Uncertainty in microscale gas damping: Implications on dynamics of capacitive MEMS switches

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Uncertainty in microscale gas damping: Implications on dynamics of capacitive MEMS switches

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Effects of uncertainties in gas damping models, geometry and mechanical properties on the dynamics of micro-electro-mechanical systems (MEMS) capacitive switch are studied. A sample of typical capacitive switches has been fabricated and characterized at Purdue University. High-fidelity simulations of gas damping on planar microbeams are developed and verified under relevant conditions. This and other gas damping models are then applied to study the dynamics of a single closing event for switches with experimentally measured properties. It has been demonstrated that although all damping models considered predict similar damping quality factor and agree well for predictions of closing time, the models differ by a factor of two and more in predicting the impact velocity and acceleration at contact. Implications of parameter uncertainties on the key reliability-related parameters such as the pull-in voltage, closing time and impact velocity are discussed. A notable effect of uncertainty is that the nominal switch, i.e. the switch with the average properties, does not actuate at the mean actuation voltage. Additionally, the device-to-device variability leads to significant differences in dynamics. For example, the mean impact velocity for switches actuated under the 90%-actuation voltage (about 150 V), i.e. the voltage required to actuate 90% of the sample, is about 129 cm/s and increases to 173 cm/s for the 99%-actuation voltage (of about 173 V). Response surfaces of impact velocity and closing time to five input variables were constructed using the Smolyak sparse grid algorithm. The sensitivity analysis showed that impact velocity is most sensitive to the damping coefficient whereas the closing time is most affected by the geometric parameters such as gap and beam thickness.

1. Introduction

Micro-electro-mechanical systems (MEMS) are widely used in automotive, communications and consumer electronics applications with microactuators, microgyroscopes and microaccelerometers being just a few examples. However, in areas where high reliability is critical, such as in aerospace and defense applications, very few MEMS technologies have been adopted so far [1]. During the last decade, the National Nuclear Security Administration (NNSA) has invested in the development of microsystems for the monitoring and control of NNSA stockpiles and new weapons systems. If MEMS are to be inserted into these high-consequence applications, they must possess assured reliability [2].

The challenges of improving the MEMS reliability primarily stem from (a) the lack of knowledge of critical physical phenomena encountered in such microdevices and (b) the high degree of uncertainty in mainstream fabrication at the microscale. The presence of both significant epistemic and aleatory uncertainties make uncertainty quantification (UQ) [3–5] a key for advancing the microsystems technology. The NNSA and national security laboratories have been engaged in the development of quantification of margins and uncertainties (QMU) [6] process for the assessment of risks in complex engineered systems. Identification, quantification, aggregation and propagation of uncertainties are integral parts of the QMU framework and methods for analysis of these uncertainties need further development [7]. The present work applies advanced deterministic sampling methods to study scaled sensitivity of a non-linear response of a microsystem subject to multiple input uncertainties. The main goal of the paper is to investigate how an epistemic uncertainty, specifically in microscale gas damping, and aleatory uncertainties in geometric and mechanical properties of microstructures influence the key dynamics related to reliability of one class of microsystems, the radio-frequency (RF) MEMS capacitive switches.

Microdevices with electrostatically actuated structures such as resonators, accelerometers and RF switches involve an interplay of mechanical, electric and fluidic phenomena. Because of peculiarities...
of the microscale the relative magnitudes of electrostatic, mechanical and fluidic forces vastly differ from those encountered in macroscale systems. The electrostatic and fluidic forces are inherently surface effects and as such play an increasingly important role compared to the inertial structural forces which are volume phenomena. The most common fluidic effect in the Microsystems is the aerodynamic drag on the structures in relative motion. The aerodynamic forces constitute the damping force in electrostatically actuated Microsystems that are often described through simple forced spring-damper models. In the discussion below we will refer to these aerodynamic forces as gas damping.

The gas damping in microdevices is due to gas flows that are dynamically dissimilar from flows elsewhere. The characteristic length in Microsystems is on the order of 1 μm and less. This is not many orders of magnitude different from the molecular mean free path which is equal to about 50 nm in one-atmosphere, room-temperature air. The related non-dimensional similarity parameter is the Knudsen number, the ratio of the mean free path to the characteristic size of the device. For MEMS switches, the characteristic size for gas flow is the gap between the movable microstructures. The gap varies dynamically from a few microns for a zero-bias to a few nanometers during contact. This leads to the dynamic change in the Knudsen number during an actuation event (as shown schematically in Fig. 1). At a maximum gap of, say, 1 μm, the Knudsen number for standard conditions is 0.05 which is in the regime of slight rarefaction, or slip flow. As the characteristic size decreases during an actuation event, the Knudsen number increases, often reaching high values, Kn > 5, especially if the device operates at low pressures.

