P \sin \theta_{2}
\end{aligned}
$$

The coordinates of $P$ are

$$
\begin{aligned}
& X_{p}=X_{0}+\Delta X \\
& Y_{p}=Y_{0}+\Delta Y
\end{aligned}
$$

The coordinates of "P" are thus determined. Only two points for each right-of-way line need be determined in this fashion. The reat of the intersection points can be determined by simultaneous solution of the equations of the property line and right-of-way line just defined. Angle $Q$ will vary depending on what quadrant the point falls in. Care must therefore be exercised in getting the proper angular relationship.

The determination of the coordinates for point "b" in the example is as follows:

$$
\begin{aligned}
\tan \theta_{1} & =\text { slope of }(7 \mathrm{~A}-7 \mathrm{~B})=\frac{Y_{2}-Y_{1}}{X_{2}-X_{1}}=\frac{1,654.74-718.92}{2,115.49-1,604.33} \\
\tan \theta_{1} & =1.83077706 \\
\theta_{1} & =61^{\circ}-211-21.32 \prime
\end{aligned}
$$

In like fashion, $\theta_{2}=1^{\circ}-5^{1}-43.13^{\prime \prime}$
For this determination $=\theta_{1}-\theta_{2}$

$$
\begin{aligned}
& \text { therefore } \varphi=60^{\circ}-15^{\prime}-38.19^{\prime \prime} \\
& \sin \varphi=\frac{100.00^{\prime}}{b-7 B} \\
& b-7 B=\frac{100.00^{\prime}}{\sin \left(60^{\circ}-15^{\prime}-38.19^{\prime \prime}\right)} \\
& b-7 B=115.17 \text { feet }
\end{aligned}
$$

$$
\begin{array}{rlrl}
\Delta X & =(b-7 B) \cos \theta_{2} & \Delta Y & =(b-7 B) \sin \theta_{2} \\
& =(115.17)\left(\cos 2^{\circ}-5^{\prime}-43.13^{\prime \prime}\right)=(115.17)\left(\sin 1^{\circ}-5^{\prime}-43.13^{\prime \prime}\right) \\
\Delta X & =115.15^{\prime} & \Delta Y & =2.20^{\prime} \\
X_{b} & =X_{7 B}-\Delta X & Y_{b} & =Y_{7 B}-\Delta Y \\
& =2115.49-115.15^{\circ} & & =1,654.74-2.20 \\
X_{b} & =2000.34 & Y_{b} & =1,652.54
\end{array}
$$

The coordinates of all the intersection lines can be determined by one of the two methods above. The following table lists the intersection coordinate points thus detemined.

| Point | X | Y |
| :---: | :---: | :---: |
| a | $1,964.71$ | $1,378.69$ |
| b | $2,000.34$ | $1,652.54$ |
| d | $2,105.84$ | $1,428.45$ |
| e | $2,076.89$ | $1,375.46$ |
| f | $1,852.54$ | $1,381.93$ |
| g | $1,712.75$ | 706.80 |
| h | $1,608.26$ | 934.69 |

The areas taken and areas remaining can be computod using area by coordinates method.

The area taken from parcel 2 is defined by points $7 A, h, f, e$, and $g$. The area taken is $I 38,632.35$ square feet.

The area remaining on the left side of parcel 2 is defined by points $h, l$, and $f$. The area remaining is $53,603.08$ square iect.

The area remaining on the right side of parcel 2 is defined by points g, e, 4, 5, and 9. The area remaining is $92,483.97$ square feet.

The area taken from parcel 1.18 defined by points $f, b, 7 B, 3, d$, and e. The area taken is $50,393.66$ square feet.

The area remaining on the left side of parcel 1 is defined by points $1,2, b$, and $f$. The area remaining is $83,633.95$ square feet.

The area remaining on the right side of parcel 1 is defined by points $e$, $d$, and 4 . The area remaining is 671.60 square feet.

The determination of bearings is quite simple using the coordinates of the points. In general, the bearing of a line equals arctangent, $\frac{X_{2}-X_{1}}{Y_{2}-Y_{1}}$ where point 1 is the occupied point and point 2 is the next point. If $\Delta X$ and $\Delta Y$ are both positive, then the bearing is in quadrant 1 and the bearing is Northeast. If $\Delta X$ is negative and $\Delta Y$ positive, then the bearing is in quadrant 2 and the bearing is Northwest. If $\Delta X$ and $\Delta I$ are both negative, then the bearing is in quadrant 3 and the bearing is Southwest. If $\Delta x$ is positive and $\Delta Y$ is negative, then the bearing is in quadrant 4 and the bearing is Southeast.

## Example:

$$
\begin{aligned}
\text { Bearing } \mathrm{f}-\mathrm{b} & =\arctan \frac{X_{b}-X_{f}}{Y_{b}-Y_{f}} \\
& =\arctan \frac{2000.34-1,852.54}{1,652.54-1,381.93} \\
& =\arctan 0.54618253
\end{aligned}
$$

$$
\text { bearing } f-b=N-28^{\circ}-38^{1}-33.35^{\prime \prime} \text { E. }
$$

The other bearings can be detemined in a like mamer. The distances between points can be detemined by the Pythagorean Theorem. The description of the property to be taken can then be written using the bearings and distances.

