The Influence of Fluctuations in the Crankshaft Speed on the Tip Seal Acceleration of the Wankel Rotary Compressor

G. R. Pennock
Purdue University

J. E. Beard
Michigan Technological University

Follow this and additional works at: http://docs.lib.purdue.edu/icec
THE INFLUENCE OF FLUCTUATIONS IN THE CRANKSHAFT SPEED ON THE TIP SEAL ACCELERATION OF THE WANKEL ROTARY COMPRESSOR

G. R. Pennock  
Associate Professor  
School of Mechanical Engineering  
Purdue University  
West Lafayette, Indiana 47907-1288

J. E. Beard  
Associate Professor  
ME-EM Department  
Michigan Technological University  
Houghton, Michigan 49931

ABSTRACT

The acceleration of the tip seal in the rotor of a Wankel rotary compressor is a function of the radii of the fixed and moving pitch circles, the number of lobes, the trochoid ratio, and the speed and acceleration of the crankshaft. The equations for the tip seal acceleration are well-known and have been used, in conjunction with the compression forces, to determine the spring forces that are necessary to maintain contact between the tip seal and the housing. The equations that are available in the literature, however, are only valid for a constant crankshaft speed; i.e., they do not account for fluctuations in the crankshaft speed. The major purpose of this paper is to derive equations that will provide insight into the tip seal acceleration when fluctuations in the crankshaft speed are included in the analysis. For purpose of illustration, the effects on the tip seal acceleration due to a cyclic fluctuation in the crankshaft speed is compared to the effects using a nominal value. The results presented in the paper can be used by the designer to detect and, thereby, avoid possible leaks in the sealing or unnecessary wear of the seals.

NOMENCLATURE

The following notation is used consistently throughout this paper:

$X_1O_1Y_1$ = fixed Cartesian reference frame (attached to the fixed pitch circle)

$X_2O_2Y_2$ = moving Cartesian reference frame (attached to the rolling pitch circle)

$O_1$ = center of the fixed pitch circle, $O_2$ = center of the rolling (or generating) pitch circle

$r_1$ = radius of the fixed pitch circle, $r_2 = (T - 1)/T$, $r_2$ = radius of the rolling pitch circle

$T$ = number of generating lobes fixed to the rolling pitch circle

$T - 1$ = number of generated lobes on the fixed pitch circle

$C$ = center of the generating pin (or arc), $r$ = radius of the generating pin

$r_C$ = radius of the epitrochoidal path of point $C = O_2C$

$e$ = length of the crank, or trochoid eccentricity $= O_2O_1 = r_2 - r_1 = r_2/T$

$P$ = pole of the rolling pitch circle (coincident with the instant center)

$\phi$ = input position (position of the crank relative to the $X_1$-axis)

$\dot{\phi}$ = crankshaft speed, $\ddot{\phi}$ = crankshaft acceleration

$\theta$ = input variable for harmonic function $= \phi/T$

$\mu$ = trochoid ratio $= r_C/r_2$ and $\eta_H$ = harmonic ratio
1. INTRODUCTION

The acceleration of the tip, or apex, seal in a Wankel rotary compressor is a function of the compressor geometry and the speed and acceleration of the crankshaft [Ansdale and Lockley, 1969; and Leemhuis and Soedel, 1978]. The equations that describe the geometry (e.g., the radius of curvature of the generated shape or housing), and the velocity and acceleration of the tip seal are well-known and can be found in most of the references listed at the end of this paper. The most common design to maintain seal contact between the rotor and the housing of the compressor is a spring [Ansdale and Lockley, 1969; and Norbye, 1971]. The designs of the tip seal and spring are based, to a large extent, on the acceleration of the rotor assuming the crankshaft rotates at a constant speed [Yamamoto, 1981]. In most practical situations, however, this assumption is not realistic even during so-called steady-state operation. For example, the crankshaft speed will fluctuate due to pulsations in the inlet or outlet pressure, changes in the flow rate, or fluctuations in the power supply.

Fluctuations in the crankshaft speed cause the center of the generating pin to accelerate and decelerate. A component of the tip seal acceleration is directed from the center of the generating pitch circle to the center of the generating pin. The existing literature does not consider the effects of this radial acceleration in the dynamic force analysis of a rotary compressor. This issue needs to be addressed because, as this paper shows, even small fluctuations in the crankshaft speed can account for significant variations in the radial component of acceleration of the center of the generating pin. For example, at certain positions of the crank the variation in the radial acceleration can account for the total angular acceleration of the pin. This paper presents equations for the tip seal acceleration due to a cyclic fluctuation in the speed of the crankshaft. Section 2 is an acceleration analysis of the Wankel rotary compressor when the acceleration of the crankshaft is approximated by a harmonic function. Section 3 presents plots and a discussion of the important observations. Finally, Section 4 presents conclusions and suggestions for future research.