Both the high Knudsen and low Knudsen regimes the gas damping for simple planar geometries is described by essentially linear models. However, in the transitional regime of moderate Knudsen numbers the damping is non-linear [8] which may result in a non-linear sensitivity of key damping parameters to input parameters such as geometry or pressure. This motivates a detailed investigation of how uncertainty in gas damping affects reliability-related dynamical parameters of such movable microstructures.

Common failure mechanisms in MEMS include both mechanical (viscoelasticity, creep) and electrical (dielectric leakage, charging, and breakdown) degradation [9]. In capacitive MEMS the most commonly observed failure mode is stiction of the metal membrane to the solid dielectric surface during contact. At contact both electrical and mechanical stresses are extremely high and a good understanding of physical mechanism of such failure has not emerged yet. One possible outcome of repeated contacts in such switches is a mechanical degradation due to changes in microstructure of crystalline material in the switch movable beams [10]. However, it has been observed that the dynamics of switch closing when the contact occurs has a significant impact on the performance and lifetime [11]. The high impact velocity before metal-dielectric contact leads to much higher local stresses that have to be absorbed by the material. The higher velocity impact eases the formation of surface and bulk defects and may lead to vaporization of the solid material.

Additionally, the lifetime of capacitive MEMS switches strongly depends on gas pressure [12]. Czarnecki et al. have demonstrated that under the same actuation, the switch tested at atmospheric pressure had a lifetime of more than a million cycles (and did not fail during the testing) whereas at 200 and 20 mbar the switches failed after 330,000 and 200 cycles, respectively. One possible explanation for such a strong pressure dependence is that the dynamics of impact is much more severe at the low-pressure conditions due to reduced gas damping. To accurately predict the impact velocity and other dynamical parameters of such switches, we develop high-fidelity simulations of gas damping under various conditions and apply them to study the stochastic dynamics of a single closing event of a typical capacitive switch.

The remainder of the paper is organized as follows. In Section 2 we present details of physical models of microscale gas damping. In Section 3, we discuss the simulations of gas damping for planar microbeams, including the code and solution verification. Section 4 presents the validation with the experimental data for gas damping and the UQ methodology. In Section 5 the gas damping models are applied for analysis of dynamics of a MEMS switch with experimentally assessed uncertainties in geometry and mechanical properties. Implications of model and parameter uncertainties on the key reliability-related parameters such as the pull-in voltage, closing time and impact velocity are discussed.

2. Microscale gas damping: physical models

The choice of a physical model to describes adequately a gas flow depends on the flow regime. A map of flow regimes in terms of Knudsen and Mach numbers and applicable governing equations are shown in Fig. 1. The microflows are predominantly incompressible with mean flow velocities on the order of a few meters at a maximum. The low-speed flows can often be described by the Reynolds equation, a simplified form of the Navier–Stokes equations with negligible convective terms. Reynolds equation is often used to describe fluidic effects in Microsystems with gas confined in long gaps. However, the Reynolds and Navier–Stokes description breaks down when the characteristic size decreases and the flow transitions to rarefied regime. The Boltzmann equation is a general form of the gas transport equation based on the kinetic theory and can be reduced to Navier–Stokes equations in the near-continuum, small Knudsen number limit.

The challenge of selecting an adequate description for gas damping in MEMS switches consists in the fact that the Knudsen number varies during the switch operation. At one-atmosphere air, the Knudsen number is typically in continuum (Kn < 0.01) and slip flow (0.01 < Kn < 0.1) regimes for a typical switch in “up-state” position corresponding to the maximum static gap. As the gap between beam and the pull-down electrode closes to “down-state”, the Knudsen number increases and results in free-molecular flow. The Boltzmann equation, although significantly more involved, offers a modeling framework that is applicable for the entire range of Knudsen numbers encountered during a switch actuation.

![Fig. 1. Map of flow regimes and applicable governing equations.](image-url)
3. Gas damping: simulation for planar microbeams

The physical model corresponds to the solution of the Boltzmann equation with an ellipsoidal-statistical Bhatnagar–Gross–Krook (ESBGK) approximation for the intermolecular collision integral. The ESBGK approximation is suitable for the low-speed flows encountered in MEMS. A quasi-steady approximation of the flow can be used since the mean velocities (on the order of 1 m/s) are much smaller than the thermal velocity of the molecular motion (on the order of 10's of m/s).

The quasi-steady two-dimensional Boltzmann-ESBGK equation for the velocity distribution function is given by

$$u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} = \frac{f_0 - f}{\tau}$$

(1)

where $u$ and $v$ are the gas molecular velocities in x- and y-direction respectively, $1/\tau$ is the collision frequency. The velocity distribution function $f(x,y,u,v)$ gives the number of molecules near $(x,y)$ with velocities near $(u,v)$. The function $f_0$ is the equilibrium distribution function corresponding to an ellipsoidal (2D-Gaussian) distribution with parameters dependent on the local gas number density, temperature and mean velocities.

