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A REVIEW ON FIXED-PERCENTAGE TOLERANCES FOR
COMPRESSOR PERFORMANCE PARAMETERS

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ABSTRACT

Applying a fixed-percentage tolerance band to compressor performance parameters is a common practice in the industry. While finding the practice reasonable, its pitfalls are reviewed from the customer's as well as the compressor manufacturer's standpoint. Energy efficiency ratio needs to be controlled separately from capacity and input power, although it is derived from the two. Depending on the purpose, different levels of fixed percentages are suggested. Statistically, a bivariate normal distribution can be assumed for typical performance parameters. A set of 102 paired bivariate data for capacity and input power from an actual production audit has been statistically analyzed to verify that actual data generally fit the suggested distribution. Correlation analysis shows a slight positive correlation between capacity and input power against the assumed theoretical randomness.

INTRODUCTION

Besides cost, delivery, quality, and reliability factors, acceptance of a compressor at any given time is judged by its performance. Criteria or parameters for compressor performance are usually capacity (BTU's or calories per hour), input watts, energy efficiency ratio (EER), sound pressure level (SPL) and vibration. Among these, EER is not an independent parameter in the sense that it is derived indirectly as the ratio of capacity to input power. Among these criteria, the first three are the most commonly used ones.

PRACTICES IN THE INDUSTRY

Product evaluation, production audit and quality control deal with these parameters for control purposes. Original equipment manufacturers use them in accepting product performance. In the compressor manufacturing industry, it is common practice to allow a fixed-percentage tolerance band to the published or nominal value of a given performance parameter. Certainly, nothing is particularly wrong with the practice and it has served the industry fairly well.

Accepting the practice as a sound one, we should establish its theoretical basis and statistical significance verified with data generated in an actual production environment.

CUSTOMERS' AND MANUFACTURERS' PERSPECTIVES

Let us examine the fixed-percentage tolerance band not just from the manufacturer's point of view, but also from the customer's. First, from the manufacturer's point of view, the fixed-percentage tolerance band is a practical control tool in product assurance. The rule is simple enough to be popularly applied. Where product model proliferation is the case it is quite a convenient control scheme. On the other hand, there are some pitfalls. One of the pitfalls is the "as long as within ± x percent" syndrome. As actual mean value deviates from the published nominal value, the risk of performance fallout increases. Another pitfall is that the fixed-percentage rule is not very flexible as nominal performance of mass produced compressors could have a cyclic trend. A normal production environment sees both short-and-long-term shifting trends. Application of the rule without respect to time perspective is not a rational approach.
Next, from customer’s point of view, it is difficult to reject product shipped as long as the product performance falls into the tolerance band. Customers could make a case based on statistical analysis of performance of product shipped, but to prove performance deficiency is realistically a difficult task. First, customers have limited time and resources or lack the capability to determine the deficiency to the required accuracy. An argument based on poor performance in an air conditioning system has an inherent limitation. Poor performance does not necessarily come from poor compressor performance.

**SCOPE OF CONTROL PARAMETERS**

Performance parameters controlled by manufacturers are usually capacity, input power, energy efficiency ratio, sound pressure level, and  vibration. Of these, the first two or three are ones looked at as product assurance control for good reasons. On the other hand, as noise gets customers’ increasing attention, sound and vibration, primarily sound level, acts as an important control parameter.

A relevant issue which arises here is whether we should also control EER as long as capacity and input power are controlled. The answer is a definite yes. First, as air conditioning system EER is scrutinized as a requirement, it is only fair for the manufacturer to control EER. One might argue that EER is not an independent parameter, that capacity and input power determine the value. However, if only capacity and power are controlled within a fixed ± 5 percent limit, otherwise unacceptable products with -10 percent of the nominal EER, could result in being accepted (Point 'A' in Figure 1).

**Fig. 1 Control Limits for Performance Parameters**

Certainly, this cannot be a satisfactory situation. EER should also be controlled as a separate parameter. The next question is concerned with the proper percentage. The same ± x percent limit, such as ± 5 percent, is reasonable, as the statistical analysis of the production data will show in later discussion.

There are, however, two points which should be discussed further. First, the three solid lines of bottom boundaries of the "control window" define the definite control limits. These are the minimum capacity and EER and the maximum watts. The other three dotted lines of the
upper boundaries may seem somewhat questionable limits. One might argue: why limit better performance? The limits should still be enforced as performance beyond these limits are practically not feasible even with a well-designed compressor and controlled manufacturing. Furthermore, the limits are also helpful as an alert to possible testing errors. If the mean performance shifts from the published nominal value as shown in Figure 1, then the window may have to be shifted. The second point of concern is with the shift of data points or the control window. The three arrows point to the general directions of shift for effects of run-in (or break-in), loose assembly fits, and mechanical wear of internal parts.

