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A New Control Approach for a Direct Expansion (DX) Air Conditioning (A/C) System with Variable Speed Compressor and Variable Speed Supply Fan

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ABSTRACT
This paper presents a dynamic mathematical model for a DX A/C system which was suitable for designing multivariable control. A multi-input multi-output (MIMO) controller is designed based on the model to simultaneously control indoor air temperature and humidity by varying the speeds of both compressor and supply fan. The experiment results show that MIMO control method is capable of simultaneously controlling indoor air temperature and humidity, thus improving the thermal comfort. It is also expected that such a MIMO feedback control will have wide applications in HVAC&R systems to achieve high performance.

Keywords: MIMO, LQG, temperature and humidity, DX A/C system,

1. INTRODUCTION
The purpose of controlling HVAC system is mainly to provide the control of the temperature and humidity in a space. However, controlling indoor humidity at an appropriate level is very important since this directly affects occupants’ thermal comfort and indoor air quality (IAQ) (Fanger, 2001). Traditionally, reheating has been the principal method for indoor humidity control in central HVAC systems. This method is inherently costly and energy inefficient since it uses a great deal of energy to overcool the air, and then more energy to reheat the air to a suitable supply temperature. However, reheating is uncommon for DX A/C systems and controlling indoor humidity at an appropriate level using a DX A/C system has been difficult. This is because the cooling coil in a DX A/C system must perform both cooling and dehumidification simultaneously. However, most DX A/C units are currently equipped with a single-speed compressor and supply fans, relying on on-off cycling compressor as a low-cost approach to maintain only indoor air dry-bulb temperature. This results in either space overcooling or an uncontrolled equilibrium indoor relative humidity (RH) level.

The recent advancement of variable speed drive (VSD) technology offers tremendous opportunities for improving indoor thermal control and energy efficiency for air conditioning. Compressor speed may be continually adjusted so as to modulate its output cooling capacity to match the actual thermal load. Furthermore, the speed of supply fan can also be altered to impact both sensible heat and latent heat transfer in a heat exchanger. Therefore it is possible to improve indoor thermal control by DX A/C systems by applying VSD technology.

Air conditioning systems have been traditionally controlled using simple single-input single-output (SISO) techniques. Two control loops for both air temperature and humidity have been treated as two independent SISO systems, with their cross-coupling effects being often ignored. However, MIMO control strategies are
able to take into account the coupling effects of multiple variables. Therefore they are more effective in simultaneously achieving multiple control objectives, such as: temperature, humidity, capacity, efficiency, etc. The performance of conventional SISO control strategies is inherently inferior to that of multi-input multi-output (MIMO) control strategies.

He et al. (1997) considered that there were strong cross-coupling effects among various operating parameters in a vapor compression refrigeration cycle, such as evaporating temperature, condensing temperature, degree of superheat, etc. Therefore, in order to improve the transient behavior of the cycle, a MIMO feedback controller was designed based on a low order lumped-parameter dynamic model (He et al., 1998). Controllability test results showed that the MIMO controller could effectively improve the operating performance and energy efficiency. Rasmussen and Alleyne (2004) presented the application of a multivariable adaptive control strategy to a general automotive air conditioning system. It was demonstrated that the MIMO-based adaptive approach was very successful in achieving multiple objectives by regulating both degree of superheat and evaporating pressure. Furthermore, a MIMO controller for air temperature in, and ventilation to multiple zones in a building with a model predictive control (MPC) strategy was developed by Yuan and Perez (2006). On the other hand, a DDC-based control algorithm developed by Li and Deng (2007a,b) used space sensible heat ratio (SHR) as a controlled variable to simultaneously control space air temperature and relative humidity. However the control algorithm was not MIMO based, but SISO based.

The organization of the paper is as follows. Section 2 describes briefly an experimental DX A/C system and its dynamic model. The design of the MIMO controller based on the dynamic model is presented in Section 3. The results of controllability tests of the MIMO controller are reported in Section 4.

2. DESCRIPTION OF THE EXPERIMENTAL DX A/C SYSTEM

The experimental DX A/C system was mainly composed of two parts, i.e., a DX refrigeration plant (refrigerant side) and an air-distribution sub-system (air side). Its simplified schematic diagram is shown in Fig. 1. The major components in the DX refrigeration plant included a variable-speed rotor compressor, an electronic expansion valve (EEV), a high-efficiency tube-louver-finned DX evaporator and an air-cooled tube-plate-finned condenser. The evaporator was placed inside the supply air duct on the air side to work as a DX air cooling coil. The nominal output cooling capacity from the DX refrigeration plant was 9.9 kW. The working fluid of the plant was refrigerant R22, with a total charge of 5.3 kg. The air side included an air-distribution ductwork, a variable-speed centrifugal supply fan, and a conditioned space. Inside the space, there were sensible heat and moisture load generating units (LGUs), simulating the cooling load in the space.