A detailed description of the ESBGK model can be found, for example, in Ref. [13].

The solver employs a finite volume method (FVM) with a second-order quadrant-splitting scheme applied in the physical space on uniform and non-uniform structured meshes. The velocity space in polar coordinates consists of 16th-order Gauss Hermite quadrature in the velocity magnitude and a second-order uniform quadrature in the velocity angles.

3.1. Code verification

The code has been verified for a one-dimensional test case of laminar flow between two parallel plates also known as Couette flow. The top plate is moving with a constant velocity of $u_w = 10$ m/s while the bottom plate is stationary. The distance between the plates is $H$. The gas, air, lies between the two plates maintained at a temperature of $T_w = 273$ K. At the continuum limit, it is known that the velocity profile in the y-direction should vary linearly from zero to $u_w$ according to the no-slip boundary condition. When the flow becomes sub-continuum, no-slip assumption is invalid. The first-order slip model is

$$\frac{Du}{H} = \frac{u_s}{u_w} = \frac{(2-\sigma)/\sigma}{Kn(u_w^2/C_0^2)}$$

where $Du$ is the velocity jump at the wall, $u_s$ is the slip velocity, $\sigma$ is the tangential momentum accommodation coefficient and is defined to represent the fraction of diffuse reflections. Thus the theoretical solution to incompressible Couette flow is

$$u(y) = \frac{u_w}{H(1+2y/2)} \left(1 + \frac{2-\sigma}{\sigma} \right)$$

(2)

The error in the value of velocity at the top wall is within 0.76% of the value predicted from theory. For the second case, which is in the transitional regime, $Kn=0.1$ and the deviation of velocity at the top wall from slip solution is 1.6%. Fig. 2 shows the comparison between theoretical velocity profiles and the profile obtained from the Boltzmann kinetic solver at two different Knudsen numbers $Kn=0.05$ and 0.1. Therefore, the implementation of wall boundary condition is proved to be correct.

3.2. Solution verification

Here we describe simulations of gas damping for planar microbeams which will then be used for analysis of dynamics of capacitive RF MEMS switches. The schematic of a flow geometry for a microbeam in an out-of-plane motion near a substrate is shown in Fig. 3. The damping on the beam resulting from the gas flow in/out of the gap between the beam and the substrate is often referred to as squeeze-beam damping (Fig. 4).
By using the symmetry, only the right half of the domain is used for simulations. The left, top, right and bottom boundaries are symmetry, pressure inlet, pressure inlet and wall boundaries, respectively, as shown in Fig. 3. Rigorous grid convergence tests were performed for physical and velocity space on both uniform and non-uniform meshes. The Richardson extrapolation [14] was used to estimate the accuracy of the solution and has shown that the numerical error is less than 3.5% for the case considered. The computed flowfields are shown in Fig. 5 for the case of air at a Knudsen number of 0.45:

\[
RE^2 \left( \frac{F(h)}{F(h/h)} \right) = \frac{r^k F(h) - F(h)}{r^k - 1} = F_{exact} + O(h^{k+2})
\]

where \( r \) is the refinement factor and \( h \) is the grid size for the initial mesh.

The grid convergence index (GCI) is 1.44% from mesh a to b and 1.04% from mesh b to c (Table 1).

3.3. Experimental validation of gas damping model

The main experimentally measured gas damping parameters for a planar beam are the damping ratio, \( \zeta \), or the quality factor, \( Q = \frac{1}{2\zeta} \). For the \( n \)th vibration mode these can be defined as

\[
Q = \frac{\rho \beta t o_n}{C_f}, \quad C_f = \frac{F}{\nu L}
\]

where \( \omega_n = \beta_n^2 \sqrt{EI/\rho \beta t^4} \), and \( b, t \) and \( L \) are the width, thickness and length of the beam respectively, \( E \) and \( I = bt^3/12 \) are the Young's modulus and area moment of inertia of the cantilever, \( \rho \), is the mass density of the structure.

4. Application: analysis of dynamics of a MEMS switch

In this section we apply the gas damping models for analysis of dynamics of a MEMS switch with experimentally measured uncertainties in geometry and mechanical properties.