2. THE ACCELERATION ANALYSIS

The center of the generating pin of a Wankel rotary compressor is denoted by point C, see Figure 1. The coordinates of point C, expressed in the fixed Cartesian reference frame \(X_1O_1Y_1\), are

\[
C_x = e \left[ -\cos \phi + \mu T \cos \left( \frac{\phi}{T} \right) \right] \quad \text{and} \quad C_y = e \left[ -\sin \phi + \mu T \sin \left( \frac{\phi}{T} \right) \right]
\]

(1)

where \(\mu = r_c/r_2\) is referred to as the trochoid ratio, and \(e = r_2/T\) is referred to as the crank.

![Figure 1. Geometry of the Wankel Rotary Compressor.](image-url)
The angle $\phi$ defines the crank position and is, henceforth, referred to as the input position. The acceleration of point C, expressed in terms of the input position [Hall, 1981], can be written as

$$\ddot{A}_C = (f_{X\dot{\phi}} + f_{Y\dot{\phi}}) \dot{\phi} + (f'_{X\dot{\phi}} + f'_{Y\dot{\phi}}) \dot{\phi}^2$$

(2)

where

$$f_X = \frac{dC_X}{d\phi}, \quad f_Y = \frac{dC_Y}{d\phi}, \quad \text{and} \quad f = + \sqrt{f_X^2 + f_Y^2}$$

(3a)

are referred to as the first-order kinematic coefficients, and

$$f'_{X\dot{\phi}} = \frac{d^2C_X}{d\phi^2} \quad \text{and} \quad f'_{Y\dot{\phi}} = \frac{d^2C_Y}{d\phi^2}$$

(3b)

are referred to as the second-order kinematic coefficients [Hall, 1981].

Differentiating Eqs. (1) with respect to the input position, the first and second-order kinematic coefficients are

$$f_X = e \left[ \sin \phi - \mu \sin \left( \frac{\phi}{T} \right) \right], \quad f_Y = e \left[ - \cos \phi + \mu \cos \left( \frac{\phi}{T} \right) \right]$$

(4a)

$$f'_{X\dot{\phi}} = e \left[ \cos \phi - \frac{\mu}{T} \cos \left( \frac{\phi}{T} \right) \right] \quad \text{and} \quad f'_{Y\dot{\phi}} = e \left( \sin \phi - \frac{\mu}{T} \sin \left( \frac{\phi}{T} \right) \right)$$

(4b)

The line of action of the spring force is denoted by the vector $\vec{r}_C$, as shown in Figure 1. In the following analysis, point C is assumed to have a constant linear velocity relative to the rotor; i.e., $\vec{r}_C = 0$. The radial acceleration of point C (i.e., the acceleration of the center of the tip seal along the line of action of the seal or spring force), can be obtained from the vector dot product

$$A_{Crad} = \vec{A}_C \cdot \vec{U}_{rc}$$

(5)

where

$$\vec{U}_{rc} = \cos \left( \frac{\phi}{T} \right) \vec{i} + \sin \left( \frac{\phi}{T} \right) \vec{j}$$

(6)

is the unit vector directed along the radial line from $O_2$ to C, see Figure 1. Substituting Eqs. (2) and (6) into Eq. (5) and performing the dot product, the radial acceleration of point C can be expressed as

$$A_{Crad} = (f_{X\ddot{\phi}} + f'_{X\dot{\phi}} \dot{\phi}^2) \cos \left( \frac{\phi}{T} \right) + (f_{Y\ddot{\phi}} + f'_{Y\dot{\phi}} \dot{\phi}^2) \sin \left( \frac{\phi}{T} \right)$$

(7)

To study the influence of the crankshaft acceleration on the radial acceleration of point C, substitute Eqs. (4) into Eq. (7) and rearrange to give

$$A_{Crad} = e \left[ \sin \left( \phi - \frac{\phi}{T} \right) \right] \ddot{\phi} + \left[ \cos \left( \phi - \frac{\phi}{T} \right) - \frac{\mu}{T} \right] \dot{\phi}^2$$

(8)