For Parcel 1:
. . . . . to "78", the true point of beginning;
thence $S-0^{\circ}-30^{\prime}-2^{n}-$ E, 86.28 leet to point " $3^{\prime \prime}$;
thence $S-4^{\circ}-15^{\prime}-10^{\prime \prime}-W, 140.40$ feet to point "d";
thence $S-28^{\circ}-38^{\prime}-\infty "-W, 60.38$ feet to point "e";
thence $N-88^{\circ}-21^{\prime}-\infty "$-W, 224.45 feet to point "f";
thence $N-28^{\circ}-33^{\prime}-30^{\prime \prime}-$ E, 308.34 feet to point "b";
thence $N=89^{\circ}-54^{\prime}-20^{\prime}-E, 115.17$ feet to point "7B", the true point of beginning.

For Parcel 2:
. . . . . to "7A" the true point of beginning;
thence $N-1^{\circ}-2^{1}-40^{\prime \prime}-E, 215.81$ feet to point "h";
thence $N-28^{\circ}-38^{\prime}-30^{\prime \prime}-E, 509.60$ feet to point "f";
thence $S-88^{\circ}-22^{1}-0011-E, 224.45$ feet to point " $e^{\prime \prime}$;
thence S - $28^{\circ}-38^{1}-30^{\prime \prime}-W, 759.63$ feet to point " $g$ ";
thence $N-84^{\circ}-40^{\prime}-00^{\prime \prime}-W, 108.90$ feet to point "7A", the true point of beginning.

It is hoped that tho above example, although not completely worked in detail, provided the interested reader with the methods necoseary to determine areas of right-of-way, bearings of lines, and descriptions o! areas taken. Again, as in Appendix B the author recommends the use of high opeod computers to accomplish the calculations.

APPENDIX D
GLOSSARY OF TERMS

## APP ANDIX D

## GLOSSARY OF TEPMS

Absolute Orientation - process in photogrametry where a stereoscopic model is brought to the desired map scale, and $1 s$ placed in its correct orientation with respect to the datum for elevations (16).

Cadastral Surveys - a survey relating to land boundaries and subdivisions, made to create units suitable for transfer or to define the limitations of title (17).

Coordinatograph - a drafting instrument having mutually perpendicular I and $Y \mathrm{arms}$ on which movable assenblies measure the distance moted (18).

Diapositive - a transparent positive on a glass plate used in a plotting instrument, a projector, or a comparator (17).

Floating Mark - a dot seen as occupying a position in the three-dimensional space formed by the stereoscopic fusion of a pair of photographs and used as a reference mark in examining or measuring the stereoscopic model (17).

Geodimeter - an electronic distance measuring instrument using light rays as the measuring agent (8).

Interior Orientation - the establishment of the principal distance and the position of the principal point of a photograph with respect to the flducial marks of the camera (17).

Kelsh Plotter - a second order stereoplotting instrument in wide use in the United States (12).

Model - the overlap area of two diapositives which is observed in the stereoplotting instrument (16).

Nistri Stereocomparator - a coordinate measuring device having stereo vision (16).

Nodal Points - two points associated with a lens system such that any ray in the object space directed toward the flrst or front point will emerge in the image space from the second or rear point and be parallel to its former direction (17).

Normal Vision - torm used when the stereoplottor operator aees the molel 28 it occurs on the ground (16).

Parallax - as used by stereoplotter operators, refers to the lack of complete coincidence of the two images as seen in the storeoplottor by the operator (16).

Photogrammetry - the science or art of obtaining reliable measarements by photography (17).

Planimetry - the plan detaile of a map (17).
Platen - surface of the tracing table on which the rays of light intersoct In a Kelsh Plotter (12).

Polar Survey - a survey when secondary points radiate from the central point with all angles boing measurod from the central point (9).

Presignalized Point - a point targeted prior to photography in such a way that the target shows on the photograph (16).

Pseudo Vision - tem used when the stereoplotter operator sees the model with relief features being reversed from what occurs on the ground (16).

Relative Orientation - the reconstruction of the same perspective conditions between a pair of photographs which existed whon the photograpis were taken (17).

Standard Residual Error - a measure of the precision of a series of observations (17).

Theodolite - a precision surveying instrument consiating of an alidade with a telescope. It is mounted on an accurately graduated circle and is equipped with necessary levels and reading devices (17).

Topography - the features of the actual surface of the earth considered collectively as to form (17).

Pracing Table - the movable unit of a Kelsh Plotter on which the platen and alevation counter are affixed (12).

Wild A-7 Autograph - a flrst order stereoplotting instrument (16).
Zeiss C8 Sterooplanigraph - a first onder stereoplotting instrument (16).


[^0]:    * Refers to references listod in Bibllography * Clossary of terms in Appendix D.

[^1]:    * For this discussion right and left refer to the Kelsh as seen in Figure 5.