4.1. Device fabrication and uncertainty measurements

The device structure is representative of a standard RF MEMS capacitive switch. The entire device is fabricated onto an oxidized silicon substrate. There are three electrodes of varying width underneath an electroplated nickel fixed-fixed beam. The beam has the dimensions of 500 \( \mu \)m length and 120 \( \mu \)m width. The beam is

Table 1
Richardson's extrapolation and error on three different meshes.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Size</th>
<th>DragN</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>100 \times 100</td>
<td>1.18017E – 3</td>
<td>3.25</td>
</tr>
<tr>
<td>b</td>
<td>140 \times 140</td>
<td>1.19653E – 3</td>
<td>1.91</td>
</tr>
<tr>
<td>c</td>
<td>180 \times 180</td>
<td>1.20469E – 3</td>
<td>1.24</td>
</tr>
<tr>
<td>( RE^2(a,b) )</td>
<td>1.21356E – 3</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>( RE^2(a,c) )</td>
<td>1.21563E – 3</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>( RE^2(a,b,c) )</td>
<td>1.21978E – 3</td>
<td>–</td>
<td></td>
</tr>
</tbody>
</table>

![Fig. 4. Normalized pressure contours and streamlines for Kn=4.0 and 0.04.](image1)

![Fig. 5. Solution verification for Kn = 0.4, g = 1.4 \( \mu \)m, P = 0.1 atm. Normalized pressure profiles for meshes a and c and from Richardson extrapolated.](image2)
about 4 μm thick and is about 3.5 μm above the electrodes. A silicon nitride film with thickness of 200 nm covers two of the electrodes directly underneath the beam. A diagram of the switch can be seen in Fig. 8. The detailed fabrication process used to make the switches can be found in [19].

The fabrication process used to create RF MEMS switches make them sensitive to the variations of the fabrication parameters. These manifest themselves as uncertainties in certain device geometrical and mechanical properties such as post-release gap height, beam thickness, residual stress and the beam stiffness. The most important variations of the fabrication parameters include the following. The thickness of the photoresist which varies with position due to the spin coating technique can lead to changes in the post-release gap height in the final device. The thickness and stiffness of the beam is directly related to the electroplating current density and duration and the etching process. The microstructure of the device is greatly affected by the seed layer deposition conditions such as sputtering gas pressure, metal deposition rate and seed layer thickness, and the electroplating chemistry such as the chemical composition, temperature, and pH of the plating solution. The variation of the microstructure of the beam determines the residual stress and the results in the change of the stiffness.

Explicitly observable geometric properties are measured optically using a confocal microscope Olympus OLS3100, which can reconstruct the 3D profile of the measured device, and allow direct
measurement of the beam length and width. The plating thickness is easily found by measuring the difference between the top of the beam anchor point to the substrate, since the anchor point fixes the beam directly to the substrate. If we further assume that the plating is uniform across the length and width of the beam, the gap height can be obtained by subtracting the plating thickness from the measured length.

Fig. 10. Histogram of experimental data for length and width of beam: (a) length and (b) width.

Fig. 11. Gaussian fit for experimental data for switch gap, thickness and Young's modulus: (a) switch gap, (b) beam thickness and (c) Young's modulus.
height of the top surface at the midpoint of the beam to the electrode beneath it.

Other parameters such as the switch stiffness, Young’s modulus, residual stress, etc. cannot be directly measured on the switch. For the purposes of this paper we developed a methodology to extract the effective Young’s modulus based on the measured displacement–voltage curve. In particular, the beam displacement is measured using the confocal microscope when a range of voltages is applied to the three electrodes while the beam is grounded. The measured geometric parameters of the device are then used to carry out the numerical electrostatic simulation and the results are fit to the experimental data to extract the effective Young’s modulus. Similarly, if the value of the Young’s modulus is fixed and assign the value of residual stress to fit the experimental data, the residual stress can be extracted. In order to compare our results of the extracted Young’s modulus to the reported average Young’s modulus for nickel, the values of the residual stress are extracted with the Young’s modulus to the reported average Young’s modulus for nickel, the residual stress to fit the experimental data, the residual stress can be extracted. In order to compare our results of the extracted Young’s modulus to the reported average Young’s modulus for nickel, the values of the residual stress are extracted with the Young’s modulus fixed to 200 GPa. The word effective is used to signify that this effective Young’s modulus is extracted, the spring constant can be calculated.

All of these parameters were measured for 12 devices at various locations of several different samples. By assuming the Young’s modulus to be 200 GPa, the mean value of the extracted residual stress of the nickel beam is 25.25 MPa with the standard deviation of 19.23 MPa, which validates our methodology of Young’s modulus extraction. The mean, standard deviation, coefficient of variation, skewness and kurtosis for the data are given in Table 3. Figs. 10 and 11 show the histogram, and Gaussian fit for the experimental data for length, width and thickness of the beam, gap-size between beam and actuation pad and Young’s modulus. Fig. 9 shows the experimental measurements of gap with respect to voltage for five different beams whose dimensions are shown in Table 2.