Note that the coefficients of the crankshaft speed and acceleration are multiplied by harmonic functions with the same period. Therefore, a closer examination is required to determine if a mean value of crankshaft acceleration will have a significant effect on the radial acceleration of point C. From Eq. (8), we define the ratio

$$\eta = \frac{\sin \left( \phi - \frac{\phi}{T} \right) \ddot{\phi}}{\left[ \cos \left( \phi - \frac{\phi}{T} \right) - \frac{\mu}{T} \right] \dot{\phi}^2}$$

(9)

which is the component of the radial acceleration of point C due to the crankshaft acceleration divided by the component of the acceleration due to the crankshaft speed. The crankshaft speed is now approximated by the general harmonic function

$$\dot{\phi} = \dot{\phi}_0 + \frac{\Delta \phi_o}{2} \left( 1 - \cos \left[ \frac{2 \pi ( \theta - \theta_i )}{\theta_f - \theta_i} \right] \right)$$

(10)

where $\dot{\phi}_0$ is the nominal value of the crankshaft speed, $\Delta \phi_o$ is the variation in the nominal value, $\theta$ is the input variable for the harmonic function, and $\theta_i$ is the initial value and $\theta_f$ is the final value of the input variable. For a simple representation of the harmonic function, the initial and final values of the input variable are assumed to correspond to the crank positions where the volume in a given pocket is a minimum. This assumption ensures that the minimum and maximum values of the crankshaft speed occur where the pocket volume is a minimum and a maximum, respectively. In other words, the crankshaft
will accelerate as the volume increases and decelerate as the volume decreases. The initial and final values of the input variable for the harmonic function are taken to be

\[ \theta_i = \frac{\pi}{T - 1} \quad \text{and} \quad \theta_f = \frac{\pi (2T + 1)}{T - 1} \]  

(11)

Therefore, the period of the harmonic function is

\[ \beta = \theta_f - \theta_i = \frac{2\pi T}{T - 1} \]  

(12)

Assuming that the input variable for the harmonic function can be replaced by the crank position; i.e., \( \theta = \phi \), then the crankshaft speed, see Eq. (10), can be approximated by

\[ \dot{\phi} = \dot{\phi}_0 + \frac{\Delta \phi_o}{2} \left[ 1 - \cos \left( \frac{2\pi (\phi - \phi_i)}{\phi_f - \phi_i} \right) \right] \]  

(13a)

Substituting Eqs. (11) and (12) into Eq. (13a), with \( \theta_i = \phi_i \) and \( \theta_f = \phi_f \), the crankshaft speed can also be written as

\[ \dot{\phi} = \dot{\phi}_0 + \frac{\Delta \phi_o}{2} \left[ 1 - \cos \left( \phi (T - 1) - \pi \frac{T}{T - 1} \right) \right] \]  

(13b)

Differentiating Eq. (13a) with respect to time, the crankshaft acceleration can be written as

\[ \ddot{\phi} = \frac{\pi \phi \Delta \phi_o}{2 (\phi_f - \phi_i)} \sin \left( \frac{2\pi (\phi - \phi_i)}{\phi_f - \phi_i} \right) \]  

(14)

Substituting Eqs. (13a) and (14) into Eq. (9), the ratio can be written as

\[ \eta_{\text{HH}} = \frac{\pi \phi \Delta \phi_o}{2 (\phi_f - \phi_i)} \sin (\phi - \frac{\phi}{\phi_f - \phi_i}) \sin \left[ \frac{2\pi (\phi - \phi_i)}{\phi_f - \phi_i} \right] \]  

(15)

\[ \left[ \cos (\phi - \frac{\phi}{\phi_f - \phi_i}) - \frac{\mu}{\phi_f - \phi_i} \right] \left[ \dot{\phi}_0 + \frac{\Delta \phi_o}{2} \left[ 1 - \cos \left( \frac{2\pi (\phi - \phi_i)}{\phi_f - \phi_i} \right) \right] \right]^2 \]

and is, henceforth, referred to as the harmonic ratio. Note that the harmonic ratio will tend toward infinity; i.e., the denominator in Eq. (15) will tend towards zero, when the crank position is

\[ \phi = (\frac{T}{T - 1}) \cos^{-1} \left( \frac{\mu}{\phi_f - \phi_i} \right) \]  

(16)

This situation was addressed by Beard, et al., 1992, and will also be discussed in the following section.