**SOUND AND VIBRATION AS CONTROL PARAMETERS**

As pointed out earlier, these parameters are getting ever-increasing attention. Applying a control limit to these parameters is a more complex matter. For example, setting a limit in overall sound pressure level (SPL) does not necessarily provide a satisfactory control of sound quality. Frequency content is often more important than the overall SPL value. It is tempting to use a fixed-percentage upper limit for these parameters too, but one must recognize that SPL represents a proportional number and a fixed-percentage cannot represent a true control limit. It is more appropriate to use a specific upper SPL limit, not a nominal SPL plus a fixed-percentage. In addition to this limit, specific controls on frequency content may be set.

**"GOOD" FIXED-PERCENTAGE AS A CONTROL LIMIT**

Accepting the fixed-percentage control limit as a popular, reasonable, and useful approach, the next question is what should the control limit be. Without taking a survey in the industry, ± 5 percent is known as a "good" limit and widely practiced. Despite the possible lack of a theoretical basis for the control limit, we regard the limit as a reasonable compromise between customer's and manufacturer's needs. The control range encompasses reasonable variations in manufacturing factors (variability in parts, machining, assembly, and testing errors). The actual production audit data in this paper proves the presumption.

The next question is if the same control limit is appropriate for both rating a compressor and providing samples for the customer's unit development. For these special purposes, one can make a good case for a tighter tolerance requirement. For compressor performance rating purpose, a good practice is to select samples with performance which approaches the mean nominal values as closely as possible. Note that the published value may not be the same as the actual nominal value. There is no sense to select intentionally a biased sample, as production is controlled within a fixed range. For a customer sample, it is important not to send a "good looking" biased sample. This will only hurt the customer and manufacturer later. Since the manufacturer must live up to the performance represented by the sample shipped, the sample should represent the nominal production performance. Thus, a tighter range, such as ± 2.5 or 3 percent is reasonable for customer sampling purposes.

**STATISTICAL ANALYSIS OF PERFORMANCE PARAMETERS**

As we are dealing with statistical performance data, it is appropriate to consider a statistical analysis of such data. Starting with capacity, input power and EER as control variables, we shall first characterize the variable, then suggest an appropriate statistical distribution function. The assumed distribution function will then be verified with actual production data.

Data to be examined - To examine an actual production situation, we shall analyze the production audit data of a rolling piston-type rotary compressor model. The 102 sets of raw-score data cover 28 weeks of production by a certain compressor manufacturer.
**Measured Performance Variables** - The data is in the form of paired capacity and input power data. The set of measured 102 pairs constitutes a bivariate data. Figure 2 depicts the data in a scatter diagram.

**Fig. 2 Performance Audit Data**

In this particular case, mean capacity and input power are slightly shifted from the rated mean values. This does not necessarily justify a change in the rated performance as one must look at production performance from long-term time perspective.

The locus of points in the scatter diagram provides an insight into the functional relationship that exists between the variables. As the EER is a resultant number from the other two parameters, basically we have bivariate data sets. The bivariate data represents data that occurs in ordered pairs. If we assume that the two variables are normally distributed (which is a reasonable assumption in light of the analysis to follow), we are dealing with a bivariate normal distribution.

Capacity and input power are interdependent on each other in that more capacity requires more power. However, the issue here is not a functional relationship between two parameters. Rather, we are interested if variations of the two parameters in products are independent or not. While it may be possible to prove it one way or the other from a theoretical standpoint, it would be more direct to test out a hypothesis statistically with actual data. While the result obtained from this set of data cannot be universally applied to other types of compressors or manufacturers, it gives a clue for statistical interferences.

Both variables are assumed to be random variables and must be free to vary. Values of one variables occur at random, depending on the production unit randomly selected in the sample. Both variables have equal status. It is convenient to think of the bivariate normal distribution as a three-dimensional surface, such as shown in Fig 3.
In the three-dimensional diagram, the exact shape of this figure, which looks like a fireman's hat, will depend on how closely the variables are related. Here, we can see that:

(i) The frequencies are concentrated in an elliptical area with the major axis inclined upward to the right. There are no very low capacity with high power nor very high capacity with very low power.

(ii) The frequencies pile up along the major axis, reaching a peak near the center of the distribution. They thin out around the edges vanishing entirely beyond the borders of the ellipse.

Assumed Distribution Function - Capacity and input power vary together in a joint distribution. If the form of the joint distribution of two variables, X and Y, is normal, the joint distribution is a bivariate normal distribution. A special joint distribution of X and Y is assumed in making statistical inferences in simple correlation analysis - a bivariate normal distribution. If we slice the surface at any level of Y or X, the shape of the resulting cross sections are normal.