### 4.2. Switch dynamics model

The equation of motion of the beam in one-dimension is [20]:

\[ M\ddot{X}(t) + c_r \dot{X}(t) + KX(t) = F_e \]

(6)

with initial conditions \( X(0) = 0, \dot{X}(0) = 0 \) where \( X \) is the displacement of the beam, \( F_e \) is the electrostatic force on the beam, \( c_r \) is the damping factor and \( M \) and \( K \) denote the effective mass and stiffness of the beam, respectively. This reduced order model provides a good representation of the dynamics of switch to uncertainties in the input quantities and the gas damping models.

In [20], the beam over the electrode is modeled as an ideal parallel-plate capacitor to approximate the electrostatic force. However, the effect of the fringing electric field and the existence of the silicon substrate and oxide layer cannot be neglected. To model the capacitance more accurately, the capacitance between the beam and the electrodes is simulated with real device structure in the 3D software Coventor for different values of the gap between the beam and the electrodes \( g = g_0 - X \), where \( g_0 \) is the initial gap height with zero-bias. The resulting data are then least-squares fitted by a fringing field coefficient \( \alpha \) as in the form of \( C = C_{pp}(1 + \alpha) \), where \( C_{pp} = \varepsilon_0 A/(g + t_d/\varepsilon_t) \) is the parallel-plate capacitance with a dielectric layer of thickness \( t_d \). The total overlap area between the beam and three electrodes is \( A = 3b_0w \). The electrostatic force can be computed by differentiating the energy stored in the capacitor with respect to the displacement of the beam:

\[ F_e = \frac{V^2 \varepsilon_0 C}{2} = \frac{V^2}{2} \left[ \frac{\varepsilon_0 A}{(g_0 - X + t_d/\varepsilon_t)^2} \right] (1 + \alpha) \]

(7)

### Table 3

Experimentally measured device uncertainties.

<table>
<thead>
<tr>
<th>Data</th>
<th>Mean (μ)</th>
<th>Std (σ)</th>
<th>COV (%)</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, ( L ) (μm)</td>
<td>509.54</td>
<td>0.70</td>
<td>0.14</td>
<td>−0.35</td>
<td>2.25</td>
</tr>
<tr>
<td>Width, ( w ) (μm)</td>
<td>122.93</td>
<td>0.56</td>
<td>0.46</td>
<td>0.56</td>
<td>3.31</td>
</tr>
<tr>
<td>Gap-size, ( g ) (μm)</td>
<td>3.49</td>
<td>0.22</td>
<td>6.3</td>
<td>0.20</td>
<td>1.75</td>
</tr>
<tr>
<td>Thickness, ( t ) (μm)</td>
<td>4.0</td>
<td>0.35</td>
<td>8.75</td>
<td>1.14</td>
<td>3.8</td>
</tr>
<tr>
<td>Effective Young’s modulus, ( E ) (GPa)</td>
<td>295.78</td>
<td>28.93</td>
<td>9.78</td>
<td>−0.10</td>
<td>1.74</td>
</tr>
<tr>
<td>Fringing field coefficient, ( \alpha )</td>
<td>1.34</td>
<td>0.1513</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>Damping coefficient, ( A )</td>
<td>10.39</td>
<td>1.04</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
</tbody>
</table>

Fig. 12. Correlated data for effective mass and stiffness for a Gaussian input distribution for switch gap, thickness and Young’s modulus: (a) effective mass and (b) stiffness.
where $V$ is the actuation voltage between the beam and the electrodes.

The effective stiffness of a fixed–fixed beam with the force evenly distributed about the center of the beam is given by [20]

$$K = \frac{32EfL}{8\left(\frac{x}{L}\right)^3 - 20\left(\frac{x}{L}\right)^2 + 14\left(\frac{x}{L}\right) - 1}$$

(8)

Given the effective stiffness of a fixed–fixed beam, the resonant frequency of the device can be calculated using

$$\omega_n = \sqrt{\frac{K}{M}} = \beta^2 \sqrt{\frac{EI}{\rho w t L^4}}$$

(9)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Impact velocity $V/V_o$</th>
<th>Closing time $t_{\text{close}}/t_{\text{close}0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective mass, $M$ (µg)</td>
<td>1.10</td>
<td>0.022</td>
</tr>
<tr>
<td>Effective stiffness, $K$ (N/m)</td>
<td>368.52</td>
<td>0.020</td>
</tr>
<tr>
<td>Pull-in voltage, $V_p$ (V)</td>
<td>123.38</td>
<td>0.5709</td>
</tr>
<tr>
<td>$l$</td>
<td>-0.022</td>
<td>3.806</td>
</tr>
<tr>
<td>$k$</td>
<td>0.020</td>
<td>4827</td>
</tr>
<tr>
<td>$k_e$</td>
<td>0.0277</td>
<td>1218</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.5709</td>
<td>-0.937</td>
</tr>
<tr>
<td>$A$</td>
<td>-1.024</td>
<td>0.9234</td>
</tr>
</tbody>
</table>

Table 5

Coefficients of variables in response surface for actuation voltage $V_{90} = 147.5$ V.