3. PLOTS AND DISCUSSION

This section presents plots for the radial acceleration of the generating pin and the harmonic ratio against the crank position and a variation in the nominal speed of the crankshaft. The harmonic ratio is also plotted against the crank position and the trochoid ratio for the same variation in the nominal crankshaft speed. The dimensions of the compressor are normalized such that the radius of the rolling pitch circle \( r_2 = 1 \) cm. Also, the number of generating lobes fixed to the rolling pitch circle is taken to be three. The nominal crankshaft speed is chosen to be 1700 RPM which corresponds to an industry standard electric motor. A 1% to 2% variation in the nominal value is assumed here; i.e., 17 RPM \( \pm 2 \) RPM. Finally, the range of values for the trochoid ratio is \( 1.3 \leq \mu \leq 3.8 \) which includes the commonly accepted values for a rotary compressor [Norbye, 1971].

Figure 2a is a three-dimensional plot of the radial component of the acceleration of point C against the input position and the trochoid ratio for a variation in the nominal crankshaft speed of 17 RPM. The plot indicates that the radial component of acceleration is negative over a significant range of the input position. The negative values imply that the acceleration is directed towards the center of the rotor. Therefore, a positive value indicates that an external force is required to maintain contact between the tip seal and the housing. Figures 2b and 2c are three-dimensional plots of the radial acceleration of point C due to \( \phi \) and \( \phi \), respectively, against the input position and the trochoid ratio. Figure 2a indicates that the component of the radial acceleration due to \( \phi \) has the same trend as Figure 2a. Figure 2b, however, is significantly different both in magnitude and period. Although the components of the radial acceleration of point C due to \( \phi \) are positive, they are much smaller in magnitude when compared to the values in Figure 2b.
Fig. 2a. Radial Acceleration of Point C against the Input Position and the Trochoid Ratio

Fig. 2b. Radial Acceleration of Point C due to the Crankshaft Speed.

Fig. 2c. Radial Acceleration of Point C due to the Crankshaft Acceleration.
When the harmonic ratio tends to infinity, the radial acceleration of point C (due to \( \phi^2 \) and \( \phi \)) is small compared to the maximum value of the radial acceleration of point C. For example, at \( \phi = 72.28^\circ \) (0\(^\circ\) \(\leq\) \(\phi\) \(\leq\) 2 \(\pi\) \(T\)) and for a trochoid ratio of 2, the radial component of the acceleration of point C due to \( \phi^2 \) and \( \phi \) is 0.001 m/s\(^2\) and -1.71 m/s\(^2\), respectively. Both values are insignificant when compared to the maximum value of the radial acceleration of point C (which is greater than 200 m/s\(^2\)). Note that this result is only valid for pseudo-steady state conditions and for the harmonic function given by Eq. (10).

Figure 3 is a three-dimensional plot of the harmonic ratio against the input position and the trochoid ratio for a variation in the nominal speed of the crankshaft equal to 17 RPM. The plot shows that, for a specified trochoid ratio, the harmonic ratio has a wide range of fluctuation throughout the cycle. This is particularly noticeable for the range 3.0 < \(\mu\) ≤ 3.8. The harmonic ratio tends to infinity when the crank position is as given by Eq. (16). Excluding this position, the maximum value of the harmonic ratio is 0.01 for a trochoid ratio of 3.01 and the minimum value is 0.001 for a trochoid ratio of 3.8.

![Fig. 3. Harmonic Ratio against the Input Position and the Trochoid Ratio (for a variation in the nominal crankshaft speed of 17 RPM).](image)

Figure 4 is a three-dimensional plot of the harmonic ratio against the input position and the variation in the nominal crankshaft speed for a trochoid ratio of 3.01. The plot shows that for a specified crank position, the influence of the crankshaft angular velocity is approximately linear as the variation in the nominal angular velocity increases.

![Fig. 4. Harmonic Ratio against the Input Position and the Nominal Crankshaft Speed (for a Trochoid Ratio of 3.01).](image)
4. CONCLUSIONS

The focus of this paper is to derive equations that will provide the designer with insight into the acceleration of the tip seal when fluctuations in the crankshaft speed are present. For purposes of illustration, the tip seal acceleration due to a cyclic fluctuation is compared to the nominal speed of the crankshaft. The results presented in this paper can be used to detect and, thereby, avoid possible leaks in the sealing or unnecessary wear of the seals. Future research will investigate the effects of different starting conditions and the effects of different harmonic functions on the radial acceleration of the center of the generating pin. The influence of the initial and final values of the harmonic function on the radial acceleration and on the harmonic function will also be investigated. Finally, the fluctuation in the crankshaft speed will be approximated by other realistic functions; for example; different harmonic, cyclic, or random functions.

REFERENCES