![Fig. 1 The schematic diagram of the experimental DX A/C system](image)
A dynamic mathematical model for the experimental DX A/C system has been developed and was previously reported (Qi and Deng, 2008). The model and its state-space representation after linearization are reproduced here, as follows:

\[
\begin{align*}
C_p \rho V_h \frac{dT_1}{dt} &= C_p \rho f(T_1 - T_2) + k_{qf} f + Q_{\text{load}} \\
\rho V \frac{dW_1}{dt} &= \rho f(W_1 - W_2) + M \\
C_p \rho V_{h1} \frac{dT_3}{dt} &= C_p \rho f(T_2 - T_3) + \alpha_1 A_1(T_w - \frac{T_1 + T_2}{2}) \\
C_p \rho V_{h2} \frac{dT_1}{dt} + \rho V_{h2} \frac{dW_1}{dt} &= C_p \rho f(T_3 - T_1) + \rho f h_{fg} (W_2 - W_1) + \alpha_2 A_2(T_w - \frac{T_1 + T_2}{2}) \\
(C_p \rho V_w) \frac{dT_w}{dt} &= \alpha_1 A_1 \left(\frac{T_1 + T_2}{2} - T_w\right) + \frac{T_1 + T_2}{2} - T_w - s \frac{V_{\text{com}}}{V_{\text{const}}} (h_{r2} - h_{r1}) \\
\frac{dW_1}{dt} &= - (2 \times 0.0198 T_1 + 0.085) \frac{dT_1}{dt} / 1000 = 0
\end{align*}
\]

Therefore the linearized dynamic model of the DX A/C system in state-space representation, which is highly suitable for designing multivariable control, can be written as:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]

where the state variables \( x = [\delta T_1, \delta T_2, \delta T_3, \delta T_w, \delta W_1, \delta W_2]^T \), output variables \( y = [\delta T_2, \delta W_2]^T \), which are the dynamic deviations from their set points, respectively. \( A \), \( B \), \( C \) were the coefficient matrices.

### 3. MIMO CONTROLLER DESIGN

The MIMO controller was designed based on the linearized dynamic model of the experimental DX A/C system, i.e. Equation (2). The main difference between a MIMO control system and a SISO control system was that the decoupled SISO control system only used one feedback signal to generate a single control input. However, the MIMO control system used two feedback signals, i.e., indoor air temperature and moisture content, to generate two signals to control both compressor speed and supply fan speed simultaneously.

Figure 2 shows the block diagram of MIMO feedback control system. The MIMO controller was designed based on the technique of Linear Quadratic Gaussian (LQG) (Stefani et al. 1994). A LQG multivariable controller was an observer-based compensator that used the Kalman filter to optimally estimate unmeasured state variables based on unmeasurement, and then used the optimal full state feedback to generate a control law. \( r \) was the reference set points of indoor air temperature and moisture content, and \( y \) the actual air temperature and moisture content inside the conditioned space.
The optimal observer is given by

$$\hat{x} = (A - LC) \cdot \hat{x} + B \cdot u + Ly$$  \(3\)

where \(\hat{x}\) is the estimate of the actual state \(x\).

The optimal feedback gain matrix \(K_1\) is calculated such that the feedback law \(u = -K_1 \cdot \hat{x}\) minimizes the cost function:

$$J = \int (y'Qy + u'Ru)dt$$  \(4\)

The weighting matrices \(Q\) and \(R\) can be selected suitably to obtain desire performance of the system.

In order to track the reference, an integrator should be included in the controller. The integrator variable was written as follows:

$$\dot{\eta} = r - y$$  \(5\)

Therefore the augmented state equations become:

$$\begin{bmatrix} \dot{x} \\ \eta \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & \eta \end{bmatrix} \begin{bmatrix} x \\ \eta \end{bmatrix} + \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} u \\ r \end{bmatrix} = A_a x_a + B_a u_a$$  \(6\)

$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ \eta \end{bmatrix} = C_a x_a$$  \(7\)

$$u = -K_1 \cdot \hat{x} - K_2 \eta = -[K_1 \ K_2] \begin{bmatrix} \hat{x} \\ \eta \end{bmatrix} = -K_a x_a$$  \(8\)

The transfer function of the LQG with integrators compensator can be expressed as

$$K(s) = K_a (sI - A + BK_a + LC)^{-1} L$$  \(9\)

**4. EXPERIMENT RESULT AND DISCUSSION**

The MIMO controller was designed around an operating point of an indoor air dry-bulb temperature of 24 \(^\circ\)C and an indoor air moisture content of 9.34 g/(kg dry air) (or an equivalent indoor relative humidity of 50%). The compressor speed and supply fan speed were all initially set at 3955 rpm and 2180 rpm, both being 50% of their maximum speeds, respectively. In order to protect the compressor and supply fan and to
avoid the speeds from changing sharply, each step change of both compressor speed and supply fan speed was set at not more than 5% of their respective maximum speeds. The time interval between step changes was 30 second in all tests. For this test, indoor air settings were 24 °C (dry-bulb temperature ) and 9.34 g/(kg dry air)( moisture content), or an equivalent indoor air wet-bulb temperature of 17 °C. These settings were expected to be maintained after the disturbances to both sensible load and latent heat loads were introduced. The MIMO controller would be enabled when the deviation between the measured indoor air temperature and its setting was greater than ±0.5 °C as a result of the disturbances.