Fig. 13. Correlated data for actuation voltage for a Gaussian input distribution for switch gap, thickness and Young’s modulus.

Fig. 14. Gap and velocity profiles for nominal switch actuated at mean actuation voltage $V_{\text{mean}} = 123.4$ V: (a) switch gap and (b) velocity.

Fig. 15. Comparison of quality factors for mode 1 [23].
In our measurement, three electrodes with width \( b_0 \) are biased with high voltage and the beam is grounded. In order to calculate the effective stiffness using Eq. (8), the three electrodes are modeled as one equivalent electrode with effective width \( b \). The term effective is utilized to emphasize that it is not sufficient to include the true physical width of all three electrodes, but rather the equivalent length that includes the effect of the fringing electric field and the existence of the silicon substrate and oxide layer. The real device structure is simulated in the 3D software Coventor to find the capacitance between the beam and the electrodes, and by substituting the simulated capacitance \( C \) back into the capacitance equation of the parallel-plate capacitance model, the effective electrode width can be determined by

\[
b = \frac{C(g + t_d/\varepsilon_i)}{\varepsilon_0 W}
\]  

(11)

By equating the applied electrostatic force with the mechanical restoring force due to the stiffness of the beam, we have \( F_e = kX \). Solving the equation for the voltage results in

\[
V = \sqrt{\frac{2kX}{\varepsilon_0 A(1 + \varepsilon_i)}}(g_0 + t_d/\varepsilon_i)
\]  

(12)

The plot of the beam displacement \( X \) versus applied voltage shows two possible values for every applied voltage, which is a result of the beam position becoming unstable when the displacement reaches certain point. By taking the derivative of the expression of \( V \) with respect to displacement \( X \) and setting that to zero, the displacement at which the instability occurs is found to be \( X = (g_0 + t_d/\varepsilon_i)/3 \) [20].

Substituting this value back into the voltage equation, the pull-in voltage for the switch can be found as

\[
V_p = \sqrt{\frac{8K}{27\varepsilon_0 A(1 + \varepsilon_i)}}(g_0 + t_d/\varepsilon_i)^{1.5}
\]  

(13)

The output pdfs of effective mass, stiffness and pull-in voltage from the Gaussian input pdfs for thickness, Young's modulus and switch gap using Eqs. (10), (8), (12) are shown in Fig. 12. Mean values of length \( L_0 \) and width \( W_0 \) were used since their coefficient of variation (cov) was less than 0.5%. Also shown in Table 4 are the mean, standard deviation, coefficient of variation, skewness and kurtosis for the effective mass, stiffness and actuation voltage. The relative uncertainty in the correlated variables, as expressed by the coefficient of variation, is much higher than the relative uncertainty in the input variables such as geometry and Young's modulus. For example, the coefficient of variation is about 28% and 17% for the stiffness and actuation voltage, respectively, whereas the coefficients of variation for the input parameters are less than 10%.

The one-dimensional mass-spring-damper system modeled using Eq. (6) is solved for the deterministic model and two values of input voltage (a) actuation voltage needed for 90% of samples to be successfully actuated, \( V_{90} = 147.5 \) V and (b) the actuation voltage needed for 99% of the samples to be successfully actuated. This is calculated from the area under the curve in Fig. 13 and is approximately \( V_{99} = 172.5 \) V. For the nominal switch at \( P = 1 \) atm, though the theoretical pull-in voltage is 123.6 V, the beam displaces only by \( 1 \mu \text{m} < 50 \mu \text{s} \) and the velocity becomes \( 0 \) as shown in Fig. 14.

Fig. 16. Effect of various gas damping models: (a) compact model, (b) Veijola's model [23] and (c) Gallis-Torczynski RE/DSMC model [24] on the impact velocity and closing time of a Ni switch actuated at 147.5 V.
Therefore, the conservative estimate of \( V_{\text{mean}} = 123.4 \) V could not be used here.

### 4.3. Uncertainty quantification method

In this section, we will briefly introduce the generalized polynomial chaos (gPC) and its application to finding the sensitivity of impact velocity and closing time with respect to input variables \( g,t,E,z,A \). An extensive review can be found in [21].