To start with, it is a reasonable assumption that the paired variables will a bivariate normal distribution function. A bivariate normal distribution means that each of two variables is distributed about the other normally. Because of the nature of the bivariate normal distribution, the values of either of the variables are distributed normally for a fixed value of the other variable. This distribution has five parameters, mean and standard deviation for each variable and the correlation coefficient, $\rho$, of which $r$ is an estimator. The parameter, $\rho$, measures the closeness of the population relation between X and Y; it determines the narrowness of the ellipse containing the major portion of the observations.

Actual Distribution Function - To prove that we are indeed dealing with a bivariate normal distribution, we must show that each of capacity, input power, and the resultant EER variables has a normal distribution. This can be readily proved by first converting the frequency distribution into a cumulative frequency distribution, then plotting it on special normal paper.
All three variables are found indeed to be normal distribution. Thus, the bivariate data meets all requirements to be bivariate normal distribution.

**Correlation analysis** - The objective of a correlational study is to determine the strength of a relationship between paired observations. The correlation indicates the extent to which values of one variable are related to values of another variable. Though the regression analysis can provide essentially the same information that a correlation analysis does, we are not interested in a regression analysis here. Correlation focuses solely on the strength of the relationship. In the correlation model we assume that both X and Y values differ from sample to sample.

We want to find a measure, namely, the coefficient of correlation here, which will show us the degree of this covariability. The correlation coefficient is an index of the degree of this covariability. The population correlation coefficient \( \rho \) and its sample estimate \( r \) are intimately connected in the bivariate normal distribution.

The computed covariability between the two variables is 0.35. This lies between no covariance (\( \rho = 0 \)) and perfect covariability (\( \rho = +1 \)), both variables varying in the same direction (the plus sign). Then sample correlation coefficient \( r = 0.35 \) gives the 95% confidence interval for \( \rho, +0.16 < \rho < +0.51 \). A positive relationship between two variables means that high values of capacity are paired with high values of input power, and low values of one with low values of the other. Correlation is a measure of linear relation only; it is of no use in describing nonlinear relations.

**Test of Hypothesis that \( \rho = 0 \)** We wish to test the hypothesis that \( \rho = 0 \) with a two-sided test and significance level of 0.05. A table of the normal distribution says that the region of significance is \( z > 1.96 \) and \( z < -1.96 \). For sample \( r = 0.346 \) for a sample batch of 102 cases,

\[
z = r \sqrt{\frac{n-2}{1-r^2}} = 0.346 \times 10 \times 0.05 = 3.48
\]

As \( z = 3.48 \) is in the region of significance, the hypothesis is not tenable and the sample estimate is significant. Thus, we must reject the hypothesis: \( \rho = 0 \).

What this means is that there is a slight positive correlation between capacity and input power. Thus, the variables measured are not perfect random variables. However, this is not too surprising in view of various factors which might affect perfect randomness.

**DETERMINANTS OF PARAMETER VARIABILITY**

Variability in performance results from multiple sources. Variations exist in part dimension, assembly clearances, fits, and testing errors. We also recognize "break-in" run as a variable.

(i) **Time Variability** - How compressor is "broken in" and how long it is run are known determinants. The facts that input power decreases with run time and capacity varies only slightly suggest that the distribution shifts horizontally in Figure 2. This will increase EER.

(ii) **Manufacturing Assembly** - The tightness of component fit-up in assembly affects drag of mating parts and gas leakage, thus affecting input power and capacity.

(iii) **Testing Errors** - It is not clear how much of the variation in the data may be due to testing errors. We should recognize that measured performance variables are subject to errors in measurement. That is, we really observe values with testing
error included. Testing errors are independently and normally distributed with their own mean zero and respective variances. It is quite possible, however, that different cases can bias the inherent random nature of testing errors.

CONCLUSION

Fixed-percentage tolerance band to compressor performance parameters are commonly used in the industry, and the practice is reasonable. The "as long as within ±x%" syndrome without considering statistical implication and manufacturing variability is not rational. A tighter tolerance band for customer samples is justified. Along with capacity and input power, energy efficiency ratio needs also be controlled, although it is derived from the two. Statistically, a bivariate normal distribution can be assumed for typical performance parameters. Characteristics of such distribution describes actual production audit data. A set of 102 paired bivariate data, capacity and input power, from an actual production audit has been statistically analyzed. The actual data generally fits well to the assumed bivariate normal distribution. Correlation analysis, however, shows a slight positive correlation between capacity and input power. This suggests that a strict randomness lacks in the production data.