Figures 3 to 5 present the controllability test results of disturbance rejection capability of the MIMO controller. In the test, at t=350 s, the disturbances were introduced, with the sensible and latent heat loads increased from 5.0 kW to 5.6 kW, and from 2.0 kW to 2.3 kW, respectively. Fig.3 shows the variation profile of indoor air temperatures. During the first 350 s of the test, indoor air temperatures were maintained at their respective set points, i.e., 24°C dry-bulb and 17°C wet-bulb before the disturbances were introduced. Afterwards, both air dry-bulb and wet-bulb temperatures increased gradually. At t=1360 s when indoor air dry-bulb temperature went over 24.5 °C, the MIMO controller was enabled. Fig.4 shows the variation profiles of compressor speed and supply fan speed. Following the action of the MIMO controller, both speeds increased. Towards the end of the test, the speeds of compressor and supply fan stabilized at around 4750 rpm and 2380 rpm, respectively. The variation profile of indoor air moisture content was presented in Fig.5. As seen from Fig.3, the MIMO controller can well control the indoor air dry- and wet-bulb temperatures to their respective set points, achieving a satisfactory performance in terms of disturbance rejection capability. At the end of test, the deviation between indoor air dry-bulb temperature and its setting was less than 0.1 °C.
5. CONCLUSIONS

A multivariable control strategy for simultaneously controlling temperature and humidity in a DX A/C system has been developed and reported in this paper. A MIMO controller has been designed based on Linear Quadratic Gaussian (LQG) technique. Experimental results show that the MIMO controller can effectively control the temperature and humidity in conditioned space by simultaneously varying compressor speed and supply fan speed in DX A/C system.

ACKNOWLEDGEMENT

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NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$, $B$, $C$</td>
<td>coefficient matrices (in equation 2)</td>
<td></td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat of air</td>
<td>kJkg$^{-1}$K$^{-1}$</td>
</tr>
<tr>
<td>$f$</td>
<td>Air volumetric flow rate</td>
<td>m$^3$s$^{-1}$</td>
</tr>
<tr>
<td>$M$</td>
<td>Moisture load in the conditioned space</td>
<td>kgs$^{-1}$</td>
</tr>
<tr>
<td>$Q_{load}$</td>
<td>Sensible heat load in the conditioned space</td>
<td>kW</td>
</tr>
<tr>
<td>$T_1$</td>
<td>Temperature of air leaving the DX evaporator</td>
<td>ºC</td>
</tr>
<tr>
<td>$T_2$</td>
<td>Air temperature in the conditioned space</td>
<td>ºC</td>
</tr>
<tr>
<td>$T_3$</td>
<td>Air temperature leaving the dry-cooling region of the DX evaporator</td>
<td>ºC</td>
</tr>
<tr>
<td>$T_w$</td>
<td>Temperature of the DX evaporator wall</td>
<td>ºC</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume of the conditioned space</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$V_{h1}$</td>
<td>Air side volume of the DX evaporator in dry-cooling region on air side</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$V_{h2}$</td>
<td>Air side volume of the DX evaporator in wet-cooling region on air side</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$V_{com}$</td>
<td>Swept volume of the rotor compressor</td>
<td>m$^3$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
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<tr>
<td>$W_1$</td>
<td>Moisture content of air leaving the DX evaporator</td>
<td>kg kg$^{-1}$ dry air</td>
</tr>
<tr>
<td>$W_2$</td>
<td>Moisture content of air conditioned space</td>
<td>kg kg$^{-1}$ dry air</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Heat transfer coefficient between air and the DX evaporator wall in dry-cooling region</td>
<td>kW m$^{-2}$ °C$^{-1}$</td>
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<tr>
<td>$\alpha_2$</td>
<td>Heat transfer coefficient between air and the DX evaporator wall in wet-cooling region</td>
<td>kW m$^{-2}$ °C$^{-1}$</td>
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<tr>
<td>$\rho$</td>
<td>Density of moist air</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$h_{fg}$</td>
<td>Latent heat of vaporization of water</td>
<td>kJ kg$^{-1}$</td>
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<tr>
<td>$h_{r1}$</td>
<td>Enthalpy of refrigerant at evaporator inlet</td>
<td>kJ kg$^{-1}$</td>
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<tr>
<td>$h_{r2}$</td>
<td>Enthalpy of refrigerant at evaporator outlet</td>
<td>kJ kg$^{-1}$</td>
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<tr>
<td>$k_{spl}$</td>
<td>Coefficient of supply fan heat gain</td>
<td>kJ m$^{-3}$</td>
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<tr>
<td>$v_s$</td>
<td>Specific volume of superheated refrigerant</td>
<td>m$^3$ kg$^{-1}$</td>
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<tr>
<td>$s$</td>
<td>Speed of compressor</td>
<td>rpm</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Compressor’s displacement coefficient</td>
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REFERENCES