The gPC expansion seeks to approximate a random function \( gPC \) via orthogonal polynomials of random variables. The \( P \) th-order gPC approximation of any random function \( u(z) \) can be obtained by

\[
u(z) \approx u_{n_z}^P(z) = \sum_{i=1}^{M} u_i \Phi_i(z), \quad M = \left( \begin{array}{l} n_z + P \\ n_z \end{array} \right)
\]  

(14)

where \( z \in \mathbb{R}^{n_z} \) is random variable, \( \{\Phi_i(z)\} \) are the \( N \)-variate orthogonal polynomials which are constructed as products of a sequence of univariate polynomials in each directions of \( z_i \), \( i = 1, \ldots, n_z \), i.e.

\[
\Phi_i(z) = \phi_{t_1}(z_1) \cdot \phi_{t_2}(z_2), \quad t_1 + \cdots + t_{n_z} \leq P
\]  

(15)

and satisfy

\[
\mathbb{E}[\Phi_j(z)\Phi_k(z)] = \int \Phi_j(z)\Phi_k(z)p(z) \, dz = \delta_{jk}
\]  

(16)

for all \( 1 \leq j,k \leq M \), and \( \mathbb{E} \) is the expectation operator. The Fourier coefficients \( \{u_i\} \) are defined as

\[
\hat{u}_i = \int u(z)\Phi_i(z)p(z) \, dz = \mathbb{E}[u(z)\Phi_i(z)], \quad 1 \leq i \leq M
\]  

(17)

and can be approximated by

\[
\hat{u}_i \approx \sum_{j=1}^{\mathcal{Q}} u(z^{(j)})\Phi_i(z^{(j)})w^{(j)}, \quad 1 \leq i \leq M
\]  

(18)

where \( \{z^{(j)},w^{(j)}\}_{j=1}^{\mathcal{Q}} \) are a set of nodes and weights of quadrature rule, and \( u(z^{(j)}) \) is the deterministic value of \( u(z) \) with fixed \( z^{(j)} \). Legendre polynomials are chosen as bases.

The advantage of this algorithm is that it uses a smaller number of runs to approximate a response surface when compared to Monte Carlo sampling. For example, a second-order gPC requires only 61 samples. Using first-order Smolyak sparse grid [22], the response surface for impact velocity and closing time was calculated based on 11 samples generated from five input variables \( t,g,E,z,A \). The response surfaces of impact velocity and closing time had a fit of \( R^2=1 \) and 0.98, respectively, and their equations are

\[
V_{\text{eff}} = 185.86 - 0.686 \cdot t + 0.728 \cdot g - 0.011688 \cdot E + 53.085 \cdot z + 12.387 \cdot A
\]  

(19)

\[
t_{\text{eff}} = -189.144 + 20.343 \cdot t + 29.572 \cdot g + 0.089 \cdot E - 14.93721 \cdot z + 1.915 \cdot A
\]  

(20)

**Fig. 17.** Effect of various gas damping models: (a) ESBGK, (b) Reynold's equation [23] and (c) Gallis-Torczyński RE/DSMC [24] on the impact velocity and closing time of a Ni switch actuated at 172.5 V.
Converting this to a function of non-dimensional parameters \( \tilde{t} = t/t_0 \) and so on, the coefficients are shown in Table 5. The negative sign of the coefficient for \( A, E \) and \( t \) in Eq. (19) suggests that the impact velocity decreases as the thickness, Young’s modulus and damping coefficient increase. The higher the magnitude of a coefficient, the greater is the sensitivity of output variable with respect to that of input parameter. Therefore, from the coefficients it can be seen that the impact velocity is most sensitive to the damping model coefficient and least sensitive to gap. Also, the closing time is most sensitive to geometric parameters \( g \) and \( t \).

4.4. Results and discussion

Effect of gas damping model: The effect of using different gas damping models in the switch dynamics simulations has been studied first. Here we compare three models: (i) a model based on unsteady Reynolds equation [23]; (ii) a model based on a modified Reynolds equation with the first-order slip boundary conditions formulated from DSMC simulations [24]; (iii) the model based on Boltzmann–ESBGK simulations described in Section 2. As shown in Fig. 15, overall good qualitative agreement has been observed for quality factors predicted by the three models.

The three models are then used for prediction of dynamics of a single switching event for the switch with mean properties (as listed in Table 3). The predictions of displacement, velocity, acceleration of the nickel fixed–fixed beam using three models for an ambient pressure of \( P = 1 \) atm are shown in Figs. 16 and 17. For the actuation voltage \( V_{90} = 147.5 \) V, we can in general observe that the profile for the switch gap from Reynolds’ equation and Gallis-DSMC match well up to \( g = 2.2 \) \( \mu \)m and then tend to deviate. The velocity profiles from compact model and Reynolds’ model have the same shape (increasing trend with time) whereas the one from Gallis-DSMC model is completely different.

Effect of ambient gas pressure: Fig. 18 shows the simulated variation of switch gap, velocity and acceleration of the Ni beam versus time at three different pressures (0.01, 0.1 and 1 atm) when the actuation voltage of \( V_{90} = 148.6 \) V is applied to the switch at hand. The switch considered in these graphs has the mean dimensions and material properties (as listed in Table 3). All simulations

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Mean</th>
<th>Std</th>
<th>COV (%)</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuation at 90% pull-in voltage (147.5 V)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closing time (( \mu )s)</td>
<td>20.73</td>
<td>8.268</td>
<td>39.88</td>
<td>1.323</td>
<td>4.41</td>
</tr>
<tr>
<td>Impact velocity (cm/s)</td>
<td>128.95</td>
<td>12.66</td>
<td>9.82</td>
<td>0.67</td>
<td>4.21</td>
</tr>
<tr>
<td>Actuation at 99% pull-in voltage (172.5 V)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closing time (( \mu )s)</td>
<td>13.0</td>
<td>5.05</td>
<td>38.90</td>
<td>2.51</td>
<td>12.55</td>
</tr>
<tr>
<td>Impact velocity (cm/s)</td>
<td>173.01</td>
<td>19.17</td>
<td>11.06</td>
<td>0.61</td>
<td>4.17</td>
</tr>
<tr>
<td>Actuation at 90% pull-in voltage gPC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closing time (( \mu )s)</td>
<td>21.45</td>
<td>9.81</td>
<td>45.73</td>
<td>0.84</td>
<td>3.22</td>
</tr>
<tr>
<td>Impact velocity (cm/s)</td>
<td>119.9</td>
<td>9.28</td>
<td>7.74</td>
<td>-0.26</td>
<td>2.76</td>
</tr>
<tr>
<td>Actuation at 99% pull-in voltage gPC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closing time (( \mu )s)</td>
<td>12.9</td>
<td>3.33</td>
<td>25.79</td>
<td>1.7</td>
<td>7.19</td>
</tr>
<tr>
<td>Impact velocity (cm/s)</td>
<td>204.79</td>
<td>30.75</td>
<td>15.01</td>
<td>1.57</td>
<td>7.31</td>
</tr>
</tbody>
</table>

Fig. 18. Effect of ambient gas pressure on the impact velocity and closing time of a nickel fixed–fixed beam actuated at 90% pull-in bias of 147.5 V: (a) switch gap. (b) velocity and (c) acceleration.
are stopped when the switch gap reaches the surface roughness of 50 nm. At that point it is assumed that at that location the switch would come in contact with a hard stop layer (e.g. a solid dielectric). Since the focus of this paper is on investigating the gas dynamics and its implications on the switch impact velocity, no modeling of contact phenomena is included.

As seen in Fig. 18a, higher pressures result in higher switching times. For example an approximate 4 × increase in switching time is observed when the pressure is increased by 100 × (from 0.01 to 1 atm). This is expected due to the dependence of the damping factor on the pressure. The pressure effect is even more pronounced on the switch impact velocity and acceleration at contact as shown in Fig. 18b and c. The impact velocity is increased by almost 6 × when the pressure is decreased by 100 ×.

Effect of device-to-device variability: Finally, we consider how the uncertainty in geometric parameters and mechanical properties affects the dynamics of such switches. Table 6 shows the mean, standard deviation, coefficient of variation, skewness, kurtosis for the output pdfs of closing time and impact velocity from simulations on a device sample size of 1000. Gaussian input pdfs were chosen for the fringing field and damping coefficients, mean and standard deviation from Table 3. Uniform input pdfs were chosen with the switch gap and effective Young’s modulus were chosen with the uncertainty in geometric parameters and mechanical properties affects the dynamics of such switches. Table 6 shows the mean, standard deviation, coefficient of variation, skewness, kurtosis for the output pdfs of closing time and impact velocity from simulations on a device sample size of 1000. Gaussian input pdfs were chosen with the mean and standard deviation from Table 3. Uniform input pdfs were chosen for the fringing field and damping coefficients, mean and standard deviation.

5. Conclusions

In this paper we have demonstrated the influence of gas damping and device-to-device variability on the closing time and impact velocity of capacitive RF MEMS switches. A number of different gas damping models spanning the range of continuum to rarefied gas flow are studied and their predictions are discussed. It is found that although both continuum models – based on the Reynolds equation – and the rarefied models – based on Boltzmann equation – give similar predictions for the closing time, the impact velocity varies by more than a factor of two. Since gas damping in typical RF MEMS switches near contact occurs in free-molecular regime which is formally not described by continuum flow theories, the rarefied flow model is employed to calculate the impact switch velocity under uncertain condition. Specifically we consider a switch fabricated and characterized at Purdue University using a typical fabrication process. The uncertainty in the gap (6.3%) and thickness (8.8%) of the structure dominate the uncertainty in its actuation voltage (17%). A conservative approach of applying the 90% actuation voltage is studied first. This case yields an average impact velocity of 129 cm/s. A more realistic approach of actuating the average switch with a voltage that would result in successful actuation of 95% of the fabricated switches is considered next. This time the average impact velocity is increased to 173 cm/s. Since a higher impact velocity results in higher damage at the contact interface, these results underline the importance of carefully considering the process-induced switch variations in the design process.

Acknowledgments